第18章　电磁波

18.1　电磁波的波动方程

$$\oiint_S \vec{D}\cdot \mathrm{d}\vec{S} = \iiint_V \rho \mathrm{d}V$$

$$\oiint_S \vec{B}\cdot \mathrm{d}\vec{S} = 0$$

$$\oint_l \vec{E}\cdot \mathrm{d}\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot \mathrm{d}\vec{S}$$

$$\oint_l \vec{H}\cdot \mathrm{d}\vec{l} = \iint_S \vec{J}_c \cdot \mathrm{d}\vec{S} + \iint_S \frac{\partial \vec{D}}{\partial t}\cdot \mathrm{d}\vec{S}$$

积分形式

电、磁分量都具有波动特征——电磁波！

当电磁波沿 x 方向传播时

$$\frac{\partial^2 E_y}{\partial x^2} = \mu\varepsilon \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 H_z}{\partial x^2} = \mu\varepsilon \frac{\partial^2 H_z}{\partial t^2}$$

结合 $\vec{D} = \varepsilon \vec{E}$

$$\vec{B} = \mu \vec{H}$$

经过复杂的推导得：

$$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

和

$$\nabla^2 \vec{H} = \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

其中 $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

拉普拉斯算符

电、磁分量都具有波动特征——电磁波！

当电磁波沿 x 方向传播时

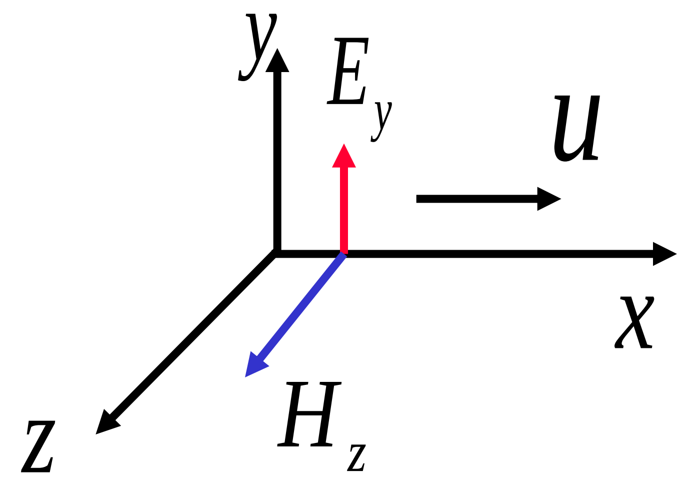
$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \mu\varepsilon \frac{\partial^2 E_y}{\partial t^2}}$$

$$\boxed{\frac{\partial^2 H_z}{\partial x^2} = \mu\varepsilon \frac{\partial^2 H_z}{\partial t^2}}$$

电磁波波速为：

$$u = \frac{1}{\sqrt{\mu\varepsilon}}$$

即：若设电场方向沿y方向，磁场必为z方向！



比较波动方程

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2 \xi}{\partial t^2}$$

电磁波波速与光矢量

真空中

$$u=\frac{1}{\sqrt{\mu_0\varepsilon_0}}=3\times10^8\,\mathrm{m/s}$$ ——光速 c

推测：光也是电磁波！

在介质中

$$u=\frac{1}{\sqrt{\mu\varepsilon}}=\frac{c}{\sqrt{\mu_r\varepsilon_r}}=\frac{c}{n}$$

$$n=\sqrt{\varepsilon_r}$$

$$n=\sqrt{\mu_r\varepsilon_r}$$ — 折射率

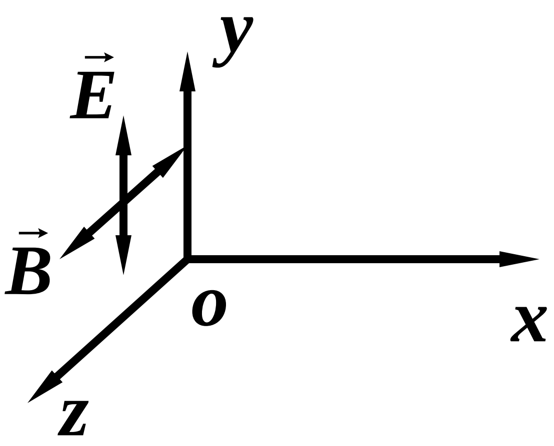
在光波段　$\mu_r=1$，与物质作用的主要是　$\vec{E}$ 矢量

$\vec{E}$ ——通常被称为光矢量！

注意：在BEC(Bose-Einstein Condensation)介质中，光的传播速度可以慢到大约为~0m/s。

18.2 电磁波的性质

一、 性质



$$\vec{E} = \vec{E}_0 \cos \omega t$$

$$\Rightarrow E = E_0 \cos \omega t$$

$$E = E_0 \cos \omega (t - \frac{x}{c})$$

沿*x*轴正向传播的平面简谐波

由： $$\frac{\partial E}{\partial x} = -\mu_0 \frac{\partial H}{\partial t} \qquad E = E_0 \cos\omega(t - \frac{x}{c})$$

$$\Rightarrow H = -\frac{1}{\mu_0}\int \frac{\partial E}{\partial x}\mathrm{d}t = \frac{E_0}{\mu_0 c}\cos\omega(t - \frac{x}{c}) = H_0 \cos\omega(t - \frac{x}{c})$$

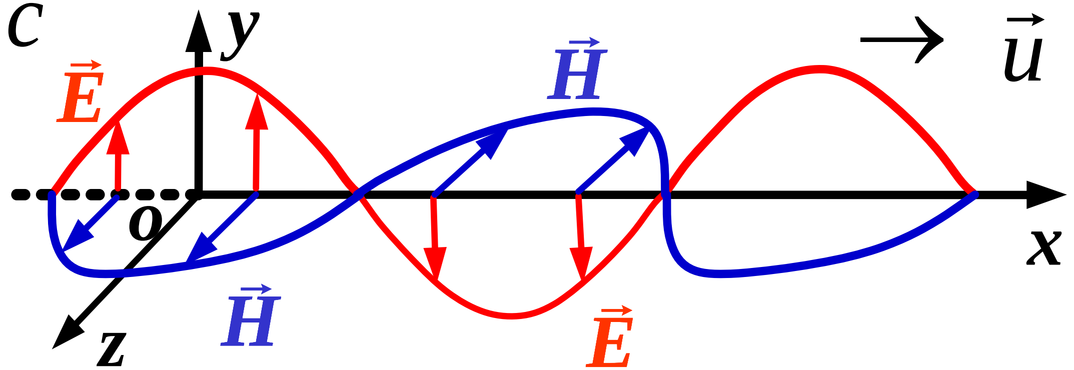
$$H_0 = \frac{E_0}{\mu_0 c} = \sqrt{\frac{\varepsilon_0}{\mu_0}}E_0 \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

沿***x***轴负向传播： $$E = E_0 \cos\omega(t + \frac{x}{c})$$

$$H = -H_0 \cos\omega(t + \frac{x}{c})$$

$$E = E_0 \cos \omega (t - \frac{x}{c})$$

$$H = H_0 \cos \omega (t - \frac{x}{c})$$



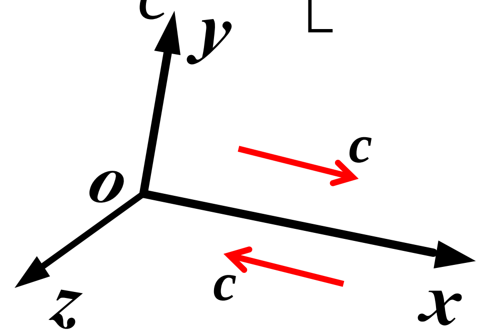
性质： 1. 横波性 $\vec{E}, \vec{H} \perp \vec{u}$

2. 偏振性 $\vec{E} \perp \vec{H},\ \vec{u} \,//\, \vec{E} \times \vec{H}$

3. $\vec{E}, \vec{H}$ 同周相 $|H| = \sqrt{\dfrac{\varepsilon_0}{\mu_0}}|E| \quad |B| = \dfrac{1}{c}|E|$

Example (Standing wave) An important example is the superposition of two similar plane waves traveling in opposite directions. Consider a wave traveling in the x direction, described by

$$\vec{E}_1 = E_0 \cos\left[\omega(t-\frac{x}{c})+\frac{\pi}{2}\right]\vec{j} \qquad \vec{B}_1 = \frac{E_0}{c}\cos\left[\omega(t-\frac{x}{c})+\frac{\pi}{2}\right]\vec{k}$$



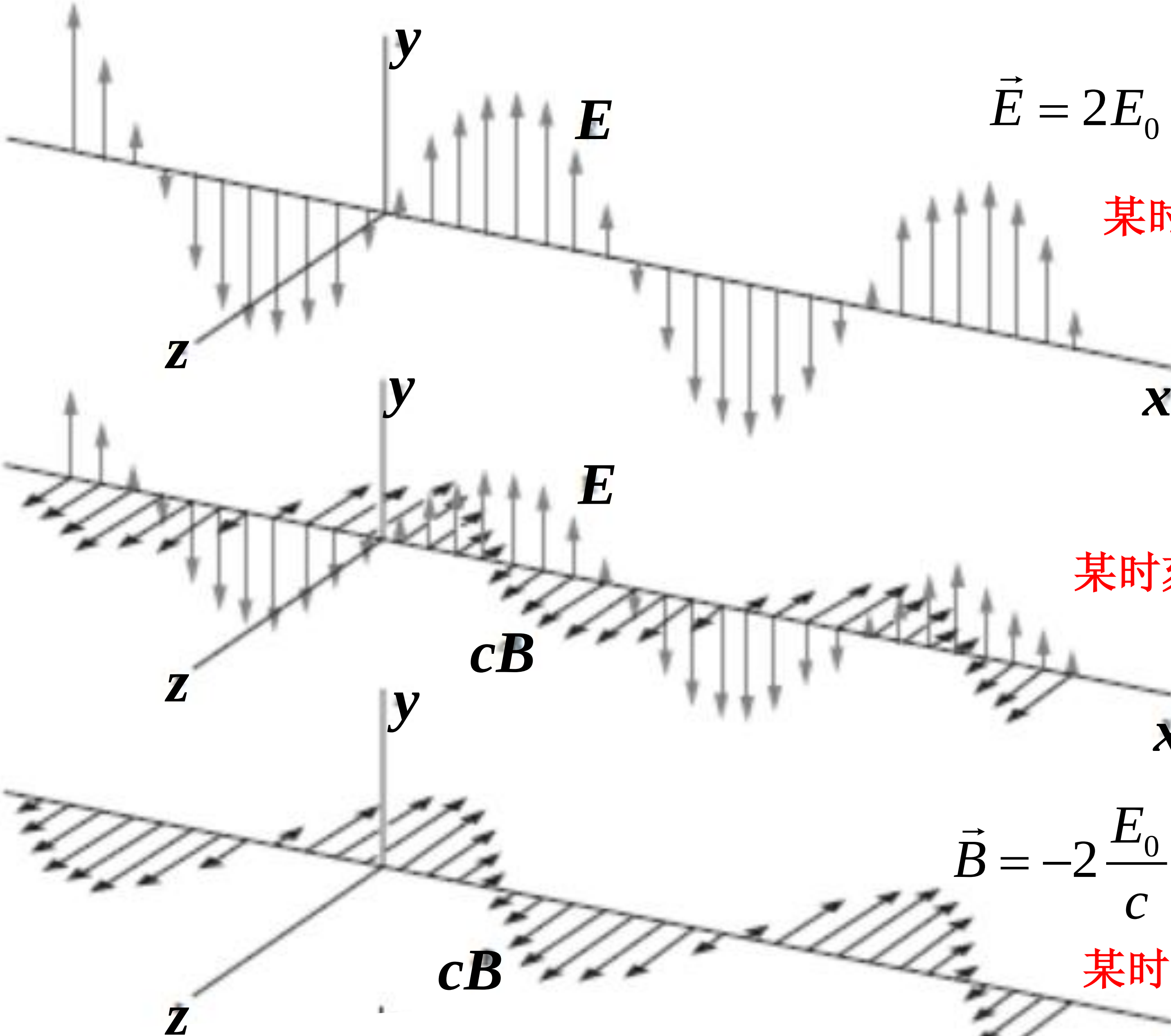
Now consider another wave:

$$\vec{E}_2 = -E_0 \cos\left[\omega(t+\frac{x}{c})+\frac{\pi}{2}\right]\vec{j} \qquad \vec{B}_1 = \frac{E_0}{c}\cos\left[\omega(t+\frac{x}{c})+\frac{\pi}{2}\right]\vec{k}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 2E_0 \sin\omega\frac{x}{c}\cos\omega t\vec{j}$$

standing wave.

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = -2\frac{E_0}{c}\cos\omega\frac{x}{c}\sin\omega t\vec{k} \qquad \frac{\omega}{c} = \frac{2\pi}{\lambda}$$



$$\vec{E} = 2E_0 \sin \omega \frac{x}{c} \cos \omega t \vec{j}$$

某时刻， $\omega t = 2n\pi$

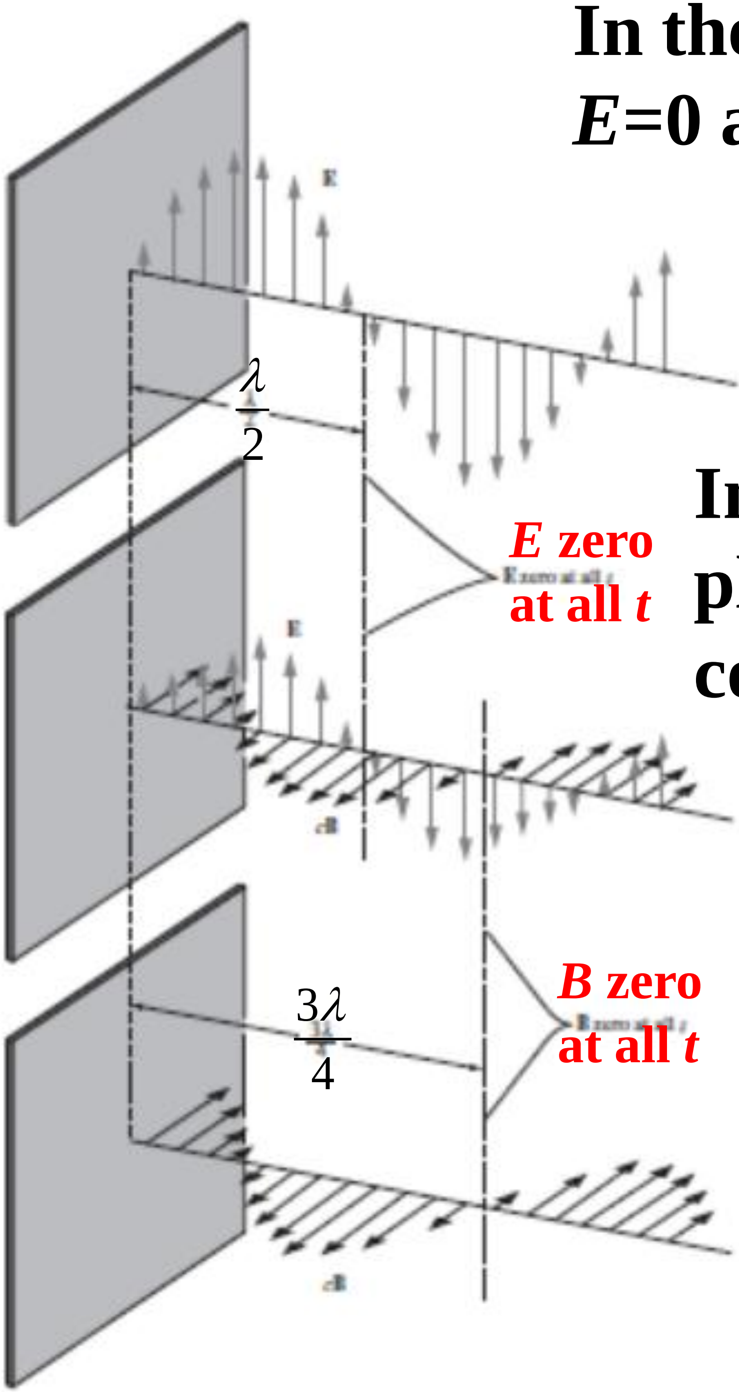
某时刻，$\omega t = 2n\pi + \dfrac{\pi}{4}$

$$\vec{B} = -2\frac{E_0}{c} \cos \omega \frac{x}{c} \sin \omega t \vec{k}$$

某时刻，$\omega t = 2n\pi + \dfrac{\pi}{2}$

In contrast with the traveling wave, the standing wave has its electric and magnetic fields “out of step” in both space and time.

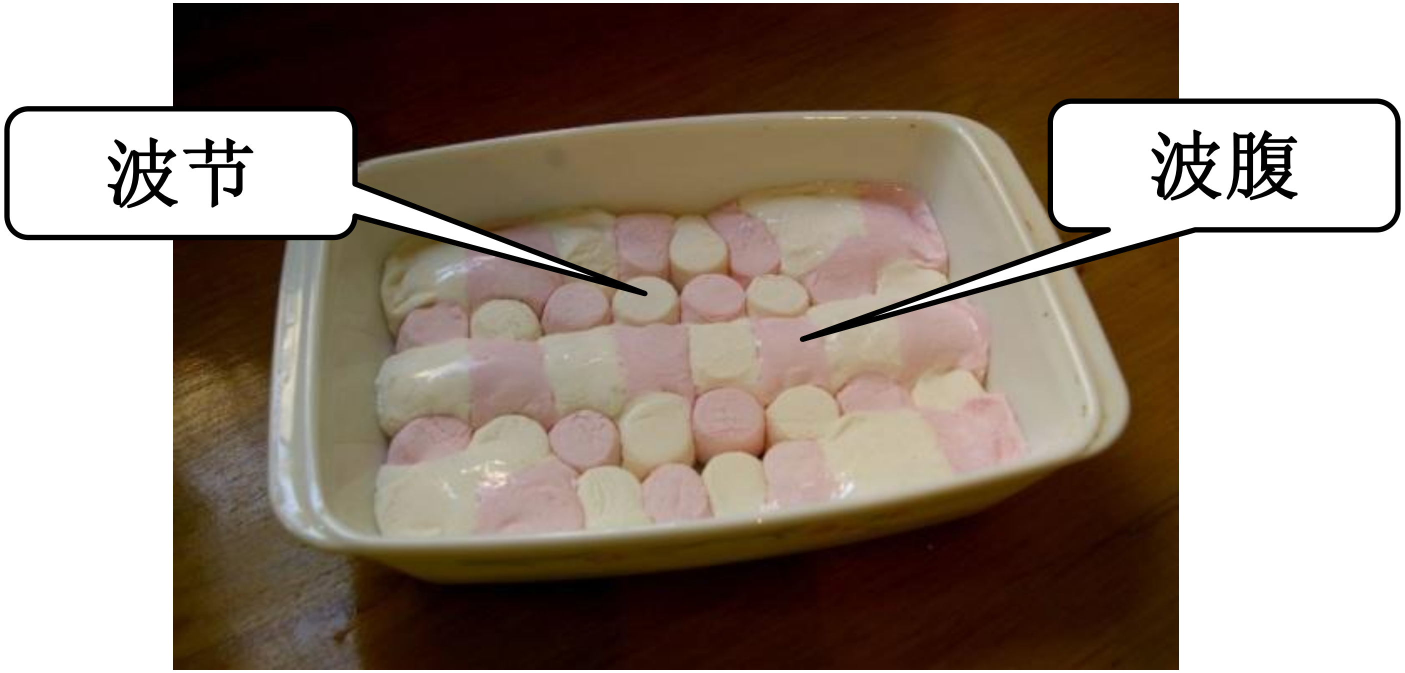
In the above standing wave, note that E=0 at all times on the plane x=0.



Imagine that we could cover the *yz* plane at x=0 with a sheet of perfectly conducting metal.

Therefore it provides a solution to the problem of a plane electromagnetic wave reflected, at normal incidence, from a flat conducting mirror.

棉花糖+微波炉


波节
波腹

二、 坡因廷矢量

电磁波的能流密度：

$$S = wu \qquad w = \frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2$$

利用：$u = \dfrac{1}{\sqrt{\mu\varepsilon}} \quad \sqrt{\varepsilon}E = \sqrt{\mu}H$

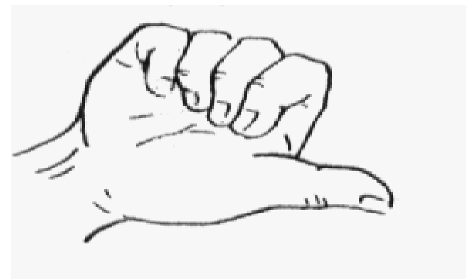
$$\frac{1}{2}\varepsilon E^2 = \frac{1}{2}\sqrt{\varepsilon}E\sqrt{\mu}H$$

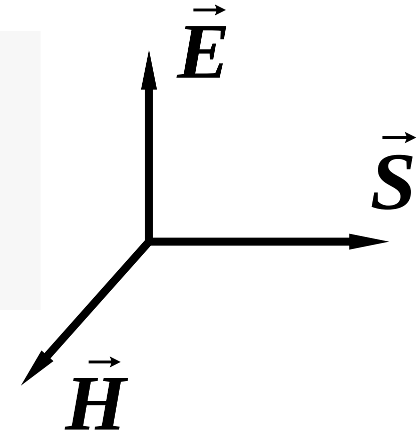
$$\frac{1}{2}\mu H^2 = \frac{1}{2}\sqrt{\mu}H\sqrt{\varepsilon}E$$

$$\Rightarrow S = EH$$

$$\vec{S} \,//\, \vec{u} \,//\, \vec{E}\times\vec{H}$$

$$\Rightarrow \vec{S} = \vec{E}\times\vec{H}$$





电磁波强度：

$$I = \bar{S} = \sqrt{\frac{\varepsilon}{\mu}}\,\overline{E^2}$$

$$= \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\,{E_0}^2 = \frac{1}{2}E_0H_0 = \bar{w}u$$

$$\Rightarrow S = EH$$

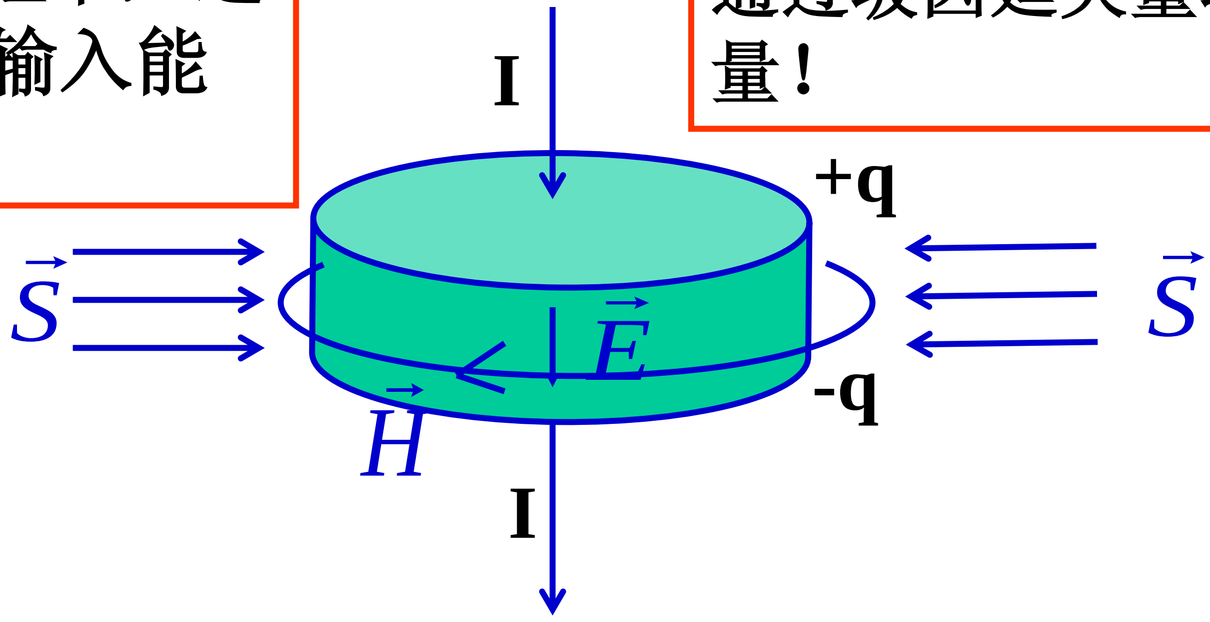
坡因延矢量举例

$\vec{S} = \vec{E} \times \vec{H}$

- 电容器充、放电

电容器充电过程中，通过坡因延矢量输入能量！

电容器放电过程中，通过坡因延矢量输出能量！



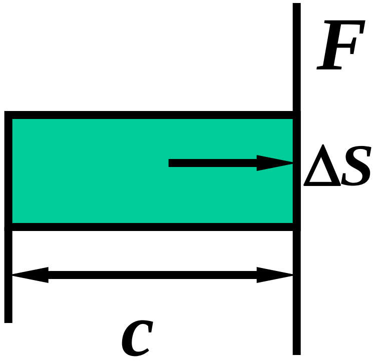
三、 辐射压强

质量密度： $$m=\frac{w}{c^2}(\because E=mc^2)$$

动量密度： $$p=\frac{E}{c}=\frac{w}{c}(\because E^2=p^2c^2+m_0{}^2c^2)$$

$$\vec{p}=\frac{\vec{S}}{c^2}=\frac{1}{c^2}\vec{E}\times\vec{H}=\frac{1}{\mu_0c^2}\vec{E}\times\vec{B}\ (w=\frac{S}{c})$$

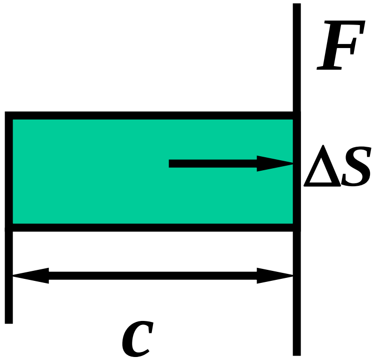
辐射压强： $$\frac{F}{\Delta S}=\frac{pc\Delta S}{\Delta S}=pc=w$$



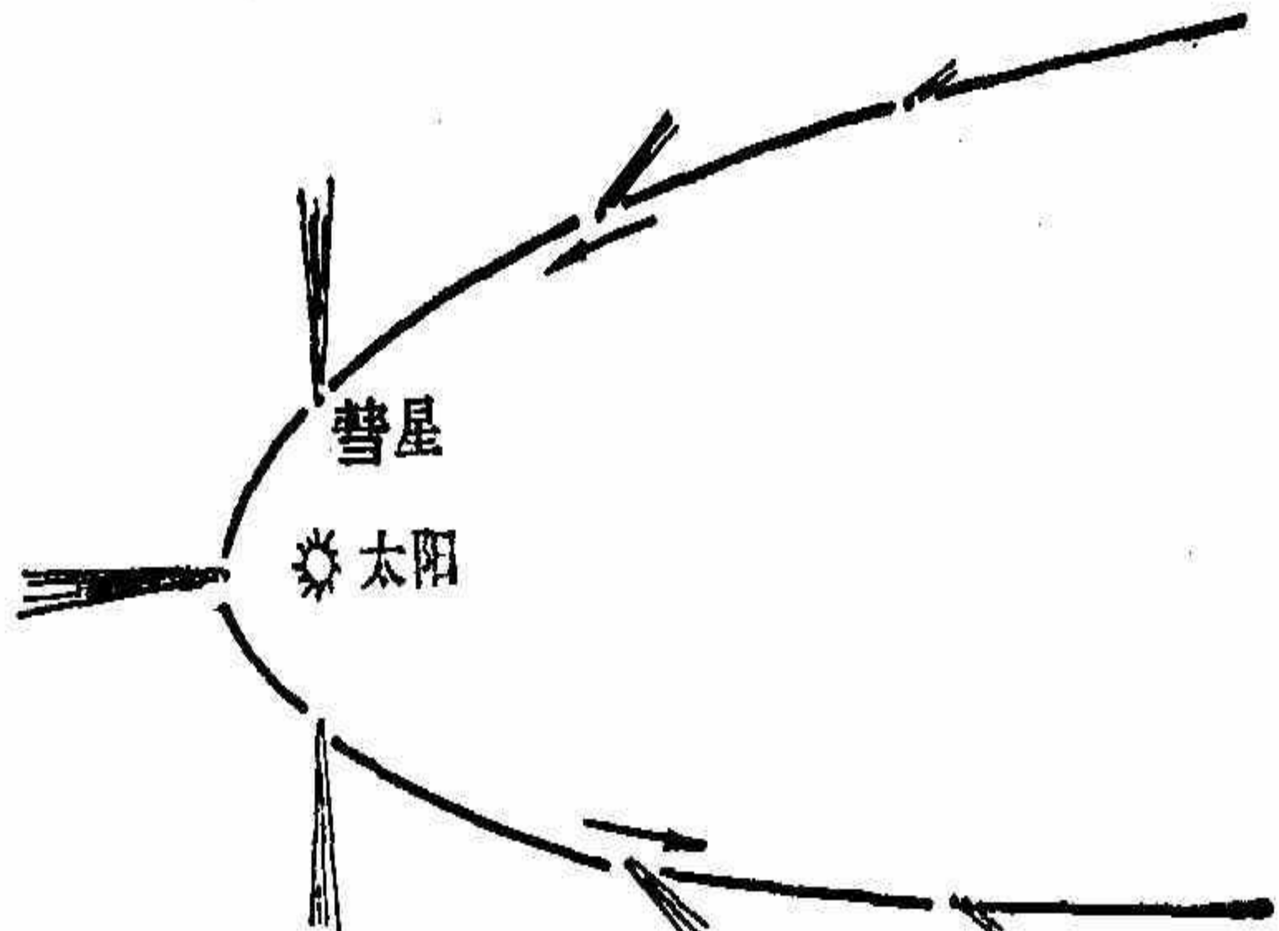
•平均压强：$$\overline{\left(\frac{F}{\Delta S}\right)} = \overline{w} = \frac{\bar{S}}{c}$$

•完全反射时：$$2\overline{\left(\frac{F}{\Delta S}\right)} = 2\overline{w} = 2\frac{\bar{S}}{c}$$

辐射压强：$$\frac{F}{\Delta S} = \frac{pc\Delta S}{\Delta S} = pc = w$$

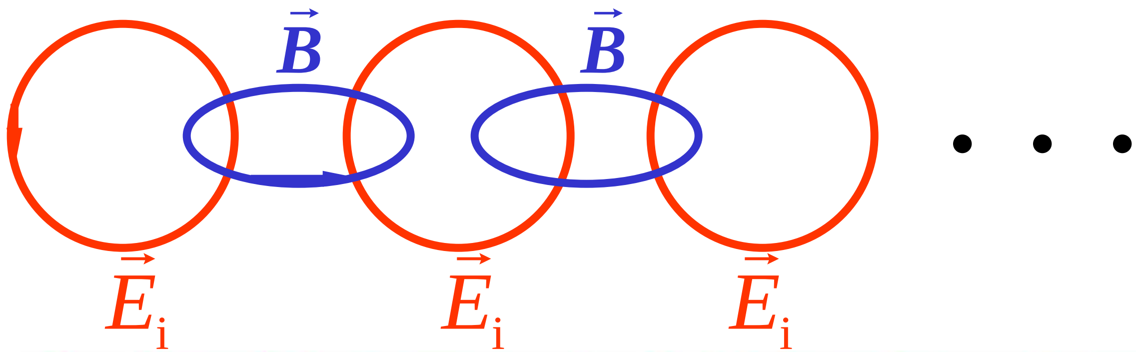


辐射压强：

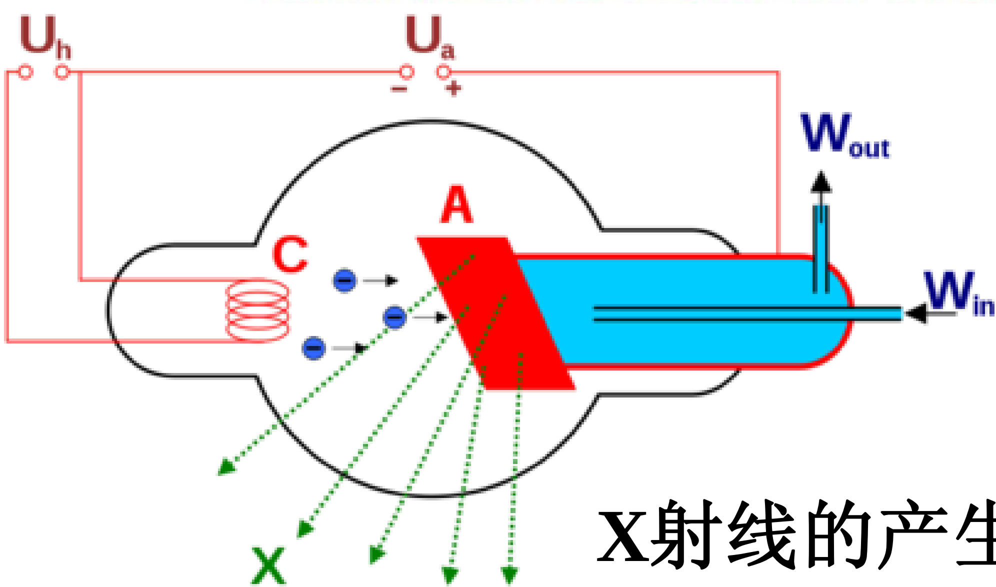


18.4 振荡电偶极子的辐射　赫兹实验

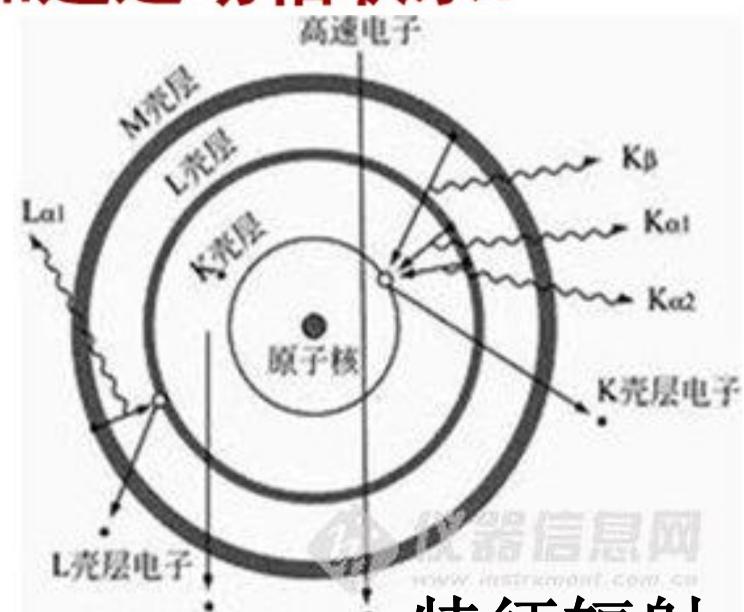
一、　电磁波的产生



电磁辐射总是和电荷的加速运动相联系。

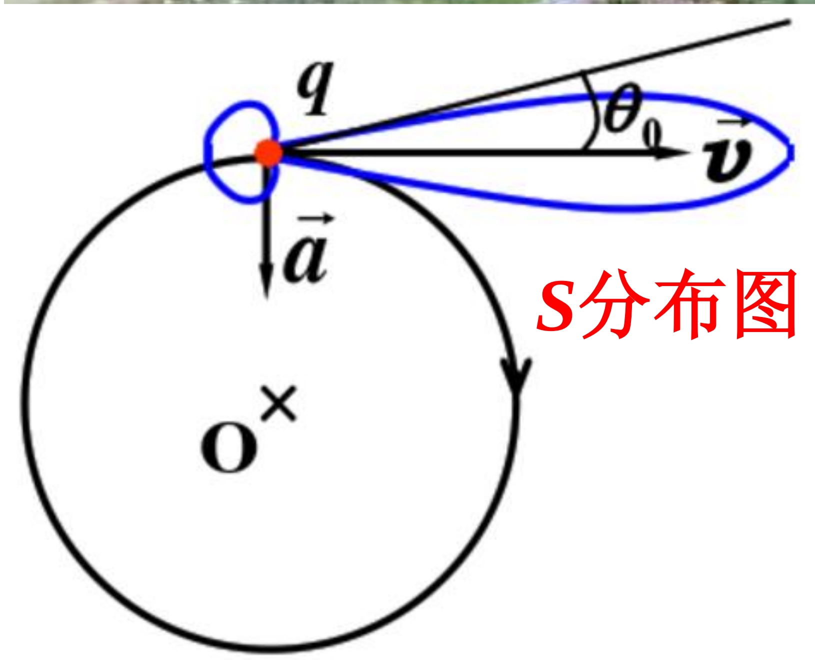


X射线的产生

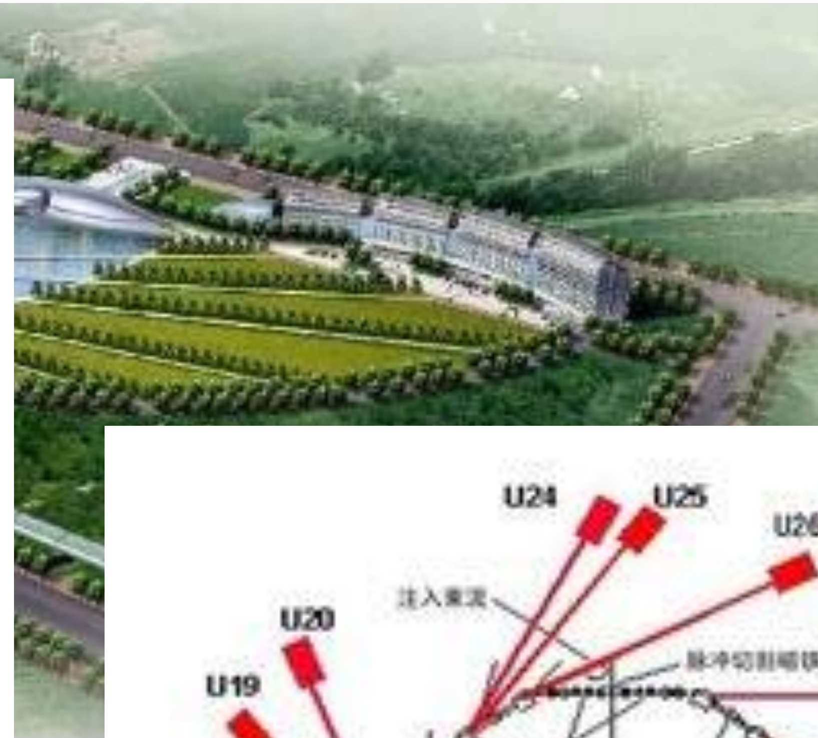


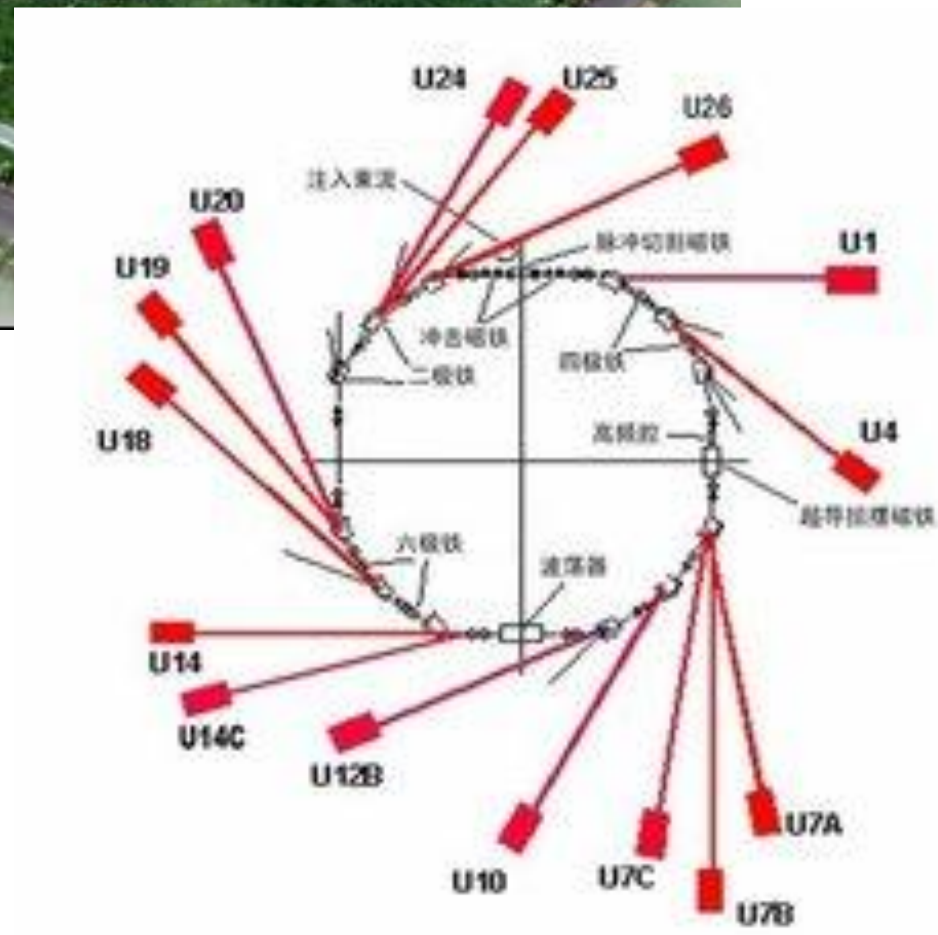
克鲁克斯管　韧致辐射

特征辐射

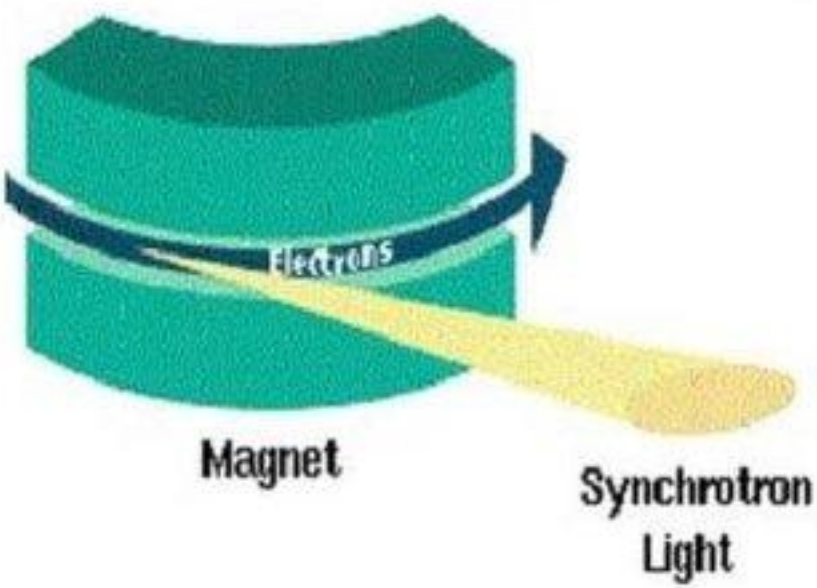


q
θ_0
$\vec{v}$
$\vec{a}$
S分布图
O



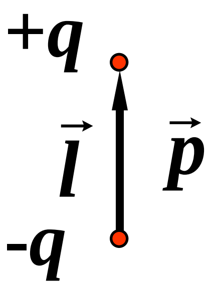


U24
U25
U26
U20
U19
U18
U1
U4
注入束流
脉冲切割磁铁
冲击磁铁
二极铁
四极铁
高频腔
超导扭摆磁铁
六极铁
波荡器
U14
U14C
U12B
U10
U7C
U7B
U7A



Electrons
Magnet
Synchrotron
Light

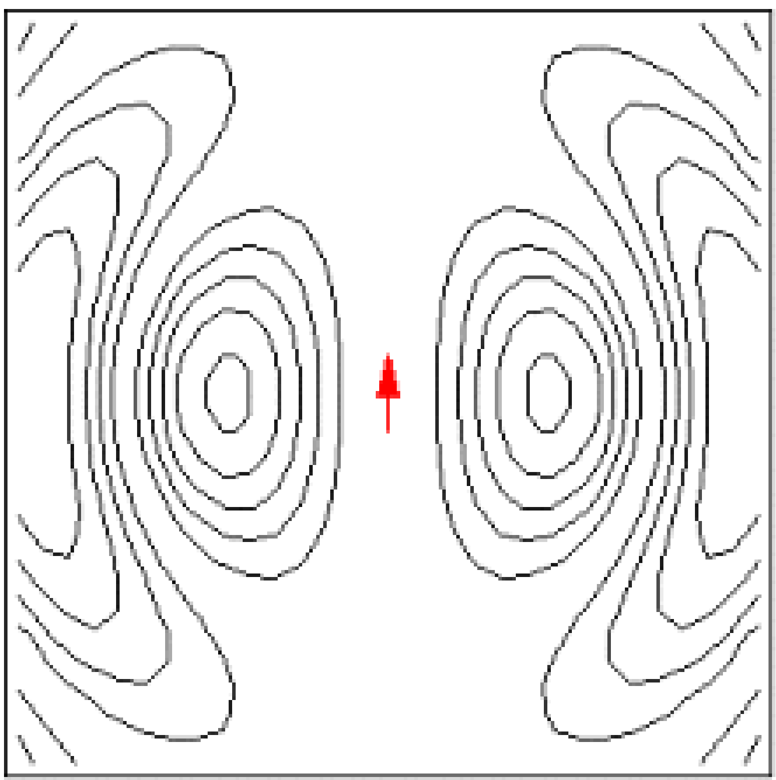
振荡电偶极子



电矩： $\vec{p} = |q|\vec{l}$

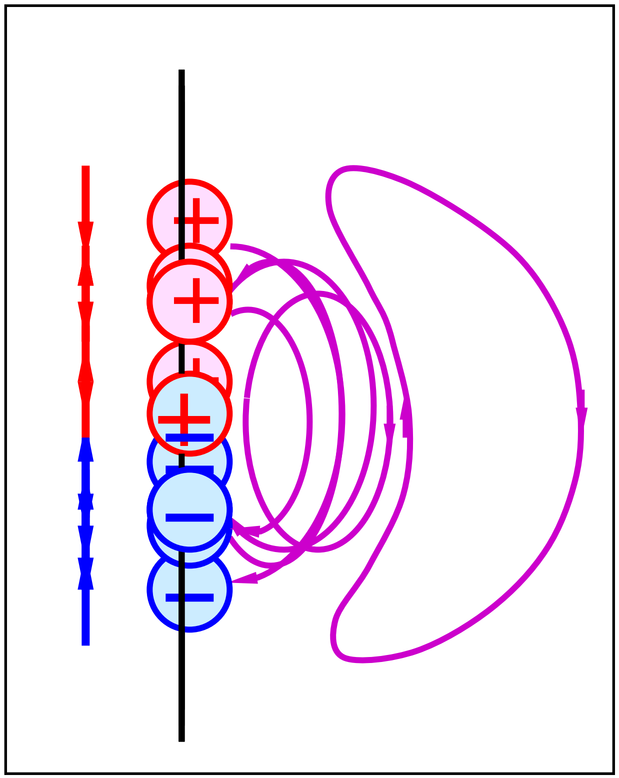
振荡电偶极子： $p = p_0 \cos \omega t$

$$= ql \cos \omega t$$

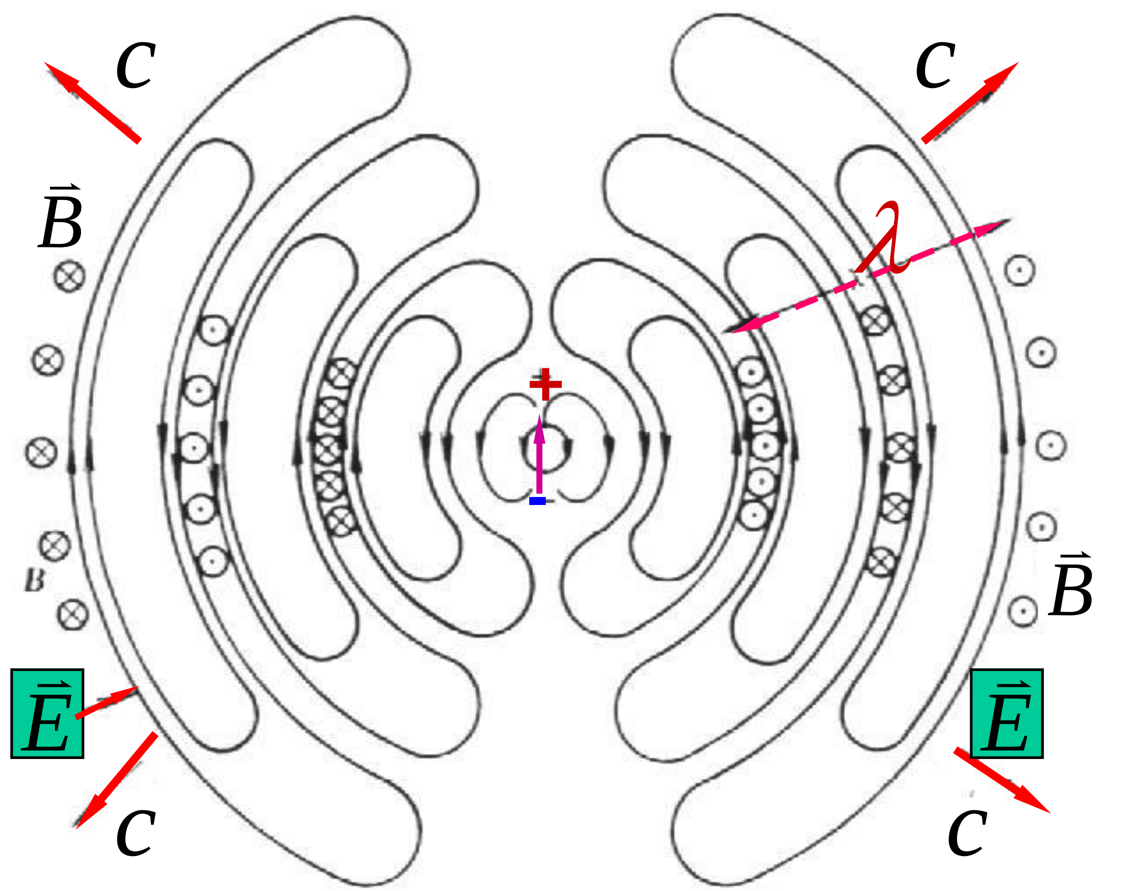


不同时刻振荡电偶极子附近的电场线

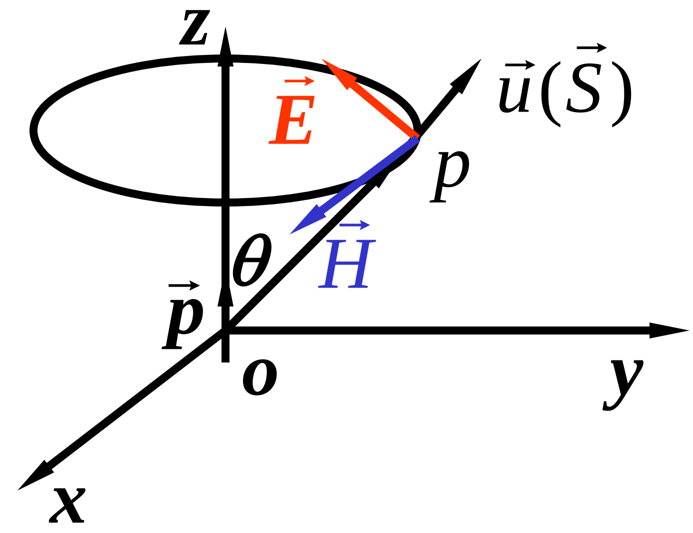
$$p = p_0 \cos \omega t$$



振荡电偶极子附近的电磁场线

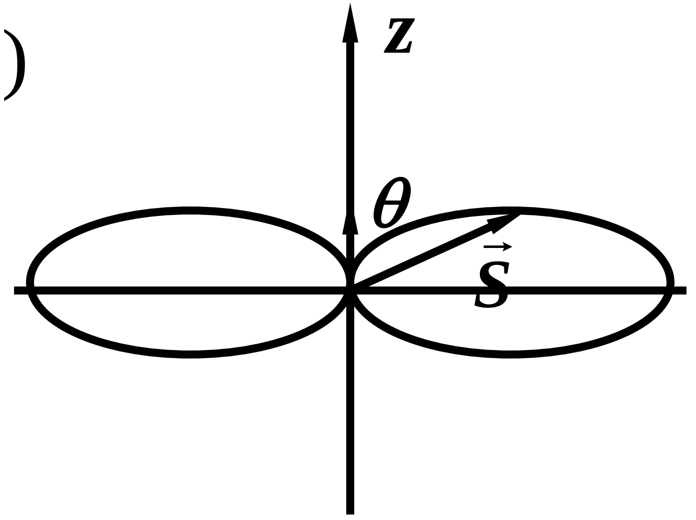


$$E=-\frac{\omega^2 p_0 \sin\theta}{4\pi\varepsilon_0 c^2 r}\cos\omega(t-\frac{r}{c})$$



$$H=-\frac{\omega^2 p_0 \sin\theta}{4\pi\varepsilon_0 c r}\cos\omega(t-\frac{r}{c})$$

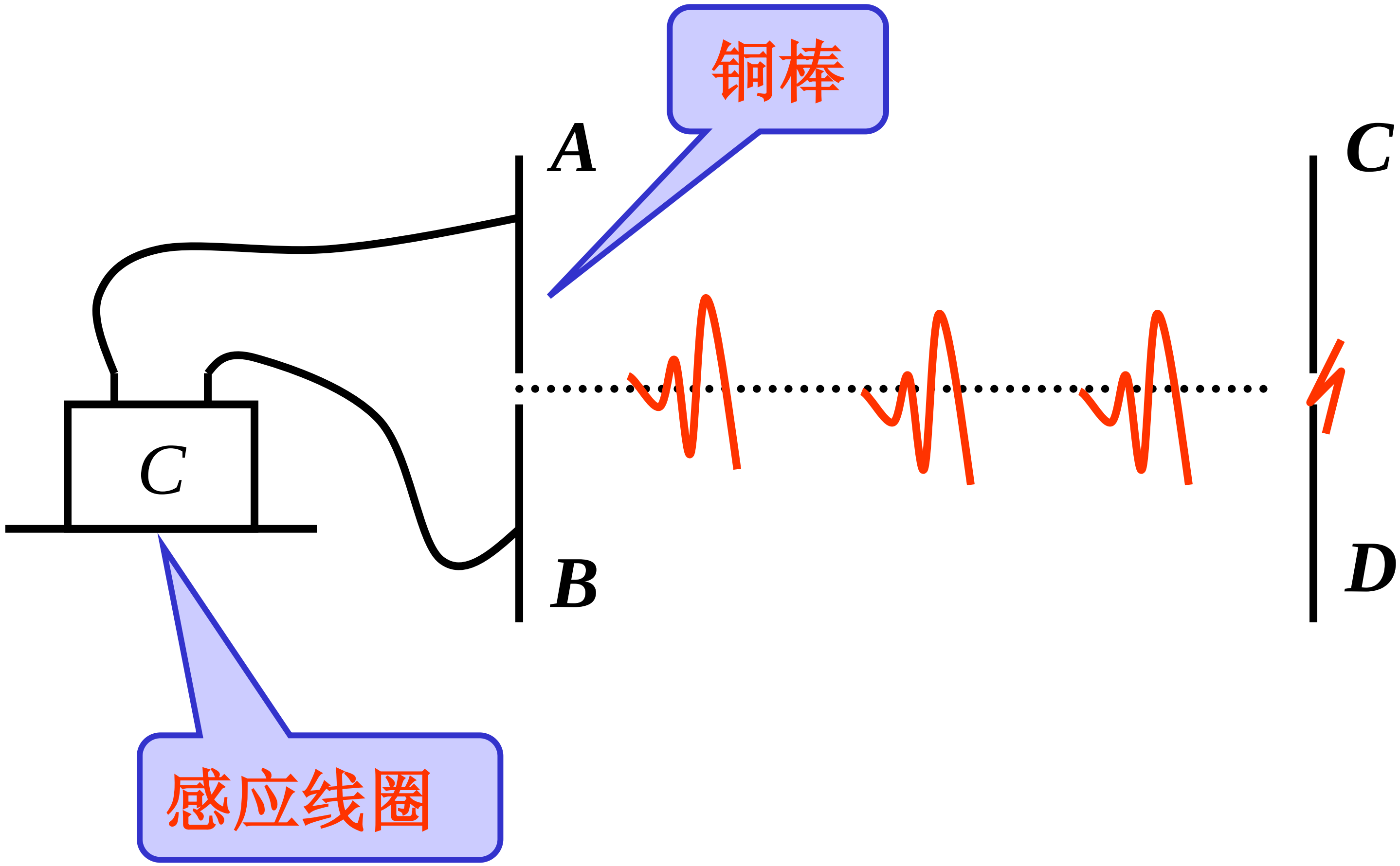
$$S=E\cdot H=\frac{\omega^4 p_0^{\ 2}\sin^2\theta}{16\pi^2\varepsilon_0 c^3 r^2}\cos^2\omega(t-\frac{r}{c})$$



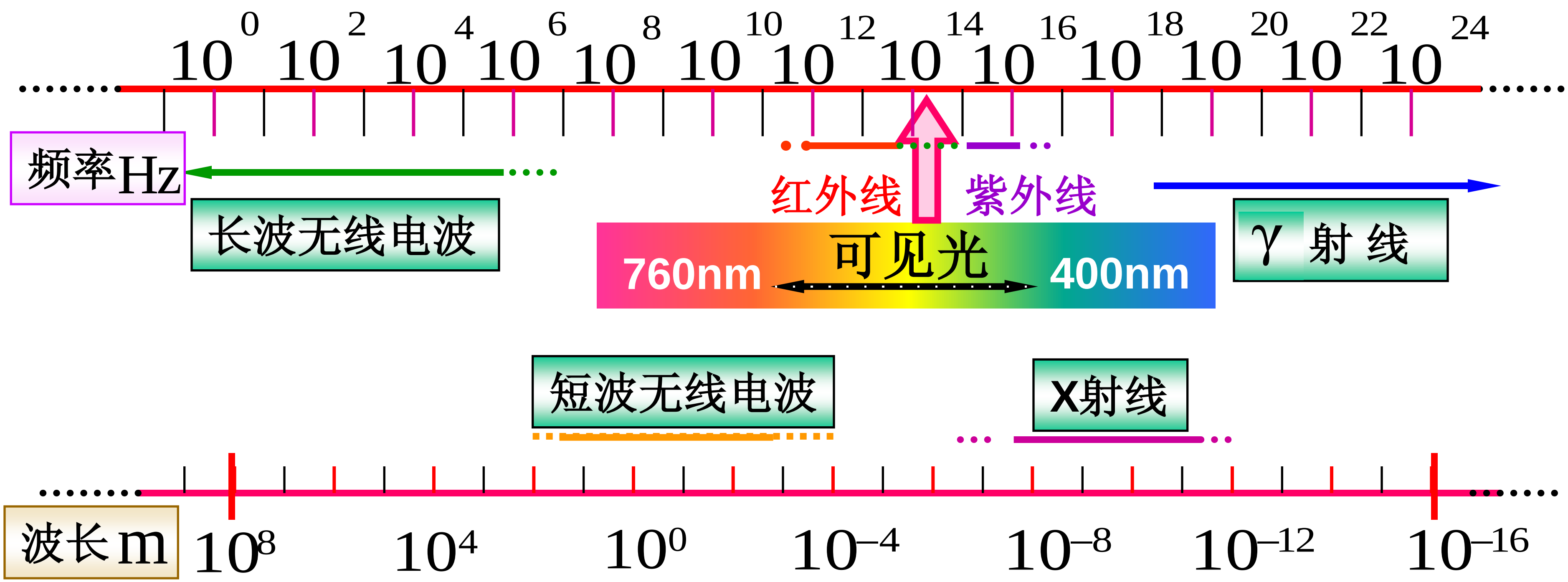
总辐射功率： $p=\int_{球面} S r^2 \sin\theta \mathrm{d}\theta \mathrm{d}\varphi$

$$\overline{p}=\frac{\omega^4 p_0^{\ 2}}{12\pi\varepsilon_0 c^3}$$

二、 赫兹实验



三、电磁波谱



无线电波	$3\times10^{4}\,\mathrm{m}\sim0.1\mathrm{cm}$	紫外光	$400\,\mathrm{nm}\sim5\mathrm{nm}$
红外线	$6\times10^{5}\,\mathrm{nm}\sim760\mathrm{nm}$	x 射线	$5\mathrm{nm}\sim0.04\mathrm{nm}$
可见光	$760\,\mathrm{nm}\sim400\,\mathrm{nm}$	γ 射线	$<0.04\mathrm{nm}$

【例】 某广播电台的平均辐射功率 $\overline{P}$ 。假定辐射出来的能流均匀地分布在以电台为中心的半个球面上, 在离电台为r 处的平均能流密度= $\overline{S}=\dfrac{\overline{P}}{2\pi r^2}$;

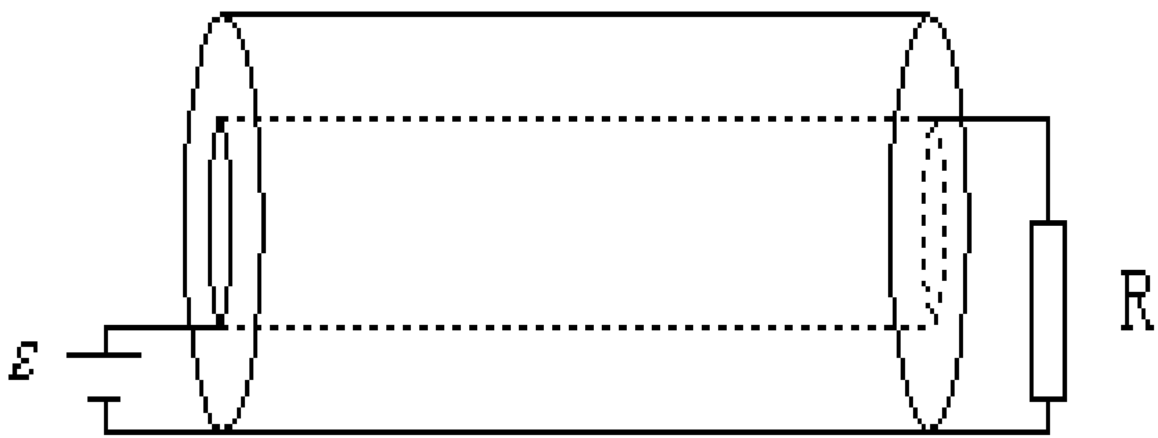
(2)在r处一个小的空间范围内电磁波可看作平面波, 该处电场强度的振幅=__________和磁场强度的振幅=________

$$E_0=\sqrt{\frac{\overline{P}}{\pi r^2\varepsilon_0 c}}$$

$$H_0=\sqrt{\frac{\overline{P}}{\pi r^2\mu_0 c}}$$

$$\overline{S}=\varepsilon_0 c E_0^2/2 \qquad \overline{S}=\mu_0 c H_0^2/2 \qquad \sqrt{\varepsilon_0}E=\sqrt{\mu_0}H,$$

[例]如图所示，同轴电缆内外半径分别为a 和b ，用来作为电源ε和电阻R的传输线，电缆本身的电阻忽略不计。
(1)试求电缆中任一点(a<r<b)处的坡印廷矢量S；
(2)试求通过电缆横截面的能流，该结果说明什么物理图象？



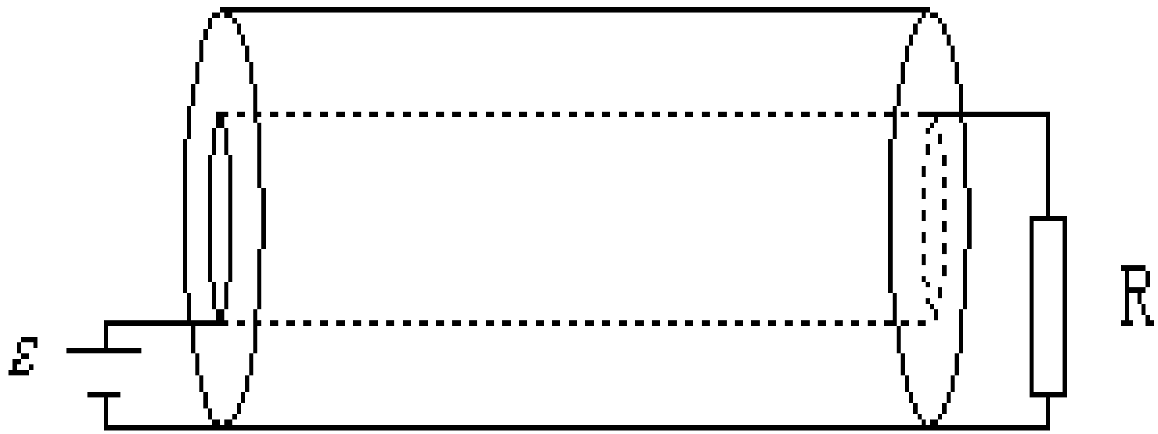
解：（1）

电场强度

$$E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

按定义：$\varepsilon=\int_a^b \vec{E}\cdot\mathrm{d}\vec{l}=\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}$

$$\Rightarrow \frac{\lambda}{2\pi\varepsilon_0} = \frac{\varepsilon}{\ln\frac{b}{a}} \quad \Rightarrow \quad E = \frac{\varepsilon}{r\ln\frac{b}{a}}$$



解：（1）

电场强度

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

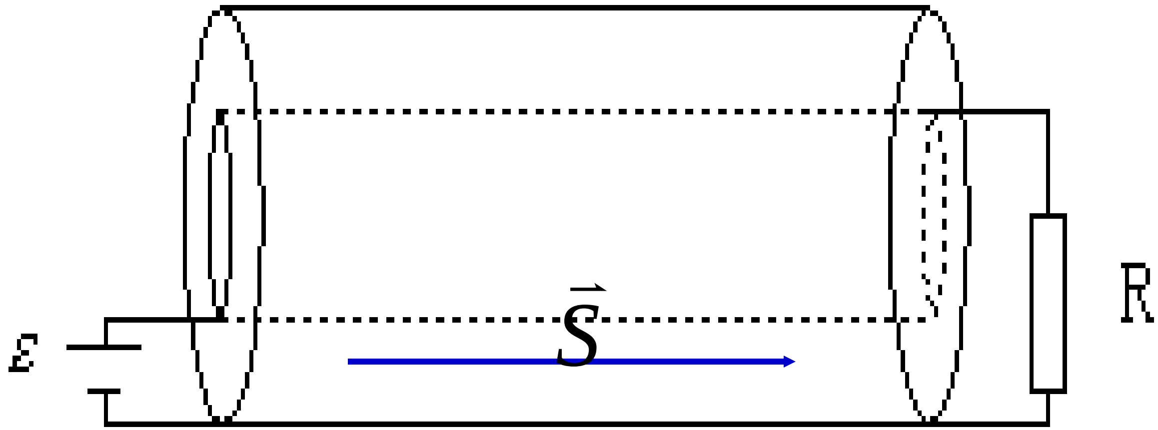
按定义：$\varepsilon = \int_a^b \vec{E}\cdot \mathrm{d}\vec{l} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}$

$$\frac{\lambda}{2\pi\varepsilon_0} = \frac{\varepsilon}{\ln\frac{b}{a}} \quad \longrightarrow \quad E = \frac{\varepsilon}{r\ln\frac{b}{a}}$$

磁场强度：

$$H = \frac{I}{2\pi r}$$

$$= \frac{\varepsilon}{2\pi r \cdot R}$$

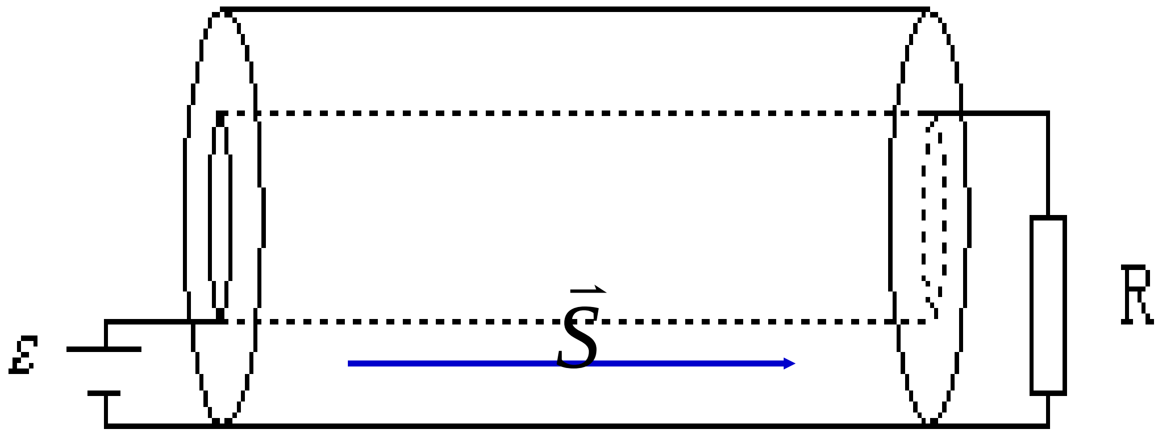


$$\longrightarrow \quad S = EH = \frac{\varepsilon^2}{2\pi r^2 R \ln\frac{b}{a}}$$

(2)通过电缆横截面的能流

$$P=\int S\cdot \mathrm{d}A=\int_a^b \frac{\varepsilon^2}{2\pi r^2 R\ln\frac{b}{a}}\cdot 2\pi r\cdot \mathrm{d}r=\frac{\varepsilon^2}{R}$$

电源通过电缆以坡因廷矢量的形式传输能量到负载。



$$S=EH=\frac{\varepsilon^2}{2\pi r^2 R\ln\frac{b}{a}}$$