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→ Lista 6

1)

a- $\|\vec{u}\| = \|(x_1, y_1, z_1)\|^2$

$\|\vec{u}\| = \sqrt{(1,1,1)\cdot(1,1,1)}$

$\|\vec{u}\| = \sqrt{1^2 + 1^2 + 1^2}$

$\|\vec{u}\| = \sqrt{3}$

b- $\|\vec{u}\|^2 = \|x\vec{i}\|^2 + \|y\vec{k}\|^2$

$\|\vec{u}\|^2 = 3^2\|i\|^2 + 4^2\|\vec{k}\|^2$

$\|\vec{u}\|^2 = 3^2 + 4^2$

$\|\vec{u}\| = \sqrt{3^2 + 4^2}$

$\|\vec{u}\| = 5$

c- $\|\vec{u}\|^2 = \|-i\|^2 + \|j\|^2$

$\|\vec{u}\| = 1 + 1$

$\|\vec{u}\| = \sqrt{2}$

d- $\|\vec{u}\| = \|(x_1, y_1, z_1)\|$

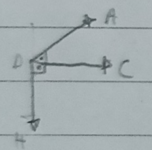
$\|\vec{u}\| = \sqrt{4^2 + 3^2 + (-1)^2}$

$\|\vec{u}\| = \sqrt{16 + 9 + 1}$

$\|\vec{u}\| = \sqrt{26}$

2)

a- $e_1 DH = \frac{\vec{DH}}{\|\vec{DH}\|} = 1$; $e_2 DC = \frac{\vec{DC}}{\|\vec{DC}\|} = 1$; $e_3 \vec{DA} = \frac{\vec{DA}}{\|\vec{DA}\|} = 1$. Além de serem vetores unitários $\vec{DH}$ e $\vec{DC}$ são perpendiculares e $\vec{DH}$ e $\vec{DA}$ também são, o que garante que os tres vetores fazem parte de uma base ortonormal.



(base "canto de parede")

b-

$\vec{u} = \vec{CD} + \vec{CB}$	$\vec{v} = \vec{DC} + \vec{CB}$	$\vec{w} = \vec{GC}$	$\vec{u} = (0, -1, 1)$
$\vec{u} = -\vec{DC} + \vec{DA}$	$\vec{v} = \vec{e_2} + \vec{DA}$	$\vec{w} = -\vec{DH}$	$\vec{v} = (0, 1, 1)$
$\vec{u} = -e_2 + e_3$	$\vec{v} = \vec{e_2} + \vec{e_3}$	$\vec{w} = -\vec{e_1}$	$\vec{w} = (-1, 0, 0)$

c-

$f_1 = \frac{\vec{u}}{\|\vec{u}\|} = \frac{-e_2+e_3}{\sqrt{(-e_2+e_3)^2}} = \frac{-e_2+e_3}{\sqrt{2}}$

$f_2 = \frac{\vec{v}}{\|\vec{v}\|} = \frac{e_2+e_3}{\sqrt{(e_2+e_3)^2}} = \frac{e_2+e_3}{\sqrt{2}}$

$f_3 = \vec{w} = -\vec{e_1}$

$f_1 \cdot f_2 = 0$

$\vec{e_1}\cdot\vec{e_2} = 0$ $\quad \vec{e_2}\cdot\vec{e_3} = 0$ $\quad \vec{e_3}\cdot\vec{e_1} = 0$ $\quad \|\vec{e_1}\|^2 = \|\vec{e_2}\|^2 = \|e_3\|^2 = 1$

$\frac{-e_2+e_3}{\sqrt{2}} \cdot \frac{e_2+e_3}{\sqrt{2}} = 0$

$\frac{1}{2}\cdot[(-e_2+e_3)\cdot(e_2+e_3)]$

$\frac{1}{2}\cdot[-e_2 - e_2\cdot e_3 + e_3\cdot e_2 + e_3\cdot e_3]$

$\frac{1}{2}\cdot[-1 + 0 + 0 + 1]$

$\frac{1}{2}\cdot 0 = 0$

$\frac{-e_2+e_3}{\sqrt{2}}\cdot(-e_1)$	$\frac{e_2+e_3}{\sqrt{2}}\cdot(-e_1)$
$\left((-e_2+e_3):\frac{1}{2}\right)\cdot(-e_1)$	$\left((e_2+e_3):\frac{1}{2}\right)\cdot(-e_1)$
$2\cdot(-e_2+e_3)\cdot(-e_1)$	$2(e_2+e_3)\cdot(-e_1)$
$2\cdot(-e_2\cdot(-e_1) + e_3\cdot(-e_1))$	$2\cdot(e_2\cdot(-e_1) + e_3\cdot(-e_1))$
$2\cdot(0+0)$	$2\cdot(0+0)$
0	0

d-

$E \to F$

$f_1 = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \quad f_2 = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad f_3 = 1 \cdot \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

$M = \begin{bmatrix} 0 & 0 & -1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$

$F \to E \quad (M^{-1}) \qquad M^{-1} = M^t$

$M^{-1} = \begin{bmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1 & 0 & 0 \end{bmatrix} = M^t = \begin{bmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1 & 0 & 0 \end{bmatrix}$ M é ortogonal

e-

$\vec{HB} = \vec{HD} + \vec{DC} + \vec{CB}$

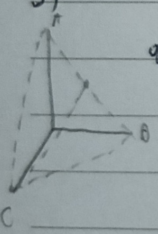
$\vec{HB} = -\vec{DH} + \vec{e_2} + \vec{e_3}$

$\vec{HB} = -e_1 + e_2 + e_3$

$\vec{HB} = (-1, 1, 1)_E$

$F = \begin{bmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+0+1 \\ -1/\sqrt{2}+1/\sqrt{2}+0 \\ 1/\sqrt{2}+1/\sqrt{2}+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \sqrt{2} \end{bmatrix}$

3)



a- $\vec{AB} = (2, 4, 3) - (5, 1, -3)$ | $\vec{BC} = (5, 1, -3) - (0, -3, 1)$ | $\vec{CA} = (2, 4, 3) - (0, -3, 1)$

$\vec{AB} = (3, -3, -6)$ | $\vec{BC} = (-5, -4, 4)$ | $\vec{CA} = (2, 7, 2)$

b- $\|\vec{AB}\| = \sqrt{(3, -3, -6) \cdot (3, -3, -6)}$ | $\|\vec{BC}\| = \sqrt{(-5, -4, 4) \cdot (-5, -4, 4)}$ | $\|\vec{CA}\| = \sqrt{(2, 7, 2) \cdot (2, 7, 2)}$

$\|\vec{AB}\| = \sqrt{9+9+36}$ | $\|\vec{BC}\| = \sqrt{25+16+16}$ | $\|\vec{CA}\| = \sqrt{4+49+4}$

$\|\vec{AB}\| = \sqrt{54}$ | $\|\vec{BC}\| = \sqrt{57}$ | $\|\vec{CA}\| = \sqrt{57}$

$\|\vec{AB}\| \cong 7,35$ | $\|\vec{BC}\| \cong 7,55$ | $\|\vec{CA}\| \cong 7,55$

Sim, é um triângulo isósceles pois os lados $\vec{BC}$ e $\vec{AC}$ são iguais

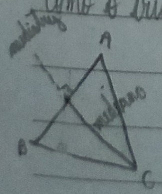
c-

$P_{mc}\vec{AB} = \left(\frac{2+5}{2}, \frac{4+1}{2}, \frac{3+(-3)}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}, 0\right)$

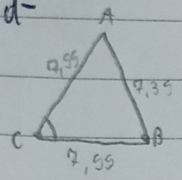
$P_{mc}\vec{BC} = \left(\frac{5+0}{2}, \frac{1+(-3)}{2}, \frac{-3+1}{2}\right) = \left(\frac{5}{2}, -1, -1\right)$

$P_{mc}\vec{CA} = \left(\frac{2+0}{2}, \frac{4+(-3)}{2}, \frac{3+1}{2}\right) = \left(1, \frac{1}{2}, 2\right)$

Como o triângulo formado é isósceles, e os lados = são $\vec{BC}$ e $\vec{CA}$ a mediana coincide com a mediatriz



d-



$\vec{CB}\cdot\vec{CA} = 5\cdot2+4\cdot7+(-4)\cdot2$
$= 10+28-8=30$

$\cos\theta = \frac{30}{\sqrt{57}\cdot\sqrt{57}} = \frac{30}{57} = 58,3°$

e- Pois esse vetores formam uma forma fechada, o que pela regra determina que nessa situação a soma será 0.

4-

a- $\vec{u}$ e $\vec{v} \in \mathbb{R}^n \to |\vec{u}\cdot\vec{v}| \leq \|\vec{u}\|\cdot\|\vec{v}\|$ (só vale se $\vec{u}$ e $\vec{v}$ forem paralelos) $\rightsquigarrow \vec{u}=\lambda\vec{v}$ ou $\vec{v}=\lambda\vec{u}$

$\vec{u}\cdot\vec{v} = \|\vec{u}\|\cdot\|\vec{v}\|\cdot\cos\theta$

$|\vec{u}\cdot\vec{v}| = \|\vec{u}\|\cdot\|\vec{v}\|\cdot|\cos\theta|$

$|\cos\theta| \leq 1$ isso garante a desigualdade

$|\cos\theta| = 1 \rightsquigarrow 0°$ ou $180°$, ou seja, $\vec{u}$ e $\vec{v}$ estão na mesma direção (ou direção oposta) o que implica:

$\vec{u} = \lambda\vec{v}$ para algum $\lambda \in \mathbb{R}$

b- $\|u+v\|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = \|\vec{u}\|^2 + 2(\vec{u}\cdot\vec{v}) + \|\vec{v}\|^2$

aplicando o item (a)

$|\vec{u}\cdot\vec{v}| \leq \|\vec{u}\|\cdot\|\vec{v}\| \rightsquigarrow 2\cdot(\vec{u}\cdot\vec{v}) \leq 2\|\vec{u}\|\cdot\|\vec{v}\|$

$\|\vec{u}+\vec{v}\|^2 \leq \|u\|^2 + 2\|\vec{u}\|\cdot\|\vec{v}\| + \|\vec{v}\|^2 = (\|\vec{u}\| + \|\vec{v}\|)^2$

$\|\vec{u}\cdot\vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$

c- $4\vec{u}\cdot\vec{v} = \|\vec{u}+\vec{v}\|^2 - \|\vec{u}-\vec{v}\|^2$

$(\|u\|^2 + (2\cdot\vec{u}\cdot\vec{v}) + \|\vec{v}\|^2) - (\|\vec{u}\|^2 - (2\cdot\vec{u}\cdot\vec{v}) + \|v\|^2)$

$\cancel{\|u\|^2} + (2\cdot\vec{u}\cdot\vec{v}) + \cancel{\|v\|^2} - \cancel{\|u\|^2} + (2\cdot\vec{u}\cdot\vec{v}) - \cancel{\|v\|^2}$

$4\vec{u}\cdot\vec{v}$

5)

a-

$\vec{v}\cdot\vec{u} = (-2,10,2)\cdot(1,0,1) = -2+0+2 = 0$

$\frac{\vec{v}\cdot\vec{u}}{\|v\|\cdot\|u\|} = \frac{0}{\|v\|\cdot\|u\|} = 0 = \frac{\pi}{2}$ rad

b- $\vec{u}\cdot\vec{v} = (-1,1,1)\cdot(1,1,1) = -1+1+1 = 1$

$\|\vec{u}\| = \sqrt{\vec{u}\cdot\vec{u}} = \sqrt{(-1,1,1)\cdot(-1,1,1)} = \sqrt{1+1+1} = \sqrt{3}$

$\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{\quad} = \sqrt{3}$

$\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|} = \frac{1}{\sqrt{3}\cdot\sqrt{3}} = \frac{1}{3} = \frac{2\pi}{9}$ rad

c- $\vec{u}\cdot\vec{v} = (3,3,0)\cdot(2,1,-2) = 6+3+0 = 9$

$\|\vec{u}\| = \sqrt{\vec{u}\cdot\vec{u}} = \sqrt{(3,3,0)\cdot(3,3,0)} = \sqrt{9+9+0} = \sqrt{18}$

$\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{(2,1,-2)\cdot(2,1,-2)} = \sqrt{4+1+4} = 3$

$\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|} = \frac{9}{3\cdot\sqrt{18}} = \frac{9}{3\cdot3\sqrt{2}} = \frac{9:9}{9\sqrt{2}:9} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{\pi}{4}$ rad

d- $\vec{u}\cdot\vec{v} = (\sqrt{3},1,0)\cdot(\sqrt{3},1,2\sqrt{3}) = 3+1+0 = 4$

$\|\vec{u}\| = \sqrt{(\sqrt{3},1,0)\cdot(\sqrt{3},1,0)} = \sqrt{3+1+0} = 2$

$\|\vec{v}\| = \sqrt{(\sqrt{3},1,2\sqrt{3})\cdot(\sqrt{3},1,2\sqrt{3})} = \sqrt{3+1+12} = 4$

$\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|} = \frac{4}{2\cdot4} = \frac{4}{8} = \frac{1}{2} = \frac{\pi}{3}$ rad

6-

a- $((x+1)\cdot(x-1)) + (1\cdot(-1)) + (2\cdot(-2)) = 0$

$(x^2 - x + x + 1) - 1 - 4 = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

b- $(x\cdot 4) + (x\cdot x) + 4\cdot 1 = 0$

$x^2 + 4x + 4 = 0$

$\Delta = 4^2 - 4\cdot 1\cdot 4$ | $x = \frac{-4 \pm 0}{2} = -2$

$\Delta = 0$

7)

a-

$$\vec{v}\cdot\vec{w} = \begin{bmatrix} i & j & k \\ 4 & -1 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{matrix} i & j \\ 4 & -1 \\ 1 & -2 \end{matrix}$$

$+k + 10i - 12j - 3i + 5j - 8k = (7i, -7j, -7k) \rightarrow u_0 = (7, -7, -7)$

$\vec{u} = k\cdot\vec{u}_0 = (7k, -7k, -7k)$ $\qquad$ $\vec{u} = \frac{1}{7}\cdot(7, -7, -7)$

$7k + (-7k) + (-7k) = -1$ $\qquad$ $\vec{u} = (1, -1, -1)$

$-7k = -1$

$k = \frac{1}{7}$

588 | 2
294 | 2
147 | 3
49 | 7
7 | 7
1

b-

$$\vec{v}\cdot\vec{w} = \begin{bmatrix} i & j & k \\ 2 & 3 & -1 \\ 2 & -4 & 6 \end{bmatrix} \begin{matrix} i & j \\ 2 & 3 \\ 2 & -4 \end{matrix}$$

$u_0 = (14, -14, 14)$

$-6k - 4i - 12j + 18i - 2j - 8k = (14i, -14j, -14k)$

$\vec{u} = k\cdot\|\vec{u}_0\|$

$3\sqrt{3} = |k|\cdot\sqrt{588}$

$3|k| = \frac{3\sqrt{7}}{14\sqrt{3}}$

$|k| = \frac{3\sqrt{3}}{14}$

52 | 2
26 | 2
1

$\|\vec{u}\| = \sqrt{\vec{u}\cdot\vec{u}}$

$3\sqrt{3} = \sqrt{u^2}$ $\qquad$ $\vec{u} = \pm\frac{3}{14}\cdot(14, -14, -14)$

$(3\sqrt{3})^2 = \vec{u}^2$ $\qquad$ $\vec{u} = \pm(3, -3, -3)$

$27 = \vec{u}^2$

$3\sqrt{3} = \vec{u}^2$

$\boxed{\vec{u} = (3, -3, -3)}$

$\vec{u}\cdot\vec{i} = (3, -3, -3)\cdot(1, 0, 0) = 3 \rightarrow > 0$ (ângulo agudo)

$\vec{u}\cdot\vec{i} = (-3, 3, 3)\cdot(1, 0, 0) = -3 \rightarrow < 0$ (ângulo obtuso)

c- $\frac{\pi}{4} = 45° = \frac{\sqrt{2}}{2}$ cos

$\cos = \frac{(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v})}{\|\vec{u}+\vec{v}\|\cdot\|\vec{u}-\vec{v}\|} = \frac{u^2 - u\cdot v + v\cdot u - v^2}{(\|u\|^2 + \|v\|^2 + 2u\cdot v)\cdot(\|u\|^2 + \|v\|^2 - 2u\cdot v)} = \frac{\|u\|^2 - \|v\|^2}{(5+1+2\cdot\frac{\sqrt{10}}{2})\cdot(5+1-2\cdot\frac{\sqrt{10}}{2})} = \frac{(\sqrt{5})^2 - (1)^2}{(6+\sqrt{10})(6-\sqrt{10})} = \frac{4}{\sqrt{6+\sqrt{10}}\cdot\sqrt{6-\sqrt{10}}} = \frac{4}{\sqrt{36-10}} = \frac{4}{\sqrt{26}} = \frac{\pi}{\sqrt{13}}$ rad

$u\cdot v = \|u\|\cdot\|v\|\cdot\cos\theta$

$u\cdot v = \sqrt{5}\cdot 1\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{10}}{2}$

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8)

a- $Proj_{\vec{u}}\vec{v} = \frac{(\vec{u}\cdot\vec{v})}{\|\vec{u}\|^2}\cdot\vec{u} = \frac{6}{11}\cdot(3,-1,1) = \left(\frac{18}{11}; -\frac{6}{11}; \frac{6}{11}\right)$

$\vec{u}\cdot\vec{v} = (3,-1,1)\cdot(1,-1,2) = 3+1+2 = 6$

$\|\vec{u}\|^2 = \left(\sqrt{(3,-1,1)\cdot(3,-1,1)}\right)^2 = \left(\sqrt{9+1+1}\right)^2 = \left(\sqrt{11}\right)^2 = 11$

b- $Proj_{u}v = \frac{(\vec{u}\cdot\vec{v})}{\|u\|^2}\cdot\vec{u} =$

$\vec{u}\cdot\vec{v} = (-3,1,0)\cdot(1,3,5) = -3+3+0 = 0$

Como o numerador é $= 0$ isso faz com que $\frac{\vec{u}\cdot\vec{v}}{\|u\|^2}\cdot\vec{u}$ seja $= (0,0,0)$

c- $Proj_{u}v = \frac{\vec{u}\cdot\vec{v}}{\|u\|^2}\cdot\vec{u} = \frac{5}{9}\cdot(-2,1,2) = \left(-\frac{10}{9}; \frac{5}{9}; \frac{10}{9}\right)$

$\vec{v}\cdot\vec{u} = (-1,1,1)\cdot(-2,1,2) = 2+1+2 = 5$

$\|\vec{u}\|^2 = \left(\sqrt{(-2,1,2)\cdot(-2,1,2)}\right)^2 = \left(\sqrt{4+1+4}\right)^2 = \left(\sqrt{9}\right)^2 = 9$

d- $Proj_{u}v = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\cdot\vec{u} = \frac{-42}{84}\cdot(-2,-4,-8) = -\frac{1}{2}\cdot(-2,-4,-8) = (1,2,4)$

$\vec{u}\cdot\vec{v} = (-2,-4,-8)\cdot(1,2,4) = -2-8-32 = -42$

$\|\vec{u}\|^2 = \left(\sqrt{(-2,-4,-8)\cdot(-2,-4,-8)}\right)^2 = \left(\sqrt{4+16+64}\right)^2 = \left(\sqrt{84}\right)^2 = 84$

9) $\vec{u} = (2,-2,1)$ $\vec{v} = (3,-6,0)$

a-

$Proj_{u}v = \frac{\vec{v}\cdot\vec{u}}{\|u\|^2}\cdot\vec{u} = \frac{18}{9}\cdot(2,-2,1) = 2\cdot(2,-2,1) = (4,-4,2)$

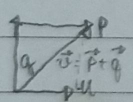
$Proj_{v}u = \frac{\vec{v}\cdot\vec{u}}{\|\vec{v}\|^2}\cdot\vec{v} = \frac{18:9}{45:9}(3,-6,0) = \frac{2}{5}\cdot(3,-6,0) = \left(\frac{6}{5}; -\frac{12}{5}; 0\right)$

$\vec{v}\cdot\vec{u} = (3,-6,0)\cdot(2,-2,1) = (6+12+0) = 18$

$\|\vec{u}\|^2 = \left(\sqrt{(2,-2,1)\cdot(2,-2,1)}\right)^2 = \left(\sqrt{4+4+1}\right)^2 = 9$

$\|v\|^2 = \left(\sqrt{(3,-6,0)\cdot(3,-6,0)}\right)^2 = \left(\sqrt{9+36+0}\right)^2 = 45$

b-



$\vec{p} \parallel \vec{u} \rightsquigarrow \vec{p} = Proj_{\vec{u}}\vec{v} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\cdot\vec{u} = (4,-4,2)$

$\vec{q}\cdot\vec{u} = 0$ $\vec{u}\cdot\vec{v} = (2,-2,1)\cdot(3,-6,0) = 18$

$\|u\|^2 = \left(\sqrt{(2,-2,1)\cdot(2,-2,1)}\right)^2 = 9$

$\vec{v} = \vec{p} + \vec{q}$

$\vec{q} = \vec{v} - \vec{p}$

$\vec{q} = (3,-6,0)-(4,-4,2) = (-1,-2,-2)$

c- $A = \|\vec{u}\times\vec{v}\| \rightarrow \sqrt{6^2+3^2+(-6)^2} = \sqrt{36+9+36} = \sqrt{81} = 9$

$$\left|\begin{matrix} i & j & k \\ 2 & -2 & 1 \\ 3 & -6 & 0 \end{matrix}\right|\begin{matrix} i & j \\ 2 & -2 \\ 3 & -6 \end{matrix}$$

$6i + 6i + 0 + 0 + 3j - 12k = (6,3,-6)$

10)

a- $\vec{u} = (3,3)$ e $\vec{v} = (5,4)$

$$\begin{bmatrix} i & j & k \\ 3 & 3 & 0 \\ 5 & 4 & 0 \end{bmatrix}\begin{matrix} i & j \\ 3 & 3 \\ 5 & 4 \end{matrix} = -15k + 0 + 0 + 0 + 0 + 12k = -3\vec{k}$$

$(0,0,3) \Rightarrow \|\vec{u}\times\vec{v}\| = \sqrt{0^2+0^2+3^2} = \sqrt{9} = 3$

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b- $\vec{u}=(7,0,-5)$ $\vec{v}=(1,2,-1)$

$$\vec{u}\cdot\vec{v}=\begin{bmatrix} i & j & k \\ 7 & 0 & -5 \\ 1 & 2 & -1 \end{bmatrix}\begin{matrix} i & j \\ 7 & 0 \\ 1 & 2 \end{matrix}$$

$\|\vec{u}\cdot\vec{v}\|=\sqrt{10^2+2^2+14^2}=\sqrt{100+4+196}=\sqrt{300}=10\sqrt{3}$

$0+10i+7j+0-5j+14k=(10,2,14)$

c- $\vec{u}=(1,-3,1)$ $\vec{v}=(1,1,4)$

$$\vec{u}\cdot\vec{v}=\begin{bmatrix} i & j & k \\ 1 & -3 & 1 \\ 1 & 1 & 4 \end{bmatrix}\begin{matrix} i & j \\ 1 & -3 \\ 1 & 1 \end{matrix}$$

$\|\vec{u}\cdot\vec{v}\|=\sqrt{(-13)^2+(-3)^2+4^2}=\sqrt{169+9+16}=\sqrt{194}$

$3k-i-4j-12i+j+k=(-13,-3,4)=$

d- $\vec{u}=(2,1,2)$ $\vec{v}=(4,2,4)$

$\vec{v}=2\vec{u}\rightarrow\vec{u}\cdot\vec{v}=0$

$\rightarrow\|\vec{u}\cdot\vec{v}\|=0$

11)

a- $\|\vec{u}\cdot\vec{v}\|^2=(\|u\|\|v\|\cdot sen\,\theta)^2=\|u\|^2\cdot\|v\|^2\cdot sen^2\theta$

$|\vec{u}\cdot\vec{v}|=(\|u\|\cdot\|v\|\cdot\cos\theta)^2=\|u\|^2\cdot\|v\|^2\cdot\cos^2\theta=\|\|\|(1-\cos^2\theta)=\|\vec{u}\|^2\,\|v\|^2\cdot sen^2\theta=$

$\|\vec{u}\cdot\vec{v}\|^2=\|u\|^2\cdot\|v\|^2-(\vec{u}\cdot\vec{v})^2$

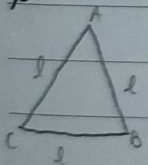
b- $\|\vec{u}\cdot\vec{v}\|^2=\|1\|^2\cdot\|5\|^2-(3)^2$

$\|\vec{u}\cdot\vec{v}\|^2=1\cdot 25-9$

$\|\vec{u}\cdot\vec{v}\|^2=16$

$\|\vec{u}\cdot\vec{v}\|=4$

c-



$\|\vec{AB}\cdot\vec{AC}\|\rightarrow\|\vec{AB}\cdot\vec{AC}\|^2=\|\vec{AB}\|^2\cdot\|\vec{AC}\|^2-(\vec{AB}\cdot\vec{AC})$

$\|AB\cdot AC\|^2=l^2\cdot l^2-(x_1x_2+y_1y_2+z_1z_2)$

$\|AB\cdot AC\|=\sqrt{l^2\cdot l^2-\|\|\|\|\|\|\|\|\|\|}$

$\|AB\cdot AC\|=l^2-\left(\sqrt{(x_1x_2+y_1y_2+z_1z_2}\right)$

12)

a- $\begin{cases}\vec{x}\cdot(2\vec{i}+3\vec{j}+4\vec{k})=9\\ \vec{x}\cdot(-i+\vec{j}-\vec{k}+2\vec{i}-2\vec{k})=0\end{cases}$

$\begin{cases}\vec{x}\cdot(2\vec{i}+3\vec{j}++4\vec{k})=9\rightarrow\vec{x}=(2,3,4)\rightarrow 2x+3y+4z=9\\ \vec{x}\cdot(i+\vec{j}-3\vec{k})=0\end{cases}$

$2x+3\cdot(2-x)+4x=9$

$x=1$

$\vec{x}=(x,y,z)$ $\vec{v}=(-1,1,-1)$

$$\begin{bmatrix} i & j & k \\ x & y & z \\ -1 & 1 & -1 \end{bmatrix}\begin{matrix} i & j \\ x & y \\ -1 & 1 \end{matrix}$$

$yk-zi+xj-yi-zj+xk$

$(-y-z)i-(-x+z)j+(x+y)\cdot k=-2i+0j+2k$

-2 $\quad 0$ $\quad 2$

$x=z=1$

$x+y=2\rightarrow y=2-x$

$y=2-1=1$

$\vec{x}=(1,1,1)$

___/___/___

$\begin{cases}\vec{x}\cdot(1,0,1)=2\cdot(1,1,-1)\\ \|\vec{x}\|=\sqrt{6}\end{cases}$ $\left[\begin{matrix} i & j & k\\ x & y & z\\ 1 & 0 & 1\end{matrix}\right] = yi-(x-z)j-yk$ $\|\vec{x}\|=\sqrt{x^2+y^2+z^2}=\sqrt{6}\to x^2+y^2+z^2=6$

$\to \vec{x}\cdot(1,0,1)=(2,2,-2)\to y=2$

$-(x-z)=2\to x-z=-2\to z=x+2$

$-y=-2$

$z=-1+2$

$z=1$

$\vec{x}=(-1,2,1)$

$\to x^2+2^2+(x+2)^2=6$

$x^2+4+x^2+2x+2x+4=6$

$2x^2+4x+8=6$

$2x^2+4x+2=0$ $\div 2$

$x^2+2x+1=0$

$\Delta=2^2-4\cdot1\cdot1=0$

$x=\frac{-2\pm0}{2}=-1$

c-

$\|\vec{x}\|=\sqrt{3}$ $\left[\begin{matrix} i & j & k\\ -3 & 0 & 3\\ 2 & -2 & 0\end{matrix}\right]\begin{matrix} i & j\\ -3 & 0\\ 2 & -2\end{matrix}$

$0+6i+0+0+6j+6k=(6,6,6)$

$\vec{x}=\|x_0\|\cdot k$

$3=\sqrt{108}\cdot k$

$k=\frac{3}{\sqrt{108}}=\frac{3}{6\sqrt{3}}=\frac{1}{2\sqrt{3}}$

108 | 2
54 | 2
27 | 3
9 | 3
3 | 3

$\|\vec{x}\|=\sqrt{x\cdot x}$

$\sqrt{3}=\sqrt{x^2}$

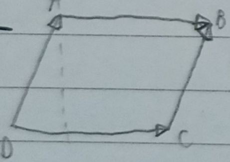
$(\sqrt{3})^2=x^2$

$3=\vec{x}$

$\vec{x}=\frac{1}{2\sqrt{3}}\cdot(6,6,6)=\left(\frac{6}{2\sqrt{3}};\frac{6}{2\sqrt{3}};\frac{6}{2\sqrt{3}}\right)\to\left(\frac{3}{\sqrt{3}};\frac{3}{\sqrt{3}};\frac{3}{\sqrt{3}}\right)$

13)

a-



$\vec{AB}=(1,1,-1)$

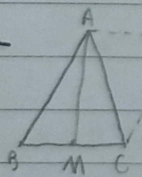
$A=(3,2,-1)$

$D=(5,3,3)$

$\vec{DA}=(3-5,2-3,-1-3)$

$\vec{DA}=(-2,-1,-4)$

b-



área $=\frac{1}{2}\cdot\|\vec{BC}\cdot\vec{CA}\|=\frac{13,45}{2}=6,725$

$\vec{BC}=-\vec{AB}+\vec{AC}=(1,-1,0)+(0,1,3)=(1,0,3)$

$\|\vec{BC}\cdot\vec{CA}\|^2=\|\vec{BC}\|^2\cdot\|\vec{CA}\|^2-(\vec{BC}\cdot\vec{CA})$

$\|\ \|^2=(\sqrt{1+0+9})^2\cdot(\sqrt{0+1+9})^2-(0+0-9)^2$

$\|\ \|^2=10\cdot10+81$

$\|\ \|^2=181$

$\|\ \|=\sqrt{181}$

$\|\vec{BC}\cdot\vec{CA}\|\cong13,45$

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14)

a-

$\vec{u}\cdot(\vec{v}\cdot\vec{w}) = (\vec{u}\cdot\vec{v})\cdot\vec{w} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & x_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \rightarrow x_3\cdot\begin{bmatrix} y_1 & y_2 \\ z_1 & z_2 \end{bmatrix} + y_3\cdot\begin{bmatrix} x_1 & x_2 \\ z_1 & z_2 \end{bmatrix} + z_3\cdot\begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}$

$x_3\cdot(-y_2z_1+y_1z_2)+y_3\cdot(-x_2z_1+x_1z_2)+z_3(-x_2y_1+x_1y_2)$

$\rightarrow x_1\cdot\begin{bmatrix} y_2 & y_3 \\ z_2 & z_3 \end{bmatrix} - y_1\begin{bmatrix} x_2 & x_3 \\ z_2 & z_3 \end{bmatrix} + z_1\begin{bmatrix} x_2 & y_2 \\ x_3 & y_3 \end{bmatrix}$

$x_1(-y_3z_2+y_2z_3)+y_1(-x_3z_2+x_2z_3)+z_1\cdot(-x_3y_2+x_2y_3)$

$\rightarrow \left|\begin{matrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{matrix}\right| \begin{matrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{matrix}$

$-(z_1y_2x_3)-(x_1z_2y_3)-(y_1x_2z_3)+(x_1y_2z_3)+(y_1z_2x_3)+(z_1x_2y_3)$

$+x_1\cdot(y_2z_3-z_2y_3)+y_1\cdot(x_3z_2-x_2z_3)+z_1\cdot(x_2y_3-x_3y_2)$

b- $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ -1 & 2 & 0 \end{bmatrix}\begin{matrix} 1 & 3 \\ 0 & 1 \\ -1 & 2 \end{matrix} \rightarrow [\vec{u}, \vec{v}; \vec{w}]$

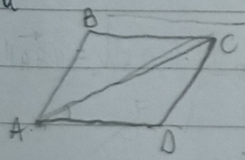
$+2+4+0+0+6+0=12$

$[\vec{v}, 2\vec{w}, \vec{u}] = 12\cdot 2 = 24$

$[\vec{u}, 3\vec{v}-2\vec{u}, \vec{w}+3\vec{u}] = 12$

15)

a-



$\vec{BE} \parallel \vec{AD} = \vec{BE} = \vec{AD} \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 2 & 2 & 2 \end{vmatrix}\begin{matrix} i & j \\ 1 & 0 \\ 2 & 2 \end{matrix}$

$0-2i-2j+0+2j+2k = -2i+2k = (-2, 0, 2)$

$\text{área} = \sqrt{(-2,0,2)^2} = \sqrt{4+4} = 8 = 2\sqrt{2}$

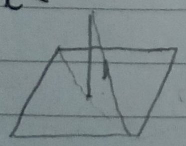
b-

$V = |(\vec{AB}\cdot\vec{AD})\cdot\vec{AE}|$

$\vec{AE} = \vec{AB} + \vec{BE} = (3, 2, 3)$

$(-2,0,2)\cdot\begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} = (-6,0,6) = \sqrt{(-6,0,6)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

c-



$h = \frac{V}{A_b} = \frac{6\sqrt{2}}{2\sqrt{2}} = 3$

___/___/___

d- $V = \frac{1}{6} \cdot |(\vec{AB} \cdot \vec{AD}) \cdot \vec{AE}|$

$V = \frac{6\sqrt{2}}{6} = \sqrt{2}$

$Ab = \frac{1}{2} \cdot \| \vec{AB} \cdot \vec{BE} \|$ $(1, 0, 1) \cdot \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} = (2, 0, 2) = \sqrt{2}$

e- $V = \frac{1}{3} \cdot Ab \cdot h$

$h = V : \frac{Ab}{3}$

$h = \frac{3V}{ab} = \frac{3\sqrt{2}}{\sqrt{2}} = 3$