Problem Set 1: Rotations
EECS C 106A/C206A, Fall 2019

Due: Monday, September 9, 2019 at 11:59 PM on Gradescope

1 Multiple Rotation Matrices

a) You are given the rotation matrices: R_{AB}, R_{CB}. Write the expression for R_{CA}.

b) You are given the rotation matrices: R_{AB}, R_{CA}. Write the expression for R_{BC}.

c) You are given the rotation matrices: R_{AB}, R_{BC}. Write the expression for R_{AA}.

d) You are given the rotation matrices: R_{AB}^{-1}, R_{BC}^{T}. Write the expression for R_{AC}.

2 Euler Angles

Consider two initially coincident reference frames, A and B. Frame B is then rotated about the Z axis by $\pi/4$ radians.

a) Sketch the coordinate frames A and B after the rotation.

b) Write the rotation matrix R_{AB} that will take a point from the B frame and represent it in the A frame.

c) Write the rotation matrix R_{BA}.

d) What are the coordinates in frame A of a point with coordinates $p_B = [0, 0, 1]^T$ given with respect to frame B?

e) What are the coordinates in frame B of a point with coordinates $p_A = [0, 0, 1]^T$ given with respect to frame A?

3 Multiple Euler Angles

A frame is rotated first about the Z axis by angle $\frac{\pi}{2}$, then about the Y axis by an angle of $\frac{\pi}{2}$, then about the mobile X axis an angle of $\frac{\pi}{2}$.

a) Draw the frame before and after the rotation. Label all axes.

b) Write the composite rotation matrix.

A frame is rotated first about the Z axis by angle $\frac{\pi}{2}$, then about the original Y axis by an angle of $\frac{\pi}{2}$, then about the original X axis an angle of $\frac{\pi}{2}$.

c) Draw the frame before and after the rotation. Label all axes.

d) Write the composite rotation matrix.

4 Properties of Rotations

Determine which of the following transformation matrix below is a valid rotation matrix. Justify your answer

a) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, d) $\begin{bmatrix} \frac{1}{2} & \sqrt{2} \\ -\sqrt{2} & 0 \end{bmatrix}$

5 Axis Angle Notation

1. Use the Rodrigues formula to show that a rotation of θ radians about the Y axis results in the same rotation matrix as the Euler Y equation.

2. Use the Rodrigues formula to find the rotation matrix for a rotation of $\frac{\pi}{4}$ about the axis given by the vector $[1, 2, 3]$.

6 Rotation Matrices in Action

Open `HW1.zip`. Inside, you should find two files: `HW1.py` and `parking_path.mat`. `HW1.py` reads the parallel parking path stored in `parking_path.mat` and displays it. However, there's a problem. `HW1.py` doesn't take the car's orientation into consideration when plotting, so nothing looks right.

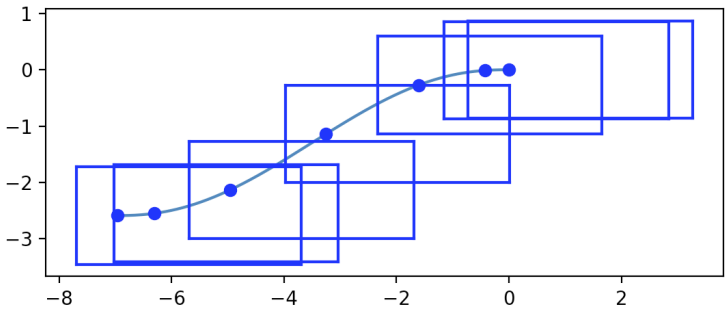

Figure 1: Cars can't move like this!

Edit the script to take this into account, and submit the corrected plot with your homework solution.