Grupo #7
Paralelo 110 - Práctico CV
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Taller Formativo #9

Recta ON: $m = \frac{1-(-1)}{(-1-0)} = 2$

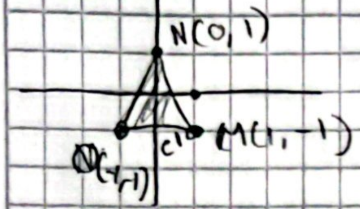
$(y-1) = 2x \rightarrow y = 2x+1$

Recta NM: $m = \frac{(-1,-1)}{(1-0)} = -2$

$(y-1) = -2x$

$y = -2x+1$

Tema #1



a) $\int_{-1}^{0}\int_{-1}^{2x+1} (-1-1)\,dy\,dx$

$\int_{-1}^{0}\int_{-1}^{2x+1} -2\,dy\,dx + \int_{0}^{1}\int_{-1}^{-2x+1} -2\,dy\,dx$

$= -2\int_{-1}^{0} [2x+1+1]\,dx + (-2)\int_{0}^{1} [-2x+1+1]\,dx$

$= -4\int_{-1}^{0} [x+1]\,dx + (-4)\int_{0}^{1} [-x+1]\,dx$

$= -4\left[\frac{x^2}{2}+x\right]_{-1}^{0} + (-4)\left[-\frac{x^2}{2}+x\right]_{0}^{1}$

$= -4\left[0-\left[\frac{1}{2}-1\right]\right] + (-4)\left[-\frac{1}{2}+1\right]$

$= -4\left[\frac{1}{2}\right] + \left[-4\left(\frac{1}{2}\right)\right] = -2-2 = \boxed{-4}$

→ literal a)

b) $\int_{C'} P\,dx + Q\,dy$ $\qquad C' = r(t) = (t,-1) \quad -1 \le t \le 1$

$x = t \quad y = -1$

$dx = dt \quad dy = 0$

$\int_{C'} y\,dx - x\,dy$

$= \int_{C'} (-1)(dt) - \cancel{(t)(0)}$

$= \int_{C'} -1\,dt = -1\int_{-1}^{1} dt = -1[1+1] = \boxed{-2}$

→ literal b

c) $-4-(-2) = -2$ //R

→ literal c

Tema #2

$z = x^2 + y^2 \qquad z = 2x \qquad r(t) = \begin{cases} z = x^2+y^2 \\ z = 2x \end{cases}$

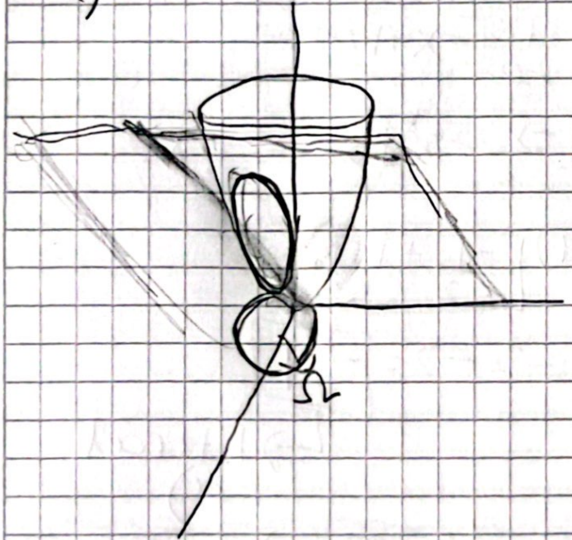
a) Grafique S y la curva $r(t)$

b) Siendo $F(x,y,z) = (z, x, y)$ calcule $\oint_C F \cdot dr$ utilizando el teorema de Stokes.

$r(t) = \begin{cases} z = x^2+y^2 \\ z = 2x \end{cases} \rightarrow 2x = x^2 + y^2 \rightarrow (x^2 - 2x + 1) + y^2 = 1$

$\rightarrow (x-1)^2 + y^2 = 1$

a)



$r\ (r\cos\theta - 1)^2 + r^2 \operatorname{sen}\theta^2 = 1$

$r^2 \cos^2\theta - 2r\cos\theta + 1 + r^2 \operatorname{sen}^2\theta = 1$

$r^2 - 2r\cos\theta \cancel{=} 0$

$r(r - 2\cos\theta) = 0$

Teorema de Stokes

$\int_r F \cdot dr = \iint_S \operatorname{rot} F \cdot N\, ds$

b) S: $z - 2x = 0$

vector normal: $(-2, 0, 1)$

$G = z - 2x = 0$

$\nabla G = (-2, 0, 1)$

$ds = r(u, v)$

$ds = \|r_u \times r_v\|\, du\, dv$

$r(x,y) = (x, y, 2x)$

$r_x = (1, 0, 2)$

$r_y = (0, 1, 0)$

$r_x \times r_y = (-2, 0, 1) - N$

$\|r_x \times r_y\| = \sqrt{5}$

$\operatorname{Rot}(F) = (P_y - N_z,\ M_z - P_x,\ N_x - M_y)$

$\operatorname{rot}(F) = (1 - 0,\ 1 - 0,\ 1 - 0) = (1, 1, 1)$

$\iint_S (1,1,1) \cdot \frac{(-2, 0, 1)}{\sqrt{5}} \cdot \sqrt{5}\ dy\,dx$

$\iint_\Omega (-2+1)\, dy\, dx = -1 \iint_\Omega dy\,dx$ → como Ω es un círculo y $\iint$ es el área saco el área del círculo de $r = 1$

$\int = -1(\pi) = -\pi$

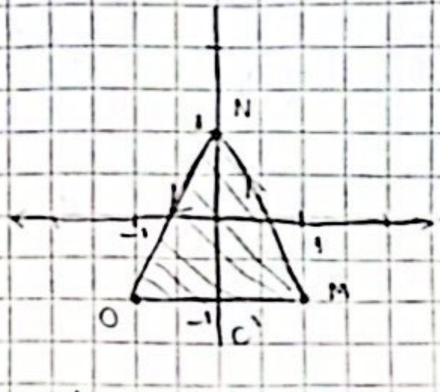
→ Respuesta

Tema 1

$F(x,y) = (\overset{M}{y}, \overset{N}{-x})$ vértices $M(1,-1)$, $N(0,1)$ y $O(-1,-1)$ sin formar un camino cerrado, con teorema de Green realice:

a) Añada C' entre $O(-1,-1)$ y $M(1,-1)$ y calcule

$$\int_{C \cup C'} P dx + Q dy.$$



$F(M,N)$ $M,N \in C^1$

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \iint_R -1 - 1 \; dA$$

$$= -2 \iint_R dy dx = -2\left(\frac{2 \cdot 2}{2}\right) = -4$$

$$dA = \frac{b \cdot h}{2}$$

b) Utilizando la definición calcule $\int_{C'} P dx + Q dy$

$$\int_{C'} P dx + Q dy \qquad C' = r(t) = (t,-1) \; ; \; -1 \leq t \leq 1$$

$$= \int_{-1}^{1} -dt - t(0) = \int_{-1}^{1} -dt = -t \Big|_{-1}^{1} = -1 - 1 = -2_{//}$$

c) $\int_{C \cup C'} P dx + Q dy - \int_{C'} P dx + Q dy$

$$\int_C P dx + Q dy = -4 + 2 = -2_{//}$$

Tema 2

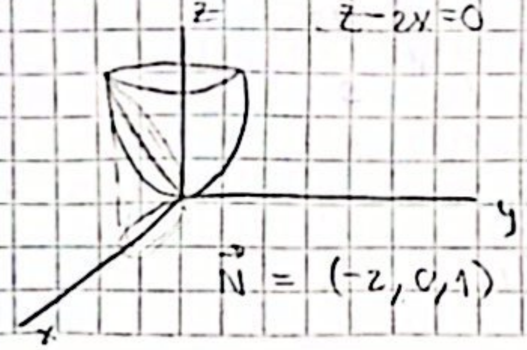
Sea S la parte del paraboloide $z = x^2 + y^2$ que queda bajo $z = 2x$, y sea $r(t)$ la curva intersección

a) Grafique S y la curva $r(t)$

b) Siendo $F(x,y,z) = (z, x, y)$ calcular $\int_C F \cdot dr$ utilizando T. de Stokes.

a)

$z - 2x = 0$



$\vec{N} = (-2, 0, 1)$

b) $\iint_S \nabla \times \vec{F} \cdot \vec{N} \, ds$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z & x & y \end{vmatrix} = (1, 1, 1)$$

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