Lista 7

1 -

a) $v = B - A = (-2,3,2) - (5,4,1) = (-7,-1,1)$

$x = 5 - 7t$

$y = 4 - 1t \to 4 - t \to t = 4 - y$

$z = 1 + 1t \to 1 + t \to t = z - 1$

$\to x = 5 - 7t$

$7t = 5 - x$

$t = \frac{5-x}{7}$

O ponto A será o P_0 e v será o vetor diretor

$\frac{5-x}{7} = 4 - y = z - 1$

$\frac{5-x}{7} = \frac{4-y}{-1} = \frac{z-1}{1} \to$ equação simétrica

b) $v = B - A = (1,0,0) - (0,-1,0) = (1,1,0)$

$x = 0 + 1t \to t = x$

$y = -1 + 1t \to t = y + 1$

$z = 0 + 0t \to z = 0$

$A = P_0$

v = vetor diretor

$\frac{x-0}{1} = \frac{y-(-1)}{1}$ e $z = 0$

$x = y + 1$ e $z = 0$

c) $v = B - A = (0,0,0) - (0,1,-1) = (0,-1,1)$

$x = 0 + 0t \to x = 0$

$y = 0 - t \to y = -t$

$z = 0 + t \to z = t$

$B = P_0$

v = vetor diretor

$x = 0$ e $\frac{y-0}{-1} = \frac{z-0}{1} \to x = 0$ e $-y = z$

d) $v = B - A = (6,1,-4) - (3,2,1) = (3,-1,-5)$

$x = 3 + 3t \to t = \frac{x-3}{+3}$

$y = 2 - t \to -t = y - 2$

$z = 1 - 5t \to t = \frac{z-1}{-5}$

$A = P_0$

v = vetor diretor

$\frac{x-3}{3} = \frac{y-2}{-1} = \frac{z-1}{-5}$

2 -

a)

ponto A $(\lambda = 0) \to x = 1 - 0 = 1$

$\to (1,0,4)$ $y = 0$

$z = 4 + 2 \cdot 0 = 4$

ponto B $(\lambda = 1) \to x = 1 - 1 = 0$

$\to (0,1,6)$ $y = 1$

$z = 4 + 2 \cdot 1 = 6$

$v_1 = (-1,1,2)$

$v_2 = -2 \cdot (-1,1,2) = (2,-2,-4)$

b)

$\begin{cases} x = 1 - \lambda \\ y = \lambda \\ z = 4 + 2\lambda \end{cases}$

ponto $P = (1, 3, -3)$

$x = 1 - \lambda$	$y = \lambda$	$z = 4 + 2\lambda$	
$1 = 1 - \lambda$	$3 = \lambda$	$-3 = 4 + 2\lambda$	$0 \neq 3 \neq -\frac{7}{2}$
$\lambda = 0$		$\lambda = -\frac{7}{2}$	$P \notin r$

ponto $Q = (-3, 4, 12)$

$x = 1 - \lambda$	$\lambda = y$	$z = 4 + 2\lambda$	
$-3 = 1 - \lambda$	$\lambda = 4$	$12 = 4 + 2\lambda$	$4 = 4 = 4$
$\lambda = 4$		$\lambda = 4$	$Q \in r$

c)

$P_0 = (1, 4, -7)$	$x = x_0 + at \rightsquigarrow x = 1 + (-1)t \rightsquigarrow x = 1 - t$
$v = (-1, 1, 2)$	$y = y_0 + bt \rightsquigarrow y = 4 + t$
	$z = z_0 + ct \rightsquigarrow z = -7 + 2t$

3)

a)

$\vec{AB} = B - A = (-5, 2, 3) - (3, 6, -7) = (-8, -4, 10)$

$\vec{AC} = C - A = (4, -7, -6) - (3, 6, -7) = (1, -13, 1)$

$\vec{AB} = \lambda \cdot \vec{AC} \rightsquigarrow$

$(-8, -4, 10) = \lambda \cdot (1, -13, 1)$

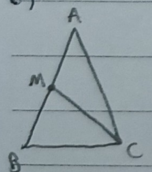
$-8 = \lambda \cdot 1 \rightsquigarrow \lambda = 8$

$-4 = \lambda \cdot (-13) \rightsquigarrow \lambda = \frac{-4}{-13} = \frac{4}{13}$

$10 = \lambda \cdot 1 \rightsquigarrow \lambda = 10$

$8 \neq \frac{4}{13} \neq 10$ $\vec{AB}$ e $\vec{AC}$ não são paralelos, e portanto A, B e C não são colineares e por isso não são vértices de um triângulo

b)



duas extremidades

$M = (3, 6, -7)$ e $(-5, 2, 3)$

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

$M = \left(\frac{3-5}{2}, \frac{6+2}{2}, \frac{-7+3}{2}\right)$

$M = (-1, 4, -2)$

$\vec{CM} = M - C = (-1, 4, -2) - (4, -7, -6) = (-5, 11, 4)$

equação vetorial $\Rightarrow P = P_0 + t\vec{v} \rightsquigarrow x = 4 - 5t$

$P_0 = C$ $\quad y = -7 + 11t \quad t \in \mathbb{R}$

$z = -6 + 4t$

4-

a) $C = (1 + \lambda, 2 + \lambda, 0 - 3\lambda) = (1 + \lambda, 2 + \lambda, -3\lambda)$

Como o triângulo é retângulo, um dos ângulos formados deve ser 90°, ou seja, o produto escalar de 2 vetores deve ser 0

$\vec{AC} = C - \vec{A} = (1 + \lambda, 2 + \lambda, -3\lambda) - (0, 1, 2) = (1 + \lambda, 1 + \lambda, -3\lambda - 2)$

$\vec{AB} = B - A = (-3,0,9) - (0,1,8) = (-3,-1,1)$

$\vec{BC} = C - B = (1+\lambda, 2+\lambda, -3\lambda) - (-3,0,9) = (4+\lambda, 2+\lambda, -3\lambda-9)$

$\vec{BA} \cdot \vec{BC} = -(-3,-1,1) \cdot (4+\lambda, 2+\lambda, -3\lambda-9) = 0$

$-3 \cdot (4+\lambda) + 1 \cdot (2+\lambda) - 1 \cdot (-3\lambda-9) = 0$

$12 + 3\lambda + 2 + \lambda + 3\lambda + 9 = 0$

$7\lambda + 23 = 0$

$7\lambda = 23$

$\lambda = \frac{23}{7}$ → um dos casos

$AB \cdot AC = 0$ → 2º caso

$CA \cdot CB = 0$ → 3º caso

b) um ponto em r

$\vec{P} = (1,0,0) + \lambda(1,1,1)$

$P = (1,0,0) + (\lambda,\lambda,\lambda)$

$P = (1+\lambda, \lambda, \lambda)$

$d(P,A) = d(P,B)$

$d(P,A)^2 = d(P,B)^2$

$(P,A)^2 = (1+\lambda-1)^2 + (\lambda-1)^2 + (\lambda-1)^2 = \lambda^2 + (\lambda-1)^2 + (\lambda-1)^2$

$\lambda^2 + (\lambda^2 - 2\lambda + 1) + (\lambda^2 - 2\lambda + 1)$

$3\lambda^2 - 4\lambda + 2$

$d(P,B)^2 = (1+\lambda-0)^2 + (\lambda-0)^2 + (\lambda-1)^2 = (1+\lambda)^2 + \lambda^2 + (\lambda-1)^2$

$= (1 + 2\lambda + \lambda^2) + \lambda^2 + (\lambda^2 - 2\lambda + 1)$

$= 3\lambda^2 + 2$

$\cancel{3\lambda^2} + \cancel{2} = \cancel{3\lambda^2} - 4\lambda + \cancel{2}$

$\lambda = 0$

$P = 0 \cdot (1+\lambda, \lambda, \lambda)$

$P = (1,0,0)$ único ponto equidistante de A e B

5-

a) $X = A + \lambda u + \alpha v$

$X = (1,2,0) + \lambda(1,1,0) + \alpha \cdot (2,3,-1)$ → equação vetorial

$X = (a,b,c)$

$a = 1 + \lambda + 2\alpha$

$b = 2 + \lambda + 3\alpha$

$c = 0 + 0\lambda - \alpha$ → $c = -\alpha$

equações paramétricas

b)

$v = (2,1,0)$

$\vec{AB} = B - A = (1,-1,-1) - (1,1,0) = (0,-2,1)$

não paralelo

$X = A + \lambda \vec{AB} + \alpha \vec{v}$

$X = (1,1,0) + \lambda \cdot (0,-2,1) + \alpha(2,1,0)$ → equação vetorial

$X = (a,b,c)$

$a = 1 + 0\lambda + 2\alpha$ → $a = 1 + 2\alpha$

$b = 1 - 2\lambda + \alpha$

$c = 0 - \lambda + 0\alpha$ → $c = -\lambda$

equações paramétricas

c) $X = A + \lambda\vec{AB} + \beta\vec{AC} = (a, b, c)$

$A = (1,0,1)$

$\vec{AB} = B - A = (2,1,-2) - (1,0,1) = (1,1,-3)$

$\vec{AC} = C - A = (1,-1,0) - (1,0,1) = (0,-1,-1)$

$X = (1,0,1) + \lambda(1,1,-3) + \beta(0,-1,1)$ ↳ eq. vetorial

$$\left.\begin{aligned} a &= 1 + \lambda + 0\beta = 1 + \lambda \\ b &= 0 + \lambda - 1\beta = \lambda - \beta \\ c &= 1 - 3\lambda - \beta \end{aligned}\right\}$$ equações paramétricas

6 -

a) $n = \vec{u} \cdot \vec{v} = (0,1,0) \cdot (1,1,1) = (1,0,-1)$ $A = (9,-1,0)$

$a = 1\cdot1 - 0\cdot1 = 1$

$b = 0\cdot1 - 0\cdot1 = 0$

$c = 0\cdot1 - 1\cdot1 = -1$

↳ $(x - z + d = 0$ (eq. parcial)

$9 - 0 + d = 0$

$d = -9$

eq. geral $= x - z - 9 = 0$

b) eq. geral $\Rightarrow ax + by + cz + d = 0$

$\vec{AB} = B - A = (-1,0,1) - (1,0,1) = (-2,0,0)$

$\vec{AC} = C - A = (2,1,2) - (1,0,1) = (1,1,1)$

$n = \vec{AB} \cdot \vec{AC} = (-2,0,0) \cdot (1,1,1) = (0,2,-2)$

$$\begin{bmatrix} i & j & k \\ -2 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{matrix} i & j \\ -2 & 0 \\ 1 & 1 \end{matrix}$$

$+0 + 0 + 2j + 0 + 0 - 2k = (0i, 2j, -2k) = (0,2,-2)$

$0x + 2y - 2z + d = 0$

$+2\cdot0 - 2\cdot1 + d = 0$

$d = 2$

eq. geral $= y - z + 1 = 0$

c) eq. geral $\Rightarrow ax + by + cz + d = 0$

$\vec{AB} = B - A = (1,-1,-1) - (1,1,0) = (0,-2,-1)$

$\vec{v} \cdot \vec{AB} = (2,1,0) \cdot (0,-2,-1) =$

$$\begin{bmatrix} i & j & k \\ 2 & 1 & 0 \\ 0 & -2 & -1 \end{bmatrix} \begin{matrix} i & j \\ 2 & 1 \\ 0 & -2 \end{matrix}$$

$0 + 0 + 2j - i + 0 - 4k = (-1, 2, -4)$

eq. parcial

$-x + 2y - 4z + d = 0$

$-1 + 2\cdot1 - 4\cdot0 + d = 0$

$-1 + 2 - 0 + d = 0$

$d = -1$

eq. geral

$-x + 2y - 4z - 1 = 0$

d)

$P_r = \vec{v} - P = (1,1,-1) - (1,-1,1) = (0,2,-2)$

$\vec{u} = P - P_r = (1,-1,1) - (0,2,-2) = (1,-3,-1)$

$n = \vec{u} \cdot \vec{v} = (4,2,4)$

$$\begin{bmatrix} i & j & k \\ 1 & -3 & -1 \\ 1 & 1 & -1 \end{bmatrix} \begin{matrix} i & j \\ 1 & -3 \\ 1 & 1 \end{matrix}$$

$3k + i + j + 3i + j + k = (4,2,4)$

eq. parcial

$4x + 2y + 4z + d = 0$

$4\cdot1 + 2\cdot(-1) + 4\cdot1 + d = 0$

$d = -10$

eq. geral

$4x + 2y + 4z - 10 = 0$

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7-

a) $4x+2y-z+5=0$

$\vec{u}\times\vec{v}=(4,2,-1)$ $d=5$

$x=s$ e $y=t$

$4s+2t-z+5=0$

$z=4s+2t+5$

$\begin{cases} x=s \\ y=t \\ z=4s+2t+5 \end{cases}$ com $s,t\in\mathbb{R}$

b) $5x-y-1=0$ → z implícito

$x=s$ e $z=t$

$5s-y-1$

$y=5s-1$

$\begin{cases} x=s \\ y=5s-1 \\ z=t \end{cases}$ com $s,t\in\mathbb{R}$

c) $z-3=0$ → y e x implícito (0y e 0x)

R: esse plano é paralelo ao plano xy, fazendo com que z sempre seja 3

$x=s$ e $y=t$

$\begin{cases} x=s \\ y=t \\ z=3 \end{cases}$ s e $t\in\mathbb{R}$

d) $y-z-2=0$ → x implícito

$x=s$ $z=t$

$y-t-2=0$

$y=t+2$

$\begin{cases} x=s \\ y=t+2 \\ z=t \end{cases}$ com s e $t\in\mathbb{R}$

8)

a-

$\begin{cases} x=1+\lambda-\mu \\ y=2\lambda+\mu \\ z=3-\mu \end{cases}$

$A\cdot(x-x_0)+B\cdot(y-y_0)+C\cdot(z-z_0)=0$

$-2(x-1)+1\cdot(y-0)+3\cdot(z-3)=0$

$-2x+y+3z-7=0$

$P_0=(1,0,3)$ quando λ e $\mu=0$

$v_1=(1,2,0)$

$v_2=(-1,1,-1)$

$n=v_1\cdot v_2=\begin{bmatrix} i & j & k \\ 1 & 2 & 0 \\ -1 & 1 & -1 \end{bmatrix}\begin{matrix} i & j \\ 1 & 2 \\ -1 & 1 \end{matrix}$ $=+2k+j-2i+0+k$

$(-2,1,3)$

b-

$\begin{cases} x=1+\lambda \\ y=2 \\ z=3-\lambda+\mu \end{cases}$

$A\cdot(x-x_0)+B\cdot(y-y_0)+C\cdot(z-z_0)=0$

$0\cdot(x-1)-1\cdot(y-2)+0\cdot(z-3)=0$

$-y+2=0 \rightarrow y=2$

$P_0=(1,2,3)$ quando λ e $\mu=0$

$v_1=(1,0,-1)$

$v_2=(0,0,1)$

$n=v_1\cdot v_2=\begin{bmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}\begin{matrix} i & j \\ 1 & 0 \\ 0 & 0 \end{matrix}$

$0+0-j+0+0+0=(0,-1,0)$

c-

$\begin{cases} x=-2+\lambda-\mu \\ y=2\lambda+2\mu \\ z=\lambda+\mu \end{cases}$

$A\cdot(x-x_0)+B\cdot(y-y_0)+C\cdot(z-z_0)=0$

$0(x+2)-2\cdot(y-0)+4\cdot(z-0)=0$

$-2y+4z=0$

$P_0=(-2,0,0)$

$v_1=(1,2,1)$

$v_2=(-1,2,1)$

$\begin{bmatrix} i & j & k \\ 1 & 2 & 1 \\ -1 & 2 & 1 \end{bmatrix}\begin{matrix} i & j \\ 1 & 2 \\ -1 & 2 \end{matrix}$

$+2k-2i-j+2i-j+2k=(0,-2,4)$

9)

a-

$\begin{cases} 1+2\lambda=-1+4\mu \\ \lambda=-1+2\mu \\ 1+3\lambda=-2+6\mu \end{cases}$

$\to 1+2\cdot(-1+2\mu)=-1+4\mu$

$1-2+4\mu=-1+4\mu$

$-1+4\mu=-1+4\mu$ ✓

$\to 1+3\cdot(-1+2\mu)=-2+6\mu$

$1-3+6\mu=-2+6\mu$

$-2+6\mu=-2+6\mu$ ✓

são retas concorrentes

$r \xrightarrow{\mu=1} \lambda=-1+2\cdot 1=1$

$x=1+2\cdot 1=3$

$y=1$

$z=1+3\cdot 1=4$

ponto de intersecção: $P=(3,1,4)$

$\vec{v}_r=(2,1,3)$

$\vec{v}_s=(4,2,6)$

$\vec{v}_s=2\vec{v}_r$ (são coincidentes e L.D)

R: a reta s está contido em r, por isso não há como se formar um plano

b-

reta r
$\begin{cases} x=1+\lambda \\ y=1+2\lambda \\ z=0+3\lambda \end{cases}$

reta s
$\begin{cases} x=2+3\mu \\ y=3+2\mu \\ z=3+\mu \end{cases}$

$\begin{cases} 1+\lambda=2+3\mu \to \lambda=1+3\mu \to \lambda=1 \\ 1+2\lambda=3+2\mu \to 1+2\cdot(1+3\mu)=3+2\mu \\ 3\lambda=3+\mu \end{cases}$

$1+2+6\mu=3+2\mu$

$3+6\mu=3+2\mu$

$\mu=0$

$3\cdot 1=3+0$

$3=3$ ✓

as retas são concorrentes

$\lambda=1$ em r

$\begin{cases} x=2 \\ y=3 \\ z=3 \end{cases}$

$P=(2,3,3)$

$\vec{v}_s=(3,2,1)$

$\vec{v}_r=(1,2,3)$

$n=\vec{v}_r\cdot\vec{v}_s$

$\begin{bmatrix} i & j & k \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{matrix} i & j \\ 3 & 2 \\ 1 & 2 \end{matrix}$

$-2k-2i-9j+6i+j+6k$

$(-4,8,4):4=(-1,2,1)$

eq. geral $=-1\cdot(x-2)+2\cdot(y-3)+1\cdot(z-3)=0$

$-x+2y-z-1=0$ //

c-

reta s

$\mu=\dfrac{x}{2}=\dfrac{y-1}{-2}=z$

$\begin{cases} x=2\mu \\ y-1=-2\mu \to y=-2\mu+1 \\ z=\mu \end{cases}$

$\begin{cases} 2-4\lambda=2\mu \to 2-4\lambda=2\cdot 11 \\ 4+5\lambda=1-2\mu \\ 11=\mu \end{cases}$

$-4\lambda=22-2$

$-4\lambda=20$

$\lambda=-5$

$\to 4+5\cdot(-5)=1-2\cdot 11$

$-21=-21$ ✓ retas concorrentes

$\lambda=-5$ em r

$\begin{cases} x=22 \\ y=-21 \\ z=11 \end{cases}$

$P=(22,-21,11)$

$\vec{v}_r=(-4,5,0)$ e $\vec{v}_s=(2,-2,1)$

$\begin{bmatrix} i & j & k \\ -4 & 5 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{matrix} i & j \\ -4 & 5 \\ 2 & -2 \end{matrix}$

$-10k+0+4j+5i+0+8k=(5,4,-2)$

eq. geral: $5\cdot(x-22)+4\cdot(y+21)-2\cdot(z-11)=\underline{5x+4y-2z-4=0}$

d-

reta r

$\lambda=\dfrac{x-2}{3}=\dfrac{y+2}{4}=z$

$\begin{cases} x=2+3\lambda \\ y=-2+4\lambda \\ z=\lambda \end{cases}$

reta s

$\mu=\dfrac{x}{4}=\dfrac{y}{2}=\dfrac{z-3}{2}$

$\begin{cases} x=4\mu \\ y=2\mu \\ z=2\mu+3 \end{cases}$

$\begin{cases} 2+3\lambda=4\mu \\ -2+4\lambda=2\mu \\ \lambda=2\mu+3 \end{cases}$

$\to 2+3\cdot(2\mu+3)=4\mu$

$2+6\mu+9=4\mu$

$\mu=-\dfrac{11}{2}$

$\to \lambda=2\cdot\left(-\dfrac{11}{2}\right)+3$

$\lambda=-11+3=-8$

$\to -2+4\cdot(-8)=2\cdot\left(-\dfrac{11}{2}\right)$

$-2+32=-11$

$-34\neq-11$

R: as retas não são concorrentes por isso não se interceptam

* São paralelas ou reversas

$\vec{v}_r=(3,4,1)$

$\vec{v}_s=(4,2,2)$

não existe $k\in\mathbb{R}$ que garanta que $(4,2,2)=k\cdot(3,4,1)$, por isso são reversas e não fazem parte do mesmo plano

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10 -

a) $\vec{v}_r = n_1 \cdot n_2$

$n_1 = (1,2,3)$

$n_2 = (1,-1,2)$

$$\left[\begin{matrix} i & j & k \\ 1 & 2 & 3 \\ 1 & -1 & 2 \end{matrix}\right] \begin{matrix} i & j \\ 1 & 2 \\ 1 & 1 \end{matrix}$$

$-2k+3i-2j+4i+3j+k=(7,1,-3)$

$P_0 = (1/3, 1/3, 0)$

$z=0$

$\begin{cases} x+2x-1=0 \\ x-y=0 \to x=y \end{cases}$

$\to y+2y-1=0$

$y=1/3$

eq. vetorial

$(1/3, 1/3, 0) + t\cdot(7,1,-3), t \in \mathbb{R}$

b) $\vec{v}_r = n_1 \cdot n_2$

$n_1 = (1,1,1)$

$n_2 = (1,1,-1)$

$$\left[\begin{matrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{matrix}\right] \begin{matrix} i & j \\ 1 & 1 \\ 1 & 1 \end{matrix}$$

$-k-i+j-i+j+k=(-2,2,0)$

$P_0 = (1/2, 1/2, 0)$

$z=0$

$\begin{cases} x+y-1=0 \to \\ x+y=0 \to x=y \end{cases}$

$x+x-1=0$

$2x=1 \to x=\frac{1}{2}$

eq. vetorial

$r = (1/2, 1/2, 0) + t\cdot(-2,2,0), t \in \mathbb{R}$

c)

$n_1 = (3,0,0)$

$n_2 = (2,0,-1)$

$$\left[\begin{matrix} i & j & k \\ 3 & 0 & 0 \\ 2 & 0 & -1 \end{matrix}\right] \begin{matrix} i & j \\ 3 & 0 \\ 2 & 0 \end{matrix}$$

$0+0+3j+0+0+0=(0,3,0)$

$:3$

$P_0 = (3,0,7)$

$\begin{cases} x=3 \\ 2x-z+1=0 \end{cases}$

$2\cdot 3 - z + 1 = 0$

$z=7$

eq. vetorial

$r = (3,0,7) + t\cdot(0,1,0), t \in \mathbb{R}$

$x=3, y=t, z=7$

d)

$n_1 = (0,1,0)$

$n_2 = (0,0,1)$

$$\left[\begin{matrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right] \begin{matrix} i & j \\ 0 & 1 \\ 0 & 0 \end{matrix}$$

$(1,0,0)$

$P_0 = (0,2,0)$

eq. vetorial

$r = (0,2,0) + t\cdot(1,0,0), t \in \mathbb{R}$

$x=t, y=2, z=0$

11 -

a)

reta r

$P_r = (1,-1,1)$

$\vec{v}_r = (-2,1,-1)$

$\begin{cases} x = 1-2\lambda \\ y = -1+\lambda \\ z = 1-\lambda \end{cases}$

reta s

$n_1 = (0,1,1)$

$n_2 = (1,1,-1)$

$$\left[\begin{matrix} i & j & k \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{matrix}\right] \begin{matrix} i & j \\ 0 & 1 \\ 1 & 1 \end{matrix}$$

$-k-i+0-i+j+0=(-2,1,-1)$

$P_0 = (3,3,0) : 3 = (1,1,0)$

$z=0$

$\begin{cases} y=3 \\ x+y=6 \end{cases}$

$x+3=6$

$x=3$

$\begin{cases} x=1-2u \\ y=1+u \\ z=-1 \end{cases}$

$\vec{v}_s = \vec{v}_r$ (paralelas)

$P_r = (1,-1,1)$

$s \to y+z=3$

$-1+1=3$

$0 \neq 3$

não são coincidentes

R: r e s são retas paralelas

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b)

reta s

$P_0 = (0,0,0)$

$\vec{v}_s = (1,2,0)$

$\begin{cases} x = \lambda \\ y = 2\lambda \\ z = 0 \end{cases}$

reta r

$u = \frac{x+1}{2} = \frac{y}{3} = \frac{z+1}{2}$

$\begin{cases} x = 2u - 1 \\ y = 3u \\ z = 2u - 1 \end{cases}$

$P_r = (-1, 0, 1)$

$\vec{v}_r = (2,3,2)$

$(2,3,2) \neq k \cdot (1,2,0)$ não existe k que atenda aos três valores de $\vec{v}_r$

R: Isso significa que as retas são concorrentes ou reversas.

$\begin{cases} 2u - 1 = \lambda \\ 3u = 2\lambda \\ 2u - 1 = 0 \rightarrow u = \frac{1}{2} \end{cases}$

$\rightarrow 3 \cdot \frac{1}{2} = 2\lambda$

$\lambda = \frac{3}{4}$

$\rightarrow 2 \cdot \frac{1}{2} - 1 = \frac{3}{4}$

$1 - 1 = \frac{3}{4}$

$0 \neq 3/4$

R: As retas não se cruzam

As retas são reversas //

c)

retas s

$P_s = (8,1,9)$

$\vec{v}_s = (2,-1,3)$

$\begin{cases} x = 2\lambda + 8 \\ y = -\lambda + 1 \\ z = 3\lambda + 9 \end{cases}$

reta r

$P_r = (3,-4,4)$

$\vec{v}_r = (1,-2,2)$

$\begin{cases} x = u + 3 \\ y = -2u - 4 \\ z = 2u + 4 \end{cases}$

$(2,-1,3) \neq k \cdot (1,-2,2)$ não existe k que atende aos três valores de $\vec{v}_s$

R: Isso significa que as retas são concorrente ou reversas

$\begin{cases} 2\lambda + 8 = u + 3 \\ -\lambda + 1 = -2u - 4 \\ 3\lambda + 9 = 2u + 4 \end{cases}$

$\rightarrow \lambda = -2u - 5 \cdot (-1)$

$\lambda = 2u + 5$

$\lambda = -5$

$\rightarrow 3 \cdot (2u + 5) + 9 = 2u + 4$

$6u + 15 + 9 = 2u + 4$

$4u = -20$

$u = -5$

$\rightarrow 2 \cdot (-5) + 8 = -5 + 3$

$-10 + 8 = -2$

$-2 = -2$

R: As retas são concorrentes

d)

reta r

$\lambda = \frac{x+1}{2} = y = z$

$\begin{cases} x = 2\lambda - 1 \\ y = \lambda \\ z = \lambda \end{cases}$

$P_r = (-1, 0, 0)$

$\vec{v}_r = (2,1,1)$

reta s

$\vec{v}_{s1} = (1,1,-3)$

$\vec{v}_{s2} = (2,-1,-2)$

$\begin{bmatrix} i & j & k \\ 1 & 1 & -3 \\ 2 & -1 & -2 \end{bmatrix} \begin{matrix} i & j \\ 1 & 1 \\ 2 & -1 \end{matrix}$

$-2k - 3i + 2j - 2i - 6j - k = (-5,-4,-3)$

$\vec{v}_s = (-5,-4,-3)$

$x + y - 3z = 1$

$(-1 + 2\lambda) + (\lambda) - 3 \cdot (\lambda) = 1$

$-1 + 2\lambda + \lambda - 3\lambda = 1$

$-1 \neq 1$

$(2,1,1) \neq k \cdot (-5,-4,-3)$ não existe k que atende aos três valores de $\vec{v}_r$.

R: Isso significa que as retas são concorrentes ou reversas.

As retas são reversas

12-

a)

reta r

$A=(1,1,0)$

$\vec{v}_r=(0,1,1)$

$\begin{cases} x=1 \\ y=1+\lambda \\ z=\lambda \end{cases}$

plano

$\vec{v}_n=(1,-1,-1)$

verificar ortogonalidade

$\vec{v}_r\cdot\vec{v}_n=0\cdot 1+1\cdot(-1)+1\cdot(-1)=-2$

$-2\neq 0$ então a reta é transversal ao plano

obter ponto de interseção

$+x-y-z=2$

$1-(1+\lambda)-\lambda=2$

$1-1-\lambda-\lambda=2$

$\lambda=-1$

$\begin{cases} x=1 \\ y=1-1=0 \\ z=-1 \end{cases}$

$P=(1,0,-1)$

b) reta r

$\lambda=\frac{x-1}{2}=y=z$

$\begin{cases} x=2\lambda+1 \\ y=\lambda \\ z=\lambda \end{cases}$

$\vec{v}=(2,1,1)$

$P_r=(1,0,0)$

plano π

$P_\pi=(3,0,1)$

$\vec{v}_u=(2,2,0)$

$\vec{v}_1=(1,0,1)$

v_n

$\left[\begin{matrix} i & j & k \\ 2 & 2 & 0 \\ 1 & 0 & 1 \end{matrix}\right]\begin{matrix} i & j \\ 2 & 2 \\ 1 & 0 \end{matrix}$

$-2k-2j+2i=(2,-2,-2):2$

$(1,-1,-1)$

$-1\cdot(x-3)+1\cdot(y-0)+1\cdot(z-1)=0$

$x-y-z+2=0$

verificar ortogonalidade

$\vec{v}_r\cdot\vec{v}_n=(2,1,1)\cdot(1,-1,-1)=2\cdot 1+1\cdot(-1)+1\cdot(-1)=0$

a reta é paralela ao plano

$x-y-z+2=0$

$1-0-0+2=0$

$-1\neq 0$ (a reta não pertence ao plano apenas é paralela a ele).

c) reta r

$\begin{cases} x-y=1 \rightarrow 2y-y=1 \rightarrow y=1 \\ x-2y=0 \rightarrow x=2y \rightarrow x=2 \end{cases}$

$\begin{cases} x=2 \\ y=1 \\ z=t \end{cases}$

$P_r=(2,1,0)$

$\vec{v}_1=(1,-1,0)$

$\vec{v}_2=(1,-2,0)$

$\left[\begin{matrix} i & j & k \\ 1 & -1 & 0 \\ 1 & -2 & 0 \end{matrix}\right]\begin{matrix} i & j \\ 1 & -1 \\ 1 & -2 \end{matrix}$

$k+0+0-2k=(0,0,-1)$

plano π

$x+y=2 \rightarrow x+y+0z=2$

$v_n=(1,1,0)$

verificar a ortogonalidade

$\vec{v}_r\cdot\vec{v}_n=0\cdot 1+0\cdot 1+1\cdot 0=0$

a reta é paralela ao plano

P_r na equação do plano π

$x+y=2$

$2+1=2$

$3\neq 2$

Não há ponto de interseção

d) reta r

$x-2y=3-2z+y \rightarrow x-3y+2z-3=0$

$x-3y+2z-3=2x-z \rightarrow 2x-y+z-3=0$

$n_1=(1,-3,2)$

$n_2=(2,-1,1)$

$\left[\begin{matrix} i & j & k \\ 1 & -3 & 2 \\ 2 & -1 & 1 \end{matrix}\right]\begin{matrix} i & j \\ 1 & -3 \\ 2 & -1 \end{matrix}$

$6k+2i-j-3i+4j-k=$

$v_r=(-1,3,5)$

$P_r=(6/5,-3/5,0)$

$z=0$

$\begin{cases} x-3y=3 \\ 2x-y=3\cdot 3 \end{cases}$

$\begin{cases} x-3y=3 \\ 6x-3y=9 \end{cases}$

$(6x-3y)-(x-3y)=9-3$

$x=6/5$

$\rightarrow 2\cdot\frac{6}{5}-y=3$

$\frac{12}{5}-y=3$

$-y=3-\frac{12}{5}$

$\rightarrow y=\frac{12}{5}-3$

$y=\frac{12-15}{5}$

$y=-\frac{3}{5}$

$\begin{cases} x=6/5-\lambda \\ y=-3/5+3\lambda \\ z=5\lambda \end{cases}$

$x=\frac{6}{5}-\frac{47}{10}=\frac{12-47}{10}=-\frac{35}{10}=-\frac{7}{2}$

$y=-\frac{3}{5}+3\cdot\frac{47}{10}=-\frac{3}{5}+\frac{141}{10}=\frac{-6+141}{10}=\frac{135}{10}=\frac{27}{2}$

$z=5\cdot\frac{47}{10}=\frac{235}{10}=\frac{47}{2}$

$P=(-7/2,\frac{27}{2},47/2)$

plano π

$P_\pi=(1,4,0)$

$\vec{v}_u=(1,1,1)$

$\vec{v}_w=(2,1,0)$

$\left[\begin{matrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{matrix}\right]\begin{matrix} i & j \\ 1 & 1 \\ 2 & 1 \end{matrix}$

$-2k-i+0+0+2j+k=(-1,2,-1)$

equação geral

$-1(x-1)+2(y-4)-1\cdot(z-0)=0$

$-x+1+2y-8-z=0$

$-x+2y-z-7=0$

$\vec{v}_r\cdot\vec{v}_n=$

$(-1)\cdot(-1)+3\cdot 2+5\cdot(-1)=2$

$2\neq 0$ a reta é transversal ao plano

$\left(\frac{6}{5}-\lambda\right)-2\cdot\left(-\frac{3}{5}+3\lambda\right)+5\lambda+7=0$

$\left(\frac{6-5\lambda}{5}\right)+\frac{6}{5}-6\lambda+5\lambda+7=0$

$\frac{12-5\lambda}{5}-\lambda+7=0$

$\frac{12-5\lambda-5\lambda+35}{5}=\frac{47-10\lambda}{5} \rightarrow 47-10\lambda=0 \rightarrow \lambda=\frac{47}{10}$

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13-

a)

$\vec{v}_r = (2, m, 1)$

$\vec{v}_{\pi 1} = (1, 2, 0)$

$\vec{v}_{\pi 2} = (1, 0, 1)$

$$\begin{bmatrix} i & j & k \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{matrix} i & j \\ 1 & 2 \\ 1 & 0 \end{matrix}$$

$-2k + 0 - j + 2i + 0 + 0$

$\vec{v}_{n\pi} = (2, -1, -2)$

$\vec{v}_r \cdot \vec{v}_{n\pi} = 2 \cdot 2 + m \cdot (-1) + 1(-2) = 0$

$4 - m - 2 = 0$

$-m = 2 - 4$

$m = 2$

m tem que ser igual a 2 para que a reta r e o plano π sejam ortogonais

b)

$\vec{v}_r = (2, m, m)$

$\vec{v}_{n\pi} = (1, -3, 1)$

$\vec{v}_r \cdot \vec{v}_{n\pi} = 2 \cdot 1 + (-3) \cdot m + 1 \cdot m = 0$

$2 - 3m + m = 0$

$-2m = -2$

$m = 1$

m tem que ser igual a 1 para que a r e o plano π sejam ortogonais

ponto genérico de r

$P_r = (n, 2, 0) \rightarrow n - 3 \cdot 2 + 0 = 1$

$\pi = (x - 3y + z) = 1 \qquad n = 7$

n tem que ser igual a 7 para que a reta r esteja contido no plano π

c) $\vec{v}_r \cdot \vec{v}_{n\pi} \neq 0$

$$\begin{cases} x = \lambda m + 1 \\ y = 2\lambda \\ z = \lambda m \end{cases}$$

$\vec{v}_r = (m, 2, m)$

$\vec{P}_r = (1, 0, 0)$

$\vec{v}_{n\pi} = (1, m, 1)$

$1 \cdot m + 2 \cdot m + 1 \cdot m \neq 0$

$3m \neq 0$

$m \neq 0$

m tem que ser qualquer valor diferente de 0 para que a reta r seja transversal ao plano π

14-

a) plano π_1

$\vec{v}_{\pi 1.1} = (1, 1, 2)$

$\vec{v}_{\pi 1.2} = (3, 3, 1)$

$P_1 = (4, 2, 4)$

$$\begin{bmatrix} i & j & k \\ 1 & 1 & 2 \\ 3 & 3 & 1 \end{bmatrix} \begin{matrix} i & j \\ 1 & 1 \\ 3 & 3 \end{matrix}$$

$-3k - 6i - j + i + 6j + 3k =$

$n_1 = (-5, 5, 0) = (-1, 1, 0)$

eq. geral

$-1 \cdot (x - 4) + 1 \cdot (y - 2) + 0 \cdot (z - 4) = 0$

$-x + y + 2 = 0$

plano π_2

$\vec{v}_{\pi 2.1} = (1, 1, 0)$

$\vec{v}_{\pi 2.2} = (0, 1, 4)$

$P_2 = (3, 0, 0)$

$$\begin{bmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix} \begin{matrix} i & j \\ 1 & 1 \\ 0 & 1 \end{matrix}$$

$0 + 0 - 4j + 4i + 0 + k$

$n_2 = (4, -4, 1)$

eq. geral

$4 \cdot (x - 3) + (-4) \cdot (y - 0) + 1 \cdot (z - 0) = 0$

$4x - 12 - 4y + 0 + z - 0 = 0$

$4x - 4y + z - 12 = 0$

$(-5, 5, 0) = k \cdot (4, -4, 1) \rightarrow$ não existe

São planos transversais

$P_i = n_1 \times n_2$

$$\begin{bmatrix} i & j & k \\ -5 & 5 & 0 \\ 4 & -4 & 1 \end{bmatrix} \begin{matrix} i & j \\ -5 & 5 \\ 4 & -4 \end{matrix}$$

$-20k + 0 + 5j + 5i + 0 + 20k = (5, 5, 0) = (1, 1, 0)$

$$y = 0 \quad \begin{cases} -x = -2 \rightarrow x = 2 \\ 4x + z = 12 \rightarrow 4 \cdot 2 + z = 12 \end{cases}$$

$z = 4$

$P = (2, 0, 4)$

$x = (2, 0, 4) + \lambda \cdot (1, 1, 0), \ \lambda \in \mathbb{R}$

__/__/__

b) plano π_1

$\vec{v}_n = (1, -1, 2)$

$x - y + 2z - 2 = 0$

plano π_2

$P = (0, 0, 1)$

$\vec{v}_{\pi_2 1} = (1, 0, 3)$

$\vec{v}_{\pi_2 2} = (-1, 1, 1)$

$$\begin{bmatrix} i & j & k \\ 1 & 0 & 3 \\ -1 & 1 & 1 \end{bmatrix} \begin{matrix} i & j \\ 1 & 0 \\ -1 & 1 \end{matrix}$$

$0 - 3i - j + 0 - 3j + k$

$(-3, -4, 1)$

eq. geral $= -3\cdot(x-0) - 4\cdot(y-0) + 1\cdot(z-1)$

$-3x - 4y + z - 1 = 0$

$(1, -1, 2) = k\cdot(-3, -4, 1) \rightarrow$ não existe

Os planos são transversais

$z = 0$

$\begin{cases} -3x - 4y = 1 \\ x - y = 2 \rightarrow x = 2 + y \end{cases}$

$-3\cdot(2+y) - 4y = 1$

$-6 - 3y - 4y = 1$

$-7y = 7$

$y = -1$

$x = 2 - 1$

$x = 1$

$P = (1, -1, 0)$

$$\begin{bmatrix} i & j & k \\ 1 & -1 & 2 \\ -3 & -4 & 1 \end{bmatrix} \begin{matrix} i & j \\ 1 & -1 \\ -3 & -4 \end{matrix}$$

$-3k + 8i - j - i - 6j - 4k = (7, -7, -7) = (1, -1, -1)$

$X = (1, -1, 0) + \lambda\cdot(1, -1, -1), \lambda \in \mathbb{R}$

c)

plano π_1

$\vec{v}_{n1} = (2, -1, 1)$

$2x - y + z + 1 = 0$

plano π_2

$\vec{v}_{n2} = (4, -2, 2)$

$4x - 2y + 2z - 9 = 0$

$(4, -2, 2) = k\cdot(2, -1, 1) \rightarrow$ existe, $k = 2$

Os planos são paralelos e distintos

coincidentes ou distintos

$P_{\pi_1} = (0, 0, 1)$

se $x = 0$ e $y = 0$, $z = 1$ em π_1

eq. geral $\pi_2 = 4x - 2y + 2z - 9 = 0$

$4\cdot0 - 2\cdot0 + 2\cdot1 - 9 = 0 \rightarrow 7 \neq 0$

d) plano π_2

$\vec{v}_{\pi_2} = (4, 40, -4) = (1, 10, -1)$

$4x + 40y - 4z - 16 = 0$

$P_{\pi_1} = (0, 1, 6)$

$4\cdot0 + 40\cdot1 - 4\cdot6 - 16 = 0$

$0 = 0$

plano π_1

$\vec{AB} = B - A = (5-0, 0-1, 1-6) = (5, -1, -5)$

$\vec{AC} = C - A = (4-0, 0-1, 0-6) = (4, -1, -6)$

$$\begin{bmatrix} i & j & k \\ 5 & -1 & -5 \\ 4 & -1 & -6 \end{bmatrix} \begin{matrix} i & j \\ 5 & -1 \\ 4 & -1 \end{matrix}$$

$6k - 5i + 30j + 6i - 20j - 5k = (1, 10, -1)$

eq. geral (A) $= 1\cdot(x-0) + 10\cdot(y-1) - 1\cdot(z-6)$

$= x + 10y - z - 4 = 0$

$(1, 10, -1) = k\cdot(1, 10, -1) \rightarrow$ existe, $k = 1$

São paralelos coincidentes

15-

a)

ponto de π_1: $P_1 = (0, 0, 0)$, $\lambda_1 = 0$ e $\mu_1 = 0$

$\vec{v}_{\pi_1 1} = (-1, m, 1)$

$\vec{v}_{\pi_1 2} = (2, 0, 1)$

$$\begin{bmatrix} i & j & k \\ -1 & m & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{matrix} i & j \\ -1 & m \\ 2 & 0 \end{matrix}$$

$-2k + 0 + j + im + 2j + 0$

$(m, 3, -2)$

ponto de π_2: $P_2 = (1, 2, 3)$, $\lambda_2 = 0$ e $\mu_2 = 0$

$\vec{v}_{\pi_2 1} = (m, 1, 0)$

$\vec{v}_{\pi_2 2} = (1, 0, m)$

$$\begin{bmatrix} i & j & k \\ m & 1 & 0 \\ 1 & 0 & m \end{bmatrix} \begin{matrix} i & j \\ m & 1 \\ 1 & 0 \end{matrix}$$

$-k + 0 + m^2 j + im + 0 + 0m$

$(m, -m^2, -1)$

$n_1 \neq k n_2$

$\begin{cases} m = km \\ 3 = -km^2 \\ -2m = -k \rightarrow k = 2m \end{cases}$

$\rightarrow m = (2m)\cdot m$

$2m^2 - m = 0$

$m(2m - 1) = 0$

$m = 0$ ou $m = 1/2$

$m = 0 \rightarrow k = 2\cdot0 = 0$

$m = 1/2 \rightarrow k = 2\cdot\frac{1}{2} = 1$

$3 = -k\cdot m^2$

$3 \neq 0$

$3 = -1\cdot\left(\frac{1}{2}\right)^2$

$3 \neq -\frac{1}{4}$

Não há valor de m para que os vetores normais sejam paralelos, ou seja, π_1 e π_2 são transversais

__/__/__

b) plano π_1

$P_{\pi_1} = (1, 1, 0)$

$\vec{v}_{\pi_1 1} = (m, 1, 1)$

$\vec{v}_{\pi_1 2} = (1, 1, m)$

$$\begin{bmatrix} i & j & k \\ m & 1 & 1 \\ 1 & 1 & m \end{bmatrix} \begin{matrix} i & j \\ m & 1 \\ 1 & 1 \end{matrix}$$

$-k - i + m^2 j + mi + j + mk$

$(m-1, -m^2+1, m-1)$

plano π_2

$2x + 3y + 2z + n = 0$

$(2, 3, 2)$

$(m-1, -m^2-1, m-1) = k(2, 3, 2)$

$$\begin{cases} m - 1 = 2k \\ -m^2 - 1 = 3k \\ m - 1 = 2k \end{cases}$$

$k = \frac{m-1}{2}$

$-m^2 - 1 = 3\left(\frac{m-1}{2}\right)$

$m^2 - 1) = \frac{3m-3}{2}$

$2m^2 - 2 = 3m - 3$

$2m^2 - 3m + 1 = 0$

→ caso 1: $m - 1 = 0 \rightarrow m = 1$

então $n_1 = (0, 0, 0)$ → não pode pois indica que os $\vec{v}$ são paralelos e não formariam um plano

→ caso 2: $m - 1 \neq 0$

$-(m+1) = \frac{3}{2}$

$-m - 1 = \frac{3}{2}$

$-m = \frac{3}{2} + 1$

$-m = \frac{3+2}{2}$

$-m = \frac{5}{2}$

$m = -\frac{5}{2}$

$\vec{v}_{n_1 \pi_1} = \left(-\frac{5}{2} - 1; -\left(\frac{5}{2} - 1\right) \cdot \left(-\frac{5}{2} + 1\right), -\frac{5}{2} - 1\right)$

$\left(-\frac{7}{2}; -\left(\frac{7}{2}\right) \cdot \left(-\frac{3}{2}\right), -\frac{7}{2}\right)$

$\left(-\frac{7}{2}; -\frac{21}{4}; -\frac{7}{2}\right)$

$n_1 = -\frac{7}{4} n_2$ para serem paralelos

P_1 de $\pi_1 = (1, 1, 0)$ não pode pertencer a π_2

$2x + 3y + 2z + n \neq 0$

$2 \cdot 1 + 3 \cdot 1 + 2 \cdot 0 + n \neq 0$

$5 + n \neq 0$

$n \neq -5$

R: n tem que ser diferente de -5 para que não haja pontos em comum e $m = -\frac{5}{2}$ para que sejam paralelos