TRIGONOMETRY

Radians

Rd	Degrees
π	180
x	y

$\rightarrow 180 \cdot x = \pi \cdot y$

General notes

- Angles are measured anticlockwise
- coterminal angles are those that share a side
  - $360 + \theta$
  - $\theta - 360$

Arcs and sectors

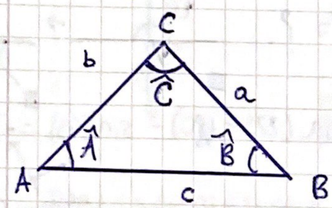
length (ℓ)

$\ell = 2\pi \frac{y^\circ}{360} r$

$\ell = \theta r \quad \rightarrow \cancel{2\pi} \frac{x}{\cancel{2\pi}} r$

Area $= \frac{1}{2} r^2 \theta \quad \rightarrow \frac{\cancel{\pi} r^2 x}{2\cancel{\pi}}$

Triangles



$\frac{S\ O}{H} \quad \frac{C\ A}{H} \quad \frac{T\ O}{A}$ — only right angle triangles

sine rule $\frac{\sin(\hat{A})}{a} = \frac{\sin(\hat{B})}{b} = \frac{\sin(\hat{C})}{c}$

cosine rule $\cos(\hat{C}) = \frac{a^2+b^2-c^2}{2ab}$

Area $= \frac{1}{2} ab \sin(\hat{C})$

any triangle

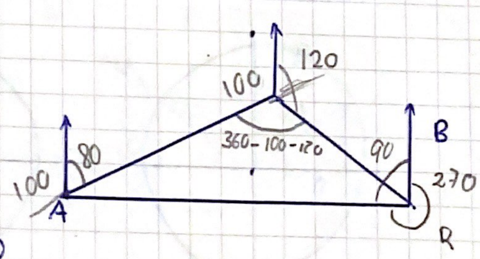
Bearings

- They are angles measured clockwise from north
- They are given in 3 figures: $30^\circ \rightarrow 030$

Remember

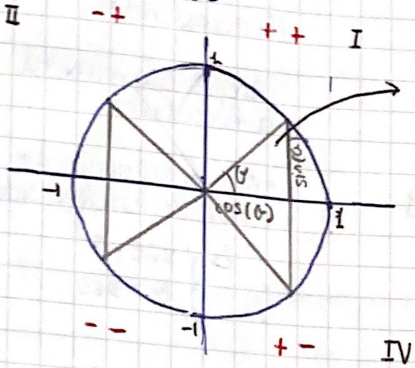
- Complete angle 360
  flat 180

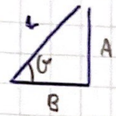
  if the bearing is cut by horizontal line $\rightarrow 90$



- When lines are extended if cut again by bearing same angle on both sides
- A from B means angle of B ... [R]

The unit circle





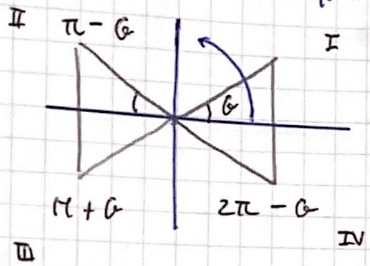
$A \to \sin(\theta) = \frac{O}{H}$

$\sin(\theta) = \frac{opp}{1} = opp$

$B \to \cos(\theta) = \frac{adj}{H} = \frac{adj}{1} = adj$

↳ here, x is positive (cos) whereas y negative (sin)

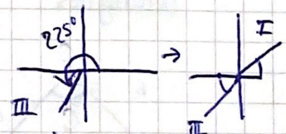
Relationship between quadrans



Angles are counted anticlockwise (α)
if clockwise $-\alpha$

⟶ Example

$\sin(225) = ?$



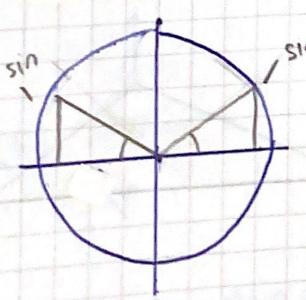
$\sin(225) = \sin(225 - 180) = \sin(45) = -\frac{\sqrt{2}}{2}$

now we add − (sin is − in III)

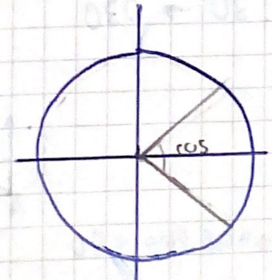
Value chart

Angle	0	30	45	60	90
Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef

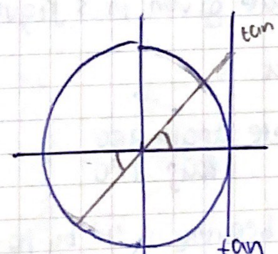
Remember



same sine



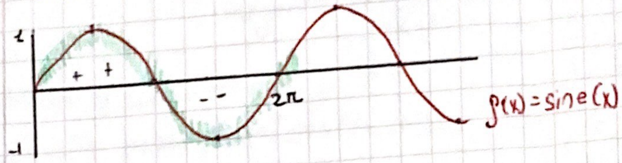
same



same tan

Trigonometric function graphs

Sine



standard

- Repeated from every 2π
- Periodic (period)
- signs coincide with h unit circle

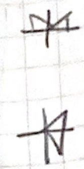
sine ODD

· $\sin(-x) = -\sin(x)$

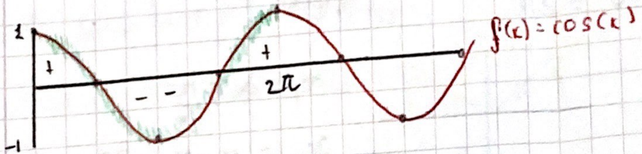
cosine Even

$\cos(-x) = \cos(x)$

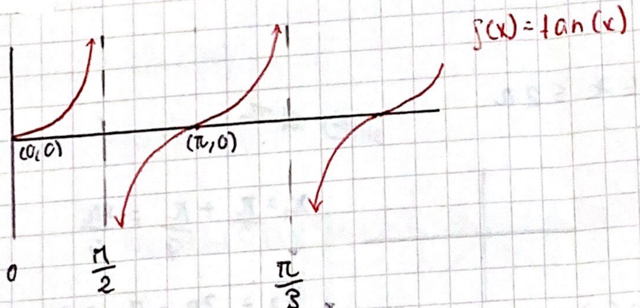
think of u.c



cosine



Tangent



TRANSFORMATIONS

(This is just to express them as a generic formula)

$$f(x) = a\sin(bx - d) + e$$

Amplitude

$$\frac{max - min}{2}$$

Frequency/cycles

$$\text{Period} = \frac{360}{\#\,cycles}$$

principle axis (moved)

$$\frac{max + min}{2}$$

to turn to cosine →

$$-\left(-\frac{\pi}{2}\right) = +\frac{\pi}{2}$$

Trigonometric identities

$1 = \sin^2\theta + \cos^2\theta$

$\mathrm{cosec}^2 = 1 + \cot^2\theta$

$\sec^2\theta = \tan^2\theta + 1$

$\sin 2\theta = 2\cos\theta\sin\theta$

$\cos 2\theta = 1 - \sin\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

NOTE

$\sec\theta = \dfrac{1}{\cos\theta}$

$\mathrm{cosec}\,\theta = \dfrac{1}{\sin\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot an = \dfrac{\cos\theta}{\sin\theta}$

solving trigonometric equations

Example

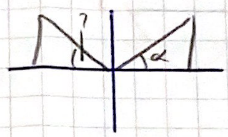
A $\sin(x) = \frac{\sqrt{3}}{2}$

Because of the chart, we know

$\sin(60) = \frac{\sqrt{3}}{2}$ $\quad 0 \le x \le 2\pi \quad$ sin is (+) so I II

$\alpha = 60 \to \frac{\pi}{3}$ rd.

↑ angle of reference (to first quad)



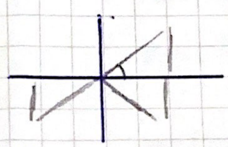
$x_1 = \alpha = \frac{\pi}{3}$

$x_2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

B $\sin(x) = -\frac{\sqrt{2}}{2}$ $\quad 0 \le x \le 2\pi$

sin(θ) IV III

$\alpha = 45 \to \frac{\pi}{4}$

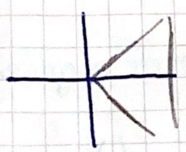


$x_1 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$x_2 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

C $\cos(x) = \frac{1}{2}$ $\quad 0 \le x \le 2\pi$

$\alpha = 60, \frac{\pi}{3}$



$x_1 = \frac{\pi}{3}$

$x_2 = 2\pi - \frac{\pi}{3}.$

If instead of x we have something else the domain also changes

$\frac{\pi}{2} < x < \pi$

$\pi < 2x < 2\pi$

arcsine
arctan I & IV

arccos I & II