

Lista 3

$$1 - \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 5 \\ -2 & -1 & 3 \end{bmatrix}, \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \rightarrow \begin{cases} x + 3y + 4z = 1 \\ x + 2y + 5z = 2 \\ -2x - y + 3z = 3 \end{cases} \quad \begin{aligned} (x-x) + (3y+2y) + (4z+5z) &= 1+2 \\ 6y + 9z &= 3 \\ (2x-2x) + (6y-y) + (8z+3z) &= 2-3 \\ 6y + 11z &= -1 \end{aligned}$$

$$\begin{aligned} (6y + 9z) - (6y + 11z) &= 3 - (-1) \\ -2z &= 4 \\ z &= -2 \end{aligned} \quad \begin{cases} x + 3 \cdot \frac{(-2)}{5} + 4 \cdot (-2) = 1 \\ x + \frac{63}{5} - 8 = 1 \\ \frac{9x+63-40}{5} = 1 \end{cases} \quad \begin{cases} 6y + 11 \cdot (-2) = -1 \\ 6y = 2, 2 - 1 \\ y = \frac{21}{5} \end{cases}$$

$$5x + 63 - 40 = 5 \\ x = -\frac{18}{5}$$

2-

$$a) \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \cdot x = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \rightarrow x = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}^{-1} \rightarrow \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} -6/10 & 4/5 \\ 4/10 & 1/5 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 6 \end{bmatrix}$$

$$-8 + 3 = -5$$

$$A_1x_1 = (-1)^{1+1} \cdot 3 = 3 \quad \begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3/5 & 4/5 \\ 1/5 & -1/5 \end{bmatrix}$$

$$A_1x_2 = (-1)^{1+2} \cdot 2 = -2$$

$$A_2x_1 = (-1)^{2+1} \cdot 4 = -4$$

$$A_2x_2 = (-1)^{2+2} \cdot 1 = 1$$

$$b) \begin{bmatrix} 1 & 3 \\ 5 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \cdot y = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} \rightarrow y = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -5+6 & -15+21 \\ 2-2 & 6-7 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & -1 \end{bmatrix}$$

$$A_1x_1 = (-1)^{1+1} \cdot 5 = 5$$

$$A_2x_1 = (-1)^{2+1} \cdot 2 = -2$$

$$A_1x_2 = (-1)^{1+2} \cdot 3 = -3$$

$$A_2x_2 = (-1)^{2+2} \cdot 1 = 1$$

$$c) \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \cdot W = \begin{bmatrix} 5 \\ 2 \\ 7 \end{bmatrix} \rightarrow W = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 2 & 3 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 5 \\ 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -8 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 5+0+0 \\ 10-2+0 \\ -40+6+7 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ -27 \end{bmatrix}$$

$$A_1x_1 = (-1)^{1+1} \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix} = -1$$

$$A_2x_1 = (-1)^{2+1} \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} = -2$$

$$A_3x_1 = (-1)^{3+1} \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix} = 8$$

$$A_1x_2 = (-1)^{1+2} \begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix} = 0$$

$$A_2x_2 = (-1)^{2+2} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = 1$$

$$A_3x_2 = (-1)^{3+2} \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = -3$$

$$A_1x_3 = (-1)^{1+3} \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 2 & 3 & 1 \end{bmatrix} = 1$$

$$A_2x_3 = (-1)^{2+3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & -1 \end{bmatrix} = 1$$

$$A_3x_3 = (-1)^{3+3} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 8 & -3 & -1 \end{bmatrix} = -1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$0+0+0-1+0+0=-1$$

32

$$a - AXB = C$$

$$X = C \cdot (AB)^{-1}$$

$$b - A(B+X) = A$$

$$B+X = A \cdot A^{-1}$$

$$X = I_n - B$$

$$c - ACXB = C$$

$$X = A^T \cdot B^{-1} \cdot I_n$$

$$d - (A \cdot B)^{-1} \cdot (AX) = C \cdot C^{-1}$$

$$(XA) \cdot (A^{-1}B^{-1}) = I_n$$

$$X \cdot A \cdot (A^{-1}) \cdot (B^{-1}) = I_n$$

$$X \cdot I_n \cdot B^{-1} = I_n$$

$$X = B^{-1} (I_n)$$

$$l - A \cdot B^T \cdot X \cdot B^{-1} = A^T$$

$$(A \cdot X) \cdot (B^T \cdot B^{-1}) = A^T$$

$$X \cdot (B^T \cdot B^{-1}) = A^T \cdot A^{-1}$$

$$X = (A^T \cdot A^{-1}) \cdot (B^T \cdot B^{-1})$$

$$f - 2AX - X = 3B$$

$$2X - X = 3B \cdot A^{-1}$$

$$X = 3B \cdot A^{-1}$$

4-

$$a) \begin{cases} 3x - 4y = 1 \\ 2x + 6y = 18 \end{cases} \quad D = \begin{bmatrix} 3 & -4 \\ 2 & 6 \end{bmatrix} \quad Dx = \begin{bmatrix} 1 & -4 \\ 18 & 6 \end{bmatrix} \quad Dy = \begin{bmatrix} 3 & 1 \\ 2 & 18 \end{bmatrix}$$

$$+8 + 18 = 26 \quad -6 \cdot 18 = -78 \quad 94 - 2 = 92$$

$$x = \frac{-78}{26} = -3, \quad y = \frac{92}{26} = 2 //$$

$$b) \begin{cases} 5x + 8y = 34 \\ 10x + 16y = 50 \end{cases} \quad D = \begin{bmatrix} 5 & 8 \\ 10 & 16 \end{bmatrix} \quad Dx = \begin{bmatrix} 34 & 8 \\ 50 & 16 \end{bmatrix} \quad Dy = \begin{bmatrix} 5 & 34 \\ 10 & 50 \end{bmatrix}$$

$$80 - 80 = 0 // \text{ impossível, pois nenhum número é divisível por 10}$$

$$c) \begin{cases} x + 2y = 5 \\ 2x - 3y = -4 \end{cases} \quad D = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix} \quad Dx = \begin{bmatrix} 5 & 2 \\ 4 & -3 \end{bmatrix} \quad Dy = \begin{bmatrix} 1 & 5 \\ 2 & -4 \end{bmatrix} \quad x = \frac{-23}{-7} //$$

$$-3 - 4 = -7 \quad -15 - 8 = -23 \quad -4 - 10 = -14 \quad y = \frac{-14}{-7} = 2 //$$

$$d) \begin{cases} x + 2y - z = 2 \\ 2x - y + 3z = 9 \\ 3x + 3y - 2z = 3 \end{cases} \quad D = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 3 & -2 \end{bmatrix} \quad Dx = \begin{bmatrix} 2 & 2 & -1 \\ 9 & -1 & 3 \\ 3 & 3 & -2 \end{bmatrix} \quad Dy = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 9 \\ 3 & 2 & 3 \end{bmatrix}$$

$$-3 - 9 + 8 + 2 + 18 - 6 = 10 \quad -3 - 18 + 36 + 4 + 18 - 27 = 0 \quad -18 - 18 + 6 + 9 - 27 + 8 = -40$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 9 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 3 \end{bmatrix} \quad x = \frac{0}{10} = 0 // \quad y = \frac{-40}{10} = 4 //$$

$$z = \frac{30}{10} = 3$$

$$-6 - 27 - 12 - 3 + 94 + 12 = 30$$

$$e) \begin{cases} 3x+2y-5z=8 \\ 2x-4y-2z=-4 \\ x-2y-3z=-4 \end{cases} \quad \left| \begin{array}{ccc|cc} 3 & 2 & -5 & 8 & 2 \\ 2 & -4 & -2 & -4 & -4 \\ 1 & -2 & -3 & 1 & -2 \end{array} \right| \quad D = -20-12+10+36-4+10 = 32 \quad \left| \begin{array}{ccc|cc} 8 & 2 & -5 & 8 & 2 \\ -4 & -4 & -2 & -4 & -4 \\ -4 & -2 & -3 & -4 & -2 \end{array} \right| \quad D_x = \left| \begin{array}{ccc|cc} 3 & -5 & 8 & 3 & -9 \\ 2 & -2 & -4 & 2 & -2 \\ 1 & -3 & -4 & 1 & -3 \end{array} \right| \quad D_y = \left| \begin{array}{ccc|cc} 3 & 2 & -5 & 3 & -9 \\ 2 & 0 & -2 & 2 & -2 \\ 1 & -3 & -4 & 1 & -3 \end{array} \right| \quad 16-36-40+24+20-18=64$$

$$D_2 = \left| \begin{array}{ccc|cc} 3 & 2 & 8 & 3 & 2 \\ 2 & -4 & -4 & 2 & -4 \\ 1 & -2 & -4 & 1 & -2 \end{array} \right| \quad X = \frac{96}{32} = 3$$

$$Y = \frac{64}{32} = 2$$

$$Z = \frac{32}{32} = 1$$

$$+32-24+16+48-8-32=32$$

$$f) \begin{cases} x+0y+3z=-8 \\ 2x-4y+0z=-4 \\ 3x-2y-5z=26 \end{cases} \quad \left| \begin{array}{ccc|cc} 1 & 0 & 3 & -8 & 0 \\ 2 & -4 & 0 & 2 & -4 \\ 3 & -2 & -5 & 3 & -2 \end{array} \right| \quad D = +36+0+0+20+0-12=44 \quad \left| \begin{array}{ccc|cc} -8 & 0 & 3 & -8 & 0 \\ -4 & -4 & 0 & -4 & -4 \\ 26-2 & -5 & 26-2 & 26-2 \end{array} \right| \quad D_x = \left| \begin{array}{ccc|cc} 1 & 3 & -8 & 3 & 3 \\ 2 & 0 & -4 & 2 & 0 \\ 3 & -5 & 26 & 3 & -5 \end{array} \right| \quad D_y = \left| \begin{array}{ccc|cc} 1 & 3 & -8 & 3 & 3 \\ 2 & 0 & -4 & 2 & 0 \\ 3 & -5 & 26 & 3 & -5 \end{array} \right| \quad +36+0+0-160+0+24=190 \quad +24-20-156+0-36+80=-108$$

$$D_2 = \left| \begin{array}{ccc|cc} 3 & 0 & -8 & 1 & 0 \\ 2 & -4 & -4 & 2 & -4 \\ 3 & -2 & -26 & 3 & -2 \end{array} \right| \quad X = \frac{180}{44} - \frac{90}{22} = \frac{45}{11}$$

$$Y = \frac{-108}{44} - \frac{54}{22} = \frac{-27}{11}$$

$$Z = \frac{196}{44} = 4$$

$$-96-8+0-204+0+32=176$$

$$g) \begin{cases} x+2y+3z=10 \\ 3x+4y+6z=23 \\ 3x+2y+3z=10 \end{cases} \quad \left| \begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 0 \\ 3 & 4 & 6 & 3 & 1 \\ 3 & 2 & 3 & 3 & 2 \end{array} \right| \quad D = +36-32-18+12+36+18=0 \quad \text{(impossível continuar, pois não é possível dividir num } n^{\circ} \text{ por 0.)}$$

5-

$$a) \begin{cases} 3x_1-4x_2=0 \cdot (-2) \\ 6x_1+8x_2=0 \end{cases} \Rightarrow \begin{cases} -6x_1+8x_2=0 \\ -6x_1+8x_2=0 \end{cases} \rightarrow \text{São equações dependentes e por isso há infinitas soluções}$$

$$b) \begin{cases} x+y+z=0 \\ 2x+3y+4z=0 \\ x+y+3z=0 \end{cases} \quad \left| \begin{array}{ccc|cc} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 4 & 0 & 0 \\ 1 & 1 & 3 & 0 & 0 \end{array} \right| \quad L_2-L_1=(2,2,4)-(1,1,1)=(1,1,3) \quad L_3-L_1=(1,1,3)-(1,1,1)=(0,0,2) \quad \left| \begin{array}{ccc|cc} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{array} \right| \quad \text{Temos duas equações lineares independentes que possuem infinitas soluções}$$

$$c) \begin{cases} x+y+2z=0 \\ x-y-3z=0 \\ x+4y=0 \end{cases} \quad \left| \begin{array}{ccc|cc} 1 & 1 & 2 & 0 & 0 \\ 1 & -1 & -3 & 0 & 0 \\ 1 & 4 & 0 & 0 & 0 \end{array} \right| \quad L_2-L_1=(0,-2,-5) \quad L_3-L_1=(0,3,-2) \quad \left| \begin{array}{ccc|cc} 1 & 1 & 2 & 0 & 0 \\ 0 & -2 & -5 & 0 & 0 \\ 0 & 3 & -2 & 0 & 0 \end{array} \right| \quad L_3+\frac{3}{2} \cdot L_2=(0,0,9.5) \quad \text{Temos 3 equações independentes com apenas uma solução}$$

6-

$$a) \begin{cases} 3x+my=2 \\ x-y=1 \end{cases} \quad \left| \begin{array}{cc|c} 3 & m & 2 \\ 1 & -1 & 1 \end{array} \right| \quad m \neq 3$$

$$b) \begin{cases} 3x-2m+2y=1 \\ mx-4y=0 \end{cases} \quad \left| \begin{array}{ccc|cc} 3 & -2m & 1 & 0 & 0 \\ m & -4 & 0 & 0 & 0 \end{array} \right| \quad \text{Não há como calcular o determinante}$$

$$c) \begin{cases} x-y+0z=2 \\ x+my+z=0 \\ -x+y-z=4 \end{cases} \quad \left| \begin{array}{ccc|cc} 1 & -1 & 0 & 2 & 1 \\ 1 & m & 1 & 0 & 1 \\ -1 & 1 & -1 & 4 & 1 \end{array} \right| \quad 0 \neq 0-1-1-m+1+0$$

$$d) \begin{cases} mx+y-2z=4 \\ x+my+z=0 \\ x-y+0z=2 \end{cases} \quad \left| \begin{array}{ccc|cc} m & 1 & -2 & 4 & 1 \\ 1 & m & 1 & 0 & 1 \\ 1 & -1 & 0 & 2 & 1 \end{array} \right| \quad \left| \begin{array}{ccc|cc} m & 1 & -2 & 4 & 1 \\ 1 & m & 1 & 0 & 1 \\ 1 & -1 & 0 & 2 & 1 \end{array} \right| \quad m \neq 1 \quad m^2 \neq -2$$

7) $x \rightarrow$ peça

$y \rightarrow$ peça c/difunto

$$\begin{cases} 6x - 2y = 750 \\ 6 \cdot (225 - y) - 2y = 750 \end{cases} \Rightarrow -8y = -600$$

$$\begin{cases} x + y = 225 \Rightarrow x = 225 - y = 150 \\ 1350 - 6y - 2y = 1750 \end{cases} \Rightarrow y = 75,5$$

8) $x \rightarrow$ Km do carro

$y \rightarrow$ Km da moto

$$\begin{cases} x + y = 540 \Rightarrow x = 540 - y \\ 0,6 \cdot (540 - y) + 0,2y = 300 \end{cases} \Rightarrow -0,4y = -24$$

$$x = 540 - 60 = 480$$

$$324 - 0,6y + 0,2y = 300 \Rightarrow y = 60$$

9)

$x \rightarrow$ nota de 10,12

$y \rightarrow$ nota de 5

$$\begin{cases} 2x + y = 92 \Rightarrow x + \frac{92-y}{2} = \frac{92-12}{2} \\ 2x + 10x + 5y = 500 \end{cases} \Rightarrow \begin{cases} 2 \cdot \left(\frac{92-y}{2}\right) + 10(92-y) + 5y = 540 \\ 18y + 920 - 10y + 5y = 540 \end{cases}$$

$$\begin{cases} 18y + 920 - 10y + 5y = 540 \\ (18y - 2y) + (920 - 10y) + 10y = 540 \end{cases} \Rightarrow \begin{cases} 18y = 1080 \\ -2y = -24 \end{cases}$$

$$y = 12$$

10)

$y \rightarrow$ Kilo

$x \rightarrow$ Uromaru

$z \rightarrow$ Tomaki

$$(x + y + z = 209 \cdot (-1)) \Rightarrow x = 209 + y (x + y) + (y + z) + (x + z) = 109 + 142 + 97$$

$$0x + y + z = 342$$

$$x + 0y + z = 97 \cdot 1$$

$$2x + 2y + 2z = 348 \cdot (2)$$

$$x + y + z = 174$$

$$(x + y + z) - (x + z) = 174 - 97$$

$$y = 77$$

$$(x + y + z) - (y + z) = 174 - 142$$

$$x = 109 - x = 32$$

$$(x + y + z) - (x - y) = 174 - 109$$

$$z = 65$$