# Syntactic dependencies correspond to word pairs with high mutual information

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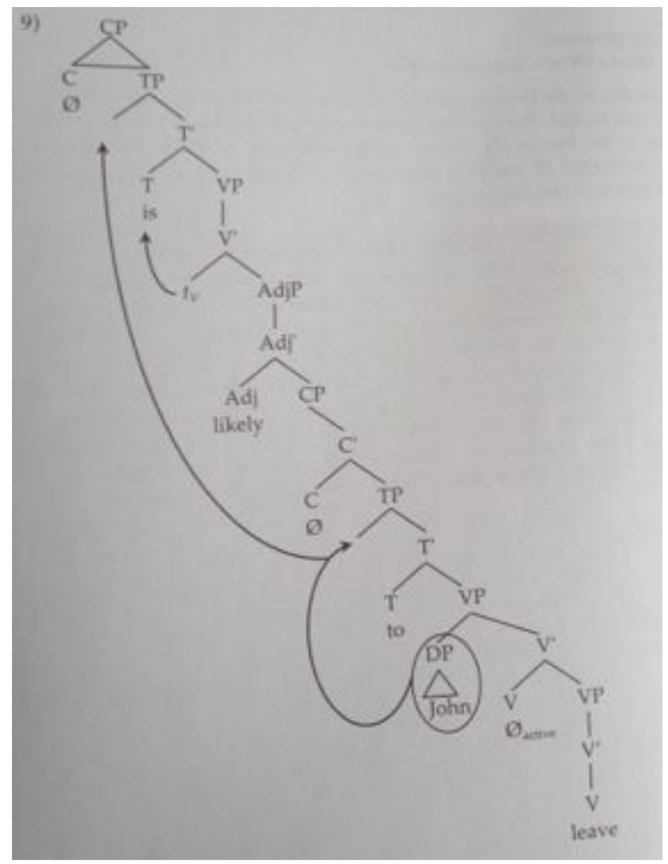

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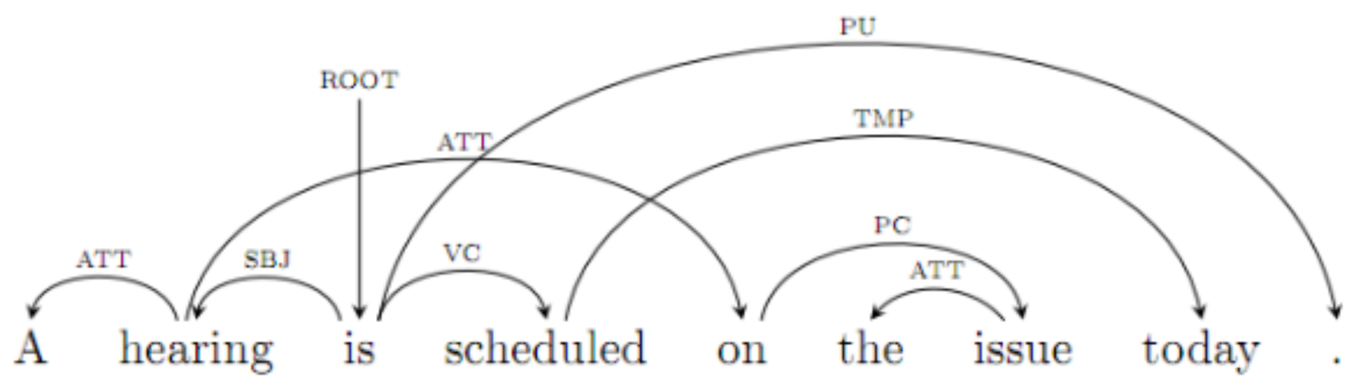

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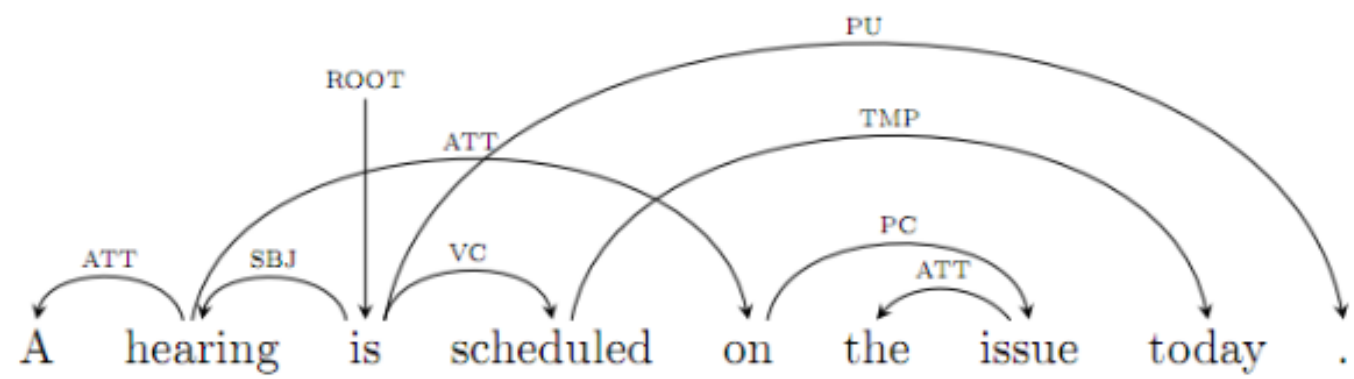


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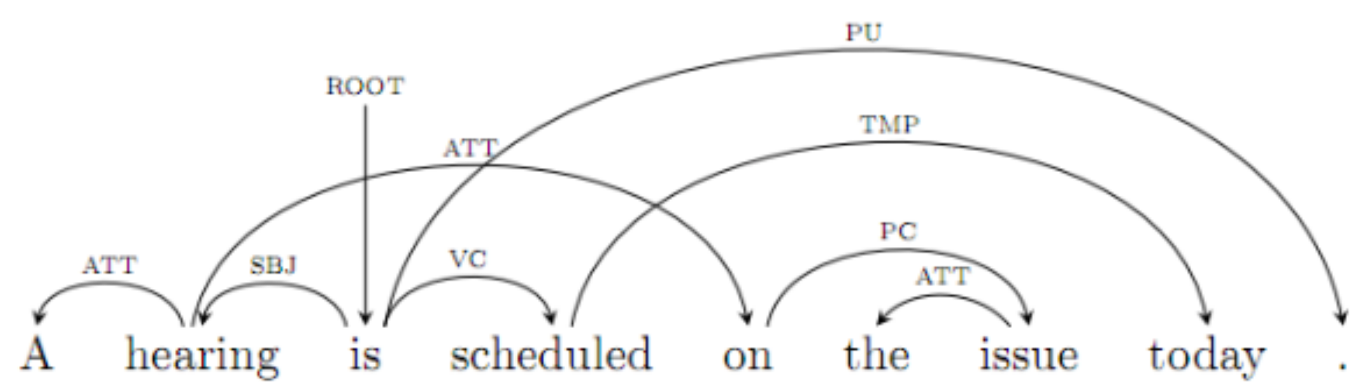


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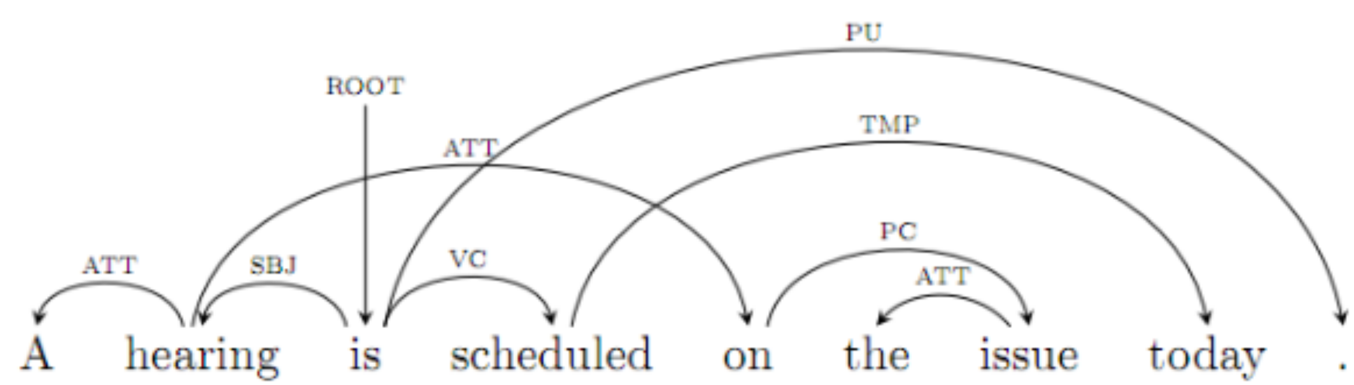


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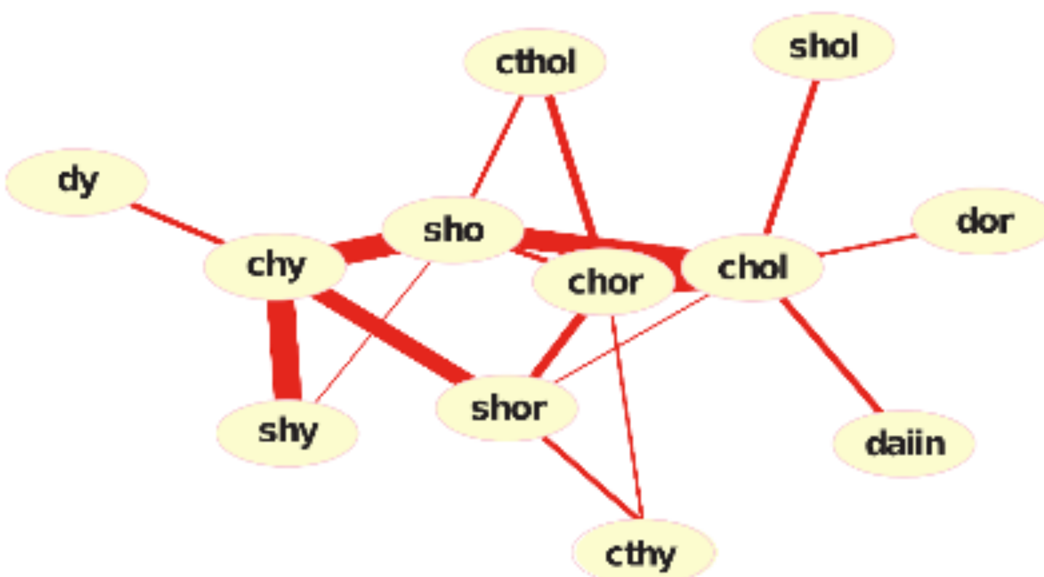


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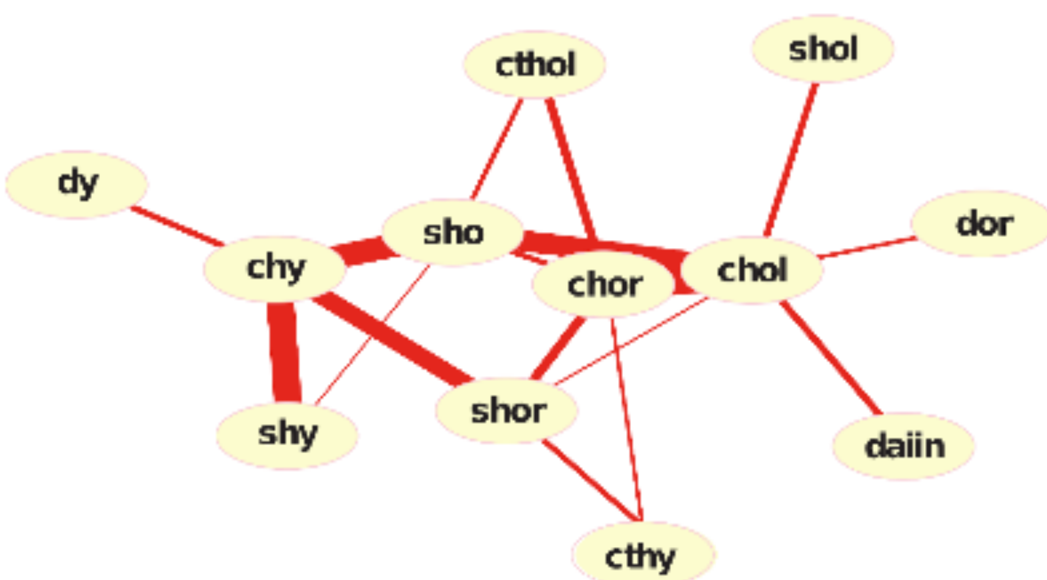


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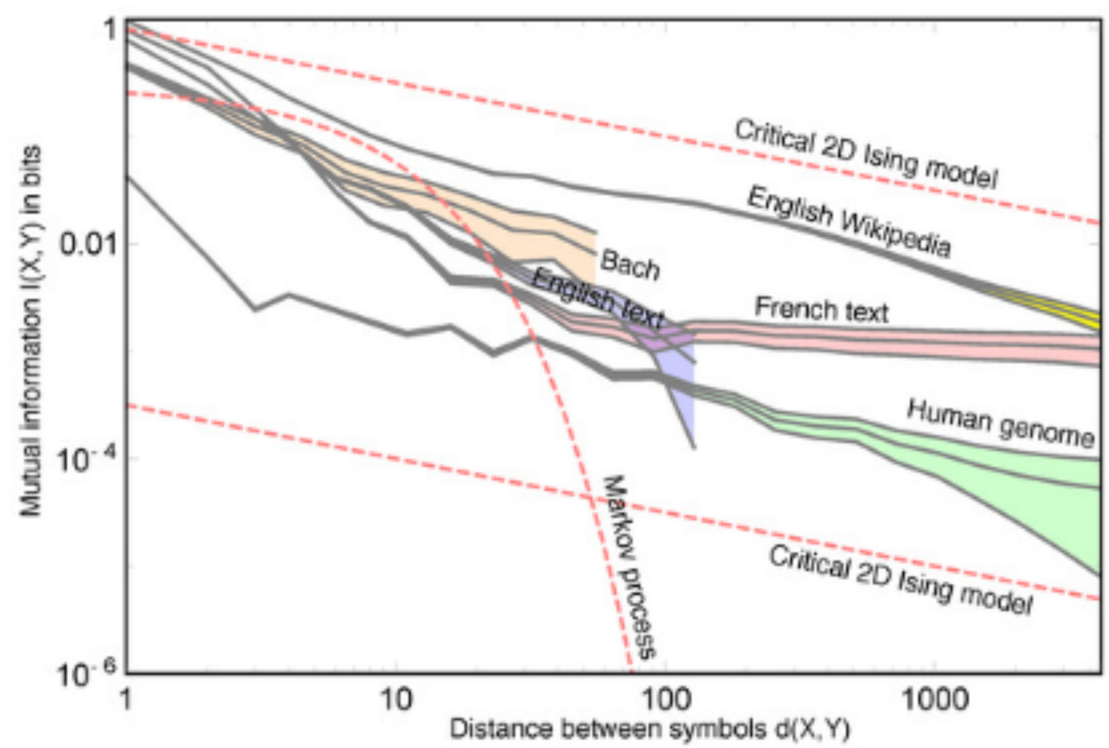


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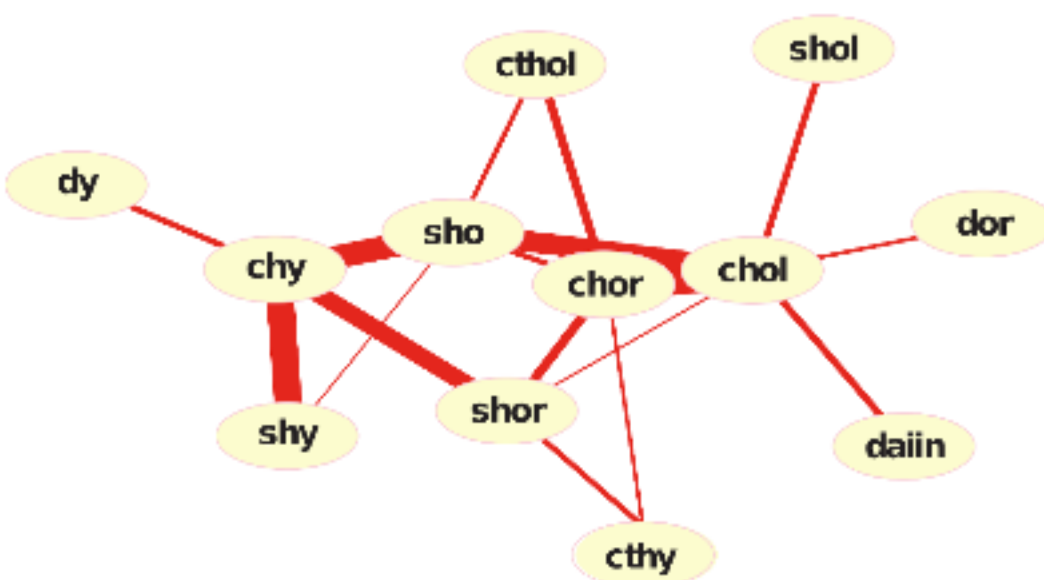


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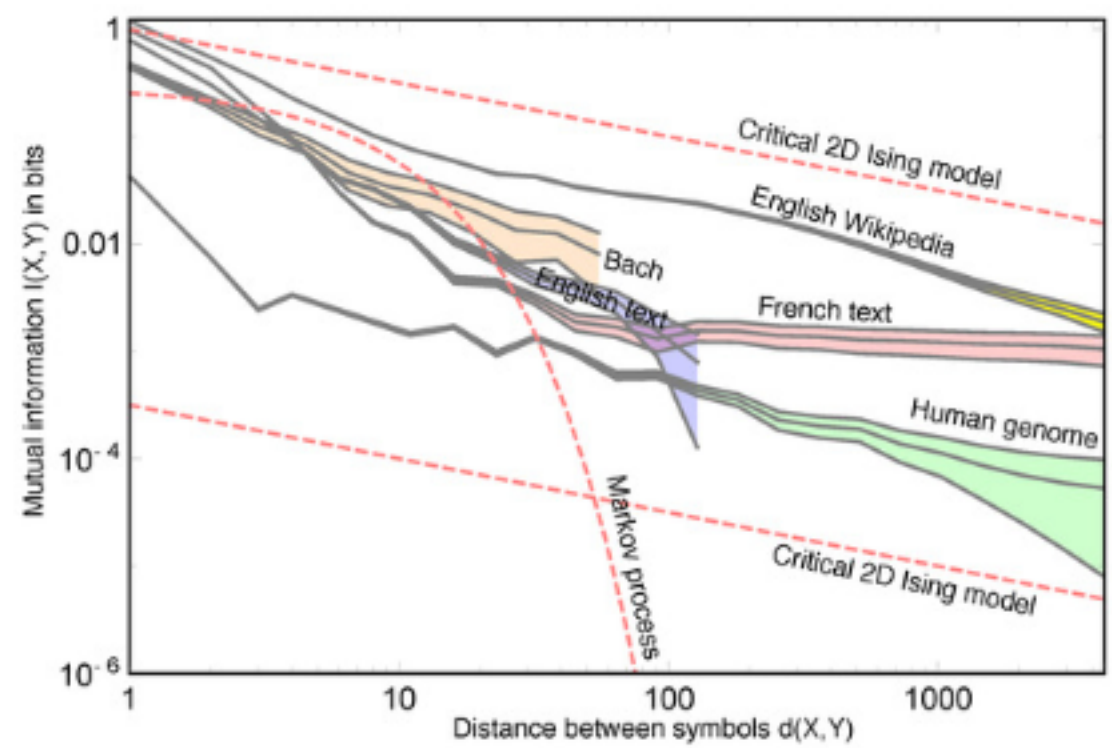


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- Goal: Characterize natural language text, as observable in corpora, as a stochastic process.

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    - Modern grammar induction (e.g. Klein & Manning, 2004, et seq.): Assume syntactic structure is the **trace of a generative process** that generated the data; try to recover the syntactic structure from statistical structure using **Bayesian inference**.

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    - Explicit or implicit in nearly all previous work on grammar induction (de Paiva Alves, 1996; Yuret, 1998; Klein & Manning, 2004, et seq.), but not yet explicitly tested at scale.
- <u>Our contribution</u>: We give **direct empirical evidence** based on a large parsed corpus, and a **new theoretical justification** based on an information-theoretic formalization of basic postulates of dependnecy grammar.

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- But in that case the MI may be hard to estimate accurately…

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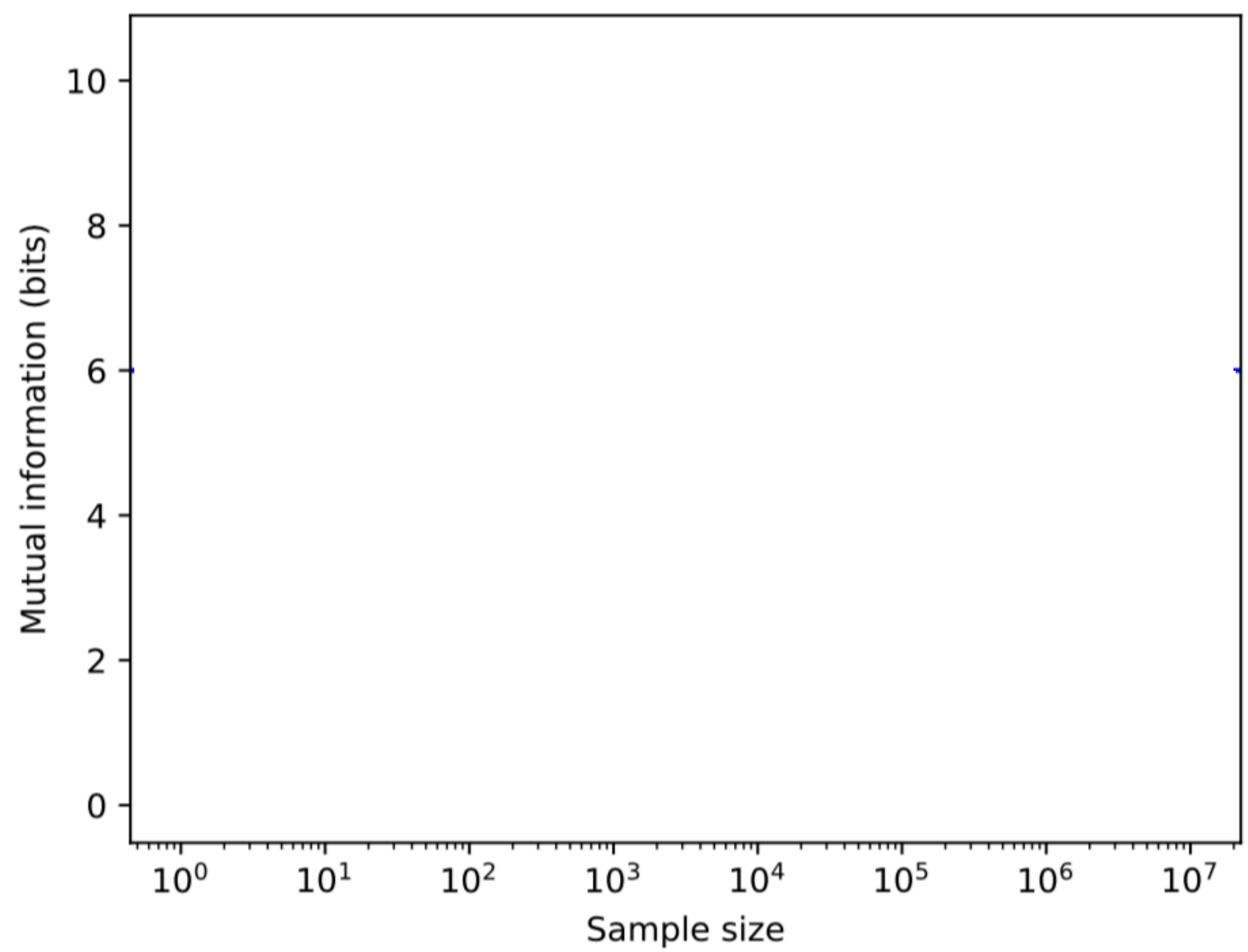

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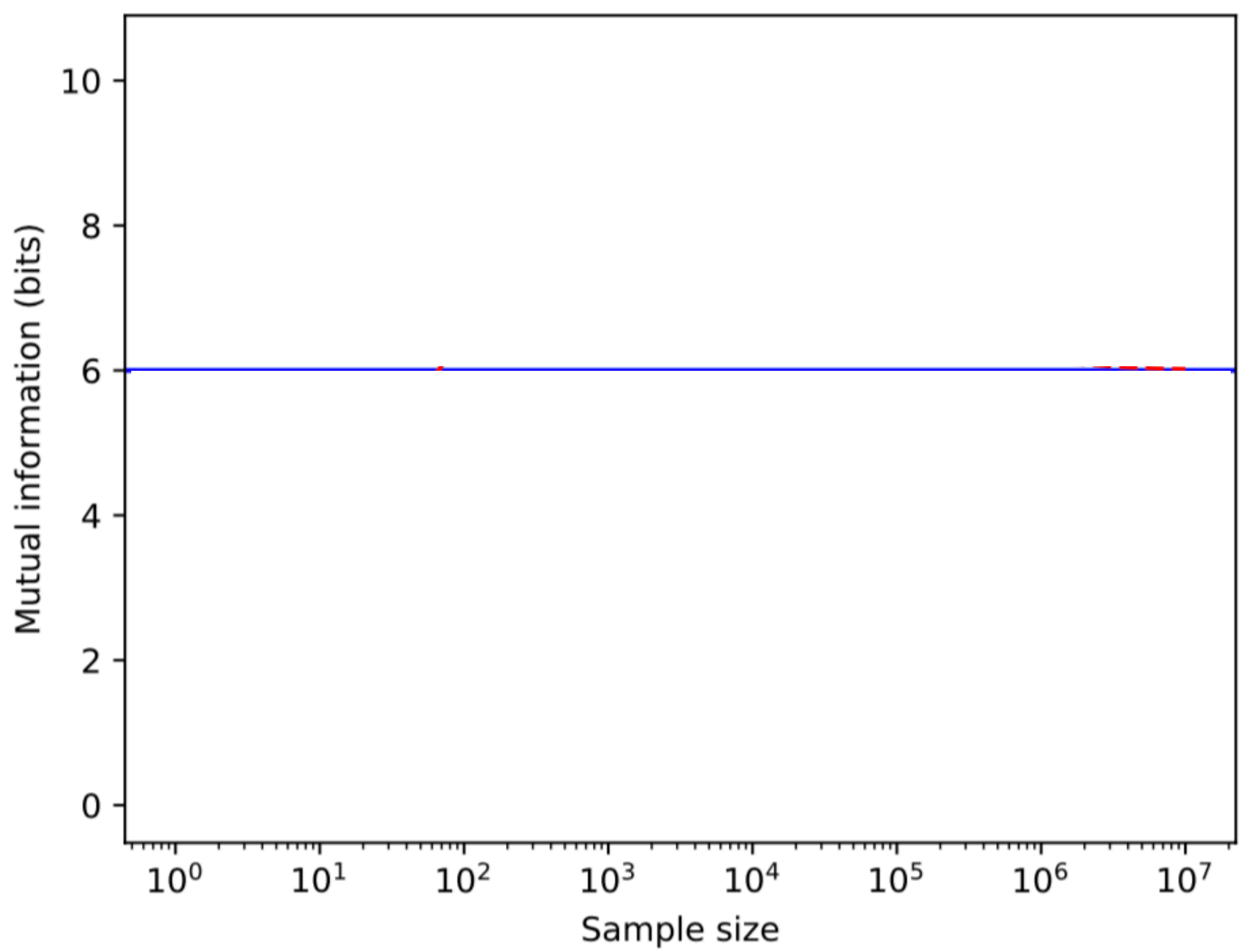

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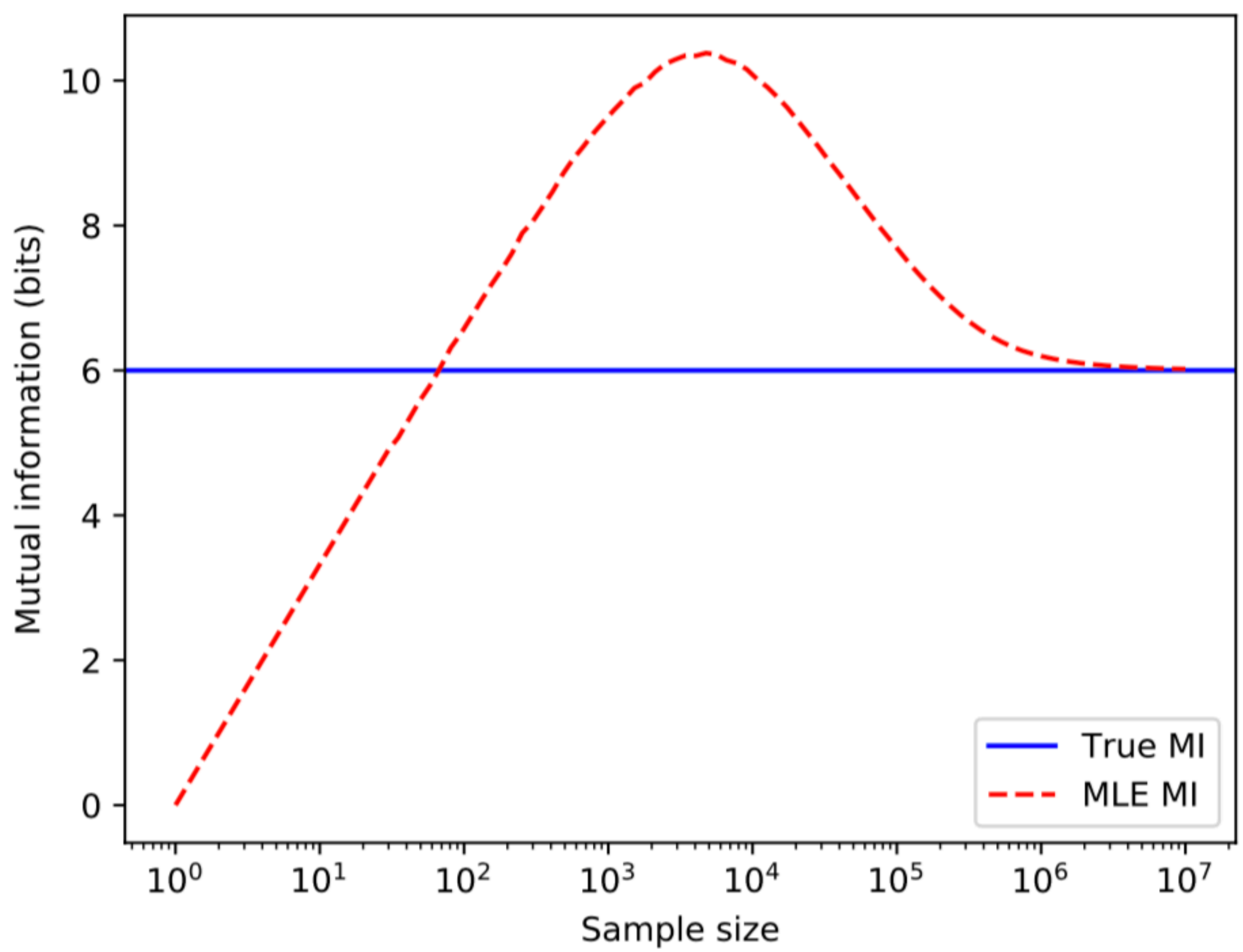

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    - Result: 320 million parsed tokens.
- For evidence for the HDMI Hypothesis from POS tags in hand-parsed UD corpora, see Futrell & Levy (2017).

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|---|---|
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| angry | |
| … | |

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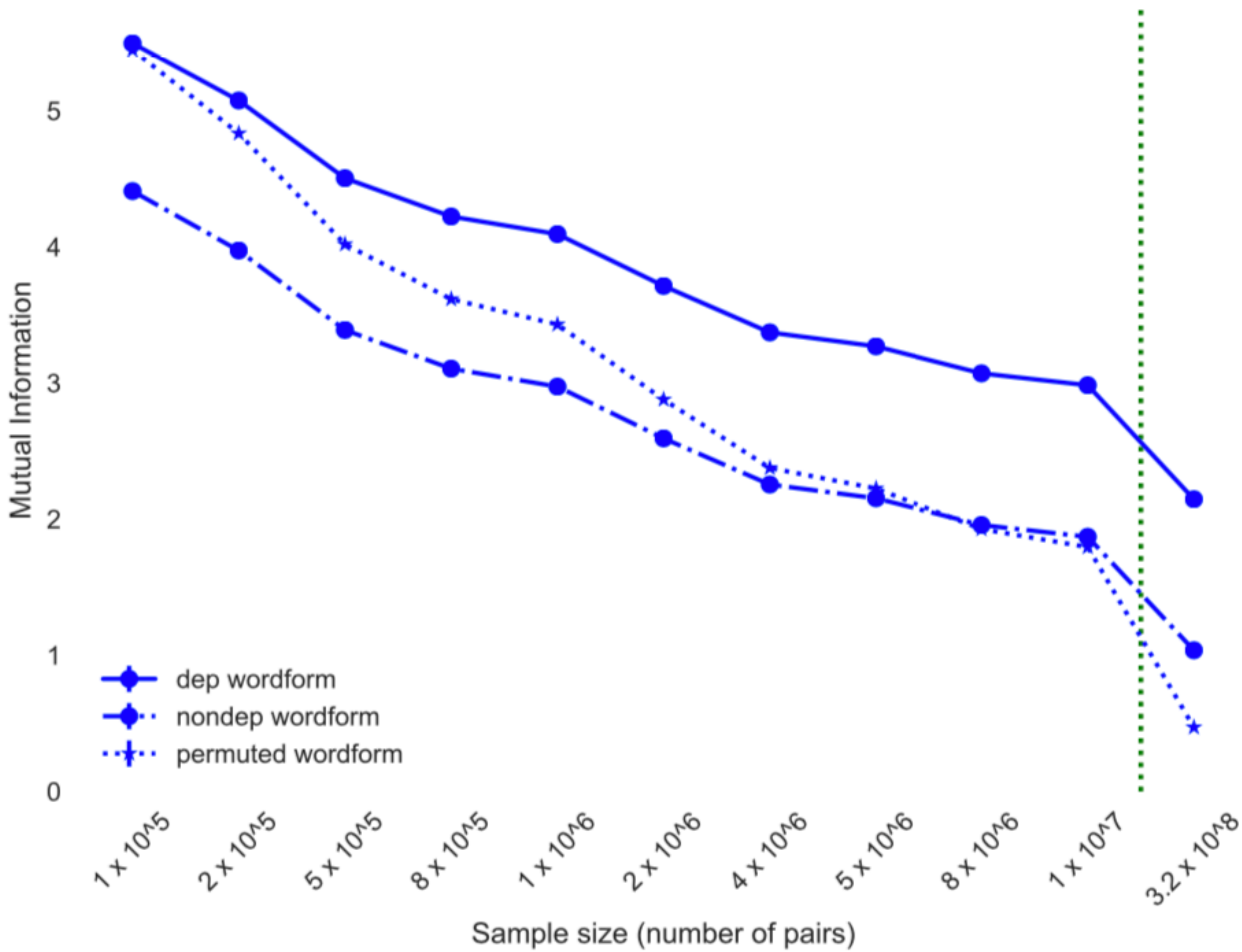
    - So we want the MI of the permuted baseline to go to zero.

# Data & Baselines Summary

| | |
|---|---|
| **dep(endency)** | MI of heads and dependents |
| **nondep** | MI of words not in a dependency relationship, matched for length with dep |
| **permuted** | MI between shuffled heads and dependents (should be zero) |

# Convergence of MLE Estimates of MI



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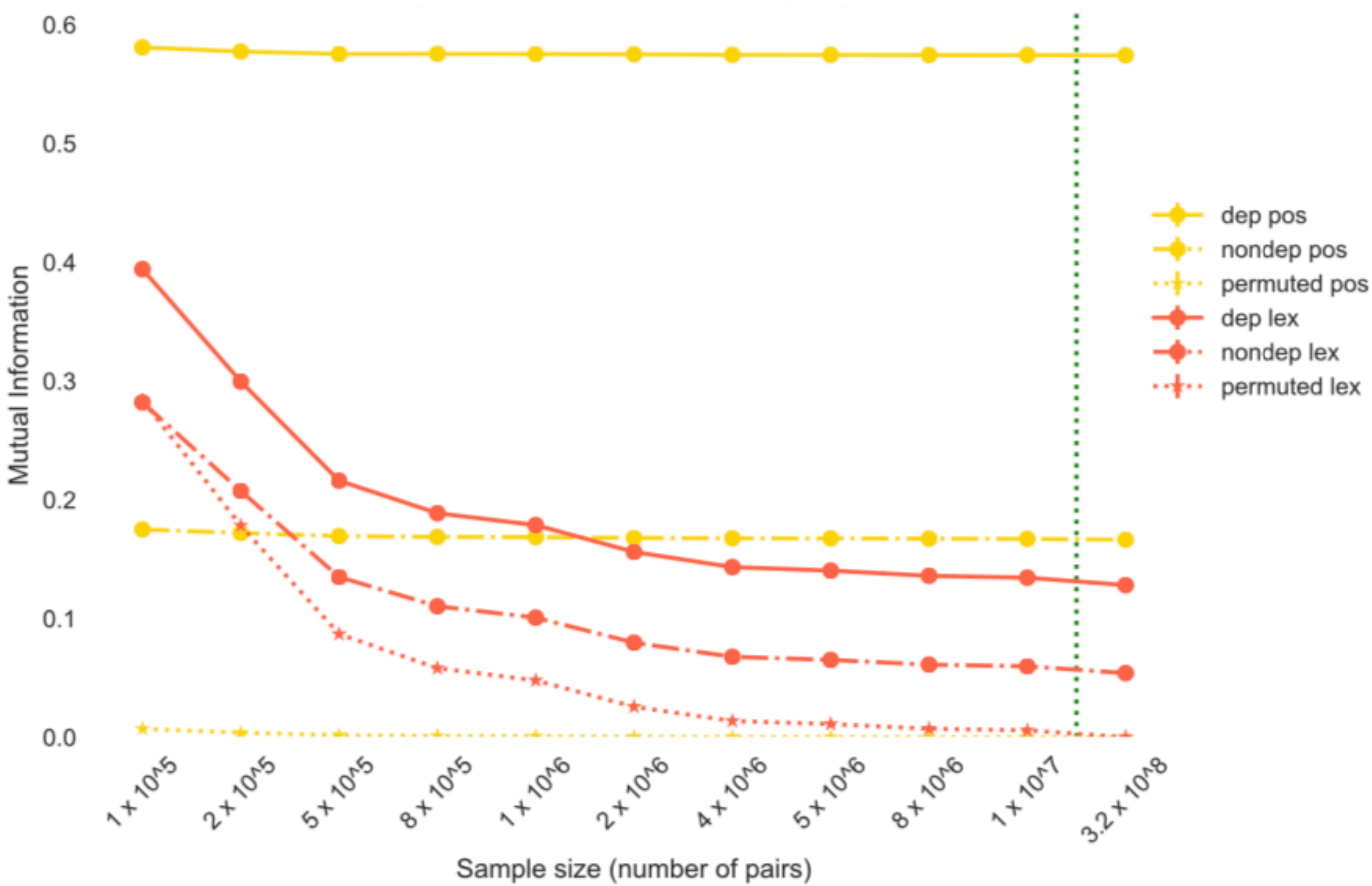
  - **Lexical clusters** derived by a spectral clustering algorithm on GloVe (Pennington et al., 2014) (certainly a lower bound on MI between wordforms).

# HDMI between POS tags and Lexical clusters



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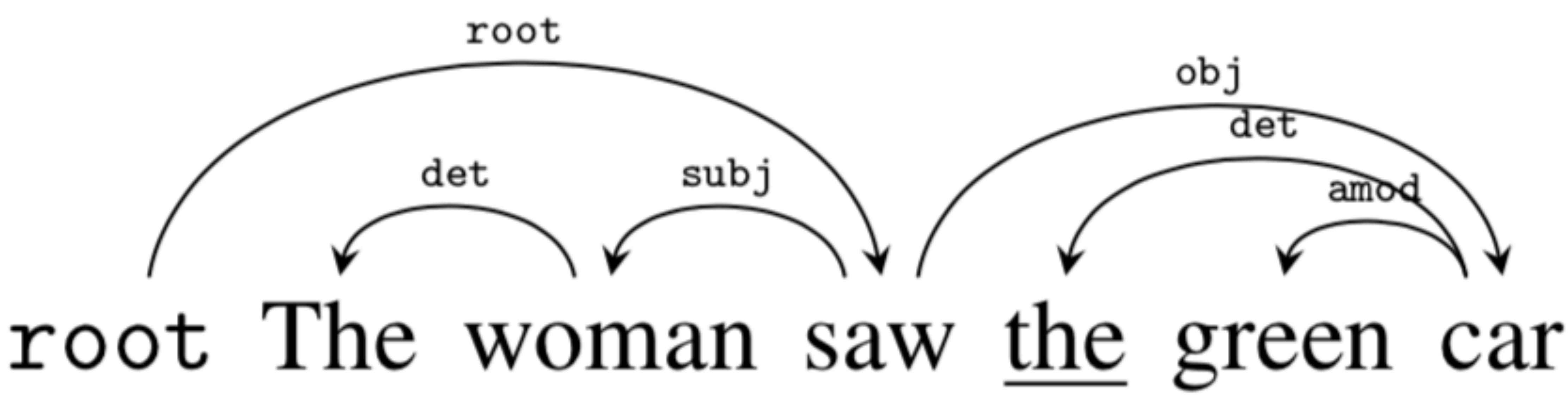

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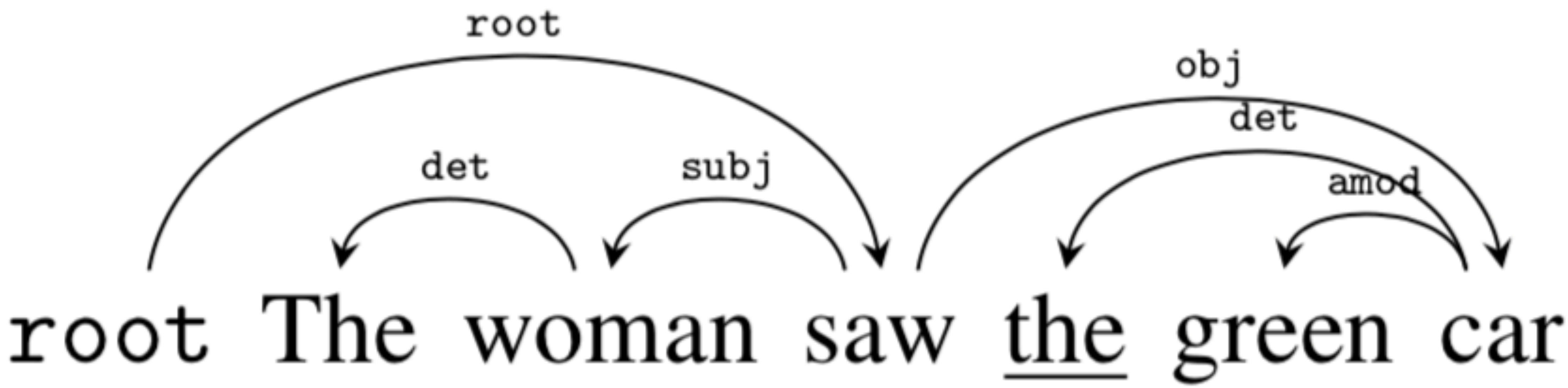

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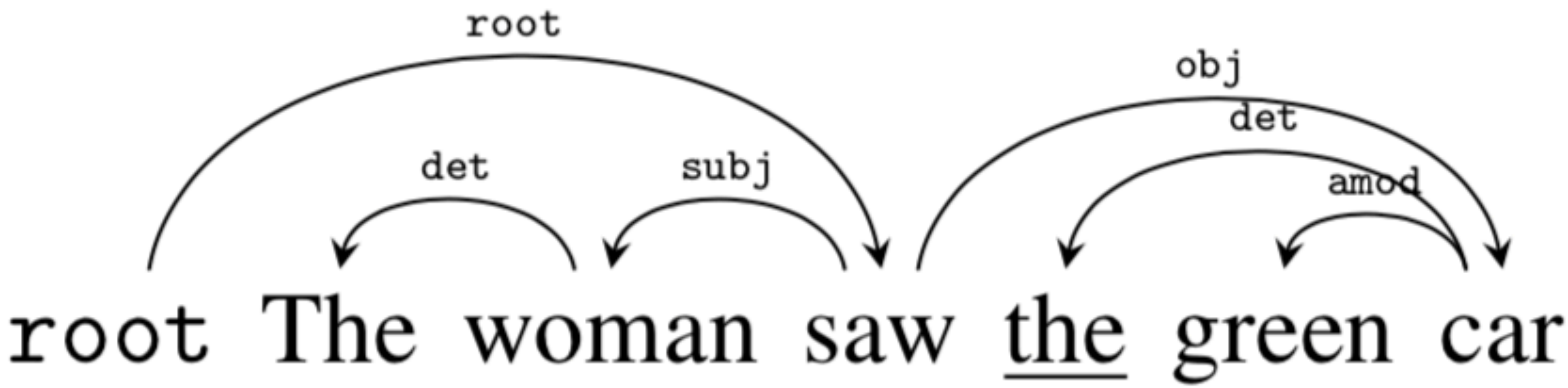


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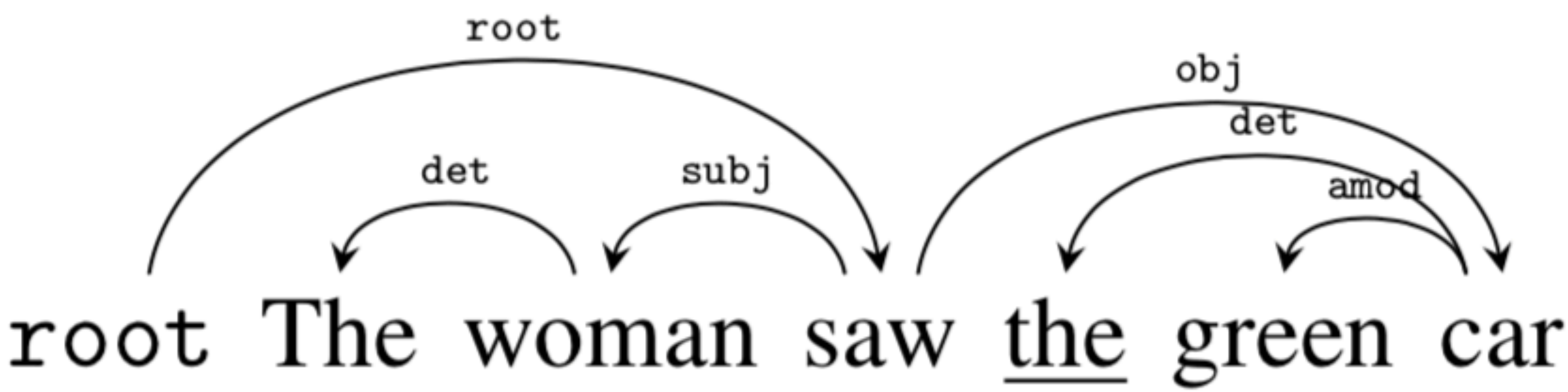


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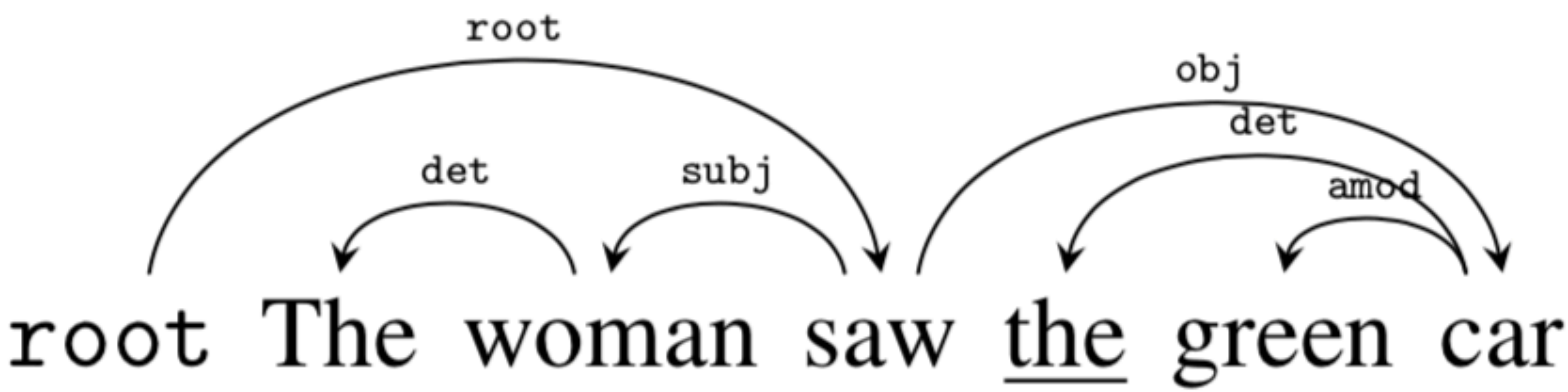


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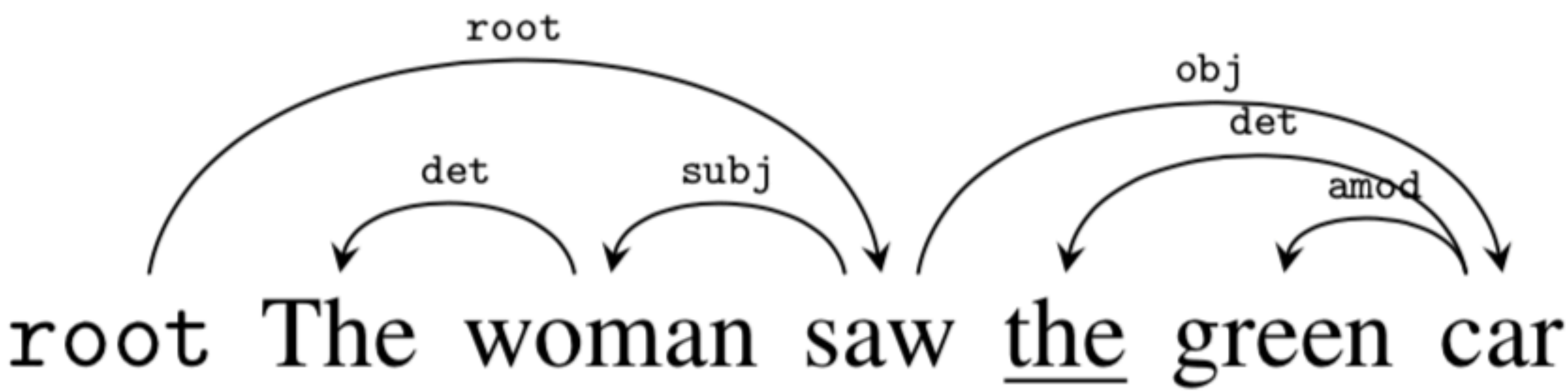


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- This is also the objective implicitly minimized in grammar induction work based on **head-outward generative models** (Eisner, 1996; Klein & Manning, 2004, et seq.)

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$$
\begin{aligned}
D_{\mathrm{KL}}(p_L(w_i|\mathbf{w}_{<i})||p_{\mathbf{t}}(w_i|t_i)) &= \mathbb{E}\left[\log \frac{p_L(w_i|\mathbf{w}_{<i})}{p_{\mathbf{t}}(w_i|t_i)}\right] \\
&= \mathbb{E}\left[\log \frac{p(\mathbf{w}_{<i}|w_i)p(w_i)}{p(\mathbf{w}_{<i})p_{\mathbf{t}}(w_i|t_i)}\right]
\end{aligned}
$$

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\begin{aligned}
\min_{\mathbf{t}} D_{\mathrm{KL}}(p_L(w_i|\mathbf{w}_{<i})||p_{\mathbf{t}}(w_i|t_i)) &= \min_{\mathbf{t}} -\mathbb{E}\left[\log \frac{p_{\mathbf{t}}(w_i|t_i)}{p(w_i)}\right] \\
&= \min_{\mathbf{t}} -I[W:T] \\
&= \max_{\mathbf{t}} I[W:T].
\end{aligned}
$$

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# Head-Dependent MI

- Introduction
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- Theoretical Arguments for HDMI
- Conclusion

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# Thanks all!

- All code is available online at https://github.com/pqian11/mi-hdmi
- Thanks to Roger Levy, Tim O'Donnell, Michael Hahn, and Ryan Cotterell for discussions.
- Thanks to the SyntaxFest reviewers for helpful comments, and thanks to the SyntaxFest and DepLing organizers!