理论力学 CAI 静力学

- 力
- 力偶
- 力系的简化
- 约束
- 力系的平衡
- 摩擦与摩擦力

力

力

- 力与力系
- 力矩

力

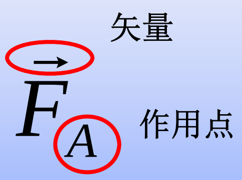
- 力与力系
- 力矩

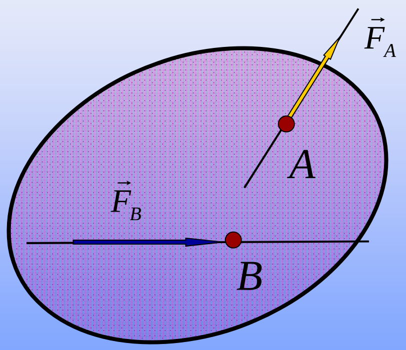
力与力系

- 力的基本性质
- 汇交力系的合成

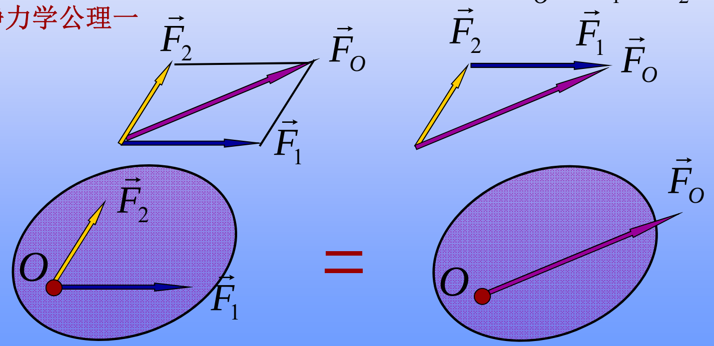
- 力是物体的相互作用
- 力作用的效果
  - 改变物体运动状态
  - 物体变形
    - 极端：破坏

- 力的性质 1
  - 力作用效果取决于力的大小、方向与作用点(三要素)



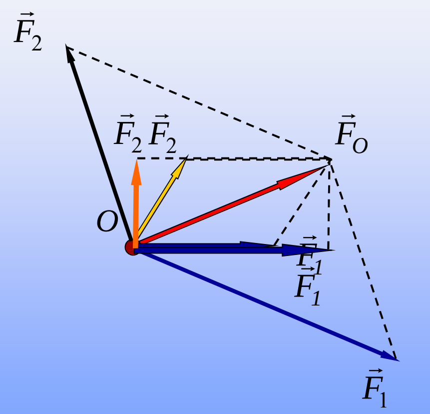


- 力的性质 **2**
  - 同一个作用点两个力的效应可由一个力等效
    - 该力称为两力的合力
    - 合力的作用点为该点
    - 合力的大小与方向符合力的平行四边形法则 $\vec{F}_O = \vec{F}_1 + \vec{F}_2$
    - 静力学公理一



上述性质表明力是一个与作用点关联的矢量

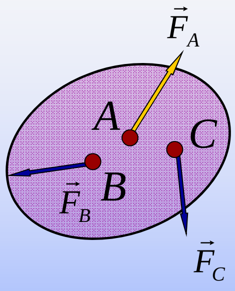
- 力（矢量）的分解



力矢量的分解不唯一

- 若干力的集合称为力系

$$\left(\vec{F}_A, \vec{F}_B, \vec{F}_C\right)$$



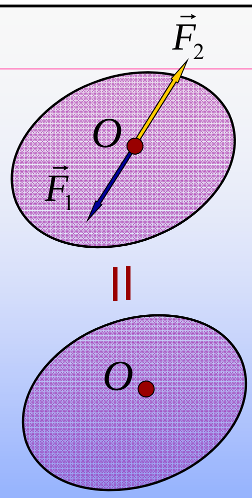
- 特殊情况的力系

$$\left(\vec{F}_1, \vec{F}_2\right)$$

同一作用点，且

$$\vec{F}_1 = -\vec{F}_2$$

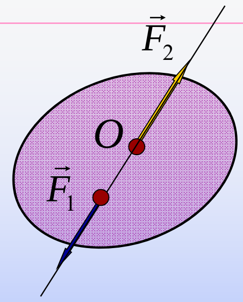
$$\vec{F}_O = \vec{F}_1 + \vec{F}_2 = \vec{0}$$



同一个作用点的两个力大小相等方向相反
其合力为零(矢量)

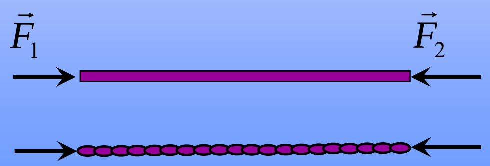
$\left(\vec{F}_1, \vec{F}_2\right)$ **平衡力系**

- 力的性质 3
  - 作用于刚体上两个力使其平衡的充分必要条件是两力处于同一直线上，且大小相等方向相反
  - 静力学公理二



$\left(\vec{F}_1, \vec{F}_2\right)$ 平衡力系

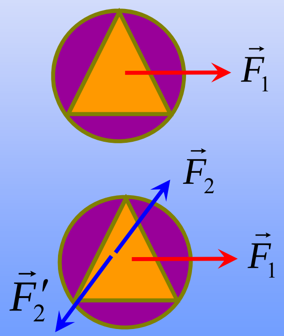
$\vec{F}_1 = -\vec{F}_2$



刚体

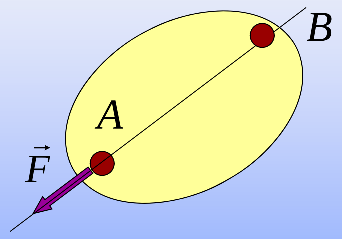
柔索？

- 力的性质 4
  - 在一个力系上加减一个平衡力系不改变原力系对刚体的作用效果
  - 静力学公理三

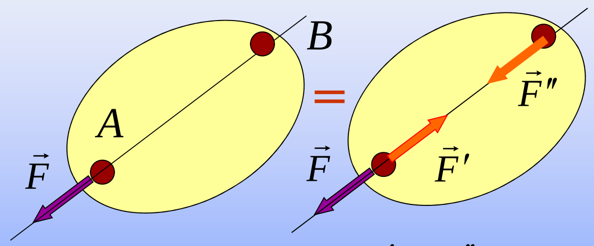


$(\vec{F}_2, \vec{F}_2')$ 平衡力系

力对刚体的作用的可传性

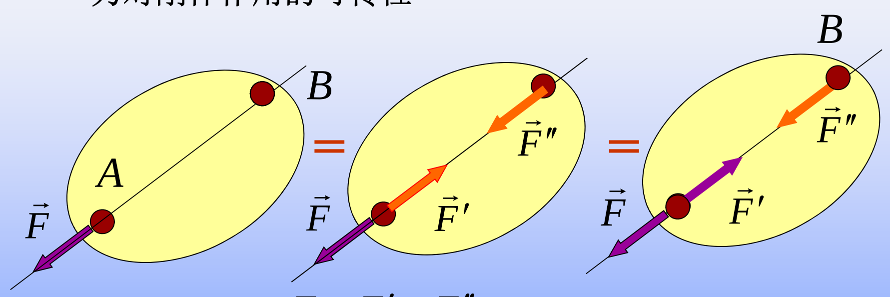


- 力对刚体的作用
  - 力对刚体作用的可传性



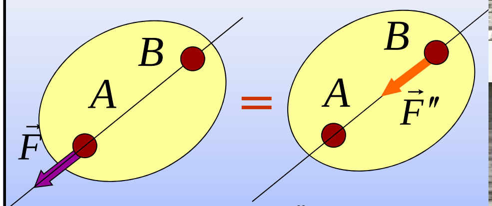
$$F = F' = F''$$

- 力对刚体的作用
  - 力对刚体作用的可传性

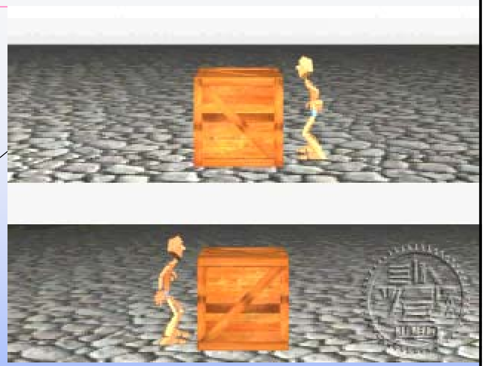


$$F = F' = F''$$

- 力对刚体作用的效果取决于力的大小、方向与作用线
- 对于刚体力是滑移矢量



$$F = F''$$



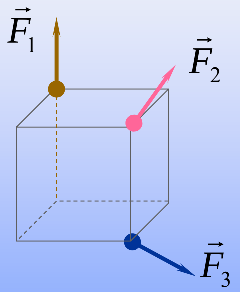
- 力对刚体作用的效果取决于力的大小、方向与作用线
- 对于刚体力是滑移矢量

- 力的性质 5
  - 作用力与反作用力同时存在
    - 大小相等方向相反
    - 沿同一条作用线
    - 作用在不同的物体上

力与力系

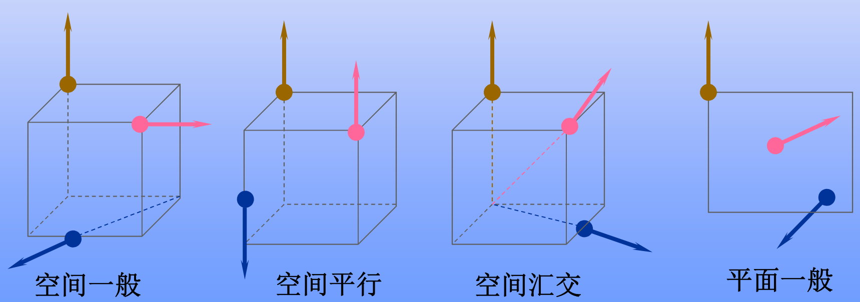
- 力的基本性质
- 汇交力系的合成

- 若干力的集合称为力系



$$(\vec{F}_1, \vec{F}_2, \vec{F}_3)$$

- 力系及其分类
  - ***n***个力的集合构成力系 $(\vec{F}_1, \vec{F}_2, \cdots, \vec{F}_n)$
  - 分类
    - 空间力系/平面力系
    - 汇交力系/平行力系/一般力系



- 力系及其分类
  - ***n***个力的集合构成力系　$(\vec{F}_1,\vec{F}_2,\cdots,\vec{F}_n)$
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    - 空间力系/平面力系
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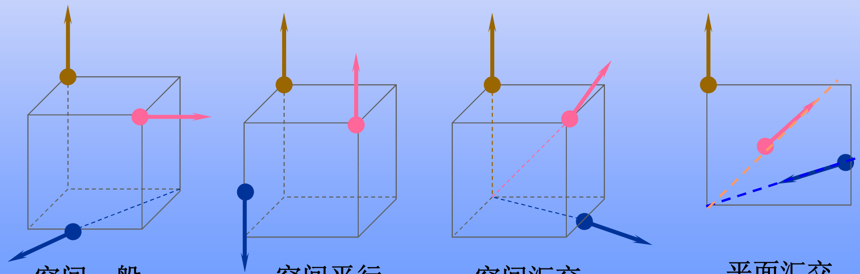

空间一般　空间平行　空间汇交　平面汇交

- 力系及其分类
  - ***n***个力的集合构成力系　$(\vec{F}_1,\vec{F}_2,\cdots,\vec{F}_n)$
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    - 空间力系/平面力系
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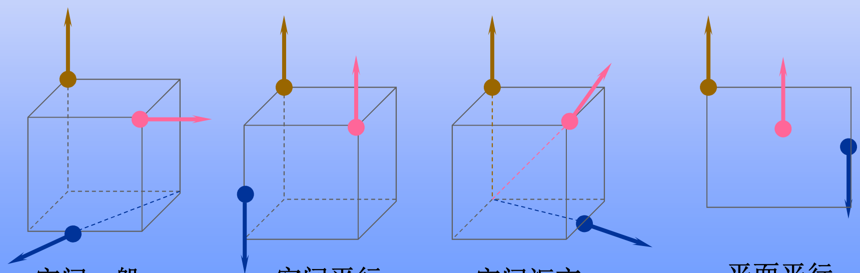
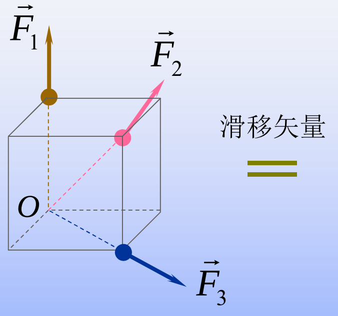
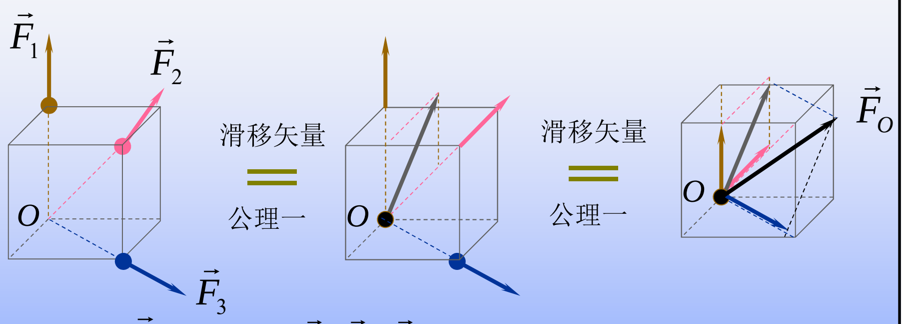


空间一般　空间平行　空间汇交　平面平行

汇交力系的合成



汇交力系的合成

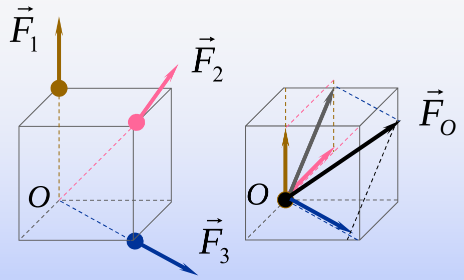


$\vec{F}_O$ 与力系 $(\vec{F}_1,\vec{F}_2,\vec{F}_3)$ 作用效果相同

$$\vec{F}_O = (\vec{F}_1,\vec{F}_2,\vec{F}_3)$$

$\vec{F}_O$ 汇交力系 $(\vec{F}_1,\vec{F}_2,\vec{F}_3)$ 的合力

- 汇交力系的合力
  - 作用点为汇交点
  - 大小与方向：
    所有力的矢量和

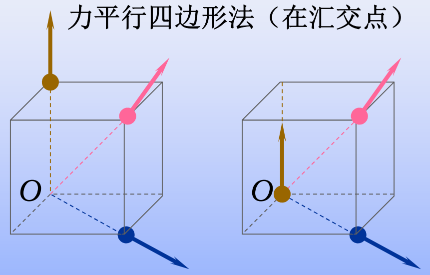


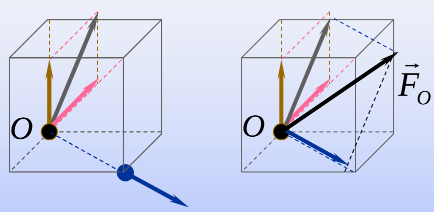
$$\vec{F}_O = \sum_{i=1}^{n} \vec{F}_i \qquad \vec{F}_O = (\vec{F}_1, \vec{F}_2, \vec{F}_3)$$

- 汇交力系的合力为定位矢量

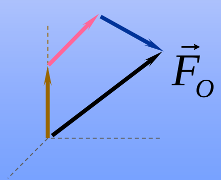
- 汇交力系的合力的计算
  - 几何法

力平行四边形法（在汇交点）





力多边形法（以汇交点为起点）



汇交力系的合力是由力多边形的起点（汇交点）指向终点

力多边形法

- 汇交力系的合力的计算
  - 解析法

定义参考基 $\vec{e} = (\vec{x} \quad \vec{y} \quad \vec{z})^{\mathrm{T}}$

$\boldsymbol{F}_O = (F_{Ox} \quad F_{Oy} \quad F_{Oz})^{\mathrm{T}}$

$\boldsymbol{F}_i = (F_{ix} \quad F_{iy} \quad F_{iz})^{\mathrm{T}}$

合力计算的坐标式

$$\vec{F}_O = \sum_{i=1}^{n} \vec{F}_i \Rightarrow \boldsymbol{F}_O = \sum_{i=1}^{n} \boldsymbol{F}_i \Rightarrow \begin{pmatrix} F_{Ox} \\ F_{Oy} \\ F_{Oz} \end{pmatrix} = \sum_{i=1}^{n} \begin{pmatrix} F_{ix} \\ F_{iy} \\ F_{iz} \end{pmatrix}$$

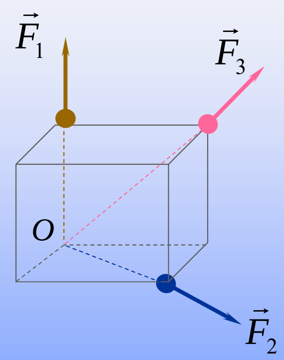
$$F_{Ox} = \sum_{i=1}^{n} F_{ix} \qquad F_{Oy} = \sum_{i=1}^{n} F_{iy} \qquad F_{Oz} = \sum_{i=1}^{n} F_{iz}$$

汇交力系合力的坐标等于力系各力对应坐标的代数和

[例]

汇交力系（$\vec{F}_1 \quad \vec{F}_2 \quad \vec{F}_3$）的作用点在边长**2m**的正六面体相应的顶点上方向如图，力的大小

$$F_1 = 3\,\mathrm{N}$$
$$F_2 = \sqrt{2}\,\mathrm{N}$$
$$F_3 = 2\sqrt{2}\,\mathrm{N}$$



求: 合力的大小与指向

[解] $F_1 = 3\,\mathrm{N}$ $F_2 = \sqrt{2}\,\mathrm{N}$ $F_3 = 2\sqrt{2}\,\mathrm{N}$

设定参考基 $\vec{\boldsymbol{e}} = (\vec{x} \quad \vec{y} \quad \vec{z})^{\mathrm{T}}$

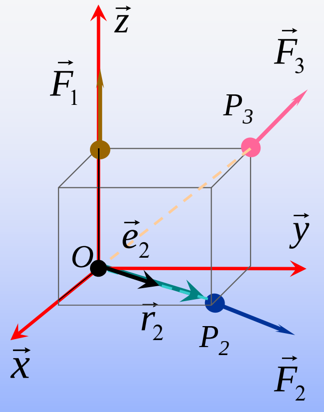
写各力的坐标阵

$\vec{F}_1 = F_1\vec{z} = 3\vec{z}$ $\boldsymbol{F}_1 = 3\boldsymbol{z} = 3(0 \quad 0 \quad 1)^{\mathrm{T}}$

$\vec{F}_2 = F_2\vec{e}_2 = \sqrt{2}\vec{e}_2$ $\boldsymbol{F}_2 = \sqrt{2}\boldsymbol{e}_2 = (1 \quad 1 \quad 0)^{\mathrm{T}}$

$$\vec{e}_2 = \frac{\vec{r}_2}{r_2} \qquad \boldsymbol{e}_2 = \frac{\boldsymbol{r}_2}{r_2} = \frac{1}{2\sqrt{2}}\begin{pmatrix}2\\2\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$$

$\boldsymbol{r}_2 = (2 \quad 2 \quad 0)^{\mathrm{T}}$ $r_2 = \sqrt{2^2 + 2^2} = 2\sqrt{2}$



边长2m的
正六面体

[解] $F_1 = 3\,\mathrm{N}$ $F_2 = \sqrt{2}\,\mathrm{N}$ $F_3 = 2\sqrt{2}\,\mathrm{N}$

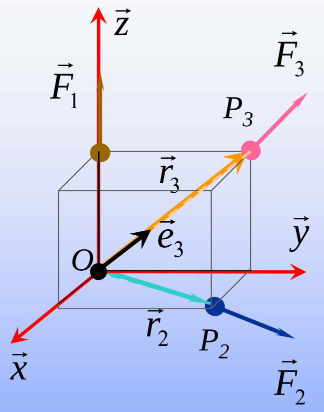
$\vec{F}_1 = F_1\vec{z} = 3\vec{z}$ $\boldsymbol{F}_1 = 3\boldsymbol{z} = 3(0 \quad 0 \quad 1)^{\mathrm{T}}$

$\vec{F}_2 = F_2\vec{e}_2 = \sqrt{2}\vec{e}_2$ $\boldsymbol{F}_2 = \sqrt{2}\boldsymbol{e}_2 = (1 \quad 1 \quad 0)^{\mathrm{T}}$

$\vec{F}_3 = F_3\vec{e}_3 = 2\sqrt{2}\vec{e}_3$ $\boldsymbol{F}_3 = 2\sqrt{2}\boldsymbol{e}_3 = 2(0 \quad 1 \quad 1)^{\mathrm{T}}$

$$\vec{e}_3 = \frac{\vec{r}_3}{r_3} \qquad \boldsymbol{e}_3 = \frac{\boldsymbol{r}_3}{r_3} = \frac{1}{2\sqrt{2}}\begin{pmatrix}0\\2\\2\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}$$

$\boldsymbol{r}_3 = (0 \quad 2 \quad 2)^{\mathrm{T}}$ $r_3 = \sqrt{2^2 + 2^2} = 2\sqrt{2}$



边长2m的
正六面体

设定参考基 $\vec{e} = (\vec{x} \quad \vec{y} \quad \vec{z})^{\mathrm{T}}$

写各力的坐标阵 $\boldsymbol{F}_1 = 3(0 \quad 0 \quad 1)^{\mathrm{T}}$

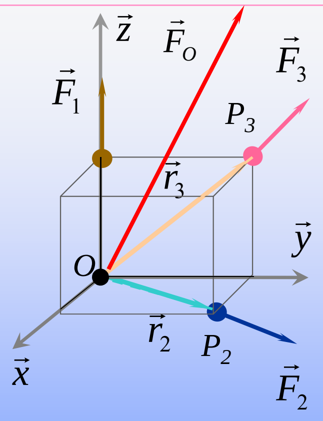
$\boldsymbol{F}_2 = (1 \quad 1 \quad 0)^{\mathrm{T}}$

$\boldsymbol{F}_3 = 2(0 \quad 1 \quad 1)^{\mathrm{T}}$

求合力的坐标阵

$$\boldsymbol{F}_O = \sum_{i=1}^{n} \boldsymbol{F}_i$$

$$= \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 = 3\begin{pmatrix}0\\0\\1\end{pmatrix} + \begin{pmatrix}1\\1\\0\end{pmatrix} + 2\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}1\\3\\5\end{pmatrix}$$



合力坐标 F_{ox}=1N F_{oy}=3N F_{oz}=5N

合力大小 $F_O = \sqrt{F_{Ox}^2 + F_{Oy}^2 + F_{Oz}^2} = \sqrt{35}\ \mathrm{N}$

参考基 $\vec{e} = (\vec{x} \quad \vec{y} \quad \vec{z})^{\mathrm{T}}$

合力坐标 F_{ox}=1N F_{oy}=3N F_{oz}=5N

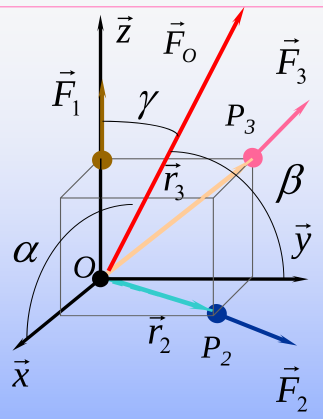
合力大小 $F_O = \sqrt{F_{Ox}^2 + F_{Oy}^2 + F_{Oz}^2} = \sqrt{35}\ \mathrm{N}$

合力方向

$$\alpha = \angle(F_O, \vec{x}) = \arccos(F_{Ox} / F_O) = 80.27^{\circ}$$

$$\beta = \angle(F_O, \vec{y}) = \arccos(F_{Oy} / F_O) = 59.53^{\circ}$$

$$\gamma = \angle(F_O, \vec{z}) = \arccos(F_{Oz} / F_O) = 32.31^{\circ}$$



$$F_O \cos\alpha = F_{Ox}$$

力与力系小结

- 力的性质
- 力对刚体的作用
  - 可传性，滑移矢量
- 力系的分类
- 汇交力系的合力
  - 几何矢量计算
  - 坐标阵计算

$$\vec{F}_O = \vec{F}_R = \sum_{i=1}^{n} \vec{F}_i$$

$$\boldsymbol{F}_O = \boldsymbol{F}_R = \sum_{i=1}^{n} \boldsymbol{F}_i$$

力

- 力与力系
- 力矩

力矩

- 力矩的定义
- 力矩的计算
- 平面问题

力矩的定义：对点的矩

力的转动效应，与力的大小及力到转轴的距离有关

力矩的定义：对轴的矩

力的转动效应，与力的大小及力到转轴的距离有关

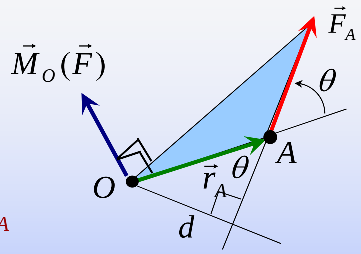
- 力对点*O* 的矩
  - 涉及力与点的一个数学计算量

    力 $\vec{F}_A$　　点O　矩心

    作用点 A　　相对矩心的矢径 $\vec{r}_A$
  - 定义

    力$\vec{F}_A$对点O的矩　$\vec{M}_O(\vec{F}_A) \overset{\text{def}}{=} \vec{r}_A \times \vec{F}_A$
  - 特征
    - 矩矢量与矩心*O*绑定
    - 垂直于力与该力作用点矢径构成的平面
    - 方向右手法则　$\vec{r}_A$　$\vec{F}_A$　$\vec{M}_O(\vec{F}_A)$
    - 大小

$$M_O(\vec{F}_A) \overset{\text{def}}{=} \left|\vec{M}_O(\vec{F}_A)\right| = F_A r_A \sin\theta \ = F_A d$$



力矩大小等于力的模与矩心到该力作用线的距离之积

- 力对点O 的矩

$$\vec{M}_O(\vec{F}_A)\overset{\text{def}}{=}\vec{r}_A\times\vec{F}_A$$

力矩方向

垂直于力$\vec{F}$ 与矢径$\vec{r}$ 构成的平面

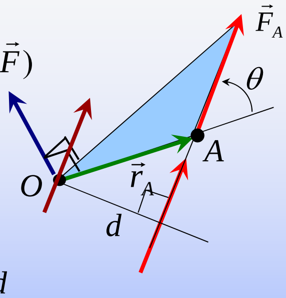
$\vec{r}$ $\vec{F}$ $\vec{M}_O(\vec{F})$ 右手法则

力矩大小

$$M_O(\vec{F})\overset{\text{def}}{=}\left|\vec{M}_O(\vec{F})\right|=Fr\sin\theta=Fd$$

作用线过矩心的力的力矩大小为零

？力在作用线上滑移，力矩大小会改变吗



- 力矩的坐标计算式

$$\vec{M}_O(\vec{F})\overset{\text{def}}{=}\vec{r}\times\vec{F}$$

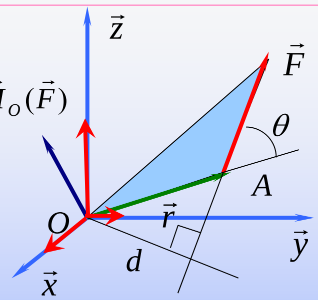
参考基 $\boldsymbol{e}=(\vec{x}\quad\vec{y}\quad\vec{z})^{\mathrm{T}}$

力矩矩阵表达式 $\boldsymbol{M}_O(\vec{F})=\tilde{\boldsymbol{r}}\boldsymbol{F}$

$$\boldsymbol{M}_O(\vec{F})=\left(M_{Ox}(\vec{F})\quad M_{Oy}(\vec{F})\quad M_{Oz}(\vec{F})\right)^{\mathrm{T}}$$

$$\boldsymbol{r}=\left(x\quad y\quad z\right)^{\mathrm{T}}\quad \boldsymbol{F}=\left(F_x\quad F_y\quad F_z\right)^{\mathrm{T}}$$

$$\begin{pmatrix}M_{Ox}(\vec{F})\\M_{Oy}(\vec{F})\\M_{Oz}(\vec{F})\end{pmatrix}=\begin{pmatrix}0&-z&y\\z&0&-x\\-y&x&0\end{pmatrix}\begin{pmatrix}F_x\\F_y\\F_z\end{pmatrix}$$



$$M_{Ox}(\vec{F})=yF_z-zF_y$$

$$M_{Oy}(\vec{F})=zF_x-xF_z$$

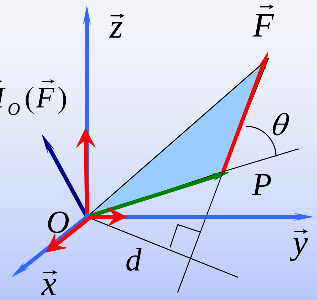
$$M_{Oz}(\vec{F})=xF_y-yF_x$$

- 力对轴的矩

参考基 $\vec{e} = (\vec{x} \quad \vec{y} \quad \vec{z})^{\mathrm{T}}$

$$\vec{M}_O(\vec{F}) \overset{\text{def}}{=} \vec{r} \times \vec{F} \Leftrightarrow \boldsymbol{M}_O(\vec{F}) = \tilde{\boldsymbol{r}}\boldsymbol{F}$$

$$\boldsymbol{M}_O(\vec{F}) = \left(M_{Ox}(\vec{F}) \quad M_{Oy}(\vec{F}) \quad M_{Oz}(\vec{F})\right)^{\mathrm{T}}$$



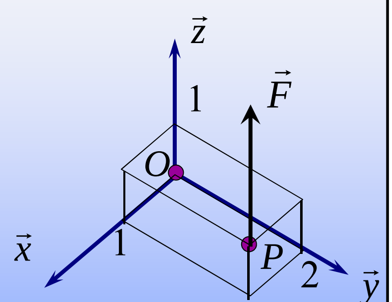
$M_{Ox}(\vec{F})$ 力对***Ox***轴的矩

$M_{Oy}(\vec{F})$ 力对***Oy***轴的矩

$M_{Oz}(\vec{F})$ 力对***Oz***轴的矩

力对轴的矩是标量，等于力对轴上任意点的矩在该轴上的投影，描述力对该轴的转动效应

[例]
如图所示，一3N的力 平行于***Oz***轴，
作用点***P***的坐标为 (1，2，1)m。

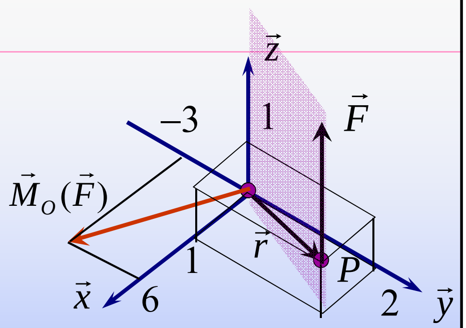


求:该力对点***O***的矩与对三轴的矩

[解] 力 与点***P***的矢径 的坐标阵

$$\boldsymbol{F} = \begin{pmatrix}0 & 0 & 3\end{pmatrix}^{\mathrm{T}}\ \mathrm{N}$$

$$\boldsymbol{r} = \begin{pmatrix}1 & 2 & 1\end{pmatrix}^{\mathrm{T}}\ \mathrm{m}$$



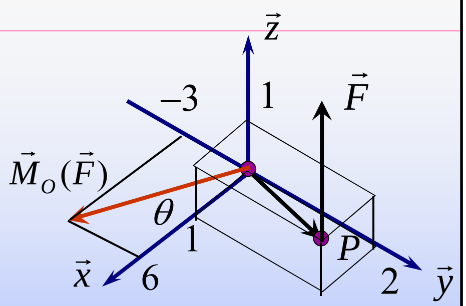
力 对点***O***矩的坐标阵

$$\boldsymbol{M}_O(\vec{F}) = \tilde{\boldsymbol{r}}\boldsymbol{F} = \begin{pmatrix}0 & -1 & 2\\ 1 & 0 & -1\\ -2 & 1 & 0\end{pmatrix}\begin{pmatrix}0\\0\\3\end{pmatrix} = \begin{pmatrix}6\\-3\\0\end{pmatrix}\ \mathrm{Nm}$$

力对***Ox***、***Oy***与***Oz***轴的矩

$$M_{Ox}(\vec{F}) = 6\,\mathrm{Nm} \qquad M_{Oy}(\vec{F}) = -3\,\mathrm{Nm} \qquad M_{Oz}(\vec{F}) = 0\ \mathrm{Nm}$$

$$\boldsymbol{M}_O(\vec{F}) = \begin{pmatrix}6\\-3\\0\end{pmatrix} \qquad \begin{aligned} M_{Ox}(\vec{F}) &= 6\\ M_{Oy}(\vec{F}) &= -3\\ M_{Oz}(\vec{F}) &= 0\end{aligned}$$



力 对点***O***矩的模

$$M_O(\vec{F}) = \sqrt{M_{Ox}^2(\vec{F}) + M_{Oy}^2(\vec{F}) + M_{Oz}^2(\vec{F})} = 6.71\ \ \text{N-m}$$

力 对点***O***矩的指向

$$\theta = \arccos(M_{Ox}(\vec{F})/M_O(\vec{F})) = 26.60^{\circ}$$

[例]

如图所示，一3N的力 平行于**Oz**轴，
作用点**P**的坐标为 (1，2，1)m。

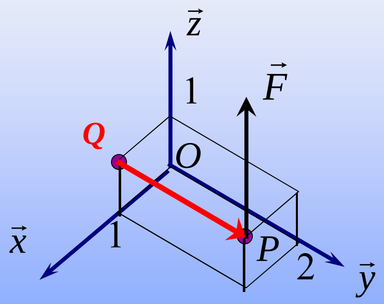
求：（1）该力对点**Q**的矩；

（2）对**y**轴的矩。

$$\underline{F} = \begin{pmatrix} 0 & 0 & 3 \end{pmatrix}^{\mathrm{T}} \mathrm{N}$$

$$\underline{r} = \begin{pmatrix} 0 & 2 & 0 \end{pmatrix}^{\mathrm{T}} \mathrm{m}$$

$$\underline{M}_Q\left(\vec{F}\right) = \begin{pmatrix} 6 & 0 & 0 \end{pmatrix}^{\mathrm{T}} N \cdot m$$

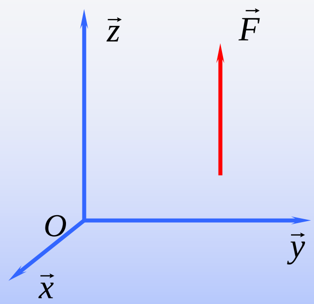


Q不是**xyz**轴上的点，不能由此得到对轴的矩

- 力对轴的矩的推论 1

$\vec{F} // \vec{z}$ $\qquad$ $\boldsymbol{F} = \begin{pmatrix} 0 & 0 & F_z \end{pmatrix}^{\mathrm{T}}$

$$M_{Oz}(\vec{F}) = xF_y - yF_x = 0$$



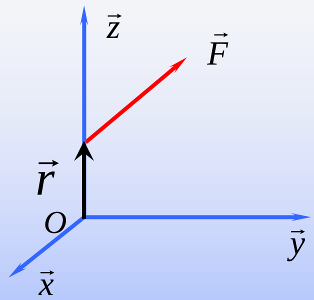
力对平行于其的轴的矩为零

- 力对轴的矩的推论 2

$$\boldsymbol{F} = (F_x \quad F_y \quad F_z)^{\mathrm{T}}$$

$$r = (\overset{0}{x} \quad \overset{0}{y} \quad z)^{\mathrm{T}}$$

$$M_{Oz}(\vec{F}) = \underline{x}F_y - \underline{y}F_x = 0$$

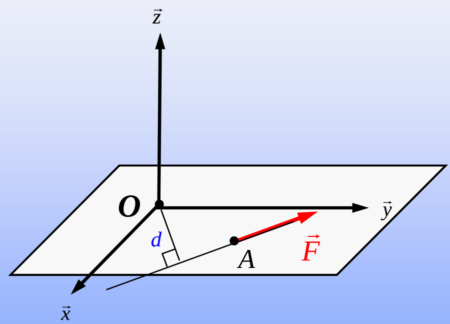


力的作用线与轴相交，对轴的矩为零

- 力对轴的矩推论 3

力与轴垂直

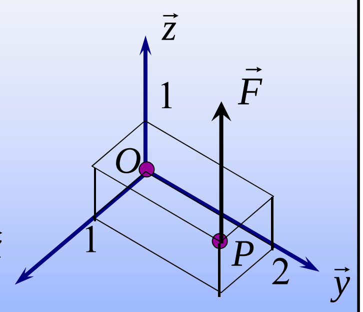
$$M_{Oz}(\vec{F}) = \pm Fd$$



正负号由右手法则判断

[例]

如图所示，一3N的力 平行于**Oz**轴，
作用点***P***的坐标为 **(1，2，1)m**。



求**:**该力对三轴的矩

可直接用推论3计算

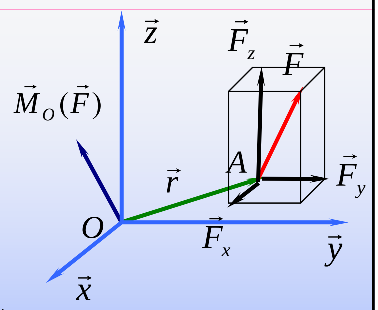
- 力对轴的矩推论 4

$$\vec{F}=\vec{F}_x+\vec{F}_y+\vec{F}_z$$

$$\vec{M}_O(\vec{F})=\vec{r}\times\vec{F}=\vec{r}\times(\vec{F}_x+\vec{F}_y+\vec{F}_z)$$

$$=\vec{r}\times\vec{F}_x+\vec{r}\times\vec{F}_y+\vec{r}\times\vec{F}_z$$

$$\vec{M}_O(\vec{F})=\vec{M}_O(\vec{F}_x)+\vec{M}_O(\vec{F}_y)+\vec{M}_O(\vec{F}_z)$$

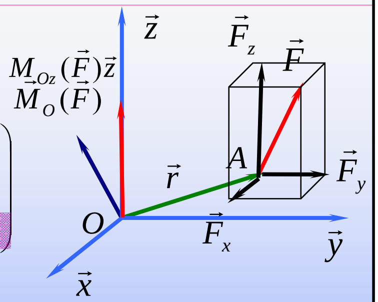


力对点的矩等于该力的三个分矢量对该点矩的矢量和

坐标式 $$\boldsymbol{M}_O(\vec{F})=\boldsymbol{M}_O(\vec{F}_x)+\boldsymbol{M}_O(\vec{F}_y)+\boldsymbol{M}_O(\vec{F}_z)$$

$$\boldsymbol{M}_O(\vec{F}) = \boldsymbol{M}_O(\vec{F}_x) + \boldsymbol{M}_O(\vec{F}_y) + \boldsymbol{M}_O(\vec{F}_z)$$

$$\begin{pmatrix} M_{Ox}(\vec{F}) \\ M_{Oy}(\vec{F}) \\ M_{Oz}(\vec{F}) \end{pmatrix} = \begin{pmatrix} M_{Ox}(\vec{F}_x) \\ M_{Oy}(\vec{F}_x) \\ M_{Oz}(\vec{F}_x) \end{pmatrix} + \begin{pmatrix} M_{Ox}(\vec{F}_y) \\ M_{Oy}(\vec{F}_y) \\ M_{Oz}(\vec{F}_y) \end{pmatrix} + \begin{pmatrix} M_{Ox}(\vec{F}_z) \\ M_{Oy}(\vec{F}_z) \\ M_{Oz}(\vec{F}_z) \end{pmatrix}$$



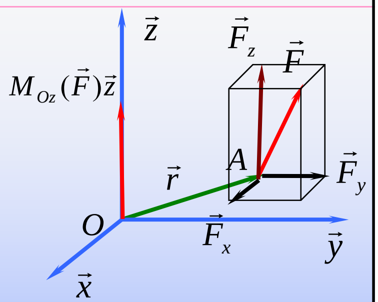
$$M_{Oz}(\vec{F}) = M_{Oz}(\vec{F}_x) + M_{Oz}(\vec{F}_y) + M_{Oz}(\vec{F}_z)$$

力对z轴的矩等于该力的三个分矢量对该轴矩的代数和

$$M_{Oz}(\vec{F}) = M_{Oz}(\vec{F}_x) + M_{Oz}(\vec{F}_y) + \cancel{M_{Oz}(\vec{F}_z)}$$

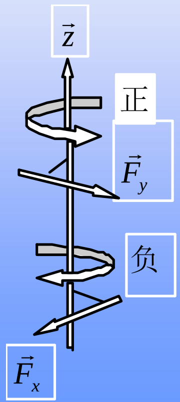
$$\vec{F}_z // \vec{z}$$

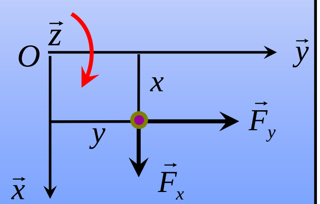
定义 $M_{Oz}(\vec{F}) = xF_y - yF_x$



$$M_{Oz}(\vec{F}_x) = -yF_x$$

$$M_{Oz}(\vec{F}_y) = xF_y$$





力对与其垂直的轴的矩等于该力与其到轴的距离的乘积，正负号右手法则决定

- 平面问题

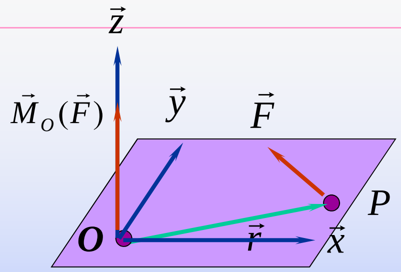
$\vec r$，$\vec F$ 平面矢量

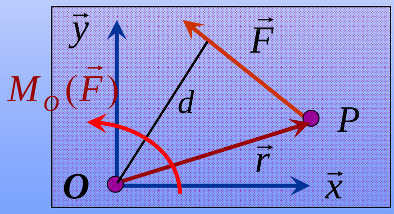
参考基 $\vec{\boldsymbol e} = (\vec x \quad \vec y)^{\mathrm T}$ 法向 $\vec z$

$$\vec M_O(\vec F) = \vec r \times \vec F \overset{\text{def}}{=} M_O(\vec F)\vec z$$

平面力对基点O的矩的大小等于该力对Oz轴的矩

$$M_O(\vec F) = \pm \boldsymbol{F}\boldsymbol{d}$$





- 平面力矩计算

$$\vec M_O(\vec F) = \vec r \times \vec F \overset{\text{def}}{=} M_O(\vec F)\vec z$$

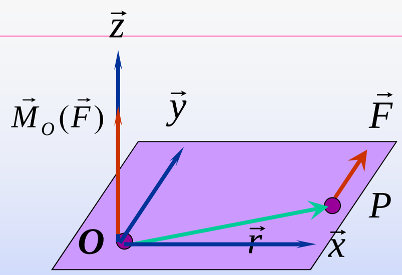
按矩阵式计算力矩

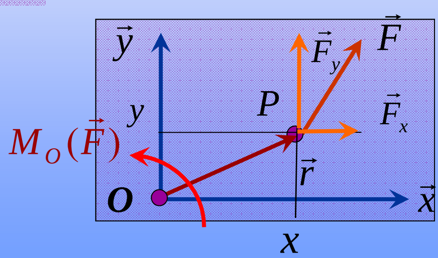
$$M_O(\vec F) = (\tilde{\boldsymbol I}\boldsymbol r)^{\mathrm T}\boldsymbol F = xF_y - yF_x$$

$\boldsymbol r = (x \quad y)^{\mathrm T}$ $\boldsymbol F = (F_x \quad F_y)^{\mathrm T}$

几何关系计算力矩

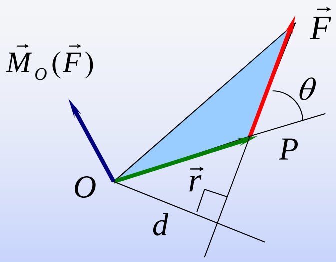
$$M_{Oz}(\vec F) = xF_y - yF_x$$





力矩小结

- 力对点*O* 的矩
  - 涉及力与点**O**的一个数学计算量
  - 定位矢量 $\vec{M}_O(\vec{F}) \overset{\text{def}}{=} \vec{r} \times \vec{F}$



- 力对轴的矩
  - 标量
  - 与对点的矩的关系
- 力矩的计算
  - 力对点的矩

$$M_{Ox}(\vec{F}) = yF_z - zF_y, M_{Oy}(\vec{F}) = zF_x - xF_z, M_{Oz}(\vec{F}) = xF_y - yF_x$$

  - 平面问题 $M_{Oz}(\vec{F}) = xF_y - yF_x$