

→ Lista 4

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\Rightarrow x = 2$
 $y = -1$

b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$ $w = 4$ $y = 2$
 $x = 3$ $z = 1$

c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \\ 1 \end{bmatrix}$ $\begin{cases} w=6 \\ x=3 \\ y=2 \\ z=1 \end{cases}$ $S = \{(w, x, y, z) \in \mathbb{R}^4 / w=6, x=3, y=2, z=1\}$
 $w+z=6 \Rightarrow y=2+2$

d) $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ $x+3z=1 \Rightarrow x=1-3z$
 $y-z=2 \Rightarrow y=2+z$
 $z=0$

e) $\begin{bmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{bmatrix} \cdot \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ -5 \end{bmatrix}$ $w-7z=8 \Rightarrow w=8+7z$ $S = \{(w, x, y, z) \in \mathbb{R}^4 / w=8+7z, x=2-3z, y=-5-z\}$
 $x+3z=2 \Rightarrow x=2-3z$
 $y+z=-5 \Rightarrow y=-5-z$
 $z=0$

f) $\begin{bmatrix} 1 & -6 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 8 \\ 0 \end{bmatrix}$ $v-6w+3z=-2 \Rightarrow v=-2+6w-3z$
 $x+4z=4 \Rightarrow x=4-4z$
 $y+5z=8 \Rightarrow y=8-5z$
 $z=0$

2-
a) $\begin{bmatrix} 3 & -2 & 1 & 3 \\ 1 & 2 & 1 & 9 \end{bmatrix} - \begin{bmatrix} 1 & -4 & 1 & 3 \\ 1 & -3 & 1 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -4 & 1 & 3 \\ 0 & 1 & 1 & 6 \end{bmatrix} \xrightarrow[L_2=(L_2+L_1)]{} \begin{bmatrix} 1 & -4 & 1 & 3 \\ 0 & 1 & 1 & 6 \end{bmatrix} \xrightarrow[x=\frac{4}{3}y, y=\frac{6}{3}=2]{} x=\frac{4}{3}y+1+\frac{4}{3}y \Rightarrow x=\frac{1}{3}+4y$
 $y=3$

b) $\begin{bmatrix} 5 & -2 & 8 & -2 & 3 & 4 \\ 10 & 16 & 16 & 16 & 16 & 16 \end{bmatrix} - \begin{bmatrix} 10 & 16 & 16 & 16 & 16 & 16 \end{bmatrix}$ impossível realizar pois a segunda equação é o dobro da primeira

c) $\begin{bmatrix} 1 & 2 & 5 \\ 2 & -3 & -4 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 5 \\ -1 & -1 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & -4 \end{bmatrix} \xrightarrow{x+2y=5} x=5-2y$

d) $\begin{bmatrix} 3 & 3 & 2 & -3 & -5 & 8 & 3 \\ 2 & -4 & -2 & 4 & -2 & 4 & 3 \\ 1 & -2 & -3 & -4 & 3 & -2 & -5 \end{bmatrix} - \begin{bmatrix} 1 & -2 & -3 & -4 \\ 2 & -4 & -2 & -4 \\ 3 & -2 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 8 & 4 & 20 \end{bmatrix} \xrightarrow[L_2=L_2-(2 \cdot R_3)]{} \begin{bmatrix} 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 8 & 0 & 16 \end{bmatrix} \xrightarrow[L_3=L_3-4L_2]{} \begin{bmatrix} 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \xrightarrow[x=3, y=1, z=1]{} x=3$

$L_2=L_2-(2 \cdot R_3)$ $L_3=L_3-4L_2$ $L_3=L_3:8$

$L_3=L_3-3R_1$ $L_3=L_3-4L_2$ $L_1=L_1+2L_3$

$L_2=L_2-(1 \cdot R_2)$ $L_1=L_1+3L_2$ $L_1=L_1+2L_3$

$L_3=[(L_3; 1) \cdot (-1)]+L_2$

$L_2=L_2-(1 \cdot R_2)$ $L_1=L_1+3L_2$ $L_1=L_1+2L_3$

$L_3=[(L_3; 1) \cdot (-1)]+L_2$

$L_2=L_2-(1 \cdot R_2)$ $L_1=L_1+3L_2$ $L_1=L_1+2L_3$

$L_3=[(L_3; 1) \cdot (-1)]+L_2$

$L_2=L_2-(1 \cdot R_2)$ $L_1=L_1+3L_2$ $L_1=L_1+2L_3$

$L_3=[(L_3; 1) \cdot (-1)]+L_2$

$L_2=L_2-(1 \cdot R_2)$ $L_1=L_1+3L_2$ $L_1=L_1+2L_3$

$L_3=[(L_3; 1) \cdot (-1)]+L_2$

$L_2=L_2-(1 \cdot R_2)$ $L_1=L_1+3L_2$ $L_1=L_1+2L_3$

$L_3=[(L_3; 1) \cdot (-1)]+L_2$

$$f) \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & -1 & 3 & 0 \\ 3 & 3 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & -5 & 5 & 5 \\ 0 & -3 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & -3 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{array}{l} x=1 \\ y=2 \\ z=3 \end{array}$$

$L_2 = L_2 - 2L_1$
 $L_3 = L_3 - 3L_1$
 $L_2 = L_2 + (-3)$
 $L_3 = L_3 + 3L_2$
 $L_1 = L_1 - 2L_2$
 $L_3 = L_3 + 3L_1$
 $L_2 = L_2 + L_3$

$$g) \begin{bmatrix} 1 & 0 & 3 & -8 \\ 2 & -4 & 0 & -4 \\ 3 & -2 & -5 & 26 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & -4 & -6 & 12 \\ 0 & -2 & -10 & 90 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & 1 & \frac{3}{2} & -3 \\ 0 & 0 & -14 & 90 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & 1 & \frac{3}{2} & -3 \\ 0 & 0 & 1 & 90 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 90 \end{bmatrix} \begin{array}{l} x=4 \\ y=3 \\ z=-4 \end{array}$$

$L_2 = L_2 - 2L_1$
 $L_2 = L_2 + (-4)$
 $L_3 = L_3 + 2L_2$
 $L_3 = L_3 + 11L_1$
 $L_2 = L_2 + L_3$
 $L_2 = L_2 - (\frac{3}{2})L_3$

$$h) \begin{bmatrix} 1 & 2 & 3 & 10 \\ 3 & 4 & 6 & 23 \\ 2 & 2 & 3 & 13 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 10 \\ 0 & -2 & -3 & -7 \\ 0 & -2 & -3 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 10 \\ 0 & 1 & \frac{3}{2} & \frac{7}{2} \\ 0 & 0 & -3 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & \frac{3}{2} & \frac{7}{2} \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{array}{l} x=3 \\ y=\frac{7}{2}-\frac{3}{2}z \\ z=t \end{array}$$

$L_2 = L_2 - 3L_1$
 $L_2 = L_2 + (-2)$
 $L_3 = L_3 + 2L_2$
 $L_3 = L_3 + 2L_1$

$$i) \begin{bmatrix} 5 & -3 & 4 & -1 & 27 \\ 2 & -1 & 3 & -2 & 19 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 4 & -1 & 27 \\ 0 & 5 & -8 & 0 & 19 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 4 & -1 & 27 \\ 0 & 1 & -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & -1 & 17 \\ 0 & 1 & -1 & 0 & 3 \end{bmatrix} \begin{array}{l} w=5 \\ y=3+t \\ z=t \\ x=11-t+s \end{array}$$

$L_2 = L_2 - 2L_1$
 $L_2 = L_2 / 5$
 $L_1 = L_1 + 3L_2$

$$3-$$

a) $\begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 8 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{array}{l} x=1-w \\ y=2 \\ z=1 \\ w= \end{array}$

$L_2 = L_2 - L_1$
 $L_3 = L_3 + 2L_2$
 $L_3 = -\frac{1}{4} \cdot [3L_3]$
 $L_3 = L_3 - 3L_1$
 $L_2 = L_2 + 3L_3$
 $L_2 = L_2 + L_3$

b) $\begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ -1 & 1 & -2 & 0 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & -1 & -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -2 & 1 \end{bmatrix} \begin{array}{l} x=y+z-w-2z \\ y=2-w \\ z=-y-2w+1 \\ w= \text{livre} \end{array}$

$L_2 = L_2 - L_1$
 $L_2 = L_2 + L_1$
 $L_2 = L_2 + L_3$
 $L_3 = L_3 - L_1$

c) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 1 & 1 & 3 & 0 \\ 1 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{array}{l} x=0 \\ y=0 \\ z=0 \\ w= \text{livre} \end{array}$

$L_2 = L_2 - L_1$
 $L_3 = L_3 - L_1$
 $L_3 = L_3 - L_2$
 $L_4 = L_4 - 2L_3$
 $L_4 = L_4 - 2L_2$
 $L_4 = L_4 - L_1$
 $L_3 = L_3 - L_1$
 $L_3 = L_3 - L_2$

d) $\begin{bmatrix} x & y & z \\ 1 & -2 & 1 & 1 & 2 \\ 2 & -5 & 1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ 0 & -1 & 1 & -4 & -5 \\ 0 & -1 & -1 & -4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & 1 & -4 & -4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 & 9 & 12 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} x=1 \\ y=1 \\ z=1 \\ w= \end{array}$

$L_2 = L_2 - 2L_1$
 $L_2 = L_2 - (-1)$
 $L_3 = L_3 + L_2$
 $L_3 = L_3 + 2L_1$
 $L_3 = L_3 - 3L_1$

$B_1 = \begin{bmatrix} 9 \\ 4 \\ 0 \end{bmatrix}$
 $B_2 = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$

5-

a) $\begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{pmatrix} + 4 \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$S = \begin{cases} x_3 = -x_1 ; -x_1 - x_3 = 5x_2 ; -x_2 = 8x_3 \end{cases}$

$\begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \cdot x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 4 & 0 & 8 \\ 0 & 4 & 8 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{pmatrix} \rightsquigarrow \begin{pmatrix} -1 & 0 & -5 \\ -1 & -1 & -1 \\ 0 & -1 & -4 \end{pmatrix}$

$\begin{pmatrix} 4x_1 \\ 4x_2 \\ 4x_3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & -5 \\ -1 & -1 & -1 \\ 0 & -1 & -4 \end{pmatrix} \rightsquigarrow \begin{array}{l} -x_1 - 5x_3 = 4x_1 \\ -x_1 - x_2 - x_3 = 4x_2 \\ -5x_3 = 6x_1 \\ x_3 = -x_1 \end{array}$

$-x_1 - x_2 - x_3 = 4x_2$

$-x_1 - x_3 = 5x_2$

$-x_2 - 4x_3 = 4x_3$

$-x_2 = 8x_3$

b)

$\begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - 2 \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$x_1 + 5x_3 = 2x_1$

$-x_1 = 5x_3$

$x_1 + x_2 + x_3 = 2x_2$

$x_2 = x_1 + x_3$

$x_2 - 4x_3 = 2x_3$

$6x_3 = x_2$

$S = \begin{cases} -x_1 = 5x_3 ; x_2 = x_1 + x_3 ; 6x_3 = x_2 \end{cases}$

6-

a) $\begin{cases} x + y + z = 2 \\ 2x + 3y + 2z = 9 \\ 2x + 3y + (a^2 - 1)z = a + 1 \end{cases}$

\rightsquigarrow S.P.D $\begin{cases} (a^2 - 1) = b \\ a + 1 = c \end{cases}$

$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 9 \\ 2 & 3 & b & c \end{bmatrix}$ posto = nº de incógnitas

$\rightsquigarrow b \neq 2 \wedge c \neq 5$ para que não haja linhas nulas

$a^2 - 1 \neq 2$ $a + 1 \neq 5$

$a \neq \pm \sqrt{3}$ $a \neq \pm 4$

\rightsquigarrow S.P.I

posto < n $b = 2$ e $c = 5$ para que haja equações

posto < 3 dependentes, e uma linha nula

$a^2 - 1 = 2$ $a + 1 = 5$

$a = \sqrt{3}$ $a = 4$

\rightsquigarrow S.I

$a + 1 \neq 5$ para que haja incompatibilidade entre a linha 2 e 3, e $a^2 - 1 = 2$

$a + 1 \neq 5$ $a^2 - 1 = 2$

b) $\begin{bmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 7 & b & c \end{bmatrix} - \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & 10 \\ 0 & -7 & a^2 - 2 & a - 14 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2 - 16 & 0 - 4 \end{bmatrix}$

$L_2 - 3L_1 \ L_3 - L_3 - UL_3 \ L_3 = L_3 - bL_2$

$a \neq \pm 4$ $a \neq \pm \sqrt{3}$

\rightsquigarrow S.P.D

posto = nº de incógnitas, $a^2 - 16 \neq 0 \rightsquigarrow a = \pm 4 \rightsquigarrow$ S.I

\rightsquigarrow S.P.I $a^2 - 16 = 0 \wedge a - 4 = 0$

posto < n

$a = 4 \ L_2 = L_2 - 4L_1 \ L_2 = L_2 - 4L_1$

posto < 3

$a^2 - 16 = 0 \wedge a - 4 \neq 0$ (contradição)

$$7) : 2 : 2 : 2 : 2$$

$$a - \left[\begin{array}{ccccc} 2 & -2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array} \right] = \left[\begin{array}{ccccc} -1 & \frac{1}{2} & 0 & 1 \\ 3 & 4 & 0 & 1 \end{array} \right] = \left[\begin{array}{ccccc} 4 & 0 & \frac{1}{2} & 1 \\ 3 & 1 & 0 & 1 \end{array} \right] = \left[\begin{array}{ccccc} 1 & 0 & \frac{1}{8} & \frac{1}{4} \\ -1 & 1 & -\frac{1}{2} & 0 \end{array} \right] = \left[\begin{array}{ccccc} 1 & 0 & \frac{1}{8} & \frac{1}{4} \\ 0 & 1 & -\frac{1}{8} & \frac{1}{4} \end{array} \right]$$

$\sqrt{2} \approx 1.41$

$$b) \left[\begin{array}{cccc|cc} 2 & -2 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{L}_1 \rightarrow L_1 + L_2} \left[\begin{array}{cccc|cc} 3 & -1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{L}_2 \rightarrow L_2 - L_1} \left[\begin{array}{cccc|cc} 3 & -1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & -2 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{L}_3 \rightarrow L_3 - L_2} \left[\begin{array}{cccc|cc} 3 & -1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 & 0 & 1 \end{array} \right] \xrightarrow{\text{L}_3 \rightarrow \frac{1}{4}L_3} \left[\begin{array}{cccc|cc} 3 & -1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{array} \right] \xrightarrow{\text{L}_1 \rightarrow L_1 - 3L_3} \left[\begin{array}{cccc|cc} 0 & -4 & 0 & -\frac{3}{4} & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{array} \right] \xrightarrow{\text{L}_1 \rightarrow -\frac{1}{4}L_1} \left[\begin{array}{cccc|cc} 0 & 1 & 0 & \frac{3}{4} & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{array} \right] \xrightarrow{\text{L}_2 \rightarrow L_2 - 2L_1} \left[\begin{array}{cccc|cc} 0 & 1 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{array} \right] \xrightarrow{\text{L}_3 \rightarrow L_3 - L_2} \left[\begin{array}{cccc|cc} 0 & 1 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \end{array} \right]$$

$$\begin{bmatrix} 1 & -1 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 1/3 & 1/6 & 1/3 & 0 \\ 0 & 0 & 1 & -1/8 & 1/4 & -3/4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 1/8 & 1/4 & 1/4 \\ 0 & 0 & 1 & -1/8 & 1/4 & -3/4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1/3/2 & 1/4 & 1/4 \\ 0 & 1 & 0 & -1/8 & 1/4 & 1/4 \\ 0 & 0 & 1 & 1/8 & 1/4 & -3/4 \end{bmatrix}$$

$L_2 = L_2 - \frac{1}{3}L_3$ $L_3 = L_3 + L_2$ $L_1 = L_1 + L_2$

$$d) \begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & -1 & 0 & 1 & 0 & \\ 2 & 1 & 0 & 0 & 0 & 1 & \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & -1 & -10 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 10 & -1 & 1 & 0 \\ 10 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & -1 & 1 \\ 10 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} =$$

$L_2 = L_2(-1)$ $L_2 = L_2 + L_3$ $L_3 = L_3 - L_1$

$$\begin{array}{c} L_2 = L_2 - L_1, \quad L_3 = L_3 + L_1 \\ \left[\begin{array}{ccccc} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 \end{array} \right] = \left[\begin{array}{ccccc} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 \end{array} \right] = \left[\begin{array}{ccccc} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & -1 & 1 & 0 \end{array} \right] = \left[\begin{array}{ccccc} 1 & 0 & 0 & -2 & -1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \\ L_3 = L_3 - L_2 \quad L_2 = L_2 + (2L_3) \quad L_3 = L_3 + (-1)L_2 \end{array}$$

$$L_1 = L_1/2 \quad L_3 = L_3 - L_1 \quad L_4 = L_4 + L_1 \quad L_2 = L_2 - \frac{1}{2}L_1$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4}u & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2}u & 0 & \frac{1}{2}u & 0 & 0 \\ 0 & 0 & -\frac{1}{2}u & 0 & \frac{1}{2}u & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \frac{1}{4}u & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2}u & 0 & \frac{1}{2}u & 0 & 0 \\ 0 & 0 & 2 & \frac{1}{2}u & -\frac{1}{2}u & -\frac{1}{4}u & \frac{1}{2}u & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \frac{1}{4}u & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2}u & 0 & \frac{1}{2}u & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{4}u & -\frac{1}{4}u & -\frac{1}{8}u & \frac{1}{2}u & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \frac{1}{4}u & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2}u & 0 & \frac{1}{2}u & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}u & -\frac{1}{2}u & -\frac{1}{4}u & \frac{1}{2}u & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$L_3 \leftarrow L_3 - \frac{1}{2}L_2$ $L_2 \leftarrow L_2 - \frac{1}{2}L_3$ $L_4 \leftarrow L_4 + 2L_3$ $L_4 \leftarrow L_4 - \frac{1}{2}$

$$\begin{array}{cccccc} 1 & 0 & 0 & \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{array}$$

8)

$$\begin{aligned}
 & x = \text{calco} \quad \begin{cases} x + 2y + 3z = 26 \\ 2x + 5y + 6z = 60 \\ 2x + 3y + 4z = 40 \end{cases} \\
 & y = \text{shorts} \\
 & z = \text{flusas}
 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 26 \\ 2 & 5 & 6 & 60 \\ 2 & 3 & 4 & 40 \end{array} \right] \xrightarrow{\begin{array}{l} L_2 - 2L_1 \\ L_3 - L_1 \end{array}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 26 \\ 0 & 1 & 0 & 8 \\ 0 & 1 & 1 & 14 \end{array} \right] \xrightarrow{\begin{array}{l} L_1 - L_2 \\ L_3 - L_2 \end{array}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 6 \end{array} \right] \xrightarrow{\begin{array}{l} L_3 - L_3/6 \\ L_3 - L_3 \end{array}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\begin{aligned}
 x + 3z = 10 \Rightarrow x = 10 - 3 \cdot (2 \cdot 1) &= 4 \\
 y = 8 & \\
 z = 1 &
 \end{aligned}$$

9)

$$\begin{aligned}
 & x = \text{audalb.} \\
 & y = \text{casacombio} \\
 & z = \text{banana split}
 \end{aligned}$$

$$\begin{aligned}
 & y = 3z \quad \begin{cases} 5x + 2y + 6z = 2200 \\ y = 3z \end{cases} \quad \begin{aligned} 5(3z - z) + 2(3z) + 6z &= 2200 \\ 10z + 6z + 6z &= 2200 \\ 22z &= 2200 \Rightarrow z = 100 \end{aligned} \\
 & y = x + z \quad \begin{cases} y = x + z \\ y = 3z \end{cases} \quad \begin{aligned} x &= y - z \\ x &= 3z - z \\ x &= 200 \end{aligned} \\
 & \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 5 & 3 & 1 & 200 \\ 1 & 1 & 0 & 0 \end{array} \right] \xrightarrow{\begin{array}{l} L_1 - L_2 \\ L_3 - L_2 \end{array}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 5 & 2 & 1 & 200 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\begin{array}{l} L_1 - L_1/5 \\ L_2 - 5L_1 \\ L_2 - L_2 \end{array}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 0 \end{array} \right]
 \end{aligned}$$

10)

$$\begin{aligned}
 & x = \text{torta} \quad \begin{cases} 40x + 30y + 10z = 7000 \\ 20x + 40y + 30z = 6000 \\ 10x + 20y + 40z = 5000 \end{cases} \quad \begin{aligned} 2(10x + 20y + 40z) - (20x + 40y + 30z) &= (5000 - 2) - 6000 \\ 50z &= 4000 \\ z &= 80 \end{aligned} \\
 & y = \text{salado} \\
 & z = \text{picante}
 \end{aligned}$$

$$\begin{aligned}
 & 2(20x + 40y + 30z) - (40x + 30y + 10z) = 12000 - 7000 \\
 & -50y - 80z = 5000 \\
 & -50y + 50 \cdot 80 = 5000 \\
 & y = 50y = 1000 \quad \therefore 150z \\
 & y = 1000 \quad \therefore x = 150z \\
 & x = 150 \cdot 80 \quad \therefore x = 12000 \\
 & x + 2 \cdot 12000 + 4 \cdot 80 = 5000 \quad \therefore x = 12000 \\
 & x = 12000 - 5000 = 7000 \\
 & x = 12000 - 12000 = 0
 \end{aligned}$$

11)

$$\begin{aligned}
 & A = x \quad \begin{cases} 2x + 3y + z = 8420 \\ x + 2y + 2z = 7940 \\ 4x + 3y + 0z = 8110 \end{cases} \quad \begin{aligned} 2(2x + 3y + z) - (x + 2y + 2z) &= (8420 - 2) - 7940 \\ 3x + 4y &= 8900 \quad (8420 - 2) - 8110 \\ 4x + 3y &= 8110 \quad \therefore y = 790 + x \\ -x + y &= 790 \quad \therefore y = 1610 \end{aligned} \\
 & B = y \\
 & C = z \quad z = 2x + 310 \\
 & 2x + 3(790 + x) + (2x + 310) = 8420 \\
 & 2x + 2370 + 3x + 2x + 310 = 8420 \\
 & 5x = 5700 \\
 & x = 820
 \end{aligned}$$

$$\begin{aligned}
 & z = 2 \cdot 820 + 310 \\
 & z = 1950
 \end{aligned}$$

maior menor = 1130