## 0.1 Inverse transform sampling

How do you generate a random sample? Hey, importing a library counts as cheating. Enter inverse transform sampling.

*Theorem* 1 (Inverse transform sampling)*.* To generate a random sample $x_1, \ldots, x_n$, generate $u_1, \ldots, u_n$ from U$(0, 1)$ and let $x_i = F_X^{-1}(u_i)$ for $i = 1, \ldots, n$.

There are two ways to understand this result.

### The CDF proof

*Proof.* If we can show that the random variable $Y = F_X^{-1}(U)$ has the same CDF as $X$, we're done. And as it turns out,

$$F_Y(y) = P(F_X^{-1}(U) < y) = P(U < F_X(y)) = F_X(y)$$

Here, we've used the fact that the CDF is monotonic and increasing. □

### The transformation proof

*Proof.* In this proof, we shall transform the random variable $Y = F_X^{-1}(U)$ and show that its PDF is $f_x(y)$.

$$f_Y(y) = f_U(F_X(y)) \cdot \frac{dF_X}{dy} = f_X(y)$$

□