EGR 321L – Spring 2024

Design Project 1: Cantilevered Beam

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I understand and have adhered to all the tenets of the Duke Community Standard in completing every part of this assignment. I understand that a violation of any part of the Standard on any part of this assignment can result in failure of this assignment, failure of this course, and/or suspension from Duke University.

Contents

1 Abstract

The objective of this project was to design, model, construct, and test a device to support a 10 lbs load cantilevered 24 inches with a maximum deflection of 1 inch at the loading point. The device was to be fixed using four $\frac{1}{4}$" 20 bolts and support the load via an underhung configuration with an eye bolt and cable assembly. Throughout the design process, various iterations were constructed with careful considerations in regards to material selection, geometry, and attachment methods to meet the specified requirements. These iterations were analyzed using Finite Element Analysis (FEA) to optimize the design for weight and performance. The constructed device successfully supported the 10 lbs load without yielding or deflecting more than 1 inch at the loading point. The device was tested to failure, and the failure load was within 15% of the predicted failure load, demonstrating the accuracy of the team's design calculations. Overall, the project was successful in meeting the design goals and requirements, and the device performed as expected during testing.

2 Design Process

2.1 Design Assumptions and Design Decision

To simplify calculations we assume a homogeneous isotropic material. We also neglected the mass of the channel connector at the end of the beam as its mass is 3.16% of the beam's weight. We did not account for the effect of the weight of the bolts and nuts in the moment of inertia calculations.

2.2 Iterative Design Process

2.2.1 Initial Discussions

Early in the semester, the team met to discuss the project. The team initially discussed the general requirements for the project to clarify that we all had the same understanding of the project and its outcomes. Failure is defined as deflecting over 1 inch. After our initial discussion, we decided that the most efficient beam should deflect less than 1 inch with 10 lbs of loading; however, it should fail soon after, at approximately 15 lbs. This decision was based on the assumption that a material that could withstand more weight would likely weigh more. Next, the team discussed the relevant equations for beam bending, which highlighted the importance of the geometry of the beam and the material.

2.2.2 First Iteration

After our first meeting the team decided to look into using wood as our material. The team found a 2x4 plywood plank and cut it into shape to determine its bending and deflecting capabilities, as well as note the most likely causes of failure of a wooden beam. After putting the beam to size and drilling holes in the correct locations, the team tested the beam by loading it at the end. Although we noted little deflection with the 10 lbs weight, the beam did not deflect much as we increased the weight and could therefore be over-engineered. Further, due to the varying grain patterns of the wood stock, it would be difficult to predict the failure of the beam. Further, the failure would most likely be from the splintering of the beam around the drill holes which would prevent the team from testing the beam multiple times. Further, the weight of the beam was also high, and would therefore not be the most efficient design.

Figure 1: First Iteration

For these reasons, the team decided to pivot away from our original wood design and move into looking at metals for our project. After discussion with the team, aluminum was chosen, as it is lightweight and strong enough to support the desired loading if designed correctly.

2.2.3 Second Iteration

The team decided to take a field trip to Home Depot to test the different cross sections of aluminum to determine which would be best suited for our project. The team quickly found that a simple flat plate of aluminum would deflect too much with minimal loading. The next cross-sections we looked into were a hollow square, an L-beam, and a C-shaped beam. After quickly testing the cross-sections with known loads in Home Depot, the team found that the L-shaped beam bent as it deflected due to the asymmetrical shape of the cross-section. Therefore, we decided against this shape due to the unpredictability of the deflection. The two remaining cross-sections are the C-beam and the hollow square. Although the hollow square was stronger and deflected less with the load, we decided to go with the C-beam as it was significantly lighter and still had a reasonable deflection.

The first iteration of the design of the C-beam consisted of a single rod attached to two of the bolts from the mount, and two L brackets to connect to the other two bolts for the mount. However, we determined that this design deflected over 1 inch at 10 lbs of loading.

2.2.4 Final Iteration

We pivoted once again to have two parallel C-shaped beams each connected to two bolts on the testing mount. These two parallel beams would be connected at the ends by a small thin plate of metal by a bolt. The metal plate would have a hole in the middle to attach the I-hook and hang the load from.

After conducting initial testing with this design the team found that the beam deflected approximately 3/4 of an inch with a load of 10 lbs, and therefore was a successful design. To further improve this design after the beam successfully supported the required load, the team shaved down the size of the end plate connecting the two shafts to reduce the overall weight and the moment at the connecting point, reduce the overall deflection, and improve the efficiency of the beam.

2.3 Final Design

The final design for our beam was constructed out of two parallel 6063 aluminum shafts with a C-shape cross-section. These two parallel shafts were connected at the end with a thin plate of aluminum to hand the load from the i-hook and evenly distribute the load between the two shafts.

Part	Qty.	Weight
C-Channel	2	149g
Plate	1	10g
Bolt	2	3g
Nut	2	1g
TOTAL		**316 g**

Table 1: Beam Weight

The final design weighed 316 grams including the nuts and bolts. It was 28 inches long, 2.5 inches wide, and 0.56 inches tall. The moment of inertia of each of the C-beams was 0.0034 in^4 and the overall moment of inertia of the system was 0.2256 in^4.

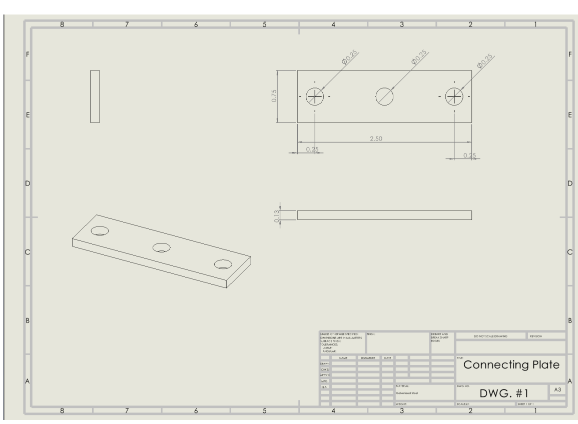

Figure 2: Channel Connector Engineering Drawing

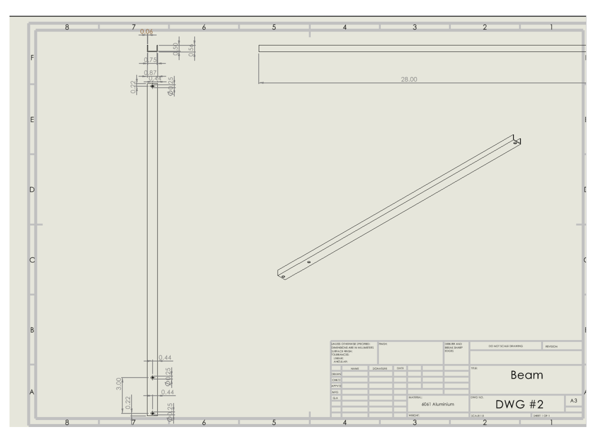

Figure 3: C-Beam Engineering Drawing

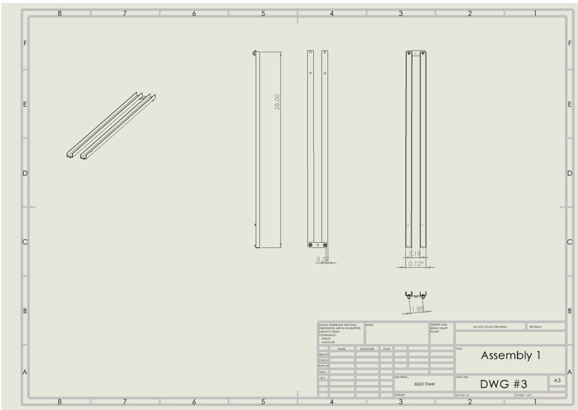

Figure 4: Assembly Engineering Drawing

Figure 5: Photo of Beam

2.3.1 Moment of Inertia Calculation

Due to the connecting plate at the end of the beam, the cross-section of the beam changes along its length. The changing cross-section would influence the moment of inertia along the length of the beam. However, to simplify the calculations the team chose to neglect the weight of the metal plate (**See Section 2.1**).

Following these assumptions the moment of inertia of one of the c-channels can be calculated using equations 1 to 4.

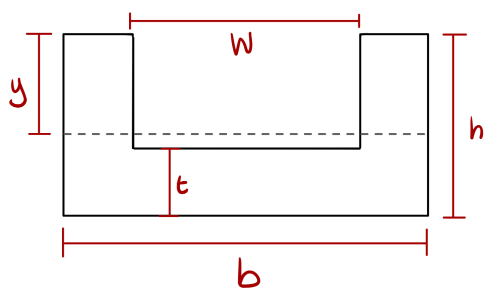

Figure 6: Diagram of Beam Cross-Section

Variable	Symbol	Value
Base Width	b	0.75 in
Thickness	t	0.0625 in
Height	h	0.5625 in
Inner Width	w	0.625 in
Neutral Axis	y	0.3705 in
Cross Sectional Area	A	0.1094 in^2

Table 2: Variables, Symbols, and Values for Calculations

$$y = h - \frac{2h^2t + wt^2}{2bh - 2w(h-t)} = 0.3705 \text{ in} \tag{1}$$

$$A = hb - w(h-t) = 0.1094 \text{ in}^2 \tag{2}$$

$$I_{c-beam} = \frac{2th^3 + wt^3}{3} - A(h-y)^2 = 0.00343613 \text{ in}^4 \tag{3}$$

$$\tag{4}$$

3 Analysis

3.1 FEA Analysis

The team conducted a Finite Element Analysis (FEA) of the beam,to obtain both the experimental maximum deflection and factor of safety. Additionally, the analysis was instrumental in pinpointing potential low stress points of the beam for future refinement. Illustrated in Figure 5 and Figure 6, the analysis revealed a maximum deflection of 0.8207 inches under a 10lbs load, with a factor of safety standing at 1.23. Initial tests unveiled minimal stress occurrences in the connecting plate's outer edges and the free section of the bolts which allowed the team to shave these down effectively decreasing the weight of the beam while maintaining its strength.

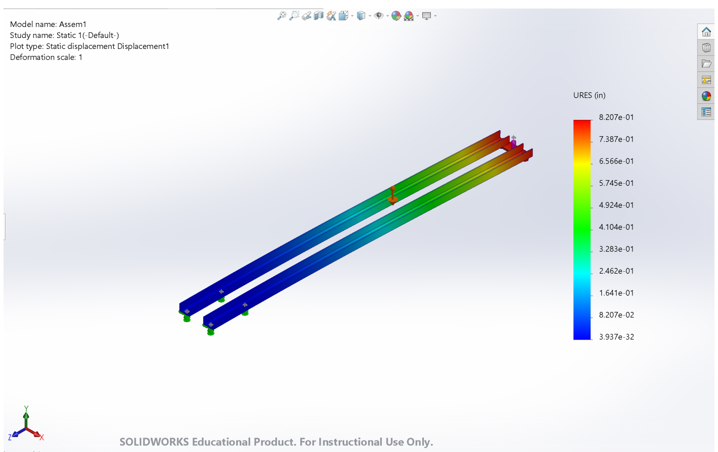

Figure 7: Beam Deflection at 10 lbs

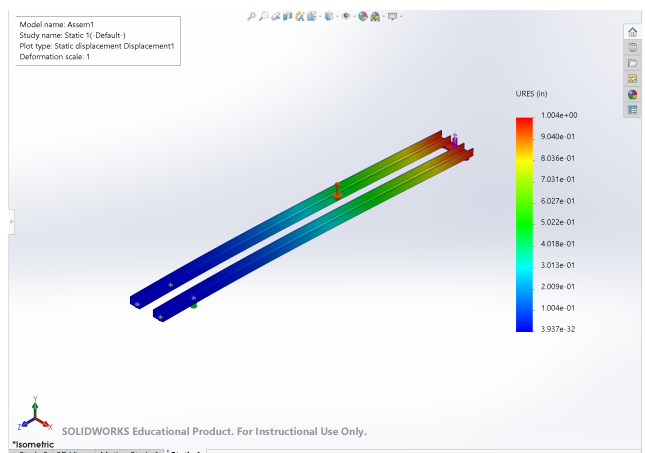

Figure 8: Beam Deflection at 12.3 lbs

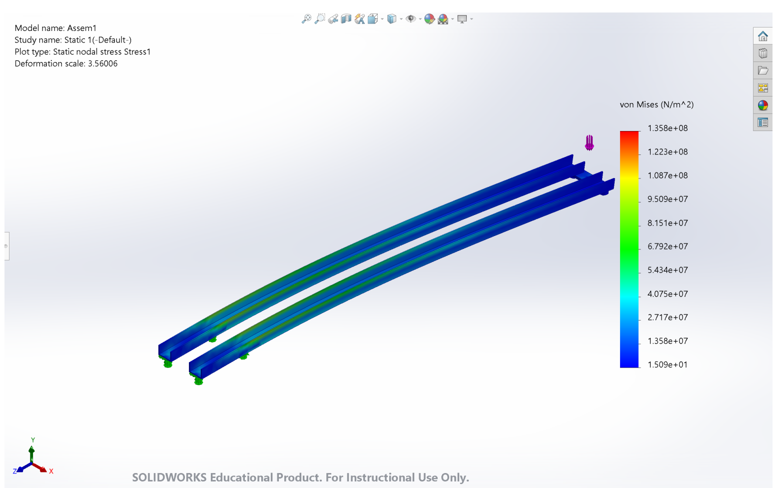

Figure 9: FEA Stress Chart

Based off of the FEA calculations, the failure load - where failure was defined to be deflection of over 1 inch - was found to be 12.3 lbs. This result can be seen in Figure 7.

3.2 Calculations

3.2.1 Superposition Deflection Calculations

Using the principle of superposition, we found that the deflection of the beam would be calculated as follows with equation 5.

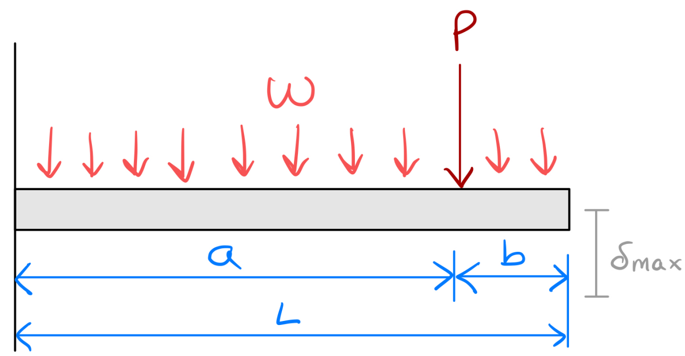

Figure 10: Diagram of Beam Deflection

$$\delta_{max} = \frac{wl^4}{8EI} + \frac{Pa^2}{6EI}(3L - a) \tag{5}$$

where w is the distributed load of the weight of the beam over the entire beams length. This equation can then be rearranged to solve for P as shown in Equation 7.

$$P = \frac{(\delta_{max} - \frac{wL^4}{8EI})(\frac{6EI}{3L-a})}{a^2} \tag{6}$$

Variable	Symbol	Value
Load	P	—
Maximum Deflection	δ_{max}	1 inch
Distributed Weight of Beam	w	0.0114 lbs/in
Length of Cantilevered Beam	L	24.5 inches
Distance to Load	a	24 inches
Young's Modulus	E	10000 ksi
Moment of Inertia	I	0.00343613 in^4

Table 3: Beam Specifications

Since our beam is constructed out of two identical parallel channels, we decided to assume that the failure load can be assumed to be double the failure load one one of the channels. Therefore, the failure load of our beam can be calculated using the equation below.

$$P = 2\left(\frac{(\delta_{max} - \frac{wL^4}{8EI})(\frac{6EI}{3L-a})}{a^2}\right) = 2(7.11286) = 14.25 \text{ lbs} \tag{7}$$

According to our beam dimensions and specifications shown above in Table 1, failure - which we defined as a deflection for 1 inch or over - would occur with an applied force of 14.25 lbs.

To arrive at these calculations we made several guiding assumptions. Firstly, as already stated above, we assumed that the failure load could be assumed to be double the failure load of one of the c-channels as the two beams were identical. In addition, as stated in 2.3.1, we neglected the connecting plate in its calculation, due to its negligible size. We also assumed a uniform thickness of the beam. Further, we assumed all bolt and support forces from the table would act where the beam begins to cantilever, leading the length of the beam being equal to the length of the cantilevered section with the remaining 3.5 inches having negligible effect of the beam deflection calculations.

3.2.2 Failure Mode & Load

To find the experimental load, the team completed five tests in which the beam was weighted at 10 lbs, 15 lbs, and 16 lbs. (**See Table 4**).

Test	**Load (lbs)**	H_0 **(in)**	H **(in)**	δ **(in)**
1	10	36.25	35.50	0.75
2	15	36.25	35.25	1.00
3	15	36.25	36.25	1.00
4	16	36.25	36.125	1.125
5	16	36.25	35.125	1.125

Table 4: Deflection

We predict that the failure mode of our beam will be due to the deflection of the beam over the maximum allowed deflection of 1 inch, as the beam will not fracture due to the loading before this point. We predict that the beam will fail when weighted with 12.3 pounds, based off of the FEA calculations. We chose to use this calculation of failure as it was an extremely accurate simulation of our beam with the correct materials and moment of inertia, whereas to compute the superposition deflection, we had to make many assumptions that would affect the failure load value. In addition, since the way we experimentally tested our failure load will be different from how it will be different from how it will be tested on test day, we decided not to consider this value in our prediction, as it is likely to be inaccurate.

FEA Caluclation	Superposition Calculation	Experimental	Group Predicted Failure Load
12.31 lbs	14.25 lbs	15 lbs	**12.31 lbs**

Table 5: Failure Load Prediction

3.2.3 Factor of Safety

The failure load on test day was found to be 12.5 lbs. The factor of safety was calculated by dividing the failure weight by 10 lbs.

$$FOS = \frac{\text{Failure Strength}}{\text{Applied Stress}} \tag{8}$$

$$FOS_{\text{beam}} = \frac{12.5}{10} = 1.25 \tag{9}$$

4 Conclusion

Through the design of various iterations and testing, the team constructed a beam capable of supporting a 10 lbs load cantilevered 24 inches with a maximum deflection of 1 inch at the loading point. Throughout the project, the team used physical testing in conjunction with multiple analytical tools to justify its design decisions. By carefully considering material selection, geometry, and attachment methods, the team navigated through various design iterations, each time refining the device's structure to optimize weight and performance. Through mathematical calculations including superposition, the factor of safety,and Finite Element Analysis (FEA), the team gained a deeper understanding of beam's deflection.

Due to the assumption of negligible effects from the channel connector, the FEA analysis contains a small margin of error. Furthermore, inaccuracies stem from the calculation of the maximum load at 1 inch, which assumes equal deflection for both beams, potentially leading to inaccuracies.

Regarding future improvements, the team believes that further iterations could be designed with an increased focus on weight and efficiency. After running a finite element analysis, the simulation revealed that there are increased stress concentration levels along the midsection of the beam. As such, one iteration of the beam that the team plans on testing is with circular holes along the initial quarter of the beam in order to lower the beam's weight.

References

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