École Centrale de Nantes

Humanoid Robots

Laboratory Report 3

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1 Introduction

In this third laboratory the goal was to optimise a trajectory of the compass walking robot 1 in order perform an optimal gait. As it is possible to see from the image, the robot is composed by two identical links of length l and mass m, having an inertia I and a distance between the centre of mass G and the hip of s. Furthermore, the robot is walking along a slope, having θ angle, which in this last laboratory will assume a 0 value. So, in this case, the black reference frame R_0, will coincide with the red one. The known quantities that have been used to test the code are the following:

- l = 0.8 m;
- m = 2 Kg;
- g = 9.81 Kg s^{-2};
- $\theta = 2\pi/180$;
- I = 0.05 Kg m^2;
- s = 0.45 m;

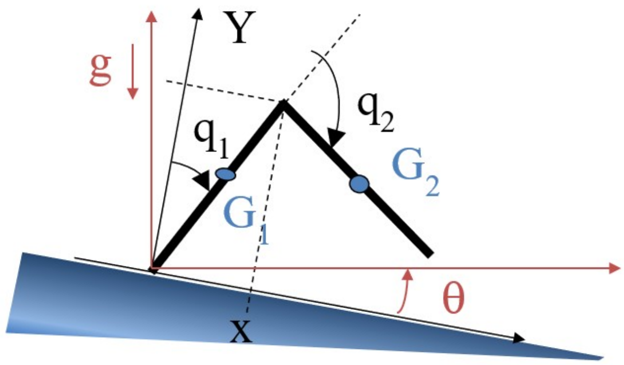

Figure 1: Compass-walking robot

2 Optimal trajectory design

In this first section the vector containing the initial value is built: is noteworthy the fact that in order to define a correct trajectory for the gait, the parameters should be optimised. The vector of initial values contains the position and velocities of the two joints at the beginning and in the middle of the period:

- initial $q_1 = \frac{-10\pi}{180}$;
- initial $\dot{q}_1 = 0$;
- initial $\dot{q}_2 = 0$;
- intermediate $q_1 = 0$;

- intermediate q_2 = -π;

In the function *fmincon*, which finds a constrained minimum of a function of several variables, these values are used, since they are passed to it through the vector *Jsolcons0* that stacks all together these parameters. The minimum of the *resol* has to be found; this passage leads to evaluate several important things, that are required in order to define the constraints for the optimization, such as the values of the trajectory, the reaction forces and the input torques. Analysing in more detail the function, is possible to see that firstly it computes the velocities after the impact using the *Impact Model* equation, defined in the first assignment; these values are extracted since will be useful in order to define the constraints vector. Secondly it is necessary to perform a relabelling of the two links, to obtain a cyclic motion. The equation used for the *Impact Model* are:

$$A(q(T))_{4x4}(\dot{q}^{+} - \dot{q}^{-}) = {J_2}^T I_2 \tag{1}$$

$$J_2\dot{q}^{+} = 0 \tag{2}$$

To evaluate the new state the following formula was applied:

$$z_{new} = E z_{old} - \begin{bmatrix} \pi \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

with matrix E equals to:

$$E = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{4}$$

The new velocities and positions values are then used to compute the coefficients $a_{0i}, a_{1i}, a_{2i}, a_{3i}, a_{4i}$ (for i = 1,2), of the polynomials trajectories. The equation from which the coefficient are computed is:

$$A = T^{-1}b \tag{5}$$

where A:

$$A = \begin{bmatrix} a_{0i} \\ a_{1i} \\ a_{2i} \\ a_{3i} \\ a_{4i} \end{bmatrix} \tag{6}$$

and T, the complete period:

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & \\ \frac{T}{2}^4 & \frac{T}{2}^3 & \frac{T}{2}^2 & \frac{T}{2} & 1 & \\ T^4 & T^3 & T^2 & T & 1 & \\ 4T^3 & 3T^2 & 2T & 1 & 0 & \end{bmatrix} \tag{7}$$

and b:

$$b = \begin{bmatrix} q_{initial} \\ \dot{q}_{initial} \\ q_{intermediate} \\ q_{final} \\ \dot{q}_{final} \end{bmatrix} \tag{8}$$

At this point is possible to perform the parametrisation of the trajectory, using the polynomials:

$$\Phi = a_{0i} + a_{1i}t + a_{2i}t^2 + a_{3i}t^3 + a_{4i}t^4 \tag{9}$$

and its first derivative:

$$\dot{\Phi} = a_{1i} + 2a_{2i}t + 3a_{3i}t^2 + 4a_{4i}t^3 \tag{10}$$

and its second derivative:

$$\ddot{\Phi} = 2a_{2i} + 6a_{3i}t + 12a_{4i}t^2 \tag{11}$$

t is each time instance in the period from 0 to T, with sampling time $\frac{T}{\Delta}$, with Δ equal to 100. Now it is possible to find the values for the input torques τ, using the inverse dynamic model:

$$A(q)\ddot{q} + h(q, \dot{q})\dot{q} + G(q) = D_\tau \tau \tag{12}$$

which in this case is possible to set equal to 0. Moreover, the ground reaction forces $\overrightarrow{R}$ can be found, considering as *2m* the total mass of the compass walking robot.

$$2m\ddot{q} = 2m\overrightarrow{g} + \overrightarrow{R} \tag{13}$$

From now on it is possible to focus on finding the constraint vector; the *constraint* function will use these constraints, that will be passed to the *fmincon* function:

- L $>$ 0.2 m;
- $R_y > 0$;
- $I_y > 0$;
- $|\frac{R_x}{R_y}| < 0.7$;
- $|\frac{I_x}{I_y}| < 0.7$;
- $|\dot{q}| < 3$ rad/s;
- $|\tau_i| < 50$ Nm;

Once this passage was done, it was necessary to optimise the trajectory; to do this task the criterion had to be defined. I used the suggested Sthenic criterion, which represent the transport cost: due to this strategy it was possible to find an optimal trajectory thanks to the optimised parameter found.

$$C = \frac{1}{L_{step}} \int_0^T \tau^T \tau \, dt \tag{14}$$

It was also suggested to evaluate this integral using the backward difference method by using a sampling period of $\Delta = \frac{T}{100}$:

$$C = \frac{1}{L_{step}} \sum_{k=1}^{N=100} \tau^2(k\Delta)\Delta \tag{15}$$

3 Main program

In the **main_optimwalk.m** program, the simulation of the first step of the compass walking robot was simulated, through the *fmincon* function. Firstly the parameters needed for the optimisation were declared and stacked in the *Jsolcons0* vector. Then in a new *Jsolcons0* vector the optimised values were saved after having called the *fmincon* function. Secondly, these new values were used in the *resol* function, and the steps explained in the *Optimal trajectory design* section were performed. Moreover, the after impact velocities were evaluated using the Impact Model and the *function_impact*, developed in the first assignment. Consequently, also the relabeling of the links was performed. Lastly, the polynomial exprpession was found, extracting the coefficients $a_{0i}, a_{1i}, a_{2i}, a_{3i}, a_{4i}$, for i = 1,2. As last step, using the already found coefficients, the angular positions were evaluated for the period from 0 to T, and an animation of the compass walking robot was simulate through the *plot_motion* function 2.

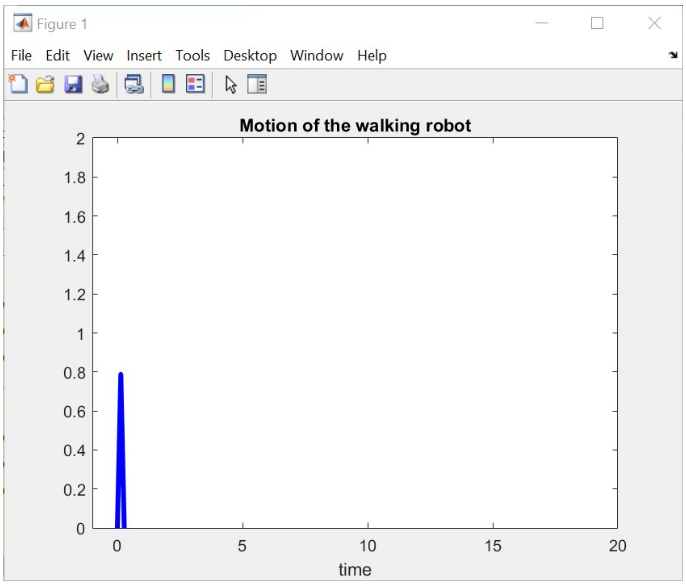

Figure 2: Step simulation of the walking robot

4 Conclusions

The main aim of this laboratory was to find the best solution for the trajectory of the compass walking robot, while respecting some constraints. Despite the time spent working on this lab, the result was not as good as expected, since I had encountered some issues. The found trajectory was not optimal at all, even if the step of the robot ends in the correct position. I could observe in the command window that, during the execution of *fmincon* function, it was unable to respect the constraints imposed by the function *constraint* and, therefore, its execution was always stopped without returning any possible optimized value, and this leads to a complete rotation around the hip during the performance of the step. Here below, I have attached some figures, in order to visualise the problem that I was talking about: the figures illustrate firstly the *q1* and *q2* behaviour 3, and then the *q1d* and *q2d* one 4. It is possible to observe how the first leg has a correct trend in both the images, whereas the second leg creates a weird curve that will leads to the incorrect rotation around the hip. I also attached the plot of the torques 5.

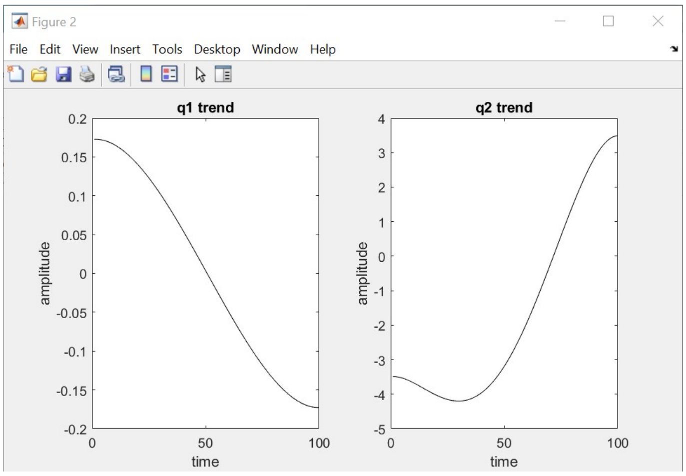

Figure 3: q1, q2

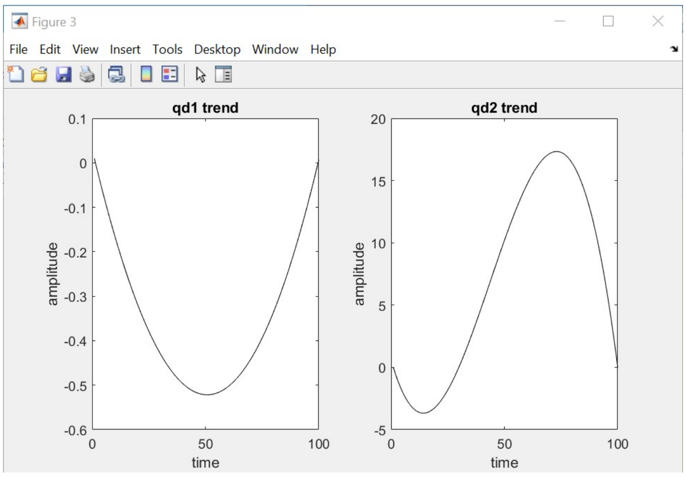

Figure 4: qd1, qd2

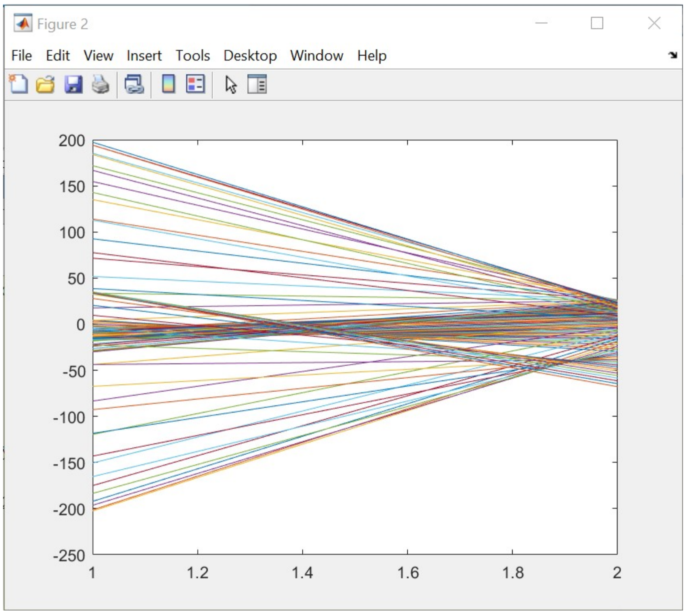

Figure 5: Torques