0.1 Induction

Mathematical induction is a proof technique mainly used to prove results about natural numbers.

The need for induction

Have a look at this formula.
$$\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$

How could you go about proving it?

For starters, you might verify the formula, plugging in a few values for n and k. Then you'll get
$$n = 3, \qquad 1 + 2 + 3 = 6 = \frac{3(3+1)}{2}$$

How about $n = 4$?
$$n = 4, \qquad 1 + 2 + 3 + 4 = 10 = \frac{4(4+1)}{2}$$

Indeed, you could work through the calculations for a bunch of other n:s.

However, your task was to prove it for every possible n. This would mean performing the same calculation an infinite number of times, which clearly isn't feasible.

The main idea

The following quote captures the main idea behind mathematical induction.

> *Mathematical induction proves that we can climb as high as we like on a ladder, by proving that we can climb onto the bottom rung (the basis) and that from each rung we can climb up to the next one (the step).*

> - R. Graham, D. Knuth, O. Patashnik, *Concrete Mathematics*

In practise

Inductive proofs all follow the same template.

Guidelines

Here are some practical guidelines.

As a rule of thumb, whenever you're asked to prove a result about natural numbers, consider an inductive proof. Induction isn't the only proof technique out there, but it might be your best bet.