CSE 107
Lab Assignment 4

In this project you will simulate a random process to be described in lecture, and which we summarize here. Suppose you have two coins for which

$$P(\text{Coin 1} = \text{heads}) = p \quad \text{and} \quad P(\text{Coin 2} = \text{heads}) = q.$$

Flip Coin 1 until the first head appears, counting the number of flips. Let N be the number of flips in this first sequence. Next, perform N flips of Coin 2, counting the number of heads. Let Y be the number of heads in this second sequence. Note that N is Geometric with parameter p. If X_i is the random variable

$$X_i = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ flip of Coin 2 is heads} \\ 0 & \text{if the } i^{\text{th}} \text{ flip of Coin 2 is tails} \end{cases}$$

then we see each X_i is Bernoulli with parameter q. We assume of course that all coin flips are independent, so the set $\{N, X_1, X_2, \ldots\}$ is independent. Observe that $Y = X_1 + X_2 + \cdots\cdots\cdots + X_N$ is the sum of a random number of independent identically distributed (iid) random variables. We will derive the mean and variance of Y, in terms of p and q, in lecture. Your goal in this assignment will be to compute both $E[Y]$ and $\text{Var}(Y)$ experimentally, for various values of p and q. Each approximate mean and variance you compute in this project will be obtained by performing 10,000 trials of the above experiment.

Your report for this project will consist of two 9×9 tables giving your estimates of the mean and variance for all combinations of values of p and q in $\{.1, .2, .3, .4, .5, .6, .7, .8, .9\}$. Your output will be formatted as below, where of course 0.000 should be everywhere replaced by your estimates of the mean and variance, respectively.

```
mean
    q:  0.1     0.2     0.3     0.4     0.5     0.6     0.7     0.8     0.9
 p   ---------------------------------------------------------------------
0.1 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.2 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.3 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.4 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.5 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.6 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.7 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.8 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.9 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000

variance
    q:  0.1     0.2     0.3     0.4     0.5     0.6     0.7     0.8     0.9
 p   ---------------------------------------------------------------------
0.1 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.2 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.3 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.4 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.5 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.6 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.7 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.8 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
0.9 |  0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000   0.000
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One would expect the mean to be large if both p is small and q is large, since this would tend to make the first sequence long, and the second sequence to contain many heads. On the other hand, if p is large and q is small, one expects the mean to be small since then the first sequence is short and the second sequence contains few heads. Similar reasoning leads one to expect the same for the table of variances. As a sanity check, observe that both of your tables have their maximum in the upper right corner and minimum in the lower left corner.

This assignment is about the same level of difficulty as lab3, but nevertheless will take some time to get right, so please start early.