Problem Set 2: Forward Kinematics
EECS C106A/C206A, BioE C106A, Fall 2019

Due: Friday, September 20th 2019 at 11:59 PM on Gradescope

1 Planar Rigid Body Transformartion

A transformation $g = (p, R) \in SE(2)$ consists of a translation $p \in \mathbb{R}^2$ and a 2×2 rotation matrix R. We represent this in homogeneous coordinates as a 3×3 matrix:

$$R = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

A twist $\hat{\xi} \in se(2)$ can be represented by a 3×3 matrix of the form:

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} \quad \hat{\omega} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \quad \omega \in \mathbb{R}, v \in \mathbb{R}$$

The twist coordinates for $\hat{\xi} \in se(2)$ have the form $\xi = (v, \omega) \in \mathbb{R}^3$. Note that v is a vector in the plane and ω is a scalar.

(a) Show that the exponential of a twist in $se(2)$ gives a rigid body transformation in $SE(2)$. Consider both the pure translation case, $\xi = (v, \omega)$, and the general case, $\xi = (v, \omega), \omega \neq 0$.

(b) Show that the planar twists which correspond to pure rotation about a point q and pure translation in a direction v are given by,

$$\xi = \begin{bmatrix} q_y \\ -q_x \\ 1 \end{bmatrix} \text{(pure rotation)} \quad \xi = \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \text{(pure translation).}$$

2 Forward Kinematics

For each of the three degree of freedom manipulators shown in Figure 1, find the forward kinematics map *symbolically*. Feel free to use a computer program if you wish. Some options are the Matlab symbolic Toolbox or SymPy. A Mathematica package `Screws` is also linked at the end of the textbook.

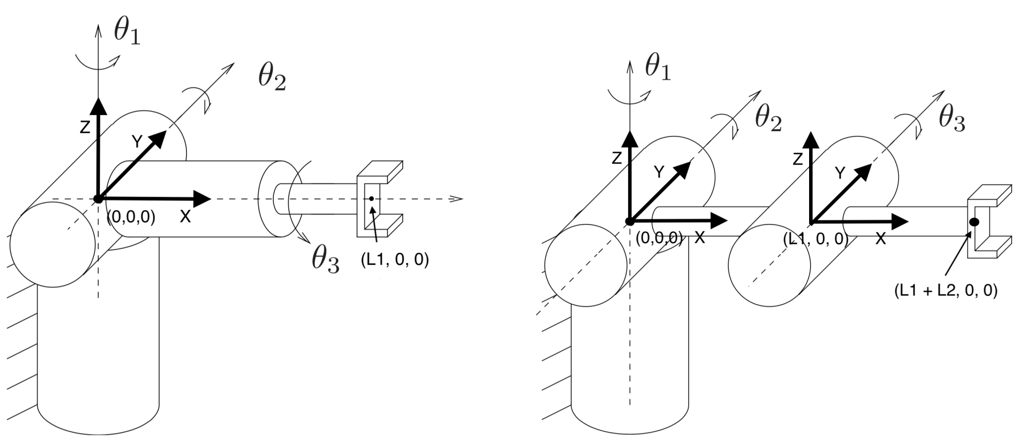

Figure 1: Two three degree of freedom manipulators

3 More Forward Kinematics

For each of the manipulators shown schematically in Figure 2, 3, find the forward kinematics map *symbolically*. Feel free to use a computer program if you wish. Some options are the Matlab symbolic Toolbox or SymPy. A Mathematica package `Screws` is also linked at the end of the textbook.

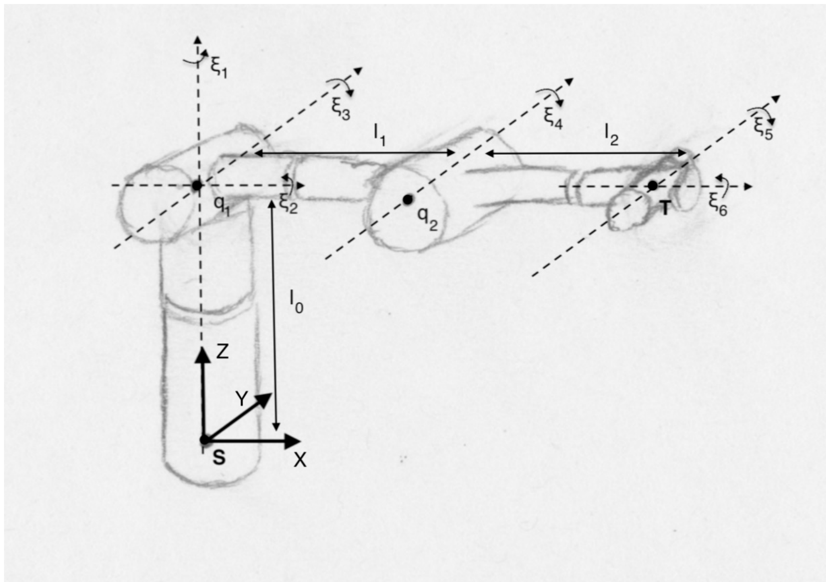

Figure 2: Inverse elbow manipulator

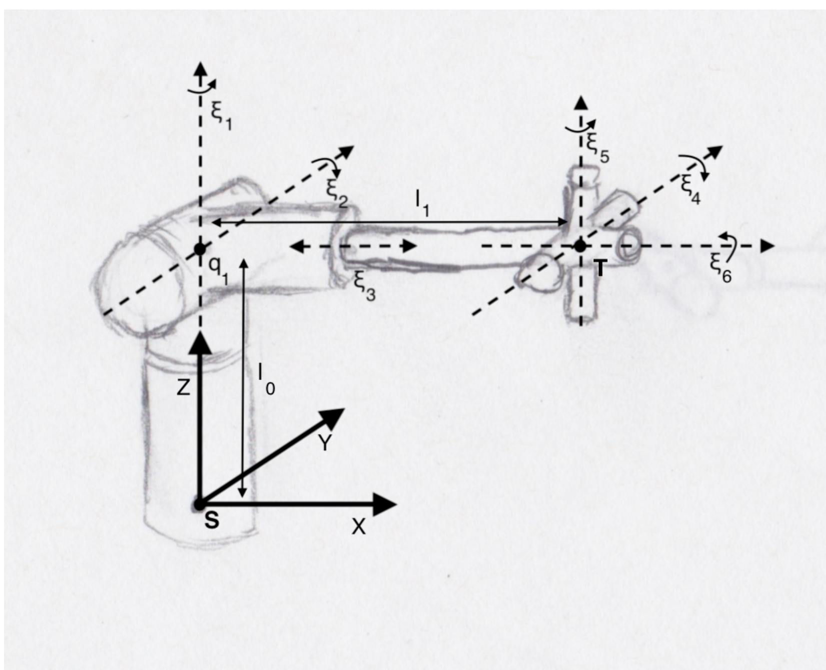

Figure 3: Stanford manipulator