Patent Protection and the Transition to Clean Technology -Preliminary version-

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Abstract

It has proven to be a challenging task for policymakers to implement sufficiently high carbon taxes and public funding of research to induce the transition to clean technology. This calls for the exploration of alternative policy tools. In this context, this paper analyzes the use of patent protection as a new policy tool to help induce the transition to clean technology. It has been argued that patent protection works as a barrier to the diffusion of clean technology, and that clean technology should therefore be exempt from patent protection. This argument, however, does not consider the role of patent protection in incentivizing innovation. We therefore propose that to induce more clean innovation, it is *dirty* technology that should be exempt from patent protection. We explore this novel idea in a in a general equilibrium model with directed technical change, and where traditional environmental policy is constrained. We find that removing patent protection on dirty technology can in fact help foster the transition to clean technology, despite the increase in demand for dirty technology in the short-run. In addition, we find potentially large welfare gains from using patent protection as a policy tool when traditional environmental policy is constrained.

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1 Introduction

Carbon taxes and research subsidies are considered the most efficient tools to induce innovation in carbon-free technologies and foster the energy transition (e.g., Fischer and Newell, 2008, Acemoglu et al., 2012, Greaker, Heggedal, and Rosendahl, 2018). However, the implementability of these policies, the economic distortions they create and the usefulness of alternative instruments still remain open questions. The rate of optimal carbon taxes and research subsidies in the existing literature largely exceeds what most administrations are willing to implement and, on top of that, the distortions associated with these policies are typically ignored. In this context, this paper analyzes the use of patent protection as a new policy tool to help induce the transition to clean technology. The policy consists of removing patent protection on technology that contributes to global warming, i.e., dirty technology. We explore this novel policy in a general equilibrium model with directed technical change, and we characterize analytically under which conditions our patent policy is effective in inducing innovation to clean technology. Numerical simulations find potentially large welfare gains from using patent protection as a policy tool when traditional climate policy is constrained.

Patent protection has been accused of slowing down the energy transition by restricting access to clean technology.[1] This has led some member states of the World Trade Organization to propose the exclusion of clean technology from international agreements on intellectual property rights.[2] This argument, however, does not consider the role of patent protection in incentivizing investments in innovation.[3] We contribute to the literature by introducing and

[1]E.g., in 2014, Elon Musk released all of Tesla's patents to the public, claiming that the patent system was partly to blame for the low market share of electric vehicles. `https://www.tesla.com/blog/all-our-patent-are-belong-you`

[2]`https://www.wto.org/english/tratop_e/trips_e/cchange_e.htm`.

[3]In addition, it is not clear whether removing patent protection on clean innovations would increase the

analyzing the novel use of patent protection as a new policy tool to induce clean innovation. Contrary to popular belief, we find that removing patent protection on dirty technology can in fact help the transition to net zero emissions.

Our model builds on Acemoglu et al. (2012) (AABH from now onwards). However, our goal is to analyze the role of patent policy and exclusivity rights on the energy transition.[4] For that reason, our model differs from AABH in some important aspects.[5] First, we take into account the monopoly distortion generated by the innovation sector.[6] This change allows us to capture the total effect on the energy transition of removing monopoly power on dirty technology. Second, we assume that unsuccessful scientists do not have monopoly rights over old technologies. This limits the extent of the monopoly distortion in the economy to a reasonable level. Third, to approximate the difficulties policymakers have faced in implementing sufficiently high levels of carbon taxes and public funding of clean innovation, we introduce efficiency losses in the transfer of public funds. Depending on the magnitude of these losses, financing climate policy, i.e., carbon tax and R&D subsidy, can become very expensive.

The main challenge of our policy is that, unlike more standard policies, its effectiveness in inducing clean innovation is not straightforward. Removing patent protection on dirty technology has opposing effects on the profitability of clean innovation relative to dirty innovation, and hence, on the transition to clean technology. On the one hand, our policy reduces the monopoly power of innovators in dirty technology, which lowers the expected profits of dirty innovation and induces innovation to the clean sector. On the other hand, however,

diffusion of clean technology to developing countries. See e.g. Hall and Helmers (2010) for an overview on the existing evidence on the link between intellectual property rights and the development and transfer of clean technology.

[4]It is important to highlight that our policy not only tackles an environmental failure, but it also corrects for a non-environmental issue, namely the monopoly distortion created by the patenting system.

[5]Other differences to the AABH model include the consideration of policy distortions, duplication effects of scientists by adding decreasing returns to the number of scientists in a sector or an alternative modeling of climate change damages.

[6]It is common in the literature to introduce production subsidies to avoid the fact that climate policy is used to correct for the monopoly distortion in the innovation sector.

our policy reduces the price of dirty innovations, which leads to an increase in the demand for dirty technology. This will push innovators back to the dirty sector and can even increase emissions in the short run.

We identify three channels of our patent policy on the relative profitability of clean innovation, which we call: i. the *direct effect of patent policy*; ii. the *indirect effect through the price channel*; and iii. the *indirect effect through the market size channel*. It is the indirect effect through the market size channel that is the counteracting force that reduces the effectiveness of our policy. We then characterize analytically under which conditions the net effect of our policy is such that it increases the relative profitability of clean innovation. The net effect depends on the elasticity of substitution between clean and dirty production. When clean and dirty production are "weak" substitutes, the price effect unambiguously dominates the market size effect. Intuitively, with a low elasticity of substitution, a reduction in the price of the dirty good will lead to only a modest increase in demand for the good. In that case, our policy is effective in inducing the transition to clean technology. We use simulations to show that our patent policy is still effective in inducing the transition for much higher levels of the elasticity of substitution.

The protection granted by patents is jointly determined by the patent length and breadth, i.e., the extent of the technological market in which the innovation has exclusivity.[7] Patent protection can therefore be reduced by limiting the length and/or breadth of patents. However, the value of a patent is to a large extent decided by its breadth as it is the breadth that restricts the patent holder's ability to raise the price of the product embodying the innovation (e.g. Gilbert and Shapiro, 1990).[8] In our model, we fix the patent length and reduce patent breadth by removing patent protection on dirty innovations.[9] As in AABH, successful

[7]While the early literature on patent protection and growth used to focus on the optimal lifetime of patents (e.g. Nordhaus, 1969, Kamien and Schwartz, 1974), the literature has more recently emphasized the importance of patent breadth (e.g. Gilbert and Shapiro, 1990, Klemperer, 1990, Gallini, 1992).

[8]In quality step models, patent length matters less (compared to expanding variety models) since patents become obsolete once there is a new innovation. This could explain why most patents elapse before their statutory length (see e.g. Lanjouw, Pakes, and Putnam, 1998, Arora, Ceccagnoli, and Cohen, 2008).

[9]We use the term "reduce" in this context because many innovations have several applications that include

innovators are granted a patent that protects them from competition for one period.[10] Only innovations in clean technology are granted patent protection.[11] We approximate the loss of patent protection on the private value of innovation by placing a cap on the mark-up that innovators can charge for their innovation in absence of a patent.[12] To support this approach, we also discuss empirical evidence and provide new evidence on the relationship between patent breadth and firm profits in the motivational evidence section.

To gain additional insight into the dynamic effect of our patent policy on the transition to clean technology, we simulate the economy for the next 400 years regarding the private value of patent protection. In line with the theoretical results, the simulation results show that, despite the increase in demand for dirty technology, our patent policy is effective in fostering the transition to clean technology. In addition, this happens for a wide range of the elasticity of substitution. We also find potentially large welfare gains from removing patent protection on dirty technology when there are efficiency losses associated with the carbon tax and clean innovation subsidy. For instance, when removing patent protection on dirty innovations causes a 60 percent reduction in the price of dirty technology, our policy can recover 44 percent of the welfare loss associated with conventional policies' distortions.

This paper relates to two strands of the literature on endogenous growth. The literature on directed technical change and the environment has emphasized the need for policies that increase the relative profitability of clean innovation. One way to induce clean innovation is by directly subsidizing R&D efforts in clean technology. However, innovation subsidies become less desirable when there are efficiency losses associated with public funding of research.

both dirty and clean technologies (and non-environmental applications). Removing patent protection on dirty innovations implies that the patent breadth of these patents is reduced to include only non-dirty applications.

[10]One period in our model corresponds to 5 years. Hence, we capture the relationship between patent protection and the private value of innovation by assuming that patents are broad but short-lived.

[11]However, the loss of patent protection does not result in complete removal of monopoly power since innovators can still protect their innovation through other channels such as secrecy or first-mover advantage. In fact, survey data has found that patents are not the main strategy for protecting innovations in many industries (e.g. Mansfield, 1986, Levin et al., 1987).

[12]This is a standard way of incorporate patent breadth in the literature (e.g. Li, 2001, Chu, 2009, Zeng, Zhang, and Fung, 2014).

Alternatively, policymakers can induce clean technology by decreasing the profitability of dirty innovation.[13] Our paper focuses on this alternative strategy to induce clean innovation.

Another strand of the literature has investigate the relationship between patent policy and growth. Optimal patent protection must balance the dynamic gain from increased innovation with the static efficiency loss caused by monopoly power (Nordhaus, 1969). In general, this optimal trade-off can be achieved by granting patents that are either short-lived but broad, or narrow but long-lived (e.g. Klemperer, 1990, O'Donoghue, Scotchmer, and Thisse, 1998). However, this literature has not considered optimal patent policy in light of climate change.

We bridge the gap between these two strands in the literature by exploring the use of patent protection as a new tool to induce clean innovation.[14] The literature has mainly proposed the combination of a carbon tax and a clean R&D subsidy as the optimal policy mix to mitigate climate change. However, it has proven to be a challenging task for policy makers to implement sufficiently high carbon taxes and public funding of clean innovation. Although some papers have considered the distortions caused by environmental policy[15], few papers have considered alternative policy tools to induce clean innovation. We contribute to the literature by characterizing analytically under which conditions removing patent protection on dirty technology is effective in inducing clean innovation. We simulate the model to quantify the potential welfare effects of using patent protection as an additional policy tool when traditional climate policy is constrained.

One paper close to ours is Gerlagh, Kverndokk, and Rosendahl (2014), which studies the optimal time path of clean energy innovation. They find that if R&D subsidies are constrained to be constant, optimal investment in clean innovation can instead be achieved by adjusting the duration of patents. However, they use a model of expanding variety where innovation is

[13] In theory, one could implement a tax on the profits from dirty patented innovations. It is, however, not straightforward how such a tax would be implemented.

[14] Some papers have studied the effect of incomplete patent protection on environmental policy, see. e.g. Greaker and Pade (2009) and Greaker, Heggedal, and Rosendahl (2018).

[15] E.g. Acemoglu et al. (2016) introduce an efficiency loss in the clean R&D subsidy and a cap on the carbon tax, while Hart (2019) combines an efficiency loss in the clean R&D subsidy with a deadweight loss in the carbon tax in the form of an enforcement cost.

always good for the environment. We are, to the best of our knowledge, the first to explore the use of patent protection as a policy tool to induce clean technical change and foster the energy transition. Our results show the relevance of simultaneously using alternative tools to trigger the energy transition and the importance of accounting for policies' distortions when evaluating climate policy.

[Paragraph about the structure of the rest of the paper.]

2 Motivational evidence

We first discuss the evolution of green, dirty and grey innovation overtime, proxied by the patenting activity. Then, we analyse the relationship between firm profits and patent breadth, providing our own estimations. We find that patents with a smaller breadth are associated with lower firm profits. This motivates the theoretical model assumption according to which removing patent protection reduces the mark-up charged by innovators.

2.1 Innovation trends

According to the US Electricity Information Administration (EIA), the electricity sector is responsible for one third of carbon dioxide emissions in the US. Despite its strong contribution to atmospheric carbon concentrations, innovation in this sector is strongly biased towards dirty technologies. Figure 1 shows the evolution of dirty, clean and grey electricity innovation in the OECD countries, measured as the ratio of patent applications over total applications at the European Patent Office (EPO).[16] One can see that dirty electricity innovation was persistently higher than clean innovation until 2005. Then, clean innovation started to converge. However, after 2010 green innovation collapsed,[17] while dirty innovation

[16] A similar pattern is found for applications filled at the United States Patent Office (USPTO).

[17] Acemoglu et al. (2019) find evidence that this decrease in green innovation could be explained by the shale gas revolution. According to this study, the decrease in the price of natural gas enhanced the incentives to innovate in dirty technologies.

continued to increase. The fact that dirty innovation still represents a large share of the current innovative activity in the electricity sector[18] evidences the need for policies to revert this trend.

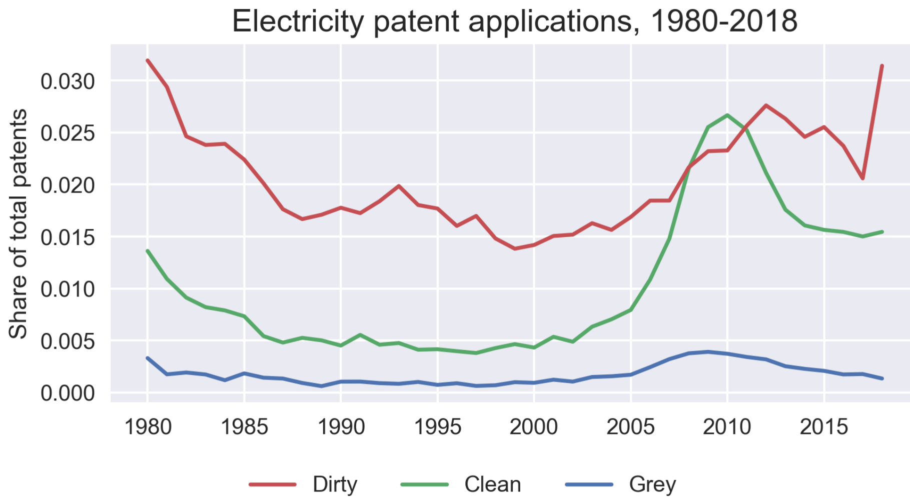

Figure 1: Based on data from PATSTAT. It includes all EPO electricity patents with an applicant or inventor from an OECD country. Patents are classified into grey, green and dirty following **?**, Table 21, which lists the CPC technological codes for each type. The authors identify grey codes as those that improve "the pollution efficiency of fossil technologies", thus, are harder to classify. When a patent has several types of CPC codes, we classify it into the type which has the higher number of codes. Draws are classified as grey.

2.2 Firm profits and patent breadth

[Should we move the details to the appendix?]Our theoretical model, detailed in the following section, features a novel patent policy that requires differentiated levels of patent protection depending on the innovation's nature. The goal of a patent is to create incentives to innovate by granting exclusive rights to innovators. Exclusivity allows patent owners to charge a higher price for their invention. The extent of the exclusivity right is defined by the length

[18]However, this is not necessarily true for every sector, for instance, the transport sector. Figure A1 in the appendix shows that green transport innovation has increased steadily since the 1990's, taking over dirty transport innovation in the early 2000's. See section A.1 in the appendix for more details.

and the breadth of a patent.[19] The vast majority of patent systems allow for a protection term of 20 years from the date of the application's filing, provided that the maintenance fees are paid throughout the period. However, the extent of patent protection is to a large degree decided by patent breadth (Gilbert and Shapiro, 1990).

Identifying the causal relationship between patent breadth and firm outcomes presents different challenges arising, for instance, from firms' quality differences. Hegde, Ljungqvist, and Raj (2021) present an attempt to overcome this challenge by restricting their analysis to US start-ups in order to exploit the quasi-random assignment of patent applications to examiners with different approval tolerance. They find causal evidence for the positive effect of patent breadth on firm growth. They measure patent breadth as the number of claims made in a patent.[20] An alternative measure is to focus on the number of technological codes that are assigned to a patent (Lerner, 1994). We complement this literature by analysing the effect of patent breadth on firm profits for all types of firms—not only start-ups—and under different measures of patent breadth. Given that our identification strategy is not as neat, despite adding relevant controls, we do not claim to provide causal evidence on the relationship of interest, rather descriptive evidence of this strong relationship.

Our analysis combines Compustat (COMPUSTAT) data on firm profits with PATSTAT data on the number of claims and technology codes of a patent. In order to match the two datasets, we use a third dataset provided by Bena et al. (2017)[21] that contains information on the firm's Compustat ID (gvkey) as well as on PATSTAT's patent number. Our sample includes firms from 45 countries during the period 1983–2019.

We use different measures of patent breadth based on the number of claims and classifi-

[19] The length of a patent is the time during which the exclusivity rights can be maintained. The breadth of a patent is the extent of the technological market in which the innovation has exclusivity, in other words, the technological region that is included in the patent.

[20] Kuhn and Thompson (2019) propose a more sophisticated measure of patent breadth that consists on the number of words of patent claims. A longer claim has a weaker breadth because the competitors must meet more conditions in order to infringe the patent.

[21] See the Bena et al. (2017)'s Internet Appendix for a detailed description of the matching procedure.

cation codes[22]: i. "Claims": the number of patent claims; ii. "Codes Long": the number of CPC codes in their long format; iii. "Codes Medium": the number of CPC codes grouped by the so-called *main groups*, which typically has five or six-digit CPC codes; and iv. "Codes Short": the number of four-digit CPC codes.

A simple regression of firms profits on patent breadth may suffer from strong endogeneity problems. For instance, firms with higher profits may be able to innovate more and to create broader innovations. In addition, the unobserved quality differences across firms can affect patent breadth and firm profits at the same time (Hegde, Ljungqvist, and Raj, 2021). In an attempt to partially address these issues we control for a rich set of fixed effects, at the firm and year level, as well as for the number of patents that a firm has on a given year.

In order to obtain a measure of the relationship between the two variables of interest, we regress profits on firm's patent breadth following

$$Profits_{ict} = \beta_0 + \beta_1 Breadth_{ict-1} + \beta 2 NumPatents_{ict-1} + \delta_i + \gamma_t + \epsilon_{ict} \tag{1}$$

where $Profits$ represent firm i, from country c in year t profits, $Breadth$ captures the firm's average number of claims or CPC codes and $NumPatents$ the total number of patents of each firm i in year t. The independent variables are lagged 1 period. δ and γ represent firm and year fixed effects, respectively.

Table 1 presents the regression results. The independent variable is in levels in columns 1 and 3-7 and is log-transformed in column 2. Column 4's dependent variable is a three-year moving average of the firm's profits. All specifications present a strong and statistically significant positive relationship. For instance, Column 2 shows that an extra approved claim is associated with 0.6 million USD additional profits. Even though this specification does not entirely solve the endogeneity issue, it motivates our assumption of a positive association

[22] A single patent can contribute to different areas of technology at the same time and all of them are documented in the patent document using codes. There are several classifications to define the technological areas and we use the Cooperative Patent International Patent Classification (CPC). Another widespread classification is the International Patent Classification (IPC). CPC is an extension of the IPC.

between patent protection and firm profits.

Table 1: Relationship between patent breadth and profits

	Profits						
	(1)	(2)	(3)	(4) MA3	(5)	(6)	(7)
Claims	0.419***	0.604***		0.665***			
	(0.09)	(0.15)		(0.15)			
Log(*Claims*)			186.836***				
			(43.23)				
CodesLong					0.909***		
					(0.20)		
CodesMedium						4.687***	
						(0.63)	
CodesShort							10.837***
							(1.89)
FE (Firm,Year)		√	√	√	√	√	√
Num. Patents	√	√	√	√	√	√	√
Observations	48974	48974	48974	44195	48974	48974	48974
Adj R^2	0.160	0.764	0.761	0.793	0.762	0.765	0.765
Countries	46	46	46	45	46	46	46

Note: Clustered standard errors on country-level in parentheses. Independent variable is lagged 1 period. Profits are measured in millions. *** p<0.01, ** p<0.05, * p<0.1.

3 The model

We consider a closed economy a la AABH. The four main differences between our model and the one in AABH are: i. the introduction of patent policy; ii. the presence of an uncorrected monopoly distortion in technology markets; iii. the fact that only successful innovators have monopoly rights over new technologies; and iv. the consideration of policy inefficiencies (distortions).[23] We therefore focus on the innovation sector and the role of patent policy, and only briefly explain the main equations and mechanisms in the model. As

[23] Other differences to AABH include intermediate input multiplicative-climate damages and decreasing returns to scientific labor.

in AABH, the model features identical households, a sector producing a final good, a clean and a dirty sector producing inputs for the final good, and a sector that supplies machines to the clean and dirty sectors. Machines are produced using the final good. The quality of machines is determined endogenously through innovation efforts, which are directed to improve the quality of machines either in the clean or dirty sector. Production of the dirty input generates emissions that accumulate in the atmosphere, which in turn has a negative effect on the level of both clean and dirty production.

3.1 Consumers and final good production

There is a representative household with a lifetime utility given by

$$U_0 = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t) \tag{2}$$

where $u(C_t)$ is the instantaneous utility of consuming a unique final good at time t, and $\rho > 0$ is the discount rate. We assume that the instantaneous utility function is increasing in C_t ($u'(C_t) > 0$), is twice differentiable and concave ($u''(C_t) < 0$), and satisfies $\underset{C_t \to 0}{lim}\, u'(C_t) = \infty$. Time is discrete and runs to infinity.

Households consume a certain quantity of a unique final good that is denoted by Y_t. Identical and perfectly competitive firms produce the final good by combining a clean and a dirty intermediate input, Y_{ct} and Y_{dt}, according to

$$Y_t = \left[Y_{ct}^{\frac{\epsilon-1}{\epsilon}} + Y_{dt}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

where ϵ is the constant elasticity of substitution between the inputs. Inputs are classified into clean or dirty depending on which technology they are produced with. For instance, in the energy sector, clean technologies are those based upon renewable energy, while dirty technologies refer to the extraction and production of fossil fuel energy, e.g. hydraulic fracturing.[24] We assume that the two inputs are gross substitutes, i.e. $\epsilon > 1$, although the exact degree of substitutability is arguable.

3.2 Intermediate goods production and the environment

Clean and dirty inputs are produced by identical and competitive firms that combine labor and a unit continuum of different machines, according to

$$Y_{jt} = \Omega(S_t) L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di \tag{4}$$

where $\alpha \in (0,1)$ and j indexes the sector, $j \in \{c, d\}$. Each machine is specific to a sector, with A_{jit} being the quality of machine i used in sector j, and x_{jit} being the quantity used of that machine. We assume drastic innovation, i.e., whenever there is an innovation in machine i in sector j, the old vintage of the machine is replaced by the new one. L_{jt} is the labor input, which is supplied inelastically. Normalizing the labor supply to 1, market clearing requires

$$L_{ct} + L_{dt} \leq 1. \tag{5}$$

We take the approach of climate damage being multiplicative to intermediate output.[25] $\Omega_t \in [0, 1]$ is the damage to production caused by an increase in atmospheric carbon concentrations, S_t, above pre-industrial levels. Under low levels of accumulated emissions, i.e., low S_t, damage to production is minimal and $\Omega_t \to 1$. Accumulated emissions evolve according to a simple difference equation

$$S_{t+1} = \xi Y_{dt} + S_t, \tag{6}$$

[24] There is no grey technology in the model, i.e. energy-efficiency technology, nor is there any neutral technology.

[25] In AABH it is trivial whether climate change reduces intermediate production or whether consumers have a dislike of climate change. However, this is no longer the case when there are efficiency losses associated with transfers of public funds. Because of the efficiency losses, overall damages from climate change will be lower if climate change reduces utility as opposed to intermediate production.

where emissions from one unit of dirty production cause an increase in carbon concentrations by ξ units.[26] By assumption, S_{t+1} can only take values in the interval $(0, \bar{S})$. The upper bound on accumulated emissions, $\bar{S}$, captures the concern that there exists some tipping point in the environmental system, that once reached, there is an environmental disaster, in which case, $\Omega_t = 0 \; \forall \; t$.

The intermediate good firm's problem is

$$\max_{L_{jt}, x_{jit}} p_{jt} Y_{jt} - w_t L_{jt} - \int_0^1 p_{jit}(1 - z_j) x_{jit} di \tag{7}$$

where p_{jt} denotes the price of the intermediate input of sector j, p_{jit} is the price of each machine of type i in sector j, while z_j is a potential subsidy on the price of machines in sector j. The wage rate, w_t must be the same across both sectors in equilibrium. The first-order condition for the optimal use of machine i results in the standard demand for the machine

$$x_{jit} = \left(\frac{\alpha \Omega(S_t) p_{jt}}{p_{jit}(1 - z_j)} \right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt}, \tag{8}$$

which is increasing in the level of technology, but decreasing in the price of the machine.

3.3 Supply of machines and patent policy

To produce one unit of a machine, ψ units of the final good must be used. There is perfect competition in machines lines where no quality improvement has taken place. Since innovation efforts were unsuccessful, old vintages of these machines are instead sold at the competitive price,

$$p_{jit}^{CO} = \psi. \tag{9}$$

[26]Notice that there could be some environmental regeneration over time caused by carbon sinks. However, a linear relationship between accumulated emissions and global warming has been found to be a a good approximation of actual climate dynamics (Dietz and Venmans, 2019, Dietz et al., 2021).

While in machine lines where innovation efforts have been successful, machines are patented and supplied by monopolists. Each monopolist maximizes profits according to

$$\max_{p_{ijt}, x_{ijt}} (p_{ijt} - \psi) x_{ijt}, \tag{10}$$

where demand for the machine, x_{jit}, is given by Eq. (8). Maximizing profits results in the unconstrained monopoly price, which is a constant markup over the marginal cost of producing the machine, ψ/α. [27]

While most of the previous literature has (implicitly) assumed that successful innovators are able to fully protect their innovations, we remove access to patent protection on dirty innovations. Motivated by the literature, we implement this patent policy by placing a cap, μ, on the mark-up charged for dirty machines. This way, the monopolist of a dirty machine must charge a lower price than the unconstrained monopoly price. Under the new patent policy, the monopolist charges the following price

$$p_{ijt}^{MO} = \begin{cases} \frac{\psi}{\alpha} & if\, j = c \\ \mu \frac{\psi}{\alpha} & if\, j = d, \end{cases} \tag{11}$$

where μ is a value in the interval $(\alpha, 1]$.

If $\mu = 1$, dirty monopolists are granted full patent protection, and can therefore charge the unconstrained monopoly price. If $\mu < 1$, dirty monopolists cannot register their innovations at the patent office, and must therefore sell the machines at a lower price to discourage imitation. Notice that $\mu = \alpha$ implies that monopolists must sell their innovations as if they have no market power, i.e., at the competitive price, in absence of patent protection. We therefore assume that $\mu > \alpha$ since innovators can instead use secrecy or their inside information to protect their innovation and generate some market power.[28] The closer μ

[27]However, notice that the social planner can equate the price of machines to the competitive price by simply setting $z_j = 1 - \alpha$.

[28]Notice that when $\mu < \alpha$, innovators must set the price below the marginal cost. We ignore this trivial

is to α, the more important patent protection is for protecting innovations and generating market power.

Combining the price of machines from Eq. (11) with the demand for machines from Eq. (8), the per-period profit of a producer of a clean machine becomes

$$\pi_{cit} = (1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{1-z_c}\right)^{\frac{1}{1-\alpha}}(\Omega(S_t)p_{ct})^{\frac{1}{1-\alpha}}A_{cit}L_{ct}. \tag{12}$$

while the per-period profit of a producer of a dirty machine becomes

$$\pi_{dit} = (\mu-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{\mu(1-z_d)}\right)^{\frac{1}{1-\alpha}}(\Omega(S_t)p_{dt})^{\frac{1}{1-\alpha}}A_{dit}L_{dt}. \tag{13}$$

The cap on the price of dirty machines, μ, enters the expression for dirty profits twice. The first term, $(\mu-\alpha)$, captures the fact that dirty monopolists must now charge a lower price in absence of patent protection, while the second term, $(\alpha/\mu(1-z_d))^{1/(1-\alpha)}$, captures the fact that a reduction in price will increase demand for dirty machines.[29]

3.4 Innovation and allocation of scientists

Scientists choose to work either in the clean or in the dirty sector. The total amount of scientists in each period is normalized to unity

$$s_{ct} + s_{dt} \leq 1, \tag{14}$$

where s_{jt} denotes the mass of scientists working in sector j. Once scientists have chosen a sector they are randomly allocated to a specific machine in that sector.[30] Their scientific

case.

[29]Notice that this latter effect of the patent policy on dirty profits would disappear if the production subsidies are used to correct for the monopoly distortion. Under the new patent policy, the required production subsidies would be $z_c = 1-\alpha$ and $z_d = 1-\frac{\alpha}{\mu}$.

[30]Alternatively, if we make the plausible assumptions that profits are increasing in the productivity of the machine, i.e. $\pi_{jit} = A_{jit}\,(p_{ijt}-\psi)\,x_{ijt}$, while the probability of innovating is decreasing in the productivity of

effort is successful with probability $\eta_j \in (0, 1)$. If successful, the innovator becomes the monopolist producer of the latest version of that machine for one period.[31]

Successful innovations cause an increase in the productivity of the technology by a factor γ, and the new productivity level is given by $(1 + \gamma)A_{jit}$. The aggregate (and average) machine quality in sector A_{jt} is denoted by

$$A_{jt} = \int_0^1 A_{jit} di. \tag{15}$$

Since scientists are unable to choose the specific machine, their decisions will be based on the average machine quality in each sector. Like Greaker, Heggedal, and Rosendahl (2018), we take into account the fact that more than one scientist might have the same successful innovation in a given period. This stepping-on-toes effect is represented by decreasing returns to scientific labor in each sector, s_{jt}^{σ}, with $\sigma \in (0, 1)$. The probability of making a new innovation is therefore given by $\eta_j s_{jt}^{\sigma}$, resulting in the following evolution of the average quality of machines in a sector

$$A_{jt} = (1 + \gamma \eta_j s_{jt}^{\sigma}) A_{jt-1}. \tag{16}$$

Using Eqs. (12) and (13), and aggregating over the quality of machines in the sector, the expected profits of a scientist engaged in clean and dirty research are given by

$$\Pi_{ct} = \eta_c s_{ct}^{\sigma-1}(1-\alpha) \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha}{1-z_c}\right)^{\frac{1}{1-\alpha}} (\Omega(S_t)p_{ct})^{\frac{1}{1-\alpha}} L_{ct}(1+\gamma)A_{ct-1} \tag{17}$$

the machine, i.e. $\frac{\eta_j}{A_{ijt}}$, scientists become indifferent with regards to which machine to choose. Mathematically, this would be equivalent to our assumption that scientists only choose sector and not the specific machine.

[31]This assumption allows us to reduce the innovation problem to a static one in the decentralized economy. However, Greaker, Heggedal, and Rosendahl (2018) demonstrate the robustness of the model to a dynamic set-up where innovators remain the incumbent until replaced by entrants.

and

$$\Pi_{dit} = \eta_d s_{dt}^{\sigma-1}(\mu-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{\mu(1-z_d)}\right)^{\frac{1}{1-\alpha}}(\Omega(S_t)p_{dt})^{\frac{1}{1-\alpha}}L_{dt}(1+\gamma)A_{dt-1}, \tag{18}$$

where $\eta_j s_{jt}$ is the average productivity of a scientist entering sector j.[32] Technical change is driven by the relative profitability of clean research, which can be expressed as

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d}\left(\frac{s_{ct}}{s_{dt}}\right)^{\sigma-1} \times \underbrace{\frac{1-\alpha}{\mu-\alpha}\left(\frac{\mu(1-z_d)}{1-z_c}\right)^{\frac{1}{1-\alpha}}}_{\text{direct patent policy effect}} \times \underbrace{\left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{1}{1-\alpha}}}_{\text{price effect}} \times \underbrace{\frac{L_{ct}}{L_{dt}}}_{\text{market size effect}} \times \underbrace{\frac{A_{ct-1}}{A_{dt-1}}}_{\text{direct productivity effect}}. \tag{19}$$

The direct effect of the patent policy (direct patent policy effect) is to reduce the price of dirty machines, which increases the relative profitability of innovating in clean technologies. This increase is mitigated by the increase in demand for dirty machines.

In addition to patent policy, technical change is driven by the standard channels in the literature by directing innovation towards the sector with the higher price (price effect), higher share of employment (market size effect), and the initially more advanced sector (direct productivity effect). However, since removing patent protection on dirty innovations affects the relative price of the clean good, and therefore also the relative labor share, patent policy will indirectly affect the relative profitability of clean innovation though the price and market size channels as well.

The following proposition puts a sign on the direct and indirect effects of patent policy on the relative profitability of clean research:

Proposition 1. *All else equal, removing patent protection on dirty innovation, i.e., $\mu < 1$, has three partial effects on the relative profitability of clean research:*

i Direct effect of patent policy: In absence of patent protection, successful innovators in

[32] Note that we divide the probability of success by the amount of scientists in a sector. Because we assume decreasing returns to the number of scientists in each sector, the solution for the allocation of scientists differs from the corner solutions found in AABH.

dirty machines must sell at a lower price. Although they now face a higher demand for their machines, expected profits from dirty innovation falls, and thus the relative profitability of clean research increases.

ii *Indirect effect through price channel: Since dirty innovators must sell their machines at a lower price, the dirty input becomes less expensive. The relative price of the clean input increases, which increases the relative profitability of clean research.*

iii *Indirect effect through market size channel: Assuming that the two inputs are substitutes, a lower price of dirty machines will lead to a re-allocation of labor to the dirty sector. The reduced market size of the clean sector will decrease the relative profitability of clean research.*

See proof of Proposition 1 in Appendix A.4. Proposition 1 states the three effects of our policy on efforts towards clean vs. dirty innovation. Although two of the effects clearly contribute to increase innovation efforts on clean technologies, the third one works in the opposite direction . Hence, the net effect is a prior ambiguous.

Inserting for the relative price and labor share of the clean good (see Appendix A.3), and assuming that there are no production subsidies to correct for the monopoly distortion (i.e., $z_j = 0$), the relative profitability of clean innovation can be expressed as a function of the share of clean scientists, average quality levels and policy instruments

$$\frac{\Pi_{ct}}{\Pi_{dt}} = (1+q_t)\frac{\eta_c}{\eta_d}\left(\frac{s_{ct}}{s_{dt}}\right)^{\sigma-1}\frac{1-\alpha}{\mu-\alpha}\mu^{\frac{1}{1-\alpha}}(1+\tau_t)^{\epsilon}\times \left(\frac{\left(\eta_c s_{ct}^{\sigma}\left(\alpha^{\frac{\alpha}{1-\alpha}}-1\right)+1\right)(1+\gamma\eta_c s_{ct}^{\sigma})}{\left(\eta_d s_{dt}^{\sigma}\left(\left(\frac{\alpha}{\mu}\right)^{\frac{\alpha}{1-\alpha}}-1\right)+1\right)(1+\gamma\eta_d s_{dt}^{\sigma})}\right)^{-\varphi-1}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi} \tag{20}$$

In addition to the carbon tax, τ_t, there is also a subsidy, q_t, given to clean innovation. From equation (20) follows the next proposition on the net effect of removing patent protection:

Proposition 2. *Suppose that $1 \leq \epsilon \leq \frac{2-\alpha}{1-\alpha}$, i.e., the energy inputs are weak substitutes. Then, all else equal, our policy will lead to an increase of the relative profitability of clean research. When $\epsilon > \frac{2-\alpha}{1-\alpha}$, however, the effect of our policy on the relative profitability of clean research is still ambiguous.*

See proof of Proposition 2 in Appendix A.4. Proposition 2 states that for low enough values of ϵ, i.e., when the two inputs are weak substitutes, our patent policy creates incentives to increase clean innovation efforts. This is because under such values the price effect dominates the market size effect, hence the net effect on the relative profitability of clean research is positive. However, when the inputs are strong substitutes this result does not necessarily hold because the market size effect now dominates the price effect. In such case, the net effect will depend on the magnitude of the direct effect and the net indirect effect, hence it is a prior ambiguous.

Note that, our patent policy can still incentivize clean innovation when ϵ is high but we cannot characterize it analytically. In addition, when ϵ is high our policy can still be welfare improving. This is because removing patent protection on dirty machines can improve welfare by reducing the monopoly distortion in the dirty sector. To gain further insight into how patent policy can help induce the energy transition, we simulate the model in section 4.

3.5 Market clearing

The equilibrium allocation of scientists in the decentralized economy is determined by equalizing the expected profits in the two sectors, i.e. $\Pi_{ct} = \Pi_{dt}$. However, the government can induce more scientists to the clean sector by giving a subsidy equal to a share, q_t, of the private value of clean innovation. In addition to public funding of clean innovation, there is also a tax levied on the price of the dirty intermediate, τ_t, i.e. a carbon tax. Both the research subsidy and the carbon tax are financed lump-sum. However, we assume that there are inefficiencies associated with such government transfers. As a result, only a share $1 - d_1$

of the clean research subsidy, and $1 - d_2$ of the carbon tax, are rebated back to consumers. The market clearing condition for final good thus becomes

$$C_t = Y_t - \psi \left(\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right) - d_1 q_t \Pi_{ct} - d_2 \tau_t p_{dt} Y_{dt}, \tag{21}$$

Since households cannot store the final good, consumption must equal production of the final good each net of the amount used up in the production of machines and the amount lost due to inefficiencies in government transfers each period.

4 Numerical analysis

4.1 Optimization problem

We assume a CRRA utility function

$$u(C_t) = \frac{C_t^{1-\nu}}{1-\nu}$$

The social planner's problem is to choose a path for the carbon tax and share of scientists in the clean sector that maximizes discounted consumption, with consumption equal to final good production, net of what is used up in producing machines and the efficiency loss in public research funding and carbon taxation. Inserting for the budget restriction from Eq. (21) into the utility function, the maximization problem becomes

$$\max_{\tau_t, s_t} U = \sum_{t=0}^{T} \frac{1}{(1+\rho)^t} \frac{(Y_t - \psi(X_{ct} + X_{dt}) - d_1 q_t \Pi_{ct} - d_2 \tau_t p_{dt} Y_{dt})^{1-\nu}}{1-\nu}$$

In the appendix, section A.3, we show how Y_t, X_{ct}, X_{dt}, Π_{ct} and Y_{dt} evolve as functions of the technology levels, accumulated emissions and policy instruments. Furthermore, the technology levels evolve as functions of the share of clean scientists, while the accumulated emissions evolve as a function of dirty production.

4.2 Parameter selection

We set one period in our model to 5 years, and we simulate the model for 80 periods, i.e., 400 years. We follow AABH by setting $\nu = 2$ to match Nordhaus' intertemporal elasticity of substitution, the machine share of national income to $\alpha = 1/3$, the per annum probability of making an innovation to $\eta_c = \eta_d = 0.02$, and the quality step to $\gamma = 1$. Without loss of generality, we normalize the cost of machines to $\psi = \alpha^2$. Although it is commonly accepted that there are decreasing returns in the innovation process, there is little empirical evidence on the exact magnitude of this effect. We follow Greaker, Heggedal, and Rosendahl (2018) and set $\sigma = 0.7$.[33]

To calibrate the efficiency loss associated with public funding of clean research, we compare the effectiveness of R&D in producing patents when the R&D is publicly or privately funded. Appendix A.2 presents the details of the estimation. We find publicly funded R&D to be 28 percent less effective in producing patents compared to privately financed R&D, and we set $d_1 = 0.28$. For the efficiency loss in the carbon tax, we use the estimate from Barrage (2014) and set $d_2 = 0.48$.

To initialize the simulations, we compute the average quality level of clean and dirty technology one period prior to the simulation runs, $A_{c,-1}$ and $A_{d,-1}$ (see Eq. (43) in Appendix A.3). The initial productivity levels are calculated using the observed production of fossil and non-fossil fuel in the world primary energy supply from the IEA. From 2012 to 2016, the world produced 392.2 and 2431.8 quadrillion Btu of energy from non-fossil and fossil energy sources, respectively. In addition to the initial production levels, we also need the initial share of scientists in the clean sector to estimate the initial average quality levels. We set $s_{c0} = 1/3$,

[33] Acemoglu et al. (2016) use a slightly lower estimate of 0.5 in their baseline analysis.

which means that two-thirds of scientists are initially engaged in dirty innovations.[34]

Accumulated emissions, S_t, evolve as a function of dirty production. An increase in atmospheric carbon concentrations cause an increase in the global mean temperature, which results in economic damages, $\Omega(S_t)$. Thus, $\Omega(S_t)$ can be written as $\Omega(\Delta(T))$, where $\Delta(T)$ is the increase in the mean temperature above its pre-industrial level in degrees Celsius. We follow AABH and define an environmental disaster as an increase in temperatures equal to $6°C$, and we calibrate the damage function to correspond to a 10% decrease in production at $3°C$ warming Van Der Wijst et al. (2023). The calibration of the carbon cycle and damage function is explained in detail in Appendix A.5.

Although it is usually assumed that clean and dirty inputs are to some extent substitutes, the literature has used a wide range of estimates on the elasticity of substitution.[35] We use a mid-range estimate of the elasticity of substitution equal to $\epsilon = 3$. However, we also check the robustness of our findings to a lower and higher elasticity of substitution of $\epsilon = 1.5$ and $\epsilon = 10$, respectively. For the discount rate, we use Nordhaus' preferred choice of 1.5 percent per annum discounting and set $\rho = 0.015$.

Finally, we simulate the economy for different values of $\mu \in (\alpha, 1]$

5 Results

The following presents the main results. We find the optimal path for the carbon tax and the clean innovation subsidy for four different scenarios. In the first scenario, dirty innovations are granted full patent protection, i.e., $\mu = 1$. In the three other scenarios, we remove access to patent protection on dirty innovations and find the optimal paths for the carbon tax and

[34] Given this initial share of clean scientists (and the initial production levels), we find that $A_{c0}/A_{d0} = 0.397$, i.e., clean technology is initially 40 percent less productive than dirty technology. Note here that the initial technology gap is not sensitive to the initial share of clean scientists. For instance, if we instead assume that $s_{c0} = 2/3$, we find that $A_{c0}/A_{d0} = 0.407$.

[35] Karydas and Zhang (2019) use a conservative elasticity of 0.7, while Greaker, Heggedal, and Rosendahl (2018) use an elasticity of 1.5 in their low substitution scenario. AABH use an elasticity of 3 in their low substitution scenario, and an elasticity of 10 in their strong substitution scenario.

clean research subsidy under different caps on the mark-up on dirty technology, that is, under different levels of policy stringency. Since $\mu \in (\alpha, 1]$, we use $\mu = 0.8$, $\mu = 0.6$, and $\mu = 0.4$. We also find the optimal paths for the carbon tax and clean innovation subsidy under a situation with no efficiency losses in environmental policy, and use the level of welfare in this scenario as a benchmark to compare the welfare gains of patent policy.

5.1 Main Results

Figure 2 shows the optimal paths for the carbon tax and subsidy to clean R&D for our baseline estimate of the elasticity of substitution, $\epsilon = 3$. Although the model has been optimized for 80 periods, i.e. 400 years, the graph shows only the first 200 years as the transition to clean technology is completed by then.

The figure also shows the transition to clean technology in the benchmark scenario when environmental policy can be financed without any losses (solid gray line). In that case, the optimal carbon tax is steadily increasing and stays at high levels (≈ 40 percent) throughout the period. There is also a gradual increase in the clean innovation subsidy, which reaches 1.5 times the private value of clean innovations after 40 years. The innovation subsidy is then gradually phased out. Despite the large carbon tax and subsidy to clean R&D, it takes more than a 100 years for scientists to have transitioned to the clean sector, and it takes double that time for dirty inputs to be completely substituted by clean inputs. The slow transition causes a substantial increase in the mean temperature.

Introducing efficiency losses in environmental policy results in a trade-off between the carbon tax and the subsidy to clean R&D. When patent policy is not used, i.e., $\mu = 1$ (dashed yellow line), the carbon tax is postponed by more than 50 years. To compensate for this delay, there is a sharp increase in the clean innovation subsidy, which now peaks at 3.5 times the private value of clean innovations. However, once the carbon tax is introduced, the clean innovation subsidy is quickly phased out.

At first, the delay in the carbon tax causes a reduction in the ratio of clean to dirty inputs compared to the benchmark. This reduction is compensated by the large subsidy to clean innovation, which speeds up the transition of scientists and thus clean productivity growth. However, once the subsidy to clean innovation is phased out, the transition of scientists slows down. The net effect is a delay in the full transition from dirty to clean innovation in the long-run.

Removing access to patent protection on dirty innovations, i.e., $\mu < 1$, allows for a similar transition of scientists to clean technology as before, but for a substantially lower clean innovation subsidy. In absence of patent protection, dirty innovators must charge a lower price for their technology, and clean innovation becomes relatively more profitable. This reduces the need for a subsidy to clean innovation.

The optimal subsidy to clean innovation becomes smaller the more stringent the patent policy is. In fact, when $\mu = 0.4$, i.e., the dirty mark-up is close to the competitive price (recall that μ is bounded below by $\alpha = 1/3$), the optimal subsidy is even smaller than in the benchmark scenario. In addition, the transition to clean technology is faster when patent policy causes a large reduction in the dirty mark-up. That means that when μ is low, there is even a reduction in the optimal carbon tax.

In general, when the elasticity of substitution is low, the economy cannot easily transition to clean technology, which results in a large increase in temperatures. When environmental policy can no longer be financed without costs, carbon taxation is postponed, while there is a sharp increase in the clean innovation subsidy. The net effect, however, is a delay in the transition of scientists to clean innovation. Removing access to patent protection on dirty innovations allows for a reduction in the optimal innovation subsidy. For large reductions in the dirty monopoly mark-up, i.e., low μ, there is even a reduction in the optimal carbon tax, and a slightly faster transition to clean innovation. These results are robust even to a lower or higher elasticity of substitution. Figures A2 and A3 in the appendix show the optimal transition to clean technology when $\epsilon = 1.5$ and $\epsilon = 10$, respectively.

Table 2 shows the loss in welfare caused by the efficiency losses in environmental policy,

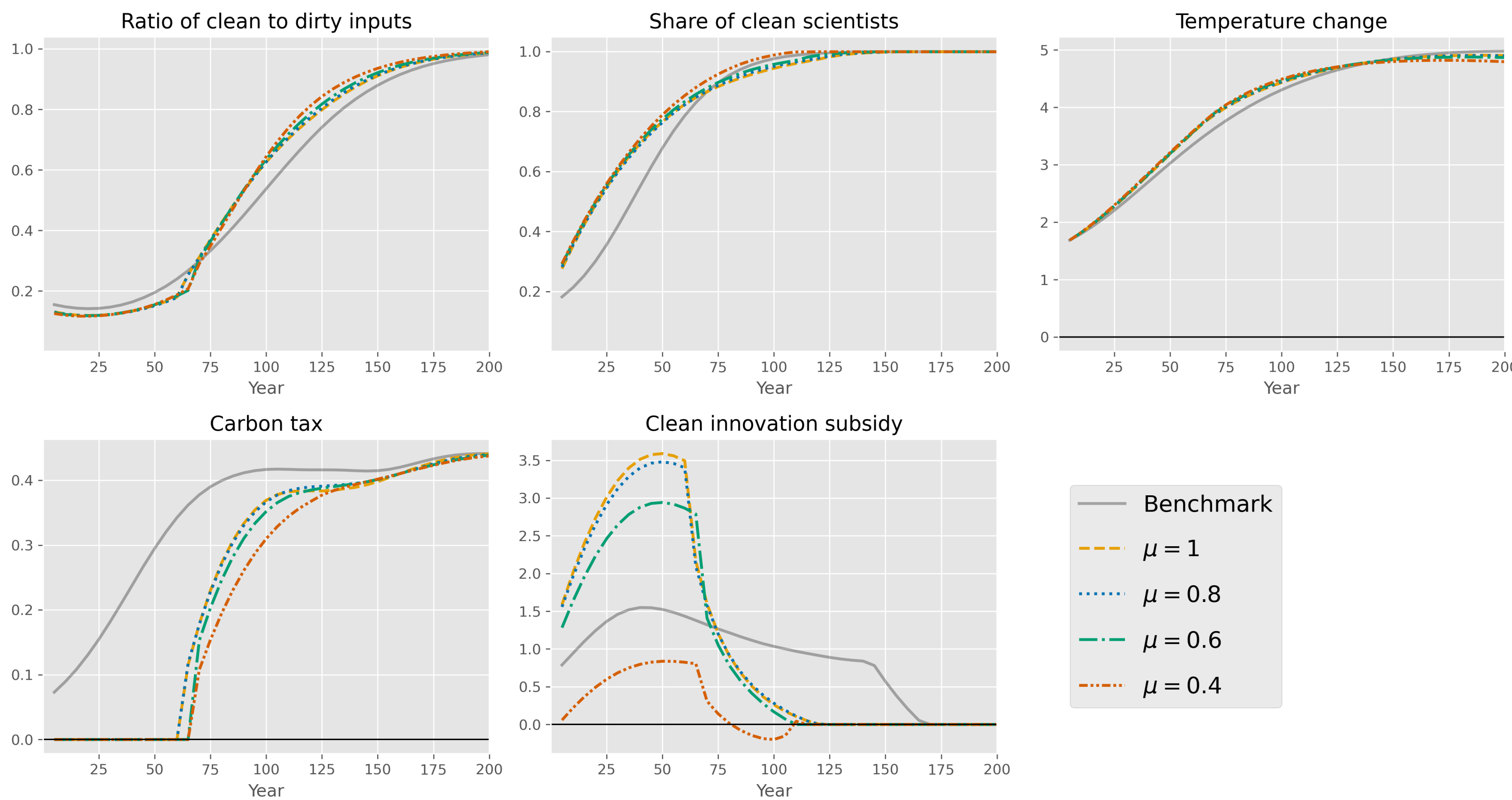

Figure 2: Simulation results for different caps on the price of dirty machines, i.e., μ. Benchmark refers to scenario without efficiency losses in the carbon tax, τ, and clean innovation subsidy, q.

and the welfare gains from combining environmental policy with patent policy. Welfare costs are measured as the equivalent percentage increase in per-period consumption that would be required to make consumers as well-off as in the benchmark scenario, i.e., when environmental policy can be financed without losses.

Column (1) in the table shows the percentage loss in consumption-equivalent welfare relative to the benchmark. When there is no use of patent policy, i.e., $\mu = 1$, consumers would require a 7.5 percent increase in per-period consumption to be equally well-off as in the benchmark scenario.[36] Removing patent protection on dirty innovations can cause substantial reduction in this welfare loss. Column (2) shows the percentage increase in welfare compared to when there is no use of patent policy. When the mark-up on dirty technology is restricted to 60 percent of the monopoly price, the welfare loss is reduced to 6.5 percent, which translates to a recovery of 13.3 percent of the welfare loss. Notice that the welfare gain from removing patent protection on dirty innovations is increasing in the reduction in the dirty mark-up.

Table 2: Welfare analysis of patent policy.

Parameter choice:		
Elasticity of substitution (ϵ)	3	
	(1)	(2)
	$\%Consumption$	$\%gain$
No patent policy ($\mu = 1$)	7.5	-
$\mu = 0.8$	7.3	3
$\mu = 0.6$	6.5	13.3
$\mu = 0.4$	4.2	44.1

Note: $\%Consumption$ refers to the percentage loss in consumption-equivalent welfare relative to the benchmark, i.e., when there are no efficiency losses in environmental policy. $\%gain$ refers to the percentage increase in consumption-equivalent welfare compared to when there is no use of patent policy, i.e., $\mu = 1$.

[36] Note that in our simulations, one period is five years.

5.2 Alternative elasticity of substitution

We test the robustness of our results to alternative levels of substitutability between clean and dirty inputs. In particular, we consider two different values for the elasticity of substitution: $\epsilon = 1.5$ (weak substitutes) and $\epsilon = 10$ (very strong substitutes). The optimal paths of economic, climate and policy variables are presented in Figures A2 and A3 in the appendix. There are three main takeaways. First, the general result persists under these different levels of the elasticity of substitution, that is, our policy reduces the stringency of standard climate policies. Second, under low levels of ϵ, the carbon tax is delayed slightly more than under higher levels of ϵ and, once it takes off, it it increases steadily overtime. This due to the fact that under low levels of substitutability, the economy requires the use of dirty inputs, at least as long as the technology levels are still low. After that, a higher restriction on these inputs is needed. Third, under high levels of ϵ, the energy transition is always faster and a modest carbon tax, combined with a higher but transitory subsidy, is enough to ensure it.

Table 3 presents the welfare analysis under $\epsilon = 1.5$ and $\epsilon = 10$. For comparisson, columns (1) and (2) reproduce the results under the baseline specification in Table 2. Under the two new cases, our policy improves consumers welfare, especially when the policy is more stringent. Under a high level of substitutability we see a large welfare improvement from our policy (column 6), although this is less relevant in "absolute" terms because the welfare loss associated to policy distortions is lower (column 5).

5.3 Exploration of the welfare recovery

In the baseline setting we have two different sources of distortions: one caused by monopoly power in the innovation sector and one caused from the efficiency losses in financing the standard environmental policies (carbon tax and research subsidy). In order to shed some light on the origin of the welfare improvements that we find with our policy, we conduct an exercise that consists on removing all distortions first and then incorporating only one of

Table 3: Welfare analysis of patent policy for different values of ϵ.

	Baseline		Robustness			
Parameter choice:						
Elasticity of substitution (ϵ)	3		1.5		10	
	(1) $\%Cons$	(2) $\%gain$	(3) $\%Cons$	(4) $\%gain$	(5) $\%Cons$	(6) $\%gain$
No patent policy ($\mu = 1$)	7.5	-	8.8	-	2.5	-
$\mu = 0.8$	7.3	3	8.6	2	2.4	0.7
$\mu = 0.6$	6.5	13.3	8	9.3	2.3	7
$\mu = 0.4$	4.2	44.1	5.9	33.4	1.2	51.4

Note: $\%Cons$ refers to the percentage loss in consumption-equivalent welfare relative to the benchmark, i.e., when there are no efficiency losses in environmental policy. $\%gain$ refers to the percentage increase in consumption-equivalent welfare compared to when there is no use of patent policy, i.e., $\mu = 1$.

them. In that sense, we define a new benchmark as a "First best" scenario in which there is no monopoly distortion[37] and no distortions to environmental policies. We first compare it to a scenario in which the monopoly distortion is present, but the environmental policies distortions are not accounted for. Finally, we incorporate our policy for different levels of stringency as before.

Table 4 presents the welfare analysis of this exercise. Once again, our benchmark scenario—the "First best", is such that all distortions are corrected for/non-existent. When the monopoly distortion is not corrected for—"No patent policy", the consumption-equivalent welfare loss is equal to 1.17% per period. Note that now the welfare loss is considerably lower than the welfare loss under the baseline scenario in Table 2. This suggests that the majority of the loss in the main results is attributable to the policy distortions, not the monopoly distortion.[38] Consistent with the main results, our policy improves welfare even when there are no policy distortions. That is because of the intrinsic nature of our policy, which removes

[37]The monopoly distortion is removed by implementing a subsidy to the use of all machines such that the demand for machines is equal to the demand under perfect competition.

[38]In fact, we confirm this by conducting an exercise in which we account for policy distortions but not for monopoly distortions. Due to space restrictions we do not present the results for this scenario.

exclusivity rights to monopolists. We take this opportunity to stress the fact that our policy, which a prior does not tackle any environmental externality directly, can be used beyond its direct area of influence (monopoly power) and induce the energy transition.

Table 4: Welfare analysis of patent policy without policy distortions (and compared to a scenario without monopoly distortions—"First best")

Parameter choice:		
Elasticity of substitution (ϵ)	3	
	(1)	(2)
	$\%Consumption$	$\%gain$
No patent policy ($\mu = 1$)	1.17	-
$\mu = 0.8$	1.01	12.93
$\mu = 0.6$	0.83	28.36
$\mu = 0.4$	0.66	43.74

Note: The "No patent policy" setting refers to a scenario where only the monopoly distortion is accounted for, that is, without standard environmental policy distortions. $\%Consumption$ refers to the percentage loss in consumption-equivalent welfare relative to the "No patent policy" scenario, i.e., when there are no efficiency losses in environmental policy. $\%gain$ refers to the percentage increase in consumption-equivalent welfare compared to when there is no use of patent policy, i.e., $\mu = 1$. The scenarios under different levels of μ do not account for policy distortions. All scenarios are compared to a "First best" scenario where the monopoly distortion has been corrected with production subsidies.

6 Extensions

[[We are currently considering which of the following extensions we should add to the paper:

- Without efficiency loss in carbon tax, but with cap on carbon tax.–¿ idea is to see no delay in carbon tax.

- A cap on Temperature at 2°C.

- Longer time periods (20 years) to match the current patent system, although in practice, most companies do not renew their patents for so long [reference]

- Spillovers between clean and dirty innovation. (Isabel has already the simulations, with a calibration of 0.75- 0-25)–¿ wEIRD RESULTS

- Lower discount rate (that would give lower temperature increase and lower delay of the carbon tax

- Case in which we correct for the monopoly distortion, i.e., add a subsidy to the use of all machines. That would be to compare results to existing literature. In this case, our patent policy has no negative effect on the energy transition, i.e., it only reduces dirty profits (APPENDIX, NOT REALLY NECESSARY)

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7 Discussion and conclusion

This paper proposes the use of a novel policy to ensure the energy transition in a context where existing studies typically ignore policy distortions and recommend unrealistically high policy rates. We introduce it in an endogenous growth model with environmental constraints to analyze its suitability. We find that our policy is able to recover a large share of the welfare loss associated with standard policy distortions.

While global coordination has been a major obstacle for the carbon tax, there is already substantial international collaboration on patent policy through the WTO, with most patent applications filed at only two patent offices (USPTO and EPO). Our suggested policy, namely to remove patent protection for dirty innovations, either globally or at local patent offices, implies an unequal treatment of technological innovations. Although this might seem difficult to implement at a first glance, in the following lines we discuss its feasibility and document a real-world example that resembles our suggested policy.

The first challenge to implement this policy lays on the patent regime. The current regime is based on a uniform system, in which all inventions are protected equally. Our policy requires, instead, differentiated protection based on the field of the invention. Even though our policy proposal is novel, both policy institutions and the law literature have already discussed the possibility of implementing a differentiated system of patent protection. For instance, OECD (2004) argue that "Economic evaluation suggests that there are further possible directions of change for patent regimes that are worth exploring. Possible avenues for economic-based reforms of patent regimes include introducing a more differentiated approach to patent protection that depends on specific characteristics of the inventions, such as their life cycle or their value (as opposed to the current uniform system)(...)". This indicates that a change in the patent regime is not unfeasible.

The second challenge lies on the practical aspect of implementation. Our policy requires that innovations on dirty energy do not get any patent protection. Hence, the patent office examiner should not grant a patent to innovations on dirty technologies. In practice, however, it is likely that innovations are not either clean or dirty, but have attributes in both sectors. In such cases, our policy can still be implemented through an evaluation of patent claims.[39] It is common practice that, during the patent application process, the patent office examiner rejects some of the claims proposed by the patent applicant. This is the case, for instance, when the technology claimed doesn't fall within the actual innovation. Our policy would require a specific examination conducted by the patent officer, to determine the nature of claims (clean vs dirty) and to reject the dirty ones.

A proper examination process should also ensure that innovators are not able to strategically rewrite their patents to make them "look" clean when they are dirty in nature. In

[39] As mentioned previously, patent claims are statements that explain the innovation and define which technology is being protected.

the eventual case when it is impossible to properly evaluate the real nature of the innovation, a mechanism should be implemented to ensure innovators reveal the real purpose of the innovations.

Although our policy has not been implemented yet, there has been some attempts to use patent regulation to enhance green innovation. These consists on speeding up the patent examination process for clean innovations, in other words, the intellectual property offices offer a fast track channel to reduce the examination process. One of them took place in the United States and was in place between 2009 and 2012. The United States Patent Office (USPTO) implemented the so-called "Green Technology Pilot Program", which conferred green innovations a special status by granting them an accelerated examination. Other national patent offices have implemented similar programs, most of which are still in place, although the adherence to this process is still limited (Dechezleprêtre, 2013).

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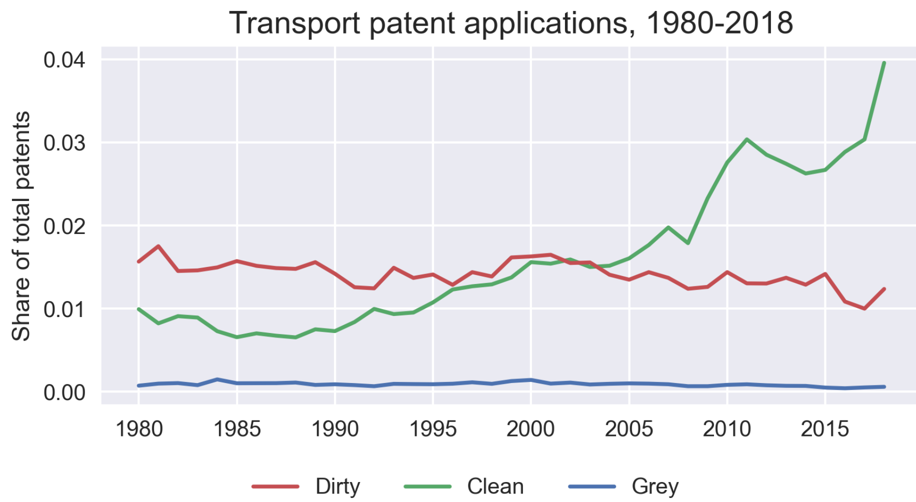

Figure A1: Based on data from OECD (2021). Patents are classified into grey, green and dirty following **?**, Table 21, which lists the CPC technological codes for each type. The authors identify grey codes as those that improve "the pollution efficiency of fossil technologies", thus, are harder to classify. When a patent has several types of CPC codes, we classify it into the type which has the higher number of codes. Draws are classified as grey.

A Appendix

A.1 Evolution of clean, dirty and grey patents

Figure A1 shows the evolution of dirty, clean and grey transport innovation in the OECD countries, measured as the ratio of patent applications over total applications at the European Patent Office (EPO). Unlike electricity innovation, clean innovation in the transport sector has been dominant since the early 2000. Dirty innovation presents a flat, or slightly decreasing, trend.

A.2 Public vs. Private R&D

Data

We use annual country level data of public and private R&D expenditure from the OECD (2020). Due to data limitations on R&D expenditures, our observational level is country-year. As a measure of innovation, we use patent data from the World Patent Statistical Database (2020).[40] In our benchmark analysis we use the universe of patent applications registered at the European Patent Office from 1981 to 2016. We focus on those patents produced in OECD countries.[41] Each patent application is assigned to the country of its applicant (or its innovator when there is no applicant).[42]

It is unreasonable to assume that all patents are of equal relevance. To account for the quality each patent we weight each application by a logarithmic transformation of the number of forward citations.[43] Each citation reflects whether the patent of interest is relevant for a later patent, either by the later patent's applicant or by a patent examiner. Each application is assigned to the year of the first filling.[44]

Empirical strategy and regression results

In order to compare the effectiveness of each type of R&D expenditure on the creation of new patents, we estimate the following model

$$Pat_{ct} = \beta_0 + \beta_1 PublicR\&D_{ct} + \beta_2 PrivateR\&D_{ct} + X_{ct} + \delta_c + \gamma_t + \epsilon_{ct} \tag{22}$$

[40] PATSTAT version: Spring 2020.

[41] More concretely, the following countries are included: AU, AT, BE, CL, CA, CN, IS, IL, CZ, DK, EE, FI, FR, DE, GR, HU, IE, IT, JP, KR, LV, LT, LU, MX, NL, NO, PL, PT, TW, NZ, SK, SI, ES, SE, CH, TR, GB, US.

[42] Whenever there are multiple applicants from different countries, we count them fractionally so that each applicant is assigned an equal share of the application.

[43] That is, each patent application is multiplied by $log(1 + \#citations)$.

[44] 85 % of the patents from 1970 to the present were registered as applications one year after the earliest filling. Therefore, there is roughly a one year gap between the earliest filing year and application filing year. Very few applications have the same year and most of the remaining ones have longer time differences.

where Pat refers to the quality-adjusted number of new patents in country c in year t, either in levels or as log-transformed. $PublicR\&D$ and $PrivateR\&D$ refer to total R&D expenditures funded by the public sector and the private sector, respectively, either in levels or as log-transformed. X includes controls such as GDP, while δ and γ account for country and time fixed effects. We wish to test whether $\beta_1 = \beta_2$, i.e. a dollar of public R&D funding has the same effect as private business R&D funding on patenting. We do so using a Wald test.

Table A1 reports our regression results. The first three columns display log-level regressions, whereas the last three columns display log-log regressions. The p-values of the Wald test are shown in the last row. All specifications control for GDP and include time and country fixed effects. Columns (1) and (4) use contemporaneous R&D expenditures, while columns (2) and (5) use a one year lag. Columns (3) and (5) measure R&D as the average expenditure over the previous three years.

Regression results show a consistent pattern in which R&D expenditures, both of public and private source, are associated with an increase in the number of new patents. More importantly, the point estimates are systematically larger for private expenditures, compared to public expenditures. Significance levels are stronger in the log-log regression. Column (6) shows that a 1 percent increase in the average public R&D expenditures for the previous three years is associated with a 0.524 percent increase in patenting. Private R&D expenditure is associate with a larger effect, with a 1 percent increase in private R&D expenditures associated with a 0.731 percent increase in patenting. We use a Wald test to determine whether the coefficients are statistically different from each other. The p-values for this test are reported in the last row "WT p-value". Throughout the table, the results for the Wald test do not allow us to reject that the coefficients are different from each other.

We choose the model specification in column (6) as our preferred specification. By using the coefficients from column (6), we estimate that there is a 28 percent efficiency loss in public funding of research $(((0.534 - 0.731)/0.731) \times 100)$.

Table A1: Relationship between patents (log citation-weighted) and R&D expenditure.

	Log-levels			Log-log		
	(1) Cont	(2) Lag	(3) M3Lag	(4) Cont	(5) Lag	(6) M3Lag
Public R&D	4.23e-6	5.01e-6**	3.71e-6	0.651***	0.628***	0.524***
	(5.8e-6)	(6.08e-6)	(5.8e-6)	(0.120)	(0.116)	(0.110)
Private R&D	1.23e-5***	1.06e-5*	6.79e-6	0.728***	0.630***	0.731***
	(4.47e-6)	(5.05e-6)	(5.24e-6)	(0.161)	(0.153)	(0.159)
Control (GDP)	√	√	√	√	√	√
FE (C)	√	√	√	√	√	√
FE (T)	√	√	√	√	√	√
Observations	934	906	706	934	906	706
Adj. R-squared	0.972	0.973	0.977	0.990	0.989	0.989
Clusters	37	37	35	37	37	35
WT p-value	0.234	0.370	0.566	0.740	0.994	0.291

Note: The dependent variable is the log-transformed number of citation-weighted patents. Clustered standard errors at the country level in parentheses. All regressions are weighted by the country average GDP. *** p<0.01, ** p<0.05, * p<0.1.

A.3 Characterization of the decentralized equilibrium

We characterize the decentralized equilibrium when there are efficiency losses associated with the funding of environmental policy, and where technology markets are distorted. To correct for the environmental externality, the government introduces both a carbon tax on dirty production and a research subsidy to clean innovation. However, for each unit of carbon tax collected and research funding given, a share of the revenue and funding is lost in the transaction.

In every period, the final producer has to pay a tax, τ_t, on the price of the dirty input. The maximization problem of the final good producer is

$$\max_{Y_{ct},Y_{dt}} Y_t - p_{ct}Y_{ct} - p_{dt}(1+\tau_t)Y_{dt}.$$

where Y_t is given by Eq. (3). Taking the ratio of the first-order conditions with respect to Y_{ct} and Y_{dt}, we obtain the relative price of the clean input

$$\frac{p_{ct}}{p_{dt}(1+\tau_t)} = \left(\frac{Y_{ct}}{Y_{dt}}\right)^{-\frac{1}{\epsilon}}, \tag{23}$$

which is decreasing in the relative supply of the good. Using the price of the final good as the numeraire, the price index of the clean and dirty good is given by

$$p_{ct}^{1-\epsilon} + (p_{dt}(1+\tau_t))^{1-\epsilon} = 1. \tag{24}$$

Recall the demand for machines in Eq. (8) by the intermediate goods producers and the price of machines in Eq. (11). Since innovators of clean machines are granted a one-period patent on their innovation they are able to charge the unconstrained monopoly price. Producers of dirty machines, on the other hand, can charge only a share μ of the monopoly mark-up. Inserting for the price of machines, demand for clean and dirty machines becomes

$$x_{cit}^{MO} = \left(\frac{\alpha^2\Omega(S_t)p_{ct}}{\psi(1-z_c)}\right)^{\frac{1}{1-\alpha}} A_{cit}L_{ct} \quad \text{and} \quad x_{dit}^{MO} = \left(\frac{\alpha^2\Omega(S_t)p_{dt}}{\mu\psi(1-z_d)}\right)^{\frac{1}{1-\alpha}} A_{dit}L_{dt}. \tag{25}$$

However, in each period there are only some machines that experience a productivity improvement. The remaining machines are instead sold at the competitive price, i.e. the marginal cost, and there is no longer a need for a production subsidy to correct for the monopoly distortion. Demand for these machines are given by

$$x_{jit}^{CO} = \left(\frac{\alpha\Omega(S_t)p_{jt}}{\psi}\right)^{\frac{1}{1-\alpha}} A_{jit}L_{jt}. \tag{26}$$

The number of machines with a successful innovation is given by $n_j s_{jt}^{\sigma}$, while the number of machines sold at the competitive price is given by $1 - n_j s_{jt}^{\sigma}$. Combining these shares with the demand for machines from Eqs. (25) and (26), and using the expression of average machine quality in Eq. (15), production of the intermediate goods can be written as

$$Y_{ct} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \Omega(S_t)^{\frac{1}{1-\alpha}} p_{ct}^{\frac{\alpha}{1-\alpha}} L_{ct} \widetilde{A_{ct}} \quad \text{and} \quad Y_{dt} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \Omega(S_t)^{\frac{1}{1-\alpha}} p_{dt}^{\frac{\alpha}{1-\alpha}} L_{dt} \widetilde{A_{dt}}, \tag{27}$$

where

$$\widetilde{A_{ct}} \equiv A_{ct} \left[\eta_c s_{ct}^{\sigma} \left(\left(\frac{\alpha}{(1-z_c)}\right)^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right]$$
$$\text{and} \quad \widetilde{A_{dt}} \equiv A_{dt} \left[\eta_d s_{dt}^{\sigma} \left(\left(\frac{\alpha}{\mu(1-z_d)}\right)^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right]. \tag{28}$$

While A_{jt} is the average quality level of machines in sector j, $\widetilde{A_{jt}}$ is the average corrected productivity level in sector j that takes into account that some machines are produced by monopolists and some are not. The average corrected productivity captures the fact that an increase in innovation efforts will not only increase the average quality of machines but also the monopoly distortion in the sector. In general, $\widetilde{A_{jt}} < A_{jt}$, with the gap increasing in the number of scientists in the sector, s_{jt}.[45] Eq. (28) shows that a reduction in μ reduces the wedge between $\widetilde{A_{dt}}$ and A_{dt}. In absence of patent protection, dirty innovators must charge a lower price for their machines, thus reducing the the monopoly distortion in the dirty sector. This leads to the following lemma:

Lemma 1. *All else given, removing patent protection of dirty technology, i.e., $\mu < 1$, reduces the monopoly distortion in the dirty sector, and thus increases the average corrected productivity of dirty technology.*

[45]However, notice that the relative effective productivity of the clean input can still be larger than its relative average quality. For simplicity, assume no machine subsidies ($z_j = 0$) and no patent policy ($\mu = 1$). Then $\frac{\widetilde{A_{ct}}}{\widetilde{A_{dt}}} > \frac{A_{ct}}{A_{dt}}$ if $s_{ct} < \frac{\eta_d^{1/\sigma}}{\eta_c^{1/\sigma} + \eta_d^{1/\sigma}}$. If $\eta_c = \eta_d$, then the relative effective productivity of the clean input is larger than its relative quality level as long as $s_{ct} < 0.5$. In this case, when the majority of scientists are in the dirty sector the monopoly distortion will be larger in that sector. The wedge between $\widetilde{A_{jt}}$ and A_{jt} will therefore be larger in the dirty sector relative to the clean sector.

Proof. $\tilde{A}_{dt}$ is decreasing in μ as one can see from its derivative wrt. μ

$$\frac{\partial\left(\left(\eta_d s_{dt}^{\sigma}\left(\left(\frac{\alpha}{\mu}\right)^{\frac{\alpha}{1-\alpha}}-1\right)+1\right)A_{dt}\right)}{\partial\mu}=-A_{dt}\eta_d s_{dt}^{\sigma}\frac{\alpha}{1-\alpha}\left(\frac{\alpha}{\mu}\right)^{\frac{\alpha}{1-\alpha}-1}\frac{\alpha}{\mu^2}<0$$

□

In addition to machines, the intermediate inputs are produced using also labor. Demand for labor is given by the first-order condition of the intermediate producers problem in Eq. (7) with respect to L_{jt}

$$(1-\alpha)p_{jt}\Omega(S_t)L_{jt}^{-\alpha}\int_0^1 A_{jit}^{1-\alpha}x_{jit}^{\alpha}di-w_t=0 \tag{29}$$

which results in the following wage rate

$$w_t=(1-\alpha)\Omega(S_t)p_{jt}\frac{Y_{jt}}{L_{jt}} \tag{30}$$

By combining the expressions of intermediate production in Eq. (27) with the fact that the wage rate in the two sectors must be equal, the relative price of the clean good can be expressed as a function of the effective productivity of clean and dirty technology

$$\frac{p_{ct}}{p_{dt}}=\left(\frac{\widetilde{A_{ct}}}{\widetilde{A_{dt}}}\right)^{-(1-\alpha)} \tag{31}$$

The relative price of the clean input is falling in the relative effective productivity of clean technology. However, notice that an increase in the share of scientists in the clean sector will have two counteracting effects on the relative effective productivity of the clean input. On the one hand, more clean scientists will increase the average quality of clean machines; but on the other hand, it will also increase the monopoly distortion in the clean sector (and reduce the monopoly distortion in the dirty sector). Whether or not an increase in the number of clean scientists leads to an increase in the relative effective productivity of the clean input depends on the initial share of clean scientists. In general, more clean scientists will reduce (increase) the relative price of the clean input when clean innovation is initially low (high).

Combining the expression of the relative price of the clean good in Eq. (31) with the price index in Eq. (24), the price of the clean and dirty good can be expressed as functions of the effective productivity and the carbon tax

$$p_{ct} = \frac{\widetilde{A_{dt}}^{1-\alpha}}{\left[\widetilde{A_{ct}}^{\varphi}(1+\tau_t)^{1-\epsilon} + \widetilde{A_{dt}}^{\varphi}\right]^{\frac{1}{1-\epsilon}}} \quad \text{and} \quad p_{dt} = \frac{\widetilde{A_{ct}}^{1-\alpha}}{\left[\widetilde{A_{ct}}^{\varphi}(1+\tau_t)^{1-\epsilon} + \widetilde{A_{dt}}^{\varphi}\right]^{\frac{1}{1-\epsilon}}} \tag{32}$$

where $\varphi = (1-\alpha)(1-\epsilon)$.

Combining the expressions for the relative price of the clean good in Eqs. (23) and (31) along with the expressions for clean and dirty production in Eq. (27), the relative labor share of the clean good can be expressed as

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{p_{ct}}{p_{dt}(1+\tau_t)}\right)^{-\epsilon} \left(\frac{\widetilde{A_{ct}}}{\widetilde{A_{dt}}}\right)^{-(1-\alpha)} \tag{33}$$

Inserting for the the prices of the clean and dirty inputs from Eq. (32), the relative labor share can be expressed as a function of the relative effective productivity of the clean good and the carbon tax

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{\widetilde{A_{ct}}}{\widetilde{A_{dt}}}\right)^{-\varphi} (1+\tau_t)^{\epsilon} \tag{34}$$

Since ϵ is positive (and assumed to be larger than one), a higher carbon tax increases the labor share used in clean production. The relative labor share of clean production is increasing in the relative effective productivity of clean machines. However, as before, an increase in clean innovation has an ambiguous effect on the relative effective productivity of clean technology. At low (high) levels of clean innovation, an increase in the number of clean scientists will increase (decrease) the relative effective productivity of the clean input, thus increasing the relative labor share in the clean sector.

Combining the relative labor share of the clean good with the market clearing condition in the labor market, i.e. $L_{ct} + L_{dt} \leq 1$, the labor shares of the clean and dirty good can be expressed as functions of effective productivity and the carbon tax

$$L_{ct} = \frac{\widetilde{A_{dt}}^{\varphi}(1+\tau_t)^{\epsilon}}{\widetilde{A_{ct}}^{\varphi} + \widetilde{A_{dt}}^{\varphi}(1+\tau_t)^{\epsilon}} \quad \text{and} \quad L_{dt} = \frac{\widetilde{A_{ct}}^{\varphi}}{\widetilde{A_{ct}}^{\varphi} + \widetilde{A_{dt}}^{\varphi}(1+\tau_t)^{\epsilon}} \tag{35}$$

Recall the expression for the profits in the clean sector relative to the dirty sector in Eq. (19). Inserting for the relative labor share and the relative price of the clean good from Eqs. (34) and (31) we obtain the following expression for the relative profit of clean innovation

$$\frac{\Pi_{ct}}{\Pi_{dt}} = (1+q_t)\frac{\eta_c}{\eta_d}\left(\frac{s_{ct}}{s_{dt}}\right)^{\sigma-1}\frac{1-\alpha}{\mu-\alpha}\left(\frac{\mu(1-z_d)}{1-z_c}\right)^{\frac{1}{1-\alpha}}(1+\tau_t)^{\epsilon}\left(\frac{\widetilde{A_{ct}}}{\widetilde{A_{dt}}}\right)^{-(1+\varphi)}\frac{A_{ct-1}}{A_{dt-1}} \tag{36}$$

where q_t is a research subsidy given to scientists in the clean sector. Inserting for the effective productivity levels and the evolution of technology from Eq. (16), and assuming that there are no production subsidies ($z_j = 0$), the relative expected profits from clean research is expressed as a function of the share of clean scientists, technology levels and the policy instruments

$$\begin{aligned}\frac{\Pi_{ct}}{\Pi_{dt}} = (1+q_t)\frac{\eta_c}{\eta_d}\left(\frac{s_{ct}}{s_{dt}}\right)^{\sigma-1}\frac{1-\alpha}{\mu-\alpha}\mu^{\frac{1}{1-\alpha}}(1+\tau_t)^{\epsilon}\times \\ \left(\frac{\left(\eta_c s_{ct}^{\sigma}\left(\alpha^{\frac{\alpha}{1-\alpha}}-1\right)+1\right)(1+\gamma\eta_c s_{ct}^{\sigma})}{\left(\eta_d s_{dt}^{\sigma}\left(\left(\frac{\alpha}{\mu}\right)^{\frac{\alpha}{1-\alpha}}-1\right)+1\right)(1+\gamma\eta_d s_{dt}^{\sigma})}\right)^{-\varphi-1}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi}\end{aligned} \tag{37}$$

Clean and dirty production are found by inserting for price and labor from Eqs. (32) and

(35) in the expressions for intermediate outputs in Eq. (27)

$$Y_{ct} = \zeta_t \frac{\widetilde{A_{ct}}\widetilde{A_{dt}}^{\alpha+\phi}(1+\tau_t)^{\epsilon}}{\left(\widetilde{A_{ct}}^{\phi}(1+\tau_t)^{1-\epsilon}+\widetilde{A_{dt}}^{\phi}\right)^{\frac{\alpha}{\phi}}\left(\widetilde{A_{ct}}^{\phi}+\widetilde{A_{dt}}^{\phi}(1+\tau)^{\epsilon}\right)}$$

$$\text{and} \quad Y_{dt} = \zeta_t \frac{\widetilde{A_{ct}}^{\alpha+\phi}\widetilde{A_{dt}}}{\left(\widetilde{A_{ct}}^{\phi}(1+\tau_t)^{1-\epsilon}+\widetilde{A_{dt}}^{\phi}\right)^{\frac{\alpha}{\phi}}\left(\widetilde{A_{ct}}^{\phi}+\widetilde{A_{dt}}^{\phi}(1+\tau)^{\epsilon}\right)} \quad (38)$$

where $\zeta_t = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\Omega(S_t)^{\frac{1}{1-\alpha}}$. Inserting for clean and dirty production in Eq. (3), we can write production of the final good as

$$Y_t = \zeta_t \frac{\widetilde{A_{ct}}\widetilde{A_{dt}}(1+\tau_t)^{\epsilon}}{\left(\widetilde{A_{ct}}^{\phi}(1+\tau_t)^{1-\epsilon}+\widetilde{A_{dt}}^{\phi}\right)^{\frac{\alpha+\epsilon(1-\alpha)}{\phi}}\left(\widetilde{A_{ct}}^{\phi}+\widetilde{A_{dt}}^{\phi}(1+\tau)^{\epsilon}\right)} \quad (39)$$

Total use of machines in sector j is the sum of demand for machines produced by a monopolist and the machines produced competitively

$$\int_0^1 x_{jit}di = \eta_j s_{jt}^{\sigma} x_{jit}^{MO} + (1-\eta_j s_{jt}^{\sigma}) x_{jit}^{CO}$$

where x_{jit}^{MO} and x_{jit}^{CO} are given by Eqs. (25) and (26). Inserting for prices and labor shares from Eqs. (32) and (35), machine use in the clean and dirty sector can be expressed as

$$x_{ct} = \left(\frac{\alpha}{\psi}\right)^{\frac{1}{1-\alpha}}\Omega(S_t)^{\frac{1}{1-\alpha}} \frac{A_{ct}\left[\eta_c s_{ct}^{\sigma}\left(\left(\frac{\alpha}{1-z_c}\right)^{\frac{1}{1-\alpha}}-1\right)+1\right]\widetilde{A_{dt}}^{1+\phi}(1+\tau)^{\epsilon}}{\left(\widetilde{A_{ct}}^{\phi}(1+\tau)^{1-\epsilon}+\widetilde{A_{dt}}^{\phi}\right)^{\frac{1}{\phi}}\left(\widetilde{A_{ct}}^{\phi}+\widetilde{A_{dt}}^{\phi}(1+\tau)^{\epsilon}\right)}$$

$$\text{and} \quad x_{dt} = \left(\frac{\alpha}{\psi}\right)^{\frac{1}{1-\alpha}}\Omega(S_t)^{\frac{1}{1-\alpha}} \frac{A_{dt}\left[\eta_d s_{dt}^{\sigma}\left(\left(\frac{\alpha}{\mu(1-z_d)}\right)^{\frac{1}{1-\alpha}}-1\right)+1\right]\widetilde{A_{ct}}^{1+\phi}}{\left(\widetilde{A_{ct}}^{\phi}(1+\tau)^{1-\epsilon}+\widetilde{A_{dt}}^{\phi}\right)^{\frac{1}{\phi}}\left(\widetilde{A_{ct}}^{\phi}+\widetilde{A_{dt}}^{\phi}(1+\tau)^{\epsilon}\right)} \quad (40)$$

Aggregating over all the machine lines in the clean sector in Eq. (12) and inserting for the price and labor share from Eqs. (32) and (35), total profits from the clean sector can be expressed as

$$\Pi_{ct} = (1-\alpha)\left(\frac{\alpha}{1-z_c}\right)^{\frac{1}{1-\alpha}} \zeta_t \frac{A_{ct}\widetilde{A_{dt}}^{\phi+1}(1+\tau)^{\epsilon}}{\left(\widetilde{A_{ct}}^{\phi}(1+\tau_t)^{1-\epsilon}+\widetilde{A_{dt}}^{\phi}\right)^{\frac{1}{\phi}}\left(\widetilde{A_{ct}}^{\phi}+\widetilde{A_{dt}}^{\phi}(1+\tau_t)^{\epsilon}\right)} \tag{41}$$

Consumption is given by the budget restriction in Eq. (21). Inserting for production of the final good, total use of machines, and profits from clean innovation from Eqs. (39), (25), (26), and (41), consumption can be expressed as a function of only the technology levels and policy instruments. The decentralized equilibrium has thus been solved as a function of clean and dirty technology levels (A_{ct} and A_{dt}), the carbon tax (τ_t), the clean research subsidy (q_t), and the patent policy parameter (μ).

Assuming that the carbon tax is initially zero and that dirty innovations are granted full protection (i.e. $\mu = 1$), using the solutions for clean and dirty production from Eq. (38), the initial effective productivity levels in the clean and dirty sectors are given by

$$\widetilde{A_{c0}} = \frac{Y_{c0}}{\zeta_0}\left(1+\left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1-\epsilon}{\epsilon}}\right)^{\frac{\alpha+\varphi}{\varphi}} \quad \text{and} \quad \widetilde{A_{d0}} = \frac{Y_{d0}}{\zeta_0}\left(1+\left(\frac{Y_{d0}}{Y_{c0}}\right)^{\frac{1-\epsilon}{\epsilon}}\right)^{\frac{\alpha+\varphi}{\varphi}} \tag{42}$$

and inserting for the effective productivity levels from Eq. (28), initial average quality of clean and dirty technology can be expressed as

$$A_{c0} = \frac{Y_{c0}}{\zeta_0\left[\eta_c s_{c0}\left(\left(\frac{\alpha}{1-z_c}\right)^{\frac{\alpha}{1-\alpha}}+1\right)-1\right]}\left(1+\left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1-\epsilon}{\epsilon}}\right)^{\frac{\alpha+\varphi}{\varphi}}$$

$$\text{and} \quad A_{d0} = \frac{Y_{d0}}{\zeta_0\left[\eta_d(1-s_{c0})\left(\left(\frac{\alpha}{1-z_d}\right)^{\frac{\alpha}{1-\alpha}}+1\right)-1\right]}\left(1+\left(\frac{Y_{d0}}{Y_{c0}}\right)^{\frac{1-\epsilon}{\epsilon}}\right)^{\frac{\alpha+\varphi}{\varphi}} \tag{43}$$

The initial average quality levels, A_{c0} and A_{d0}, are pinned down by the initial production of the two inputs, Y_{c0} and Y_{d0}, and by the initial share of scientists in the clean input sector, s_{c0}. For given initial technology levels, the model can be simulated by updating productivity according to Eq. (16) and the environmental quality according to Eq. (6). The allocation of scientists is implicitly pinned down by Eq. (37).

A.4 Proofs

1. Proof of Proposition 1

Equation 19 captures the three partial effects on the relative profitability of clean research, where $\frac{p_{ct}}{p_{dt}}$ and $\frac{L_{ct}}{L_{dt}}$ also depend on μ. We proceed with the proof of each partial effect individually:

1. Direct patent policy effect: given that $\mu \in (\alpha, 1)$, an increase in μ decreases the direct patent policy effect term, $\frac{1-\alpha}{\mu-\alpha}\mu^{\frac{1}{1-\alpha}}$ because

$$\frac{\partial\left[\frac{1-\alpha}{\mu-\alpha}\mu^{\frac{1}{1-\alpha}}\right]}{\partial\mu} = \frac{1-\alpha}{\mu-\alpha}\mu^{\frac{1}{1-\alpha}}\left[-\frac{1}{\mu-\alpha}+\frac{1}{\mu-\mu\alpha}\right] < 0$$

In other words, the direct patent policy effect is increasing in our policy—a lower μ implies higher relative clean profits.

2. Price effect: Combining equations (31) and (28) the price effect term becomes

$$\frac{p_{ct}}{p_{dt}} = \left(\frac{\tilde{A}_{ct}}{\tilde{A}_{dt}}\right)^{-(1\text{-}\alpha)} = \left(\frac{\left(\eta_c s_{ct}^{\sigma}\left(\alpha^{\frac{\alpha}{1-\alpha}}-1\right)+1\right)A_{ct}}{\left(\eta_d s_{dt}^{\sigma}\left(\left(\frac{\alpha}{\mu}\right)^{\frac{\alpha}{1-\alpha}}-1\right)+1\right)A_{dt}}\right)^{-(1-\alpha)}$$

An increase in μ decreases the price effect term because $\tilde{A}_{dt}$ is decreasing in μ (see proof of Lemma A.3). In other words, the price effect is increasing in our policy—a lower μ implies higher relative clean profits.

3. Market size effect: from equation (34) the market size effect term becomes

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{\tilde{A}_{ct}}{\tilde{A}_{dt}}\right)^{-(1-\alpha)(1-\epsilon)} (1+\tau_t)^{\epsilon}$$

For any $\epsilon > 1$, an increase in μ increases the market size effect term because $\tilde{A}_{dt}$ is decreasing in μ (see proof of Lemma A.3). In other words, the market size effect is decreasing in our policy—a lower μ implies a lower relative clean profits.

□

2. Proof of Proposition 2

Equation (37) in the appendix expresses the relative expected profits of clean research as a function of the share of scientists, technology levels and the policy instrument. Its derivative with respect to μ determines the effect of our policy on clean relative profits. The derivative of equation (37) wrt μ is given by

$$
\frac{\partial\left(\frac{\Pi_{ct}}{\Pi_{dt}}\right)}{\partial\mu}=\frac{\partial\left(\frac{\Pi_{ct}}{\Pi_{dt}}=\frac{\eta_c}{\eta_d}\left(\frac{s_{ct}}{s_{dt}}\right)^{\sigma-1}\frac{1-\alpha}{\mu-\alpha}\mu^{\frac{1}{1-\alpha}}\left(1-\tau\right)^{\epsilon}\left(\frac{\left(\eta_c s_{ct}^{\sigma}(\alpha^{\frac{\alpha}{1-\alpha}}-1)+1\right)(1+\eta_c s_{ct}^{\sigma}\gamma)}{\left(\eta_d s_{dt}^{\sigma}((\frac{\alpha}{\mu})^{\frac{\alpha}{1-\alpha}}-1)+1\right)\left(1+\eta_d s_{dt}^{\sigma}\gamma\right)}\right)^{-1\text{-}\varphi}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi}\right)}{\partial\mu}
$$

$$
=\frac{\partial\left(\frac{\Pi_{ct}}{\Pi_{dt}}=\kappa\frac{1-\alpha}{\mu-\alpha}\mu^{\frac{1}{1-\alpha}}\left(\frac{\eta_d s_{dt}^{\sigma}((\frac{\alpha}{\mu})^{\frac{\alpha}{1-\alpha}}-1)+1}{\eta_c s_{ct}^{\sigma}(\alpha^{\frac{\alpha}{1-\alpha}}-1)+1}\right)^{1+\varphi}\right)}{\partial\mu}
$$

$$
=\underbrace{\kappa}_{>0}\underbrace{\left[-\frac{(1-\alpha)}{(\mu-\alpha)^2}\mu^{\frac{1}{1-\alpha}}+\frac{1}{1-\alpha}\mu^{\frac{1}{1-\alpha}-1}\frac{1-\alpha}{\mu-\alpha}\right]}_{<0}\underbrace{\left(\frac{\eta_d s_{dt}^{\sigma}(\left(\frac{\alpha}{\mu}\right)^{\frac{\alpha}{1-\alpha}}-1)+1}{\eta_c s_{ct}^{\sigma}(\alpha^{\frac{\alpha}{1-\alpha}}-1)+1}\right)^{1+\varphi}}_{\underbrace{\kappa}_{>0}}+
$$

$$
+\underbrace{\kappa}_{>0}\left[(1+\varphi)\underbrace{\left(\frac{\eta_d s_{dt}^{\sigma}(\left(\frac{\alpha}{\mu}\right)^{\frac{\alpha}{1-\alpha}}-1)+1}{\eta_c s_{ct}^{\sigma}(\alpha^{\frac{\alpha}{1-\alpha}}-1)+1}\right)^{\varphi}}_{>0}\underbrace{(-1)\frac{\frac{\alpha}{1-\alpha}\eta_d s_{dt}^{\sigma}\left(\frac{\alpha}{\mu}\right)^{\frac{\alpha}{1-\alpha}-1}\frac{\alpha}{\mu^2}}{\eta_c s_{ct}^{\sigma}(\alpha^{\frac{\alpha}{1-\alpha}}-1)+1}}_{<0}\right]\underbrace{\frac{1-\alpha}{\mu-\alpha}\mu^{\frac{1}{1-\alpha}}}_{>0}
$$

where $\kappa\equiv\frac{\eta_c}{\eta_d}\left(\frac{s_{ct}}{s_{dt}}\right)^{\sigma-1}(1-\tau)^{\epsilon}\left(\frac{1+\eta_c s_{ct}^{\sigma}\gamma}{1+\eta_d s_{dt}^{\sigma}\gamma}\right)^{-1\text{-}\varphi}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi}$. Under $\alpha\in(0,1)$, $\mu\in(\alpha,1]$, $s_{jt}\in[0,1]$ we can determine the signs of almost every element of the derivative, as indicated in the previous expression. The sign of the term $(1+\varphi)$ determines the sign of the second element in the summation, hence the effect our policy on the relative profitability. One can distinguish three cases:

- If $\epsilon=\frac{2-\alpha}{1-\alpha}$, then $\left(1+\underbrace{(1-\alpha)(1-\epsilon)}_{\equiv\varphi}\right)=0$. Hence, the derivative of the relative clean profits wrt. μ is negative, that is, a more stringent policy induces efforts towards clean innovation.

- If $\epsilon<\frac{2-\alpha}{1-\alpha}$, then $(1+(1-\alpha)(1-\epsilon))>0$. Hence, the derivative of the relative clean profits wrt. μ is negative, that is, a more stringent policy induces efforts towards clean innovation.

- If $\epsilon > \frac{2-\alpha}{1-\alpha}$, then $(1+(1-\alpha)(1-\epsilon)) < 0$. Hence, the second element in the derivative is positive, that is, a more stringent policy induces an ambiguous effect towards clean innovation.

This proofs the proposition.

$\square$

A.5 Calibration of carbon cycle and damage function

The evolution of accumulated emissions, also known as the carbon cycle is given by Eq. (6). The increase in CO2 concentrations from one unit of dirty production is given by ξ, which we estimate from the observed value of Y_d and annual CO2 emissions between 2016 and 2020. In this period, global CO2 emissions from energy consumption were 166.77 GtCO2, while global consumption of fossil fuel energy were 2392.11 exajoules. Hence, we set $\xi = 166.77/2392.11$.

An increase in accumulated emissions, S_t, causes an increase in the global mean temperature, $\Delta(T)$, which results in economic damage. The relationship between CO2 emissions and global warming can be approximated by a linear function in accumulated emissions (Dietz and Venmans, 2019, Dietz et al., 2021).[46] We therefore map emissions to temperature change by using the following linear function

$$\Delta(T_t) = \Delta(T_0) + TCRE \times S_t$$

where $\Delta(T_t)$ is the increase in global mean temperature at time t and $\Delta(T_0)$ is the increase in temperatures that we have already experienced at the beginning of our simulations. $TCRE$ is the transient climate response to cumulative carbon emissions, which measures the change in temperatures from an additional unit of accumulated CO2 emission. Dietz et al.

[46] See Dietz and Venmans (2019) for an explanation of the underlying geophysics of this relationship

(2021) suggests a $TCRE$ of $1.7°C$ per trillion tons of cumulative carbon emissions (TtC). We increase the estimate by 10 % to take into account the warming from non-CO2 greenhouse gases ((Allen, 2016)). This results in a $TCRE$ of $1.87°C$/TtC, which corresponds to $0.00051°C$/GtCO2. Since our simulations starts after 2020, we assume global warming of $1°C$ at the start of our simulation period.

We follow AABH and relate global warming to economic damages by using the following function

$$\Omega(\Delta(T)) = \frac{(\Delta_{disaster} - \Delta(T))^{\lambda} - \lambda\Delta_{disaster}^{\lambda-1}(\Delta_{disaster} - \Delta(T))}{(1-\lambda)\Delta_{disaster}^{\lambda}},$$

where $\Delta(T)$ is the increase in the global mean temperature above its pre-industrial level given accumulated emissions at time t. We follow AABH and set the disaster temperature, $\Delta_{disaster}$, equal to $6°C$., which corresponds to a maximum carbon budget of $\bar{S}$, of 9,804 GtCO2. When the carbon budget is used up, damages are a 100% of GDP. In a scenario of $3°C$ temperature increase by 2100, Van Der Wijst et al. (2023) found global damages to be in the range of 10% to 12% of GDP. We therefore match our damage function to correspond to a 10 % reduction in production at $3°C$ warming, which leads to a value of $\lambda = 0.5958$.

A.6 More numerical results

Table A2: Parameter values for the carbon cycle and damage function

Parameter	**Description**	**Value**
Y_{c0}	Consumption of renewable, nuclear and hydro-power energy in the world from 2016 to 2020 in exajoules, from BP.	464.26
Y_{d0}	Consumption of fossil fuel energy in the world from 2016 to 2020 in exajoules, from BP.	2392.11
$emission_0$	World emissions from energy production from 2016 to 2020 in GtCO2, from BP.	166.77
$TCER$	Transient climate response to cumulative carbon emissions; increase in temperature ($°C$) per unit increase in accumulated emissions (GtCO2).	0.00051
$\Delta(T_0)$	Global warming in Celsius above pre-industrial levels in 2020.	1
$\bar{S}$	Disaster level of accumulated emissions in GtCO2.	$\frac{\Delta_{disaster}-\Delta(T_0)}{TCER} = 9,804$
λ	Parameter to match the AABH damage function to a 10% decrease in production at $3°C$ warming.	0.5958
ξ	Increase in accumulated emissions (GtCO2) from one unit of dirty production.	$\frac{emission_0}{Y_{d0}} = 0.0697$

Note:

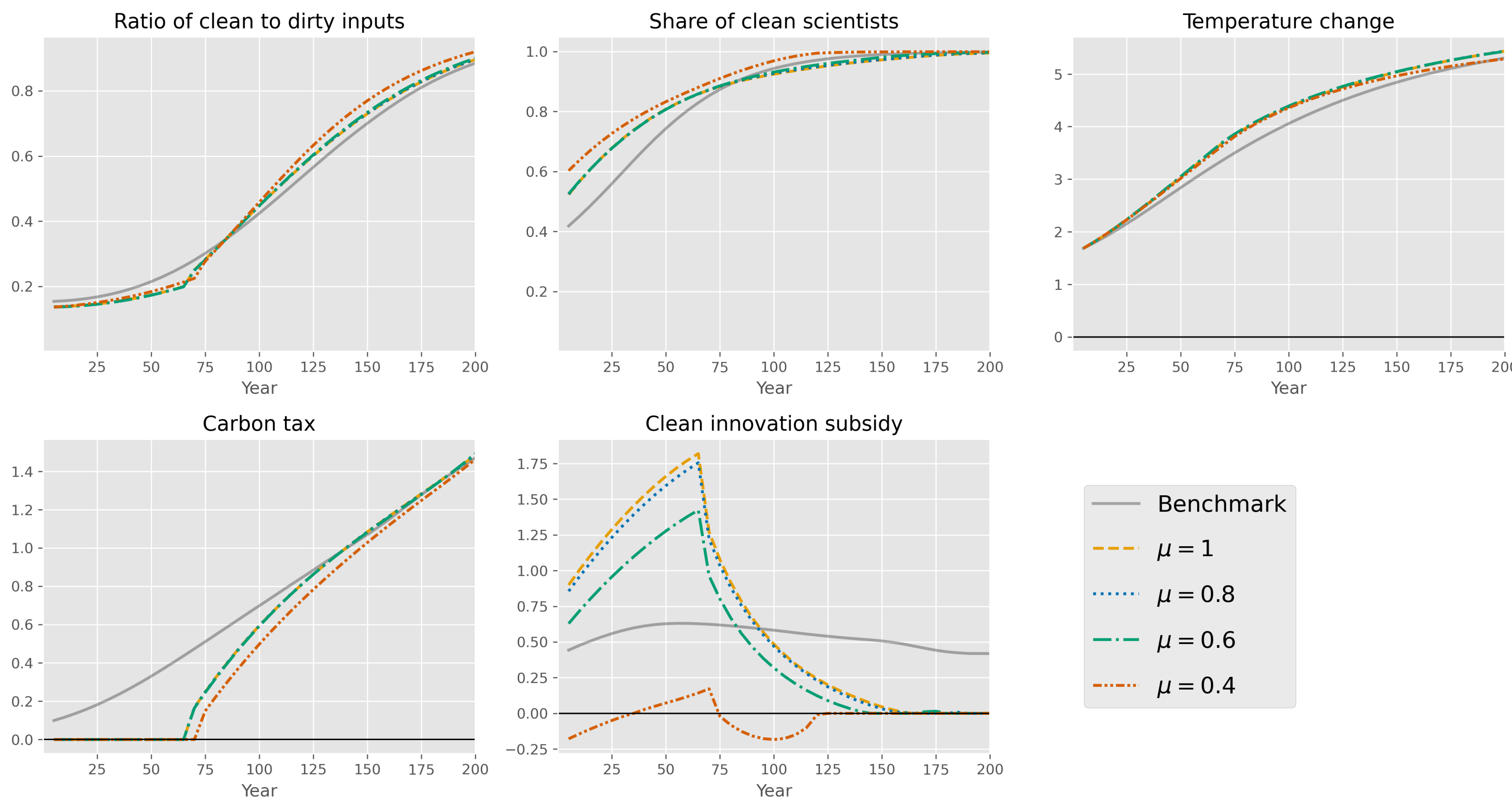

Figure A2: Simulation results for different caps on the price of dirty machines, i.e., μ, when $\epsilon = 1.5$. Benchmark refers to scenario without efficiency losses in the carbon tax, τ, and clean innovation subsidy, q.

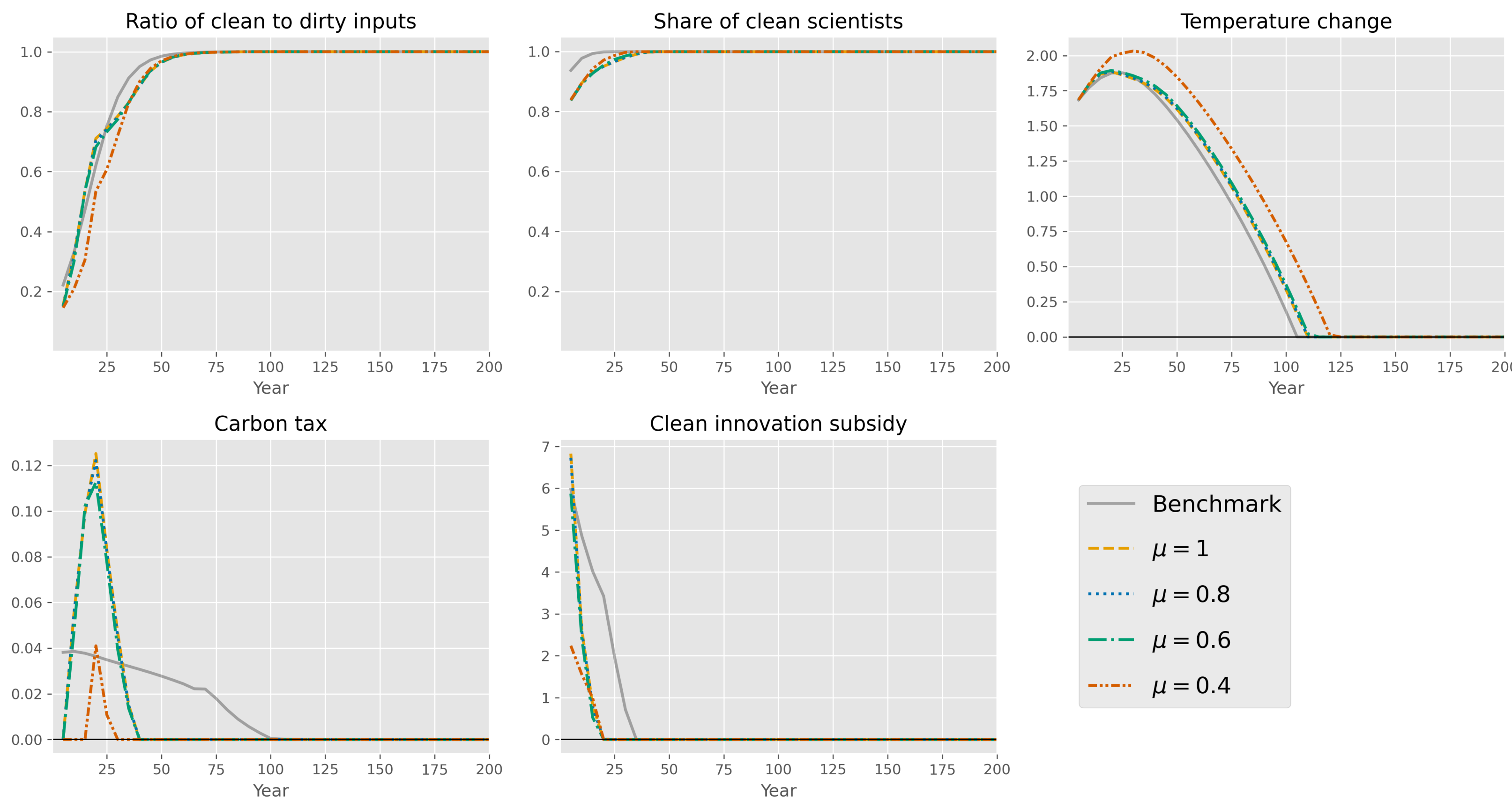

Figure A3: Simulation results for different caps on the price of dirty machines, i.e., μ, when $\epsilon = 10$. Benchmark refers to scenario without efficiency losses in the carbon tax, τ, and clean innovation subsidy, q.