→ Lista 8

1-

a) reta s

$\begin{cases} x = \lambda/3 \\ y = \frac{\lambda+36}{2} \\ z = \lambda \end{cases}$ $z=0$

$P_s = (0, 8, 0)$

$\vec{v}_s = \left(\frac{1}{3}; \frac{1}{2}; 1\right)$

reta r

$P_r = \left(-5; \frac{2}{3}; 0\right)$

$\vec{v}_r = \left(\frac{1}{2}, 1, 1\right)$

$\text{sen}\,\alpha = \frac{\|\vec{v}_s \cdot \vec{v}_r\|}{\|\vec{v}_s\| \cdot \|\vec{v}_r\|} = \frac{\sqrt{41}}{12} : \left(\frac{7}{6} \cdot \frac{3}{2}\right) = \frac{\sqrt{41}}{12} : \frac{21}{12} = \frac{\sqrt{41}}{12} \cdot \frac{12}{21} = \frac{\sqrt{41}}{21}$

$\begin{bmatrix} i & j & k \\ 1/3 & 1/2 & 1 \\ 1/2 & 1 & 1 \end{bmatrix} \begin{matrix} i & j \\ 1/3 & 1/2 \\ 1/2 & 1 \end{matrix}$

$\|\vec{v}_s\| = \sqrt{\frac{1}{3}^2 + \frac{1}{2}^2 + 1^2} = \sqrt{\frac{1}{9} + \frac{1}{4} + 1} = \sqrt{\frac{49}{36}} = \frac{7}{6}$

$\|\vec{v}_r\| = \sqrt{\frac{1}{2}^2 + 1^2 + 1^2} = \sqrt{\frac{1}{4} + 1 + 1} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

$-1/4\,k - i - 1/3\,j + 1/2\,i + 1/2\,j + 1/3\,k = (1/2, -1/6, -1/12)$

$\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{12}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{36} + \frac{1}{144}} = \sqrt{\frac{36+4+1}{144}} = \sqrt{\frac{41}{144}} = \frac{\sqrt{41}}{12}$

b) reta s

$\begin{cases} x = t+1 \\ y = t \\ z = 4 \end{cases}$ $y = t$

$x - y + 3 = 4$

$x - y = 1$

$P_s = (1, 0, 4)$

$\vec{v}_s = (1, 1, 0)$

reta s

$P_r = (1, 1, 0)$

$\vec{v}_r = (0, -1, 1)$

$\text{sen}\,\alpha = \frac{\|\vec{v}_s \cdot \vec{v}_r\|}{\|\vec{v}_s\| \|\vec{v}_r\|} = \frac{\sqrt{3}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{3}}{2}$

$\begin{bmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{matrix} i & j \\ 1 & 1 \\ 0 & -1 \end{matrix}$

$0 + 0 - j + i + 0 - k$

$(1, -1, -1)$

$\|\vec{v}_s\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$

$\|\vec{v}_r\| = \sqrt{0^2 + (-1)^2 + 1^2} = \sqrt{2}$

$\|\vec{v}_s \cdot \vec{v}_r\| = \sqrt{1^2 + (-1)^2 + (-1)^2} = \sqrt{3}$

c) reta r

$\begin{cases} x = 7 - 3z \\ y = 0 \end{cases}$ $z = t$

$\begin{cases} x = 7 - 3t \\ y = 0 \\ z = t \end{cases}$

$P_r = (7, 0, 0)$

$\vec{v}_r = (-3, 0, 1)$

reta s

$\begin{cases} x - 4y - 2z = 5 \\ y = 0 \end{cases}$ $z = t$ $\to P_s = (5, 0, 0)$

$\begin{cases} x = 5 + 2t \\ y = 0 \\ z = t \end{cases}$

$\vec{v}_s = (2, 0, 1)$

$\cos\alpha = \frac{|\vec{v}_s \cdot \vec{v}_r|}{\|\vec{v}_s\| \cdot \|\vec{v}_r\|} = \frac{|2\cdot(-3) + 0\cdot 0 + 1\cdot 1|}{\sqrt{5} \cdot \sqrt{10}} = \frac{|-6 + 0 + 1|}{\sqrt{50}} = \frac{|-5|}{5\sqrt{2}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\|\vec{v}_s\| = \sqrt{2^2 + 0^2 + 1^2} = \sqrt{5}$

$\|\vec{v}_r\| = \sqrt{(-3)^2 + 0^2 + 1^2} = \sqrt{10}$

d) reta r

$\begin{cases} x = t \\ y = -2t + 1 \\ z = 3t \end{cases}$

$P_r = (0, 1, 0)$

$\vec{v}_r = (1, -2, 3)$

reta s

$v_{s1} = (3, 1, -5)$

$v_{s2} = (1, -2, 3)$

$\begin{bmatrix} i & j & k \\ 3 & 1 & -5 \\ 1 & -2 & 3 \end{bmatrix} \begin{matrix} i & j \\ 3 & 1 \\ 1 & -2 \end{matrix}$

$-k - 10i - 9j + 3i - 5j - 6k$

$\vec{v}_s = (-7, -14, -7) = (-1, -2, -1)$

$\cos\alpha = \frac{|v_s \cdot v_r|}{\|v_s\| \cdot \|v_r\|} = \frac{|-1\cdot 1 + (-2)\cdot(-2) + 3\cdot(-1)|}{\sqrt{6} \cdot \sqrt{14}} = \frac{0}{\sqrt{84}} = 0$

$\|v_s\| = \sqrt{(-1)^2 + (-2)^2 + (-1)^2} = \sqrt{6}$

$\|v_r\| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}$

2-

reta r

$P_r = (0, 2+\lambda, 0)$

$\vec{v}_r = (0, 1, 0)$

reta s

$P_s = (1, 2, \mu)$

$\vec{v}_s = (0, 0, 1)$

$\vec{PQ} = Q - P = (1, 2, \mu) - (0, 2+\lambda, 0) = (1, -\lambda, \mu)$

$\cos = \frac{|\vec{PQ} \cdot \vec{v}_r|}{\|PQ\| \cdot \|v_r\|} = \frac{1\cdot 0 + 1\cdot(-\lambda) + \mu\cdot 0}{\sqrt{1 + \lambda^2 + \mu^2}}$ $\to \frac{\sqrt{2}}{2} = \frac{-\lambda}{\sqrt{\lambda^2 + \mu^2 + 1}}$

$-\lambda = \frac{\sqrt{2} \cdot \sqrt{\lambda^2 + \mu^2 + 1}}{2}$

$-\frac{2\lambda}{\sqrt{2}} = \sqrt{\lambda^2 + \mu^2 + 1}$

$-\sqrt{2}\lambda = \sqrt{\lambda^2 + \mu^2 + 1}$

$\|\vec{PQ}\| = \sqrt{1^2 + (-\lambda)^2 + \mu^2} = \sqrt{1 + \lambda^2 + \mu^2}$

$\|v_r\| = \sqrt{0^2 + 1^2 + 0^2} = 1$

$\|v_s\| = \sqrt{0^2 + 0^2 + 1^2} = 1$

$P = (0, 2 + (-\sqrt{2}), 0)$

$Q = (1, 2, \pm 1)$

$\frac{1}{2} = \frac{|1\cdot 0 - \lambda\cdot 0 + 1\cdot\mu|}{\sqrt{1 + \lambda^2 + \mu^2}}$

$\frac{1}{2} = \frac{\mu}{\sqrt{1 + \lambda^2 + \mu^2}}$

$\mu = \frac{\sqrt{1 + \lambda^2 + \mu^2}}{2}$

$2\mu = \sqrt{1 + \lambda^2 + \mu^2}$

$2 \cdot \left(-\frac{\sqrt{2}\lambda}{2}\right) = \sqrt{1 + \lambda^2 + \left(-\frac{\sqrt{2}\lambda}{2}\right)^2}$

$-\sqrt{2}\lambda = \sqrt{1 + \lambda^2 + \frac{2}{4}\lambda^2}$

$(\sqrt{2}\lambda)^2 = \left(\sqrt{1 + \frac{3}{2}\lambda^2}\right)^2$

$2\mu = -\sqrt{2}\lambda$

$\mu = -\frac{\sqrt{2}\lambda}{2}$

$\mu = -\frac{\sqrt{2}}{2} \cdot (\sqrt{2}) = 1$

$\to 1 + \frac{3}{2}\lambda^2 = 2\lambda^2$

$1 = \frac{1}{2}\lambda^2$

$\lambda^2 = 2$

$\lambda = \pm\sqrt{2}$

__/__/__

3-

a) reta r

$\begin{cases} x=0 \\ y=t \\ z=t \end{cases}$ $y=t$

$v_r=(0,1,1)$

v_n do plano π

$0x+0y+1z=0$

$v_n=(0,0,1)$

$v_r \cdot n_\pi = 0\cdot 0+0\cdot 1+1\cdot 1=1$

$\|v_r\|=\sqrt{0^2+1^2+1^2}=\sqrt{2}$

$\|n_\pi\|=\sqrt{0^2+0^2+1^2}=1$

$\operatorname{sen}\alpha=\frac{|1|}{\sqrt{2}\cdot 1}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2}=\frac{\pi}{4}$ rad

b) reta r

$\begin{cases} x=-\lambda \\ y=\lambda \\ z=2\lambda+1 \end{cases}$

$\vec{v}_r=(-1,1,2)$

v_n do plano π

$2x-y+z0=0$

$v_n=(2,-1,0)$

$\vec{v}_r\cdot v_n=2\cdot(-1)+1\cdot(-1)+2\cdot 0=-3$

$\|\vec{v}_r\|=\sqrt{(-1)^2+1^2+2^2}=\sqrt{6}$

$\|\vec{v}_n\|=\sqrt{2^2+(-1)^2+0^2}=\sqrt{5}$

$\operatorname{sen}=\frac{|-3|}{\sqrt{6}\cdot\sqrt{5}}=\frac{3}{\sqrt{30}}\cdot\frac{\sqrt{30}}{\sqrt{30}}=\frac{3\sqrt{30}}{30}=\frac{\sqrt{30}}{10}$

c) reta r

$P_r=(1,0,0)$

$\vec{v}_r=(1,1,-2)$

plano π

$x+y-z-1=0$

$\vec{v}_n=(1,1,-1)$

$\vec{v}_r\cdot\vec{v}_n=1\cdot 1+1\cdot 1+(-1)\cdot(-2)=4$

$\|\vec{v}_r\|=\sqrt{1^2+1^2+(-2)^2}=\sqrt{6}$

$\|\vec{v}_n\|=\sqrt{1^2+1^2+(-1^2)}=\sqrt{3}$

$\operatorname{sen}=\frac{|4|}{\sqrt{6}\cdot\sqrt{3}}=\frac{4}{\sqrt{18}}\cdot\frac{\sqrt{18}}{\sqrt{18}}=\frac{4\sqrt{18}}{18:2}=\frac{2\sqrt{18}}{9}=\frac{6\sqrt{2}}{9:3}=\frac{2\sqrt{2}}{3}$

4-

plano π_1

$n_{\pi_1}=(1,1,1)$

plano π_2

$n_{\pi_2}=(1,-1,0)$

$v\cdot v_{n_1}=(a,b,c)\cdot(1,1,1)$

$=a+b+c$ (eq.1)

$v\cdot n_2=(a,b,c)\cdot(1,-1,0)$

$=a-b$ (eq.2)

$\|v\|=\sqrt{a^2+b^2+c^2}$

$\|n_2\|=\sqrt{1^2+(-1)^2+0^2}=\sqrt{2}$

$a+b+c \Rightarrow c=-a-b$

$a^2+b^2+c^2 \Rightarrow 4\cdot(a-b)^2$

$\operatorname{sen}\alpha=\frac{|v\cdot n_2|}{\|v\|\cdot\|n_2\|}$

$\frac{\sqrt{2}}{2}=\frac{|a-b|}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{2}}$

$\frac{1}{2}=\frac{|a-b|}{\sqrt{a^2+b^2+c^2}}$

$\frac{1}{4}=\frac{(a-b)}{a^2+b^2+c^2}$ eq(3)

$a^2+b^2+(-a-b)^2=4(a-b)^2$

$a^2+b^2+(a+b)^2=4\cdot(a^2-2ab+b^2)$

$a^2+b^2+(a^2+2ab+b^2)=4a^2-8ab+4b^2$

$2a^2+2b^2+2ab=4a^2-8ab+4b^2$

$2a^2+2b^2-10ab=0 \;:2$

$a^2+b^2-5ab=0$

$a^2+b^2-5ab=0 \;:b^2$

$\left(\frac{a}{b}\right)^2+1-5\cdot\left(\frac{a}{b}\right)=0$ $x=a/b$

x^2-5x+1

$\Delta=21$

$x_1=\frac{5+\sqrt{21}}{2}$

$x_2=\frac{5-\sqrt{21}}{2}$

$b=2$

caso 1: $\frac{a}{2}=\frac{5-\sqrt{21}}{2} \Rightarrow a=5-\sqrt{21}$

caso 2: $\frac{a}{2}=\frac{5+\sqrt{21}}{2} \Rightarrow a=5+\sqrt{21}$

caso 1: $c=-a-b$

$c=-(5-\sqrt{21})-2$

$c=-7+\sqrt{21}$

caso 2: $c=-a-b$

$c=-(5+\sqrt{21})-2$

$c=-7-\sqrt{21}$

possível $v_1=(5+\sqrt{21},2,-7-\sqrt{21})$

possível $v_2=(5-\sqrt{21},2,-7+\sqrt{21})$

$\|v_1\|=\sqrt{(5-\sqrt{21})^2+2^2+(-7+\sqrt{21})^2}$

$\sqrt{(25-10\sqrt{21}+21)+4+(49-14\sqrt{21}+21)}$

$\sqrt{46-10\sqrt{21}+4+70-14\sqrt{21}}$

$\sqrt{120-24\sqrt{21}}$

$\|v_2\|=\sqrt{(5+\sqrt{21})^2+2^2+(-7-\sqrt{21})^2}$

$=\sqrt{(25+10\sqrt{21}+21)+4+(49+14\sqrt{21}+21)}$

$=\sqrt{46+10\sqrt{21}+4+70+14\sqrt{21}}$

$=\sqrt{120+24\sqrt{21}}$

$\left(\frac{5+\sqrt{21}}{\sqrt{120+24\sqrt{21}}};\frac{2}{\sqrt{120+24\sqrt{21}}};\frac{-7-\sqrt{21}}{\sqrt{120+24\sqrt{21}}}\right)$ ou $\left(\frac{5-\sqrt{21}}{\sqrt{120-24\sqrt{21}}};\frac{2}{\sqrt{120-24\sqrt{21}}};\frac{-7+\sqrt{21}}{\sqrt{120-24\sqrt{21}}}\right)$

5-

a)

$n_{\pi_1} = (2,1,-1)$

$n_{\pi_2} = (1,-1,3)$

$|n_{\pi_1} \cdot n_{\pi_2}| = (2,1,-1)\cdot(1,-1,3) = 2-1-3 = -2|$

$\|n_{\pi_1}\| = \sqrt{2^2+1^2+(-1)^2} = \sqrt{6}$

$\|n_{\pi_2}\| = \sqrt{1^2+(-1)^2+3^2} = \sqrt{11}$

$\cos\theta = \frac{|-2|}{\sqrt{11}\cdot\sqrt{6}} = \frac{2}{\sqrt{66}} \cdot \frac{\sqrt{66}}{\sqrt{66}} = \frac{2\sqrt{66}}{66} = \frac{\sqrt{66}}{33}$

b)

$n_{\pi_2} = (1,1,1)$

$n_{\pi_1} = (0,-1,0)$

$$\begin{bmatrix} i & j & k \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{matrix} i & j \\ 1 & 0 \\ 1 & 0 \end{matrix}$$

$0+0+0+0-j=0$

$(0,-1,0)$

$|n_{\pi_1} \cdot n_{\pi_2}| = (0,-1,0)\cdot(1,1,1) = -1$

$\|n_{\pi_1}\| = \sqrt{0^2+(-1)^2+0^2} = 1$

$\|n_{\pi_2}\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\cos\theta = \frac{|-1|}{1\cdot\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

c)

$n_{\pi_1} = (0,-1,1)$

$$\begin{bmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{matrix} i & j \\ 1 & 0 \\ 1 & 1 \end{matrix}$$

$0+0-j+0+0+k=$

$n_{\pi_2} = (0,0,-1)$

$$\begin{bmatrix} i & j & k \\ -1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{matrix} i & j \\ -1 & 2 \\ 0 & 1 \end{matrix}$$

$0+0+0+0+0-k$

$|n_{\pi_1} \cdot n_{\pi_2}| = (0,-1,1)\cdot(0,0,-1) = |-1|$

$\|n_{\pi_1}\| = \sqrt{0+(-1)^2+1^2} = \sqrt{2}$

$\|n_{\pi_2}\| = \sqrt{0^2+0^2+(-1)^2} = 1$

$\cos\theta = \frac{|-1|}{\sqrt{2}\cdot 1} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

6-

plano π_1

$n_{\pi_1} = (2,-1,1)$

plano π_2

$A_{\pi_2} = (1,2,3)$

$n_{\pi_2} = (1,-2,1)$

$|n_{\pi_1} \cdot n_{\pi_2}| = (2,-1,1)\cdot(1,-2,1) = 2+2+1 = |5|$

$\|n_{\pi_1}\| = \sqrt{2^2+(-1)^2+1^2} = \sqrt{6}$

$\|n_{\pi_2}\| = \sqrt{1^2+(-2)^2+1^2} = \sqrt{6}$

$\cos\theta = \frac{|5|}{\sqrt{6}\cdot\sqrt{6}} = \frac{5}{6}$

7-

a) r: $t = x-1 = 2y = z$

$$\begin{cases} x = t+1 \\ y = t/2 \\ z = t \end{cases}$$

$P = (t+1, t/2, t)$

$d(P,A)^2 = d(P,B)^2$

$P-A = (t+1-1, t/2-1, t-0) = (t, t/2-1, t)^2 = t^2 + (t/2-1)^2 + t^2$

$P-B = (t+1-0, t/2-1, t-1) = (t+1, t/2-1, t-1)^2 = (t+1)^2 + (t/2-1)^2 + (t-1)^2$

$t^2 + (\cancel{t/2-1})^2 + t^2 = (t+1)^2 + (\cancel{t/2-1})^2 + (t-1)^2$

$2t^2 = (t+1)^2 + (t-1)^2$

$\cancel{2t^2} = t^2 + 2t + 1 + t^2 - 2t + 1$

$0 = 1$ impossível

R: Não há pontos na reta r que equidistam de A e B

b)

$P_r = (4\lambda, 2\lambda, 4-3\lambda)$

$d(P,A)^2 = d(P,B)^2$

$P-A = (4\lambda-2, 2\lambda-2, 4-3\lambda-5) = (4\lambda-2, 2\lambda-2, -3\lambda-1)$

$P-B = (4\lambda-0, 2\lambda-0, 4-3\lambda-1) = (4\lambda, 2\lambda, 3-3\lambda)$

$d(P,A) = (4\lambda-2)^2 + (2\lambda-2)^2 + (-3\lambda-1)^2$

$d(P,B) = 4\lambda^2 + 2\lambda^2 + (-3\lambda+3)^2$

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$(4\lambda-2)^2+(2\lambda-2)^2+(-3\lambda-1)^2=(4\lambda)^2+(2\lambda)^2+(3-3\lambda)^2$

$(16\lambda^2-16\lambda+4)+(4\lambda^2-8\lambda+4)+(9\lambda^2+6\lambda+1)=16\lambda^2+4\lambda^2+(9-18\lambda+9\lambda^2)$

$(16+4+9)\lambda^2+(-16-8+6)\lambda+(4+4+1)=(16+4+9)\lambda^2-18\lambda+9$

$29\lambda^2-18\lambda+9=29\lambda^2-18\lambda+9$ (V)

R: Todos os pontos da reta equidistam de A e B.

c)

$P=(2+\lambda,\ 3+\lambda,\ -3+\lambda) \rightsquigarrow (2+3,\ 3+3,\ -3+3)=(5,6,0)$ //

$P-A=(1+\lambda,\ 2+\lambda,\ -3+\lambda)$

$\|PA\|^2=(1+\lambda)^2+(2+\lambda)^2+(-3+\lambda)^2=(1+2\lambda+\lambda^2)+(4+4\lambda+\lambda^2)+(9-6\lambda+\lambda^2)=3\lambda^2+14$

$P-B=(\lambda,\ 1+\lambda,\ -7+\lambda)$

$\|PB\|^2=(\lambda)^2+(1+\lambda)^2+(-7+\lambda)^2=\lambda^2+1+2\lambda+\lambda^2+49-14\lambda+\lambda^2=3\lambda^2-12\lambda+50$

$3\lambda^2+14=3\lambda^2-12\lambda+50$

$+12\lambda=50-14$

$12\lambda=50-14$

$12\lambda=36$

$\lambda=3$

8-

a)

$P=(-2,0,1)$

$P_r=(1,-2,0)$

$\vec{v}_r=(3,2,1)$

$P_rP=P-P_r=(-2-1,\ 0-(-2),\ 1-0)=(-3,2,1)$

$$P_rP\cdot\vec{v}_r=\begin{bmatrix} i & j & k \\ -3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}\begin{matrix} i & j \\ -3 & 2 \\ 3 & 2 \end{matrix}$$

$-6k-2i+3j+2i+3j-6k$

$(0,6,-12)$

$\|P_rP\cdot\vec{v}_r\|=\sqrt{0^2+6^2+(-12)^2}=\sqrt{180}=6\sqrt{5}$

$\|\vec{v}_r\|=\sqrt{3^2+2^2+1^2}=\sqrt{14}$

$d=\dfrac{\|P_rP\cdot\vec{v}_r\|}{\|\vec{v}_r\|}=\dfrac{6\sqrt{5}\cdot\sqrt{14}}{\sqrt{14}}\cdot\dfrac{\sqrt{14}}{\sqrt{14}}=\dfrac{6\sqrt{70}}{14}=\dfrac{3\sqrt{70}}{7}$ //

b)

$\lambda=\dfrac{x-2}{4}=\dfrac{y}{-3}=\dfrac{1-z}{2}$

$\begin{cases} x=4\lambda+2 \\ y=-3\lambda \\ z=1-2\lambda \end{cases}$ $\quad P_r=(2,0,1)$ $\quad \vec{v}_r=(4,-3,-2)$

$P=(1,-1,4)$

$P_rP=P-P_r=(1-2,\ -1-0,\ 4-1)=(-1,-1,3)$

$$P_rP\cdot\vec{v}_r=\begin{bmatrix} i & j & k \\ -1 & -1 & 3 \\ 4 & -3 & -2 \end{bmatrix}\begin{matrix} i & j \\ -1 & -1 \\ 4 & -3 \end{matrix}$$

$4k+9i-2j+2i+12j+3k$

$(11,10,7)$

$\|P_rP\cdot\vec{v}_r\|=\sqrt{11^2+10^2+7^2}=\sqrt{270}=3\sqrt{30}$

$\|\vec{v}_r\|=\sqrt{4^2+(-3)^2+(-2)^2}=\sqrt{29}$

$d=\dfrac{3\sqrt{30}}{\sqrt{29}}\cdot\dfrac{\sqrt{29}}{\sqrt{29}}=\dfrac{3\sqrt{870}}{29}$

c)

$\lambda=x=2y-3=2z-1$

$\begin{cases} x=\lambda \\ y=(\lambda+3)/2 \\ z=(\lambda+1)/2 \end{cases}$ $\quad P_r=(0,\ \frac{3}{2},\ \frac{1}{2})$ $\quad \vec{v}_r=(1,\ \frac{1}{2},\ \frac{1}{2})$

$P=(0,-1,0)$

$P_rP=P-P_r=(0-0,\ -1-\frac{3}{2},\ 0-\frac{1}{2})=(0,\ -\frac{5}{2},\ -\frac{1}{2})$

$$P_rP\cdot\vec{v}_r=\begin{bmatrix} i & j & k \\ 0 & -5/2 & -1/2 \\ 1 & 1/2 & 1/2 \end{bmatrix}\begin{matrix} i & j \\ 0 & -5/2 \\ 1 & 1/2 \end{matrix}$$

$+5/2\,k+1/4\,i+0-5/4\,i-1/2\,j+0$

$(-3/4,\ -1/2,\ 5/2)$

$\|P_rP\cdot\vec{v}_r\|=\sqrt{\left(-\frac{3}{4}\right)^2+\left(-\frac{1}{2}\right)^2+\frac{5}{2}^2}=\sqrt{\frac{9}{16}+\frac{1}{4}+\frac{25}{4}}=\sqrt{\frac{9+4+25}{16}}=\sqrt{\frac{38}{16}}$

$\|\vec{v}_r\|=\sqrt{1^2+\frac{1}{2}^2+\frac{1}{2}^2}=\sqrt{1+\frac{1}{4}+\frac{1}{4}}=\sqrt{\frac{4+1+1}{4}}=\sqrt{\frac{6}{4}}=\sqrt{\frac{3}{2}}$

$d=\sqrt{\frac{38}{16}}\cdot\sqrt{\frac{3}{2}}=\dfrac{\sqrt{38}}{\sqrt{16}}\cdot\dfrac{\sqrt{2}}{\sqrt{3}}=\dfrac{\sqrt{76}}{\sqrt{48}}$ //

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9-

$\pi_1 \Rightarrow x+y=2 \rightarrow y=2-x$

$\pi_2 \Rightarrow x=y+z \rightarrow x=(2-x)+z$

$x=2-x+z$

$z=2x-2$

$\begin{cases} x=2-t \\ y=t \\ z=2-2t \end{cases}$ equação paramétrica da reta r de interseção

$P_t=(2-t,\ t,\ 2-2t)$

reta s $z=\lambda$

$\begin{cases} x=\lambda+1 \\ y=\lambda+1 \\ z=\lambda \end{cases}$ $P=(1,1,0)$ $\vec{v}_s=(1,1,1)$

$(\lambda+1,\ \lambda+1,\ \lambda)$

$d=\dfrac{\|\vec{AP}\times\vec{v}_s\|}{\|\vec{v}_s\|} \rightarrow \sqrt{\dfrac{14}{3}}=\dfrac{\sqrt{14}\cdot|t-1|}{\sqrt{3}}$

$P=(2-t,\ t,\ 2-2t)$ | $\vec{AP}=P-A=(2-t-1,\ t-1,\ 2-2t-0)$ | $\vec{AP}\times\vec{v}_s=\begin{bmatrix} i & j & k \\ 1-t & -1+t & 2-2t \\ 1 & 1 & 1 \end{bmatrix}\begin{matrix} i & j \\ 1-t & -1+t \\ 1 & 1 \end{matrix}$

$A=(1,1,0)$ | $(1-t,\ -1+t,\ 2-2t)$

$-(-1+t)k-(2-2t)i-(1-t)j+(-1+t)i+(2-2t)j+(1-t)k=(3t-3,\ -t+1,\ 2-2t)$

$\|\vec{AP}\times\vec{v}_s\|=\sqrt{(3t-3)^2+(-t+1)^2+(2-2t)^2}=\sqrt{9(t-1)^2+(t-1)^2+4(1-t)^2}=\sqrt{(9+1+4)\cdot(t-1)^2}=(t-1)\cdot\sqrt{14}$

$\|\vec{v}_s\|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

$\rightarrow \dfrac{\sqrt{14}}{\sqrt{3}}=\dfrac{\sqrt{14}\cdot|t-1|}{\sqrt{3}} \rightarrow |t-1|=\pm 1$ $\rightarrow t=2 \rightarrow P_1=(2,0,2)$; $\rightarrow t=0 \rightarrow P_2=(0,2,-2)$

10-

a) plano π

$\vec{v}_1=(1,0,0)$

$\vec{v}_2=(-1,0,3)$

$\begin{bmatrix} i & j & k \\ 1 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}\begin{matrix} i & j \\ 1 & 0 \\ -1 & 0 \end{matrix}$

$0+0-3j+0+0+0=(0,-3,0)$

eq. plano

$0\cdot(x-1)-3\cdot(y-0)+0\cdot(z-0)=0$

$-3y=0$

$y=0$

$d=\dfrac{|0\cdot 1+1\cdot 3+0\cdot 4+0|}{\sqrt{0^2+1^2+0^2}}=\dfrac{|3|}{\sqrt{1}}=\dfrac{3}{1}=3$ //

b)

$P=(0,0,-6)$

$\pi: x-2y-2z-6=0$

$d=\dfrac{|1\cdot 0+(-2)\cdot 0+(-2)\cdot(-6)-6|}{\sqrt{1^2+(-2)^2+(-2)^2}}=\dfrac{|0+0+12-6|}{\sqrt{9}}=\dfrac{|6|}{3}=2$ //

c)

$P=(1,1,1)$

$\pi: 2y-x+2z-3=0$

$d=\dfrac{|2\cdot 1+1\cdot(-1)+2\cdot 1-3|}{\sqrt{2^2+1^2+1^2}}=\dfrac{|2-1+2-3|}{\sqrt{3}}=\dfrac{0}{\sqrt{3}}=0$

R: Significa que o ponto P pertence ao plano π

11-

$\begin{cases} x=\lambda \\ y=2-\lambda \\ z=2\lambda-2 \end{cases}$

$P_\lambda=(\lambda,\ 2-\lambda,\ 2\lambda-2)$

$\pi \Rightarrow x-2y-z-1=0$

$D=\sqrt{6}$ //

$\sqrt{6}=\dfrac{|1\lambda+(-2)\cdot(2-\lambda)+(-1)\cdot(2\lambda-2)-1|}{\sqrt{1^2+(-2)^2+(-1)^2}}=\dfrac{|\lambda-4+2\lambda-2\lambda+2-1|}{\sqrt{6}}=\dfrac{|\lambda-3|}{\sqrt{6}}$

$\sqrt{6}=\dfrac{|\lambda-3|}{\sqrt{6}}$

$6=|\lambda-3|$

$6=\lambda-3$ ou $-6=\lambda-3$

$\lambda=9$ // $\lambda=-3$ //

$\lambda=9$: $\begin{cases} x=9 \\ y=-7 \\ z=16 \end{cases}$ $(9,-7,16)$ //

$\lambda=-3$: $\begin{cases} x=-3 \\ y=5 \\ z=-8 \end{cases}$ $(-3,5,-8)$ //

Kit

12 -

a) reta r

$P_r = (2,1,0)$

$\vec{v}_r = (1,-1,1)$

reta s

$2x - y - 1 = 0 \rightarrow y = -1 + 2x$

$x + (2x-1) + z = 0$

$z = -3x + 1$

$x = t$

$$\begin{cases} x = t \\ y = 2t - 1 \\ z = -3t + 1 \end{cases}$$

$P_s = (0,-1,1)$ e $\vec{v}_s = (1,2,-3)$

$(1,-1,1)$ e $(1,2,-3)$ não são multiplos um de outro, ou seja, não são paralelas

$$\begin{cases} t_r = t_s \rightarrow = t \\ 1 - t_r = 2t_s - 1 \rightarrow 1 - t = 2t - 1 \rightarrow t = 2/3 \\ t_r = 1 - 3t_s \rightarrow t = 1 - 3t \rightarrow t = 1/4 \end{cases}$$

$2/3 \neq 1/4$, ou seja, são reversas

$P_rP_s = P_s - P_r = (0,-1,1) - (2,1,0) = (-2,-2,1)$

$v_r \cdot v_s = \begin{bmatrix} i & j & k \\ 1 & -1 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{matrix} i & j \\ 1 & -1 \\ 1 & 2 \end{matrix}$

$= (1,4,3)$

$\|v_r \cdot v_s\| = \sqrt{1^2 + 4^2 + 3^2} = \sqrt{26}$

$P_rP_s \cdot (v_r \cdot v_s) = (-2,-2,1) \cdot (1,4,3) = -2 \cdot 1 + (-2) \cdot 4 + 1 \cdot 3 = -7$

$D = \frac{|-7|}{\sqrt{26}} = \frac{7}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = \frac{7\sqrt{26}}{26}$

b) reta r

$$\begin{cases} x = 3\lambda - 4 \\ y = 4\lambda \\ z = -2\lambda - 5 \end{cases}$$

$P_r: (-4,0,-5)$

$\vec{v}_r: (3,4,-2)$

reta s

$P_s: (21,-5,2)$

$\vec{v}_s: (6,-4,1)$

$\vec{v}_r$ e $\vec{v}_s$ não são multiplos, ou seja, r e s não são paralelas

$$\begin{cases} -4 + 3\lambda_r = 21 + 6\lambda_s \rightarrow 3\lambda_r - 6\lambda_s = 25 \\ 4\lambda_r = -5 - 4\lambda_s \rightarrow 4\lambda_r + 4\lambda_s = -5 \\ -5 - 2\lambda_r = 2 - \lambda_s \rightarrow -2\lambda_r + \lambda_s = 7 \end{cases}$$

$(4\lambda_r + 4\lambda_s = -5) - (-2\lambda_r + 4\lambda_s = 28) \rightarrow 22\lambda_r = -33$

$\lambda_r = -\frac{33^{:3}}{12^{:3}} = -\frac{11}{4}$

$-2 \cdot \left(-\frac{11}{4}\right) + \lambda_s = 7$

$\frac{22}{4} + \lambda_s = 7$

$\lambda_s = 7 - \frac{22}{4} = 7 - \frac{11}{2} = \frac{3}{2}$

$3 \cdot \left(-\frac{11}{4}\right) - 6 \cdot \left(\frac{3}{2}\right) = 25$

$-\frac{33}{4} - \frac{18}{2} = 25$

$-\frac{69}{4} \neq 25$ portanto r e s são reversas

$P_rP_s = P_s - P_r = (21-(-4), -5-0, 2-(-5)) = (25,-5,7)$

$\begin{bmatrix} i & j & k \\ 3 & 4 & -2 \\ 6 & -4 & -1 \end{bmatrix} \begin{matrix} i & j \\ 3 & 4 \\ 6 & -4 \end{matrix}$

$-24k - 8i + 3j - 4i - 12j - 12k = (-12, -9, -36) : (-3)$

$n = (4,3,12)$

$\|n\| = \sqrt{4^2 + 3^2 + 12^2} = \sqrt{169} = 13$

$(P_rP_s \cdot n) = (25 \cdot 4 + (-5) \cdot 3) + 7 \cdot 12) = 169$

$D = \frac{|169|}{13} = 13$

c) reta r

$$\begin{cases} x = -2t + 1 \\ y = t/2 \\ z = t \end{cases}$$

$t = 0$

$P = (1,0,0)$

$v_r = (-2, 1/2, 1)$

reta s

$P_s = (0,0,2)$

$v_s = (-4,1,2)$

$v_s = 2 \cdot v_r$ então as retas são paralelas

$P_rP_s = P_s - P_r = (0,0,2) - (1,0,0) = (-1,0,2)$

$\begin{bmatrix} i & j & k \\ -1 & 0 & 2 \\ -4 & 1 & 2 \end{bmatrix} \begin{matrix} i & j \\ -1 & 0 \\ -4 & 1 \end{matrix}$

$0 - 2i + 2j + 0 - 8j - k = (-2,-6,-1)$

$\|P_rP_s \cdot v_s\| = \sqrt{(-2)^2 + (-6)^2 + (-1)^2} = \sqrt{41}$

$\|v_s\| = \sqrt{(-4)^2 + 1^2 + 2^2} = \sqrt{21}$

$D = \frac{\sqrt{41}}{\sqrt{21}} = \sqrt{\frac{41}{21}}$

13 -

a) reta r

$\vec{v}_r = (3,3,3)$

$P_r = (1,9,4)$

plano π

$v_1 = (1,0,0)$

$v_2 = (0,1,0)$

$\|v_1 \cdot v_2\| = (0,0,1)$

$\vec{v}_r \cdot n_\pi = (3 \cdot 0 + 3 \cdot 0 + 3 \cdot 1) = 3 \neq 0 \rightarrow$ reta não é paralela ao plano então a distância é $= 0$, já que eles se intersectam

Kit

b) reta r

$$\begin{cases} x-y+z=0 \\ 2x+y-z=3 \end{cases}$$

$3x=9$

$x=1$ $\rightarrow 1-y+z=0$

$z=y+1$

$(1, y, y+1) \rightarrow y=\lambda$

$$\begin{cases} x=1 \\ y=\lambda \\ z=\lambda-1 \end{cases}$$

$\vec{v}_r=(0,1,1)$

$P_r=(1,0,-1)$

plano π

$y-z=4 \rightarrow y-z-4=0$

$n_\pi=(0,1,-1)$

$\vec{v}_r \cdot n_\pi=(0\cdot 0+1\cdot 1+(-1)\cdot 1)=0 \rightarrow$ reta é paralela ao plano

$$d=\frac{|0-(-1)-4|}{\sqrt{1^2+0^2+(-1)^2}}=\frac{|-3|}{\sqrt{2}}=\frac{3}{\sqrt{2}}$$ //

c) reta r

$\lambda=z$

$$\begin{cases} x=\lambda+2 \\ y=\lambda+3 \\ z=\lambda \end{cases}$$

$\vec{v}_r=(1,1,1)$

$\vec{P}_r=(2,3,0)$

plano π

$n_\pi=(2,1,-3)$

$\vec{v}_r \cdot n_\pi=(1\cdot 2+1\cdot 1+(-3)\cdot 1)=0 \rightarrow$ reta r é paralela ao plano π

$$d=\frac{|2\cdot 2+1\cdot 3+0\cdot(-3)-10|}{\sqrt{2^2+1^2+(-3)^2}}=\frac{|3|}{\sqrt{14}}=\frac{3}{\sqrt{14}}\cdot\frac{\sqrt{14}}{\sqrt{14}}=\frac{3\sqrt{14}}{14}$$

14-

a) plano π_1

$n_{\pi_1}=(2,-1,2)$

$2x-y+2z+0=0$

plano π_2

$n_{\pi_2}=(4,-2,4)$

$4x-2y+4z-21=0$

$2\cdot n_{\pi_1}=n_{\pi_2}$, então os vetores são paralelos

$(4x-2y+4z-21=0)/2 \rightarrow 2x-y+2z-\frac{21}{2}=0 \rightarrow n_{\pi_1}=n_{\pi_2}$

$$d=\frac{|D_1-D_2|}{\sqrt{2^2+(-1)^2+2^2}}=\frac{|0-(-21/2)|}{\sqrt{9}}=\frac{21}{2}:3=\frac{21}{2}\cdot\frac{1}{3}=\frac{21:3}{6:3}=\frac{7}{2}$$ //

b) plano π_1

$2x+2y+2z-5=0$

$n_{\pi_1}=(2,2,2)$

plano π_2

$\vec{v}_{\pi_2 \cdot 1}=(-1,0,3)$

$\vec{v}_{\pi_2 \cdot 2}=(1,1,0)$

$$n_{\pi_2}=\begin{bmatrix} i & j & k \\ -1 & 0 & 3 \\ 1 & 1 & 0 \end{bmatrix}\begin{matrix} i & j \\ -1 & 0 \\ 1 & 1 \end{matrix}$$

$0-3i+0+0+3j-k$

$(-3,3,-1)$

$k\cdot n_{\pi_1}\neq n_{\pi_2}$, não existe $k\in\mathbb{R}$ que satisfaça a proposição, o que indica que os planos não são paralelos e se intersectam em algum ponto

$d=0$ //

c) plano π_1

$x+y+z=0$

$n_{\pi_1}=(1,1,1)$

plano π_2

$2x+y+z+2=0$

$n_{\pi_2}=(2,1,1)$

$k\cdot n_{\pi_1}\neq n_{\pi_2}$, não existe $k\in\mathbb{R}$ que satisfaça a proposição, o que indica que os planos não são paralelos e se intersectam em algum ponto.

$d=0$ //

15)

reta r	reta s
$\begin{cases} x+z-5=0 \\ y=1 \end{cases}$	$P_s=(4,1,1)$ $\vec{v}_s=(4,2,-3)$

se $x=0 \Rightarrow z=5 \rightsquigarrow P_{r1}=(0,1,5)$

se $x=1 \Rightarrow z=4 \rightsquigarrow P_{r2}=(1,1,4)$

$v_r=P_{r2}-P_{r1}=(1,1,4)-(0,1,5)=(1,0,-1)$

$v_r \neq v_s \cdot \lambda$, as retas não são paralelas

$P_r=(0,1,5)$
$\begin{cases} x=\lambda \\ y=1 \\ z=5-\lambda \end{cases}$

$P_s=(4,1,1)$
$\begin{cases} x=4+4\lambda \\ y=1+2\lambda \\ z=1-3\lambda \end{cases}$

$y \Rightarrow 1+2\lambda=1$
$2\lambda=0$
$\lambda=0$

$x_s=4, y_s=1, z_s=1$

$y_r=1$, $x+z=4+1=5$ (satisfaz as equações paramétricas de r)

P de intersecção de r e s $=(4,1,1)$

eq. do plano:

$$n = v_s \cdot v_r = \begin{bmatrix} i & j & k \\ 1 & 0 & -1 \\ 4 & 2 & -3 \end{bmatrix} \begin{matrix} i & j \\ 1 & 0 \\ 4 & 2 \end{matrix}$$

$-0+2i+3j+0-4j+2k=(2,-1,2)$

$\pi=(4,1,1)+\lambda(2,-1,2)$

$2\cdot(x-4)-1\cdot(y-1)+2\cdot(z-1)$

$2x-y+2z-9=0$

$$D=\frac{|D_1-D_2|}{\sqrt{n\pi}} \rightsquigarrow 2=\frac{|-9-D_2|}{\sqrt{2^2+(-1)^2+2^2}}$$

$$2=\frac{|-9-D_2|}{\sqrt{9} \rightsquigarrow 3}$$

$|-9-D_2|=6$

$\Rightarrow -9-D_2=6$
$-D_2=15$
$D_2=-15$

$\Rightarrow -9-D_2=-6$
$-D_2=3$
$D_2=-3$

$2x-y+2z-15$ e $2x-y+2z-3$