AOE/ME 4434/5434 (Adv) Introduction to CFD
Fall 2012
Instructor: Dr. Chris Roy

Homework #5
Problems for Section 6
Due Friday, Nov. 9, 2012 at 2:30 pm ET

The 1D unsteady equation is given by: $\frac{\partial T}{\partial t} - \alpha \frac{\partial^2 T}{\partial x^2} = f(x)$. Write a computer program (C, C++, Fortran, MATLAB, etc.) to solve the following explicit discretization of this heat equation

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} - \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2} = f(x_i).$$

Assume a thermal diffusivity $\alpha = 9.71\times10^{-5}$ m^2/s for all cases and use local time stepping to obtain the steady-state solutions.

1. Perform a code verification study using the steady-state manufactured solution given by $T(x) = 300 + 200\sin\left(\frac{3\pi x}{2L}\right)$ K, $L = 1$ m where *f(x)* above is set to the appropriate manufactured solution source term. Examine evenly-spaced grids of 5, 9, 17, 33, 65, and 129 nodes for $0 \le x \le 1$ m (with boundary conditions determined from the above manufactured solution) and compute the observed order of accuracy. Make sure to monitor and demonstrate iterative convergence by examining norms of the steady-state iterative residual: $\Re_i^n = \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2} + f(x_i)$

2. Compute numerical solutions assuming the bar is heated from 0.25 m $\le x \le$ 0.75 m with the source term *f(x)* = $+0.5(0.25 - |x - .5|)$ K/s (for x outside this range, *f(x)* = 0). Use the mesh sizes given above to examine the steady-state solution (monitor and demonstrate iterative residual convergence as discussed above). Obtain estimates of the numerical uncertainty in the steady-state solution using the Grid Convergence Index. Use initial conditions of *T(x, t=0)* = 300 K and boundary conditions as follows:

$$T = 300 \text{ K} \quad \text{at} \quad x = 0$$
$$\frac{\partial T}{\partial x} = -200 \text{ K/m} \quad \text{at} \quad x = 1 \text{ m}$$

   For the derivative boundary condition at x = 1 m, examine the use of both first- and second-order accurate upwind finite differences to determine the value of the temperature at the boundary.

(over)

Use no more than 5 pages for your writeup, and make sure to include a copy of your code as an appendix to your homework submission. Although you can use whatever plotting software you like, an example solution output file suitable for reading into Tecplot is shown below. Note that the values in this sample output file are for example purposes only and are not correct. Finally, I recommend that you choose the same programming language that you will use for the semester project. Recall that Fortran and C/C++ are recommended since they will run much faster than MATLAB.

Fortran Code to Output Tecplot Format Output File:

```
open(50,file='solution.dat',status='unknown')
write(50,*) 'TITLE = "Solution"'
write(50,*) 'variables="x(m)""T(K)""Texact(K)"'
write(50,*) 'ZONE T="ZONE 001"'
write(50,*) ' I=',Num_Nodes,', J=1, K=1 '
write(50,*) ' DATAPACKING=POINT'
write(50,*) ' DT=(DOUBLE DOUBLE DOUBLE )'
do i = 1,Num_Nodes
  Write(50,*) x(i), Temperature(i), Temp_Exact(i)
enddo
close(50)
```

Tecplot Formatted Output File:

```
TITLE = "Solution"
 variables="x(m)""T(K)""Texact(K)"
 ZONE T="ZONE 001"
  I=           9, J=1, K=1
  DATAPACKING=POINT
  DT=(DOUBLE DOUBLE DOUBLE )
    0.00000000000           300.000000000           300.000000000
    0.125000000000          415.118996883           411.114046604
    0.250000000000          491.683833023           484.775906502
    0.375000000000          504.135442989           496.157056081
    0.500000000000          448.524815081           441.421356237
    0.625000000000          343.844048269           339.018064403
    0.750000000000          225.624860592           223.463313527
    0.875000000000          133.962240724           133.706077539
    1.00000000000           100.000000000           100.000000000
```