→ Lista 9

01)

a- $\vec{BF}$

$\vec{b} + \vec{BF} = \vec{f}$

$\vec{BF} = \vec{f} - \vec{b}$

b- $\vec{AG}$

$\vec{CG} = \vec{BF} = \vec{f} - \vec{b}$

$\vec{AG} = \vec{c} + \vec{f} - \vec{b}$

c- $\vec{AE}$

$\vec{AE} = \vec{AF} + \vec{FE}$

$\vec{AE} = \vec{f} - \vec{b}$

f- $\vec{AB} + \vec{FG}$

$\vec{b} + \vec{f} - \vec{b}$

$\vec{f}$

g- $\vec{AD} + \vec{HG}$

$\vec{AD} = \vec{BF},\ \vec{HG} = \vec{AB}$

$\vec{f} - \vec{b} + \vec{b}$

$\vec{f}$

d) $\vec{BG}$

$\vec{BG} = \vec{BF} + \vec{FG}$

$\vec{BG} = \vec{f} - \vec{b} + \vec{f} - \vec{b}$

$\vec{BG} = 2\vec{f} - 2\vec{b}$

e- $\vec{HB}$

$\vec{HB} = \vec{BG} + \vec{GH}$

$\vec{HB} = (2\vec{f} - 2\vec{b}) - \vec{b}$

$\vec{HB} = 2\vec{f} - 2\vec{b} - \vec{b}$

$\vec{HB} = 2\vec{f} - 3\vec{b}$

h- $\vec{HF} + \vec{AG} - \vec{EF}$ → $\vec{FAB} = \vec{b}$

$\vec{f} + \vec{c} + \vec{f} - \vec{b} - (\vec{b})$

$\vec{A} + \vec{c} + \vec{f} - \vec{b} - \vec{b}$

$-2\vec{b} + 2\vec{f} + \vec{c}$

i- $2\vec{AD} - \vec{FG} - \vec{BH} + \vec{GH}$

$2\cdot(\vec{f} - \vec{b}) - (\vec{f} - \vec{b}) - \vec{b}$

$2\vec{f} - 2\vec{b} - \vec{f} + \vec{b} - \vec{b}$

$-2\vec{b} + \vec{f}$

02)

a- $\vec{DF} = \vec{DC} + \vec{CO} + \vec{OF}$

$\vec{DF} = \vec{DC} + 2\vec{DE}$

b- $\vec{DA} = \vec{DO} + \vec{OA}$

$\vec{DA} = \vec{DC} + \vec{DC}$

$\vec{DA} = 2\vec{DC}$

c- $\vec{DB} = \vec{DC} + \vec{CO} + \vec{OB}$

$\vec{DC} + \vec{DE} + \vec{DC}$

$2\vec{DC} + \vec{DE}$

d- $\vec{DO} = \vec{DC} + \vec{CO}$

$\vec{DO} = \vec{DC} - \vec{DE}$

e- $\vec{EC} = \vec{EO} + \vec{OC}$

$\vec{EC} = \vec{DC} + \vec{DE}$

f) $\vec{EB} = \vec{EO} + \vec{OB}$

$\vec{EB} = 2\vec{DC}$

g) $\vec{OB} = \vec{DC}$

h- $\vec{AF} = \vec{AO} + \vec{OF}$

$\vec{AF} = -\vec{DC} + \vec{DE}$

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3) $\vec{OD} = \vec{d}$ $\vec{OE} = \vec{e}$

a- $\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} + \vec{OE} + \vec{OF}$

$-\vec{d} - \vec{e} + \vec{d} + \vec{d} + \vec{e} + \vec{e}$

$\vec{d} + \vec{e}$

$-\vec{e} + \vec{e} = 0$

b- $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EF} + \vec{FA}$

0

c- $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EF}$

$-\vec{d} + \vec{d} + \vec{e} + \vec{d} - \vec{d}$

$+\vec{e}$

d- $\vec{OA} + \vec{OB} + \vec{OD} + \vec{OE}$

$-\vec{d} - \vec{e} + \vec{d} + \vec{e}$

0

e- $\vec{OC} + \vec{AF} + \vec{EF}$

$-\vec{d} + \vec{e} - \vec{d}$

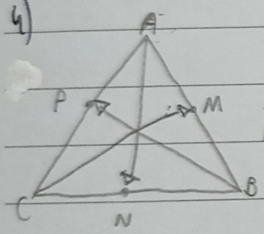
$-2\vec{d} + \vec{e}$

f- $\vec{AF} + \vec{DE}$

$\vec{e} - \vec{e}$

0

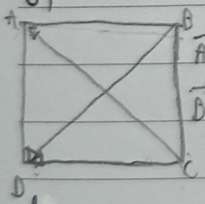
4)



$\vec{AB} = \vec{b}$ $\vec{BP} + \vec{b} = \frac{\vec{c}}{2}$ $\vec{AN} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c}$ $\vec{CM} + \vec{c} = \frac{\vec{b}}{2}$

$\vec{AC} = \vec{c}$ $\vec{BP} = \frac{\vec{c}}{2} - \vec{b}$ $\vec{CM} = \frac{\vec{b}}{2} - \vec{c}$

5)



$\vec{AD} = 5\vec{u}$ $AB = 2\vec{v}$ a- $\vec{CD}$? $\vec{BD}$? $\vec{CA}$?

$\vec{BC} = 3\vec{u}$

$\vec{CA} = \vec{AB} + \vec{BC}$ $\vec{BD} = -\vec{AB} + \vec{AD}$ $\vec{CD} = +\vec{CB} - \vec{BA} + \vec{AD}$

$\vec{CA} = 2\vec{v} + 3\vec{u}$ $\vec{BD} = -2\vec{v} + 5\vec{u}$ $\vec{CD} = 2\vec{u} - 2\vec{v}$

b-

$\vec{AD}$ e $\vec{BC}$ são colineares (paralelos), atendendo a discrição do trapézio (um par de lados opostos é paralelo).

6)

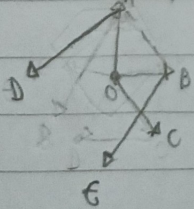
$\vec{a} = \vec{OA}$

$\vec{b} = \vec{OB}$

$\vec{c} = \vec{OC}$

$\vec{AD} = \frac{1}{4}\vec{c}$

$\vec{BE} = \frac{5}{6}\vec{a}$



$\vec{DE} = -\vec{AD} + \vec{OA} + \vec{OB} + \vec{BE}$

$\vec{DE} = -\frac{1}{4}\vec{c} - \vec{a} + b + \frac{5}{6}\vec{a}$

$\vec{DE} = -\frac{1}{4}\vec{c} - a + \frac{5}{6}a + \vec{b}$

$\vec{DE} = -\frac{1}{4}\vec{c} - \frac{1}{6}\vec{a} + \vec{b}$

7)

$\vec{OA} = \vec{a} + 2\vec{b}$ $\qquad \vec{AC} = \vec{OC} - \vec{OA}$ $\qquad BC = OC - OB$

$\vec{OB} = 3\vec{a} + 2\vec{b}$ $\qquad \vec{AC} = (5\vec{a} + x\vec{b}) - (\vec{a} + 2\vec{b})$ $\qquad BC = (5\vec{a} + x\vec{b}) - (3\vec{a} + 2\vec{b})$

$\vec{OC} = 5\vec{a} + x\vec{b}$ $\qquad \vec{AC} = 4\vec{a} + (x-2)\cdot\vec{b}$ $\qquad BC = 2\vec{a} + (x-2)\cdot\vec{b}$

$\alpha\vec{AC} + \beta\vec{BC} = 0$

$\alpha\cdot[4\vec{a} + (x-2)\vec{b}] + \beta\cdot[2\vec{a} + (x-2)\vec{b}] = 0$

$(4\alpha + 2\beta)\vec{a} + (\alpha\cdot(x-2) + \beta\cdot(x-2))\vec{b} = 0$

$$\begin{cases} 4\alpha + 2\beta = 0 \,/\, 2\alpha + \beta = 0 \to \alpha = -\frac{\beta}{2} \Rightarrow \alpha = \frac{-2}{-2} = 1 \\ \alpha(x-2) + \beta(x-2) = 0 \to (\alpha+\beta)\cdot(x-2) = 0 \to \text{se } x = 2 \to \vec{AC} = 4\vec{a},\ \vec{BC} = 2\vec{a} \text{ - múltiplos} \end{cases}$$

$\left(-\frac{\beta}{2} + \frac{\beta}{1}\right)$

$\left(\frac{-\beta + 2\beta}{2}\right)$

$\left(\frac{\beta}{2}\right)\cdot(x-2) = 0$

$\frac{\beta x}{2} - \frac{2\beta}{2} = 0$

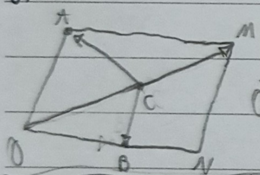
$\beta x - 2\beta = 0$

$x = \frac{2\beta}{\beta}$

$x = 2$

Se eles são múltiplo um do outro com $x = 2$, então eles são paralelos e por isso, L.D.

8)



$\vec{OB} = \frac{1}{n}\vec{ON}$

$\vec{OC} = \frac{1}{1+n}\vec{OM}$

$\vec{BC} = \vec{BO} + \vec{OC}$

$\vec{BC} = -\vec{OB} + \vec{OC}$

$\vec{BC} = -\frac{1}{n}\vec{ON} + \frac{1}{1+n}\vec{OM}$

$\vec{CA} = \vec{CO} + \vec{OA}$

$\vec{CA} = -\vec{OC} + \vec{OA}$

$\vec{CA} = -\frac{1}{1+n}\vec{OM} + \vec{NO} + \vec{OM}$

$\vec{CA} = -\frac{1}{1+n}\vec{OM} - \vec{ON} + \vec{OM}$

$\vec{CA} = \frac{-1 + (1+n)}{1+n}\vec{OM} - \vec{ON}$

$\vec{CA} = -\vec{ON} + \frac{n}{1+n}\vec{OM}$

$\vec{BC} \,//\, \vec{CA} =$

Conclusão: $\vec{CA} = n\vec{BC}$, ou seja, são L.D.

9)

Para que $\vec{u}$ e $\vec{v}$ sejam base para um V^2 entende-se que ambos são L.i

$2\vec{u} + \vec{v} \neq 0$ $\qquad \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} \neq 0$

$\vec{u} - 2\vec{v} \neq 0$

$-4 - 1 = -5$

$-5 \neq 0$, provando que ambos os vetores são L.i

10)

a- $\alpha\cdot(\vec{u} + \vec{v}) + \beta\cdot(\vec{u} - \vec{v} + \vec{w}) + \gamma(\vec{u} + \vec{v} + \vec{w}) = 0$

$\alpha\vec{u} + \alpha\vec{v} + \beta\vec{u} - \beta\vec{v} + \beta\vec{w} + \gamma\vec{u} + \gamma\vec{v} + \gamma\vec{w} = 0$

$\vec{u}\cdot(\alpha + \beta + \gamma) + \vec{v}(\alpha - \beta + \gamma) + \vec{w}(\beta + \gamma) = 0$

$$\begin{cases} \alpha + \beta + \gamma = 0 \to \alpha - \gamma + \gamma = 0 \\ \alpha - \beta + \gamma = 0 \\ \beta + \gamma = 0 \\ \to \beta = -\gamma = 0 \end{cases}$$

$\alpha = 0$

$0 - (-\gamma) + \gamma = 0$

$\gamma = 0$

$\alpha = \beta = \gamma = 0$

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b-

$\vec{t} = a\vec{u} + b\vec{v} + c\vec{w}$

$\vec{t} = k_1\vec{v}_1 + k_2\vec{v}_2 + k_3\vec{v}_3 \rightarrow$ todo $k_i = 0$ para que todos os vetores analisados $(\vec{v}_1, \vec{v}_2, \vec{v}_3)$ sejam L.i

$\vec{t} = 0$

$\vec{t} + \vec{u} = \vec{u}$

$\vec{t} + \vec{v} = \vec{v}$ $\quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix} \neq 0$

$\vec{t} + \vec{w} = \vec{w}$ $\quad 1 \neq 0$

11)

a-

$\vec{AB} = (b_1 - a_1, b_2 - a_2, b_3 - a_3)$
$\vec{AB} = (1-1, 0-3, -1-2)$
$\vec{AB} = (0, -3, -3)$

$\vec{BC} = (c_1 - b_1, c_2 - b_2, c_3 - b_3)$
$\vec{BC} = (1-1, 1-0, 0-(-1))$
$\vec{BC} = (0, 1, 1)$

$\vec{CA} = (a_1 - c_1, a_2 - c_2, a_3 - c_3)$
$\vec{CA} = (1-1, 3-1, 2-0)$
$\vec{CA} = (0, 2, 2)$

b- $\vec{AB} + \frac{2}{3}\vec{BC}$
$(0,-3,-3) + \frac{2}{3}(0,1,1)$
$(0,-3,-3) + (0, \frac{2}{3}, \frac{2}{3})$
$(0+0, -3+\frac{2}{3}, -3+\frac{2}{3})$
$(0, -\frac{7}{3}, -\frac{7}{3})$

c- $C + \frac{1}{2}\vec{AB}$
$(1,1,0) + \frac{1}{2}\cdot(0,-3,-3)$
$(1,1,0) + (0, -\frac{3}{2}, -\frac{3}{2})$
$(1+0, 1-\frac{3}{2}, 0-\frac{3}{2})$
$(1, -\frac{1}{2}, -\frac{3}{2})$

d- $A - 2\vec{BC}$
$(1,3,2) - 2\cdot(0,1,1)$
$(1,3,2) - (0,2,2)$
$(1-0, 3-2, 2-2)$
$(1,1,0)$

12)

a- $\{(2,3),(0,2)\}$ = L.i
$\begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}$ $\quad 4 \neq 0$
$4 + 0 = 4$

b- $\{(3,0),(-2,0)\}$ = L.D
$\begin{bmatrix} 3 & -2 \\ 0 & 0 \end{bmatrix}$
$0 + 0 = 0$

c- $\{(\overset{a_1}{2},\overset{b_1}{3},\overset{c_1}{4}),(\overset{a_2}{0},\overset{b_2}{3},\overset{c_2}{3})\}$ = L.i
$(2,3,4)$ e $(0,3,3)$ não são multiplos, então não são paralelos.

d- $\{(1,-1,2),(1,1,0),(1,-1,1)\}$ = L.i
$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} \begin{matrix} 1 & 1 \\ -1 & 1 \\ 2 & 0 \end{matrix}$ $\quad -2 \neq 0$
$-2+0+1+1-2+0=-2$

e- $\{(1,-1,1),(-1,2,1),(-1,2,2)\}$ = L.i
$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix} \begin{matrix} 1 & -1 \\ -1 & 2 \\ 1 & 1 \end{matrix}$ $\quad 1 \neq 0$
$+2-2-2+4-2+1=1$

f- $\{(1,0,1),(0,0,1),(2,0,5)\}$ = L.D
$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 1 & 1 & 5 \end{bmatrix} \begin{matrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{matrix}$ $\quad 0 = 0$
$0+0+0+0+0+0=0$

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13)

a- $\vec{w} = \lambda \vec{u} + \theta \vec{v}$

$\vec{w} = \lambda \cdot (2, -1) + \theta (1, -1)$

$\vec{w} = (2\lambda + \theta, -\lambda - \theta)$

$\vec{w} = 2\vec{u} - 3\vec{v}$

$\begin{cases} 2\lambda + \theta = 1 \\ -\lambda - \theta = 1 \end{cases} \rightarrow \lambda + \theta = -1 \rightarrow 2 + \theta = -1$

$(2\lambda + \theta) - (\lambda + \theta) = 1 - (-1)$ $\quad \theta = -3$

$2\lambda + \theta - \lambda - \theta = 2$

$\lambda = 2$

b-

$\vec{z} = \lambda \cdot \vec{a} + \beta \cdot \vec{b} + \theta \cdot \vec{c}$

$z = \lambda \cdot (1, 1, 1) + \beta \cdot (0, 1, 1) + \theta \cdot (1, 1, 0)$

$z = (\lambda + \theta, \lambda + \beta + \theta, \lambda + \beta)$

$z = 2\vec{a} + 1 \cdot \vec{b} - 1 \cdot \vec{c}$

$\begin{cases} \lambda + \theta = 1 \\ \lambda + \beta + \theta = 2 \\ \lambda + \beta = 3 \end{cases}$

$\lambda + \theta = 1 \rightarrow 2 + \theta = 1 \rightarrow \theta = -1$

$\lambda + \beta = 3 \rightarrow 4 - 2\beta + \beta = 3 \rightarrow -\beta = -1 \rightarrow \beta = 1$

$(\lambda + \theta) - (\lambda + \beta + \theta) - (\lambda + \beta) = 1 - 2 - 3$

$\lambda + \theta - \lambda - \beta - \theta - \lambda - \beta = -4$

$-\lambda - 2\beta = -4$

$\lambda + 2\beta = 4$

$\lambda = 4 - 2\beta \rightarrow 4 - 2 \cdot 1 \rightarrow 2$

14)

a- $\vec{u} = (1, m-1, m)$ $\quad \vec{v} = (m, 2n, 4) \rightarrow \vec{v} = k \cdot \vec{u}$

$(m, 2n, 4) = k \cdot (1, m-1, m)$

$m = k \cdot 1 = k$

$2n = k \cdot (m-1) = m \cdot (m-1) \rightarrow 2n = 2 \cdot (2-1)$ $\quad 2n = -2 \cdot (-2-1)$

$n = 1$ $\quad n = 3$

$4 = k \cdot m = m \cdot m \Rightarrow m^2 = 4 \Rightarrow m = \pm 2$

b- $\vec{u} = (1, m, n+1)$ $\quad \vec{v} = (m, n+1, 8)$

$(m, n+1, 8) = k \cdot (1, m, n+1)$

$m = k \cdot 1 \Rightarrow m = k$

$n + 1 = k \cdot m \rightarrow n + 1 = m \cdot m \rightarrow n = m^2 - 1 \rightarrow n = 2^2 - 1 \rightarrow n = 3$

$8 = k \cdot (n+1) \rightarrow m \cdot (m^2 - 1 + 1) = 8$

$m^3 - \cancel{m} + \cancel{m} = 8$

$m = 2$

15) $\begin{bmatrix} m & -1 & m^2+1 \\ m^2+1 & m & 0 \\ m & 1 & 1 \end{bmatrix} \begin{matrix} m & -1 \\ m^2+1 & m \\ m & 1 \end{matrix} \neq 0$

Não importa o valor do m já que no final o det sempre será $\neq 0$.

$-(m^2+1) + 0 + (m^2+1) + m^2 + 0 + (m^4+1) \neq 0$

$2m^2 + 1 \neq 0$

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16)

a-

f_1, f_2 e f_3 não são multiplos, o que indica que são não paralelos e L.i

$(1,1,-1)\cdot[(1,1,0)\cdot(1,0,1)] \neq (0,0,0)$

$(1,1,-1)\cdot(1,0,0) \neq (0,0,0)$

$(1,0,0) \neq (0,0,0) \rightarrow$ são não coplanares

b-

$(2,3,7) = 2\cdot(1,1,0)_B + 3\cdot(1,0,1)_B + 7\cdot(1,1,-1)$

$= (2+3+7),(2+0,7),(0+3-7)$

$(12,9,-4)_B$

c- $a\cdot(1,1,0) + b\cdot(1,0,1) + c\cdot(1,1,-1) = (2,3,7)$

$$\begin{cases} a+b+c=1 \\ a+c=3 \rightarrow a=3-c \rightarrow 3-(-9)=12 \\ b+c=7 \rightarrow b=c+7 \rightarrow -9+7=-2 \end{cases}$$

$\rightarrow 3-c+c+7+c=1$

$c=-9$

$\vec{u}=(12,-2,-9)_c$