Assignment 6: The Linear Quadratic Regulator

Isabella Rangel

April 9, 2025

Problem 4

$$
\begin{aligned}
\hat{A} &= \begin{bmatrix} 1 & 0.0802 \\ 0 & 0.6310 \end{bmatrix} \\
B &= \begin{bmatrix} 0.0034 \\ 0.0631 \end{bmatrix} \\
\tau &= 0.1 \quad \text{(seconds)} \\
k_1 &= 20 \quad \text{(number of simulation steps).}
\end{aligned}
$$

Design Parameters Initial Values:

$$
\begin{aligned}
\alpha &= 2 \quad \text{(weight for } \theta^2\text{)} \\
\beta &= 1 \quad \text{(weight for } \dot{\theta}^2\text{)} \\
\rho &= 0.5 \quad \text{(weight for } u^2\text{)} \\
\gamma &= 5 \quad \text{(terminal cost weight for } \theta^2\text{)} \\
\delta &= 2 \quad \text{(terminal cost weight for } \dot{\theta}^2\text{)}
\end{aligned}
$$

Total cost of the closed-loop simulation: **0.4004**

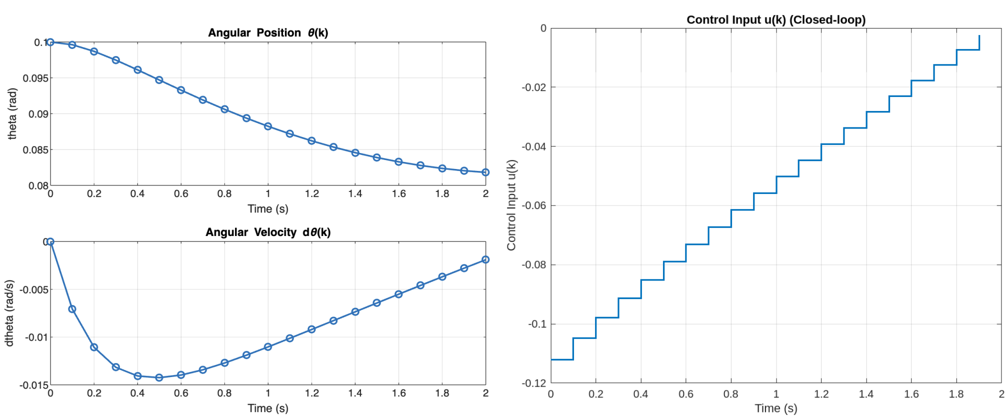


Table 1: Feedback Gains K_k at Each Time Step

Step k	$K_k(1)$	$K_k(2)$
0	1.1210	0.3553
1	1.0762	0.3458
2	1.0296	0.3359
3	0.9814	0.3257
4	0.9315	0.3151
5	0.8800	0.3042
6	0.8269	0.2930
7	0.7722	0.2814
8	0.7160	0.2695
9	0.6584	0.2573
10	0.5995	0.2448
11	0.5393	0.2320
12	0.4781	0.2191
13	0.4158	0.2060
14	0.3526	0.1928
15	0.2888	0.1798
16	0.2246	0.1675
17	0.1603	0.1572
18	0.0964	0.1519
19	0.0335	0.1594

Table 2: Riccati Matrices P_k for $k = 0, \ldots, 20$

Step k	$P_k(1,1)$	$P_k(1,2) = P_k(2,1)$	$P_k(2,2)$
0	40.0430	7.3625	3.1167
1	38.6910	7.0759	3.0559
2	37.2877	6.7784	2.9929
3	35.8335	6.4702	2.9275
4	34.3290	6.1512	2.8599
5	32.7750	5.8218	2.7901
6	31.1727	5.4821	2.7181
7	29.5235	5.1326	2.6440
8	27.8291	4.7734	2.5679
9	26.0917	4.4052	2.4898
10	24.3135	4.0284	2.4100
11	22.4972	3.6437	2.3286
12	20.6457	3.2517	2.2457
13	18.7622	2.8532	2.1617
14	16.8503	2.4492	2.0773
15	14.9136	2.0408	1.9935
16	12.9560	1.6293	1.9130
17	10.9817	1.2167	1.8421
18	8.9947	0.8052	1.7965
19	6.9994	0.3983	1.8156
20	5.0000	0	2.0000

Problem 5

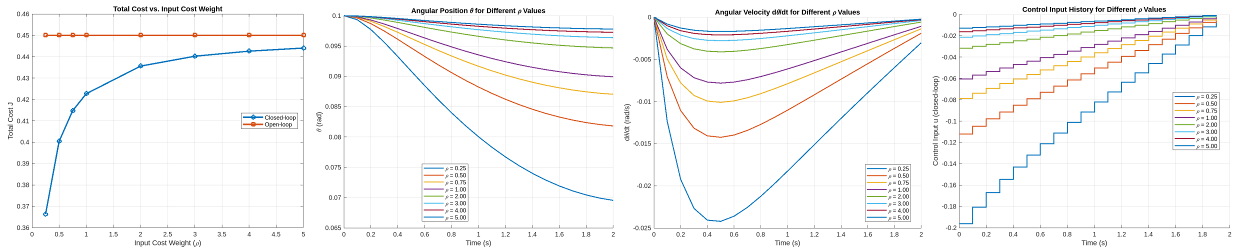
Here are some findings:

- ρ increase causes higher closed-loop cost.
- α increase causes increasing control effort and cost.
- β and δ have a smaller impact.
- γ has very small impact.

Changing ρ

Table 3: Total Costs for Varying Values of ρ

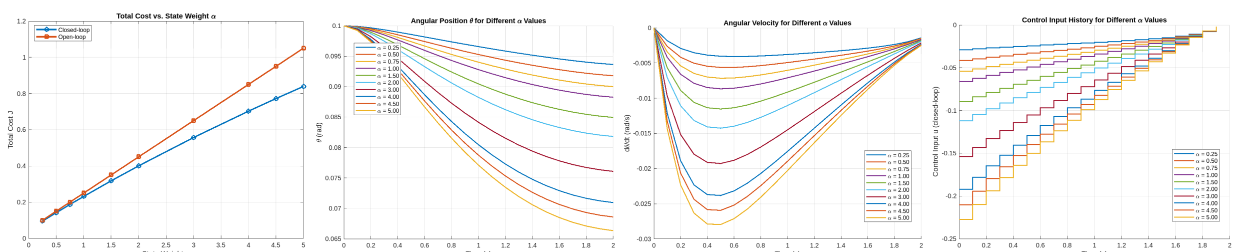
ρ	Closed-Loop Cost	Open-Loop Cost	Cost Improvement
0.25	0.36633	0.4500	0.083668
0.50	0.40043	0.4500	0.04957
0.75	0.41477	0.4500	0.035226
1.00	0.42268	0.4500	0.027322
2.00	0.43560	0.4500	0.01440
3.00	0.44022	0.4500	0.0097762
4.00	0.44260	0.4500	0.0074001
5.00	0.44405	0.4500	0.0059532



Changing α

Table 4: Total Costs for Varying Values of α

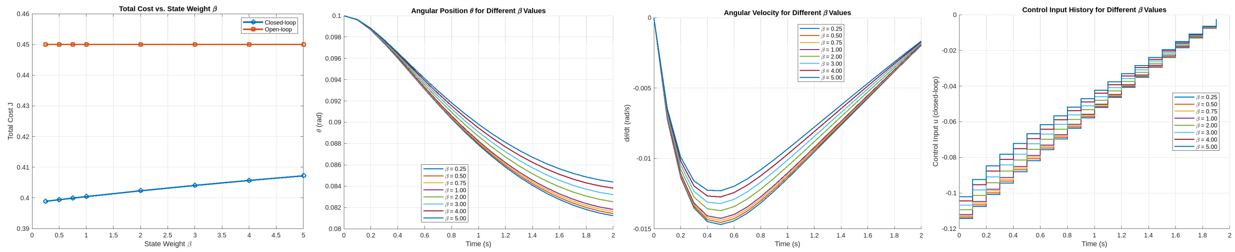
α	Closed-Loop Cost	Open-Loop Cost	Cost Improvement
0.25	0.095258	0.1000	0.0047424
0.5	0.14168	0.1500	0.0083221
0.75	0.18709	0.2000	0.012914
1.00	0.23153	0.2500	0.018472
1.5	0.31768	0.3500	0.032324
2.00	0.40043	0.4500	0.04957
3.00	0.55683	0.6500	0.093173
4.00	0.70259	0.8500	0.14741
4.50	0.77196	0.9500	0.17804
5.00	0.83922	1.0500	0.21078



Changing β

Table 5: Total Costs for Varying Values of β

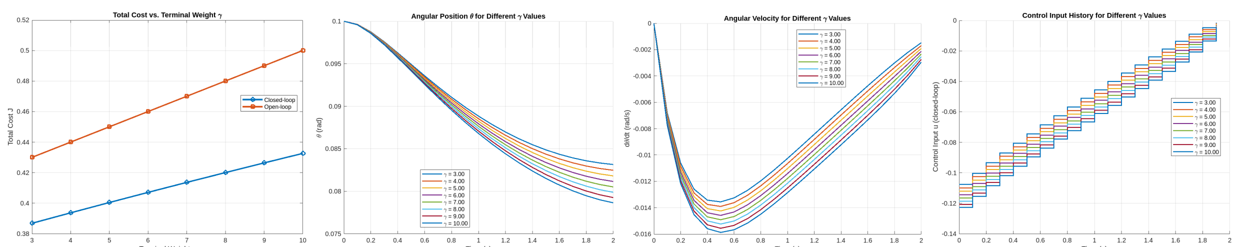
β	Closed-Loop Cost	Open-Loop Cost	Cost Improvement
0.25	0.3989	0.45	0.051096
0.5	0.39942	0.45	0.050577
0.75	0.39993	0.45	0.050068
1	0.40043	0.45	0.04957
2	0.40232	0.45	0.047676
3	0.40408	0.45	0.045925
4	0.40570	0.45	0.044301
5	0.40721	0.45	0.042791



0.1 Changing γ

Table 6: Total Costs for Varying Values of γ

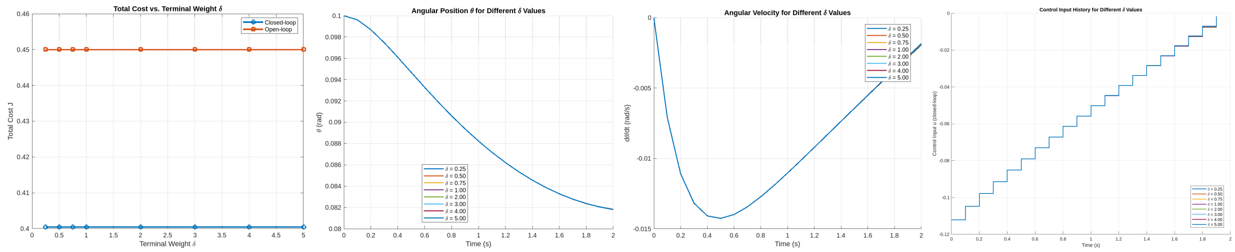
γ	Closed-Loop Cost	Open-Loop Cost	Cost Improvement
3	0.38682	0.43	0.043175
4	0.39368	0.44	0.046318
5	0.40043	0.45	0.04957
6	0.40707	0.46	0.05293
7	0.41360	0.47	0.056396
8	0.42004	0.48	0.059964
9	0.42637	0.49	0.063632
10	0.43260	0.50	0.067398



Changing δ

Table 7: Total Costs for Varying Values of δ

δ	Closed-Loop Cost	Open-Loop Cost	Cost Improvement
0.25	0.40042	0.4500	0.049577
0.50	0.40042	0.4500	0.049576
0.75	0.40043	0.4500	0.049575
1.00	0.40043	0.4500	0.049574
2.00	0.40043	0.4500	0.04957
3.00	0.40043	0.4500	0.049567
4.00	0.40044	0.4500	0.049563
5.00	0.40044	0.4500	0.04956



Problem 8

Assume that you only want to penalize (or drive to zero) the angular position and do not care about the angular velocity. However, assume that you are able to observe or measure both the angular position and the angular velocity.

- β **(weight on** $\dot{\theta}^2$**):** β becomes irrelevant in the cost function.
- δ **(terminal weight on** $\dot{\theta}^2$**):** δ also loses impact. Terminal cost calculations ignore $\dot{\theta}$.
- α **and** γ **(weights on** θ^2 **and terminal** θ^2**):** These parameters become more dominant. They affect closed-loop cost.

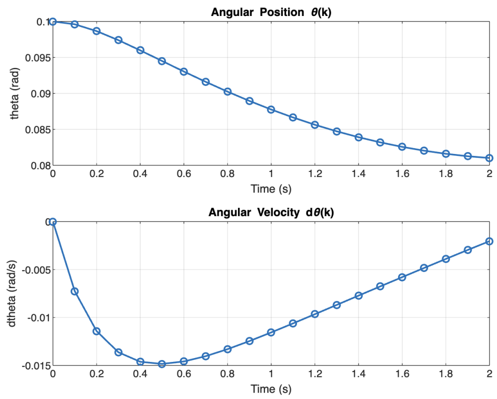
Total cost of the closed-loop simulation: **0.3984**

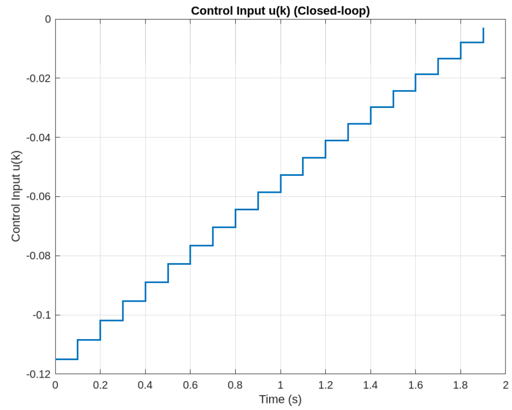
Table 8: Feedback Gains $K_k = [K_{k1} \quad K_{k2}]$

Step k	K_{k1}	K_{k2}
0	1.1501	0.2376
1	1.1045	0.2277
2	1.0572	0.2174
3	1.0080	0.2066
4	0.9570	0.1955
5	0.9043	0.1841
6	0.8499	0.1722
7	0.7937	0.1600
8	0.7360	0.1474
9	0.6768	0.1345
10	0.6161	0.1213
11	0.5541	0.1078
12	0.4909	0.0940
13	0.4266	0.0801
14	0.3615	0.0660
15	0.2957	0.0519
16	0.2296	0.0379
17	0.1636	0.0244
18	0.0981	0.0121
19	0.0340	0.0027

Table 9: Riccati Matrices P_k

Step k	$P_k(1,1)$	$P_k(1,2) = P_k(2,1)$	$P_k(2,2)$
0	39.8365	7.4985	1.5345
1	38.5103	7.2094	1.4715
2	37.1313	6.9088	1.4060
3	35.6996	6.5968	1.3380
4	34.2159	6.2735	1.2675
5	32.6809	5.9390	1.1946
6	31.0957	5.5935	1.1194
7	29.4617	5.2375	1.0418
8	27.7806	4.8712	0.9620
9	26.0546	4.4952	0.8801
10	24.2860	4.1101	0.7962
11	22.4775	3.7165	0.7106
12	20.6323	3.3151	0.6233
13	18.7537	2.9070	0.5347
14	16.8452	2.4933	0.4452
15	14.9109	2.0751	0.3553
16	12.9548	1.6543	0.2660
17	10.9812	1.2328	0.1791
18	8.9946	0.8138	0.0983
19	6.9994	0.4010	0.0322
20	5.0000	0	0.0000

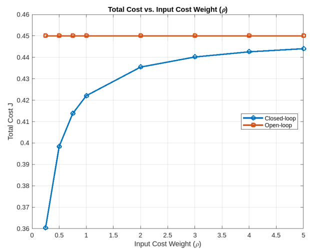


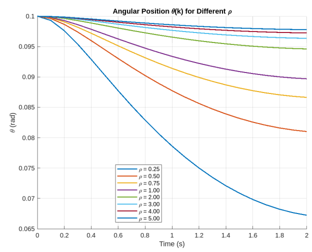


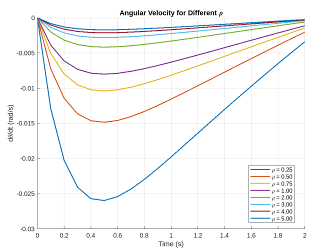
Changing ρ

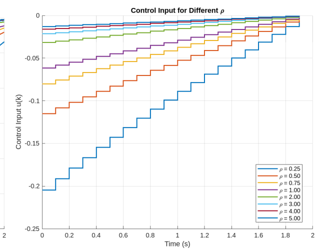
Table 10: Total Costs for Varying Values of ρ

ρ	Closed-Loop Cost	Open-Loop Cost	Cost Improvement
0.25	0.36032	0.45	0.089683
0.50	0.39837	0.45	0.051635
0.75	0.41374	0.45	0.036259
1.00	0.42206	0.45	0.027940
2.00	0.43543	0.45	0.014570
3.00	0.44015	0.45	0.0098543
4.00	0.44256	0.45	0.0074449
5.00	0.44402	0.45	0.0059822





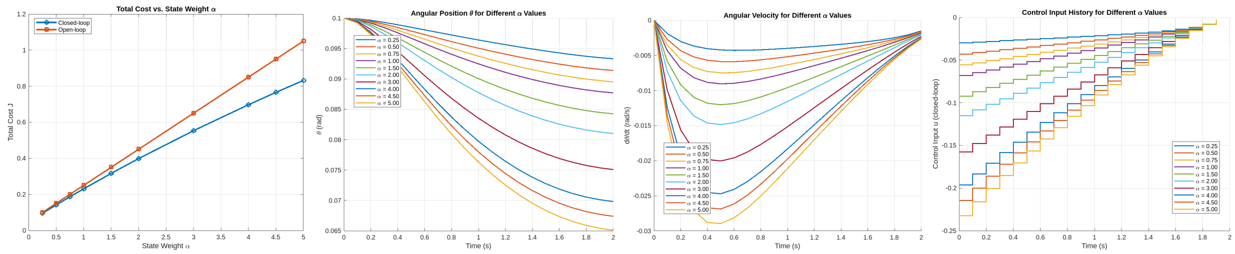




Changing α

Table 11: Total Costs for Varying Values of α

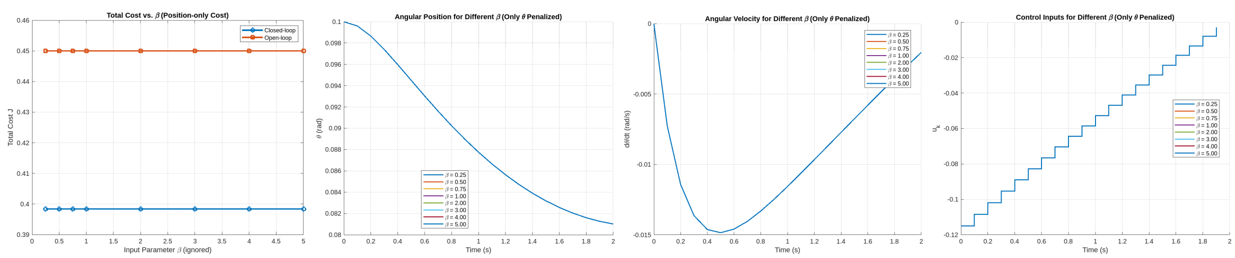
α	Closed-Loop Cost	Open-Loop Cost	Cost Improvement
0.25	0.095026	0.10	0.0049737
0.5	0.14128	0.15	0.0087152
0.75	0.18649	0.20	0.0135080
1	0.23070	0.25	0.0193030
1.5	0.31628	0.35	0.0337200
2	0.39837	0.45	0.0516350
3	0.55319	0.65	0.0968100
4	0.69716	0.85	0.1528400
4.5	0.76559	0.95	0.1844100
5	0.83188	1.05	0.2181200



Changing β

Table 12: Total Costs for Varying Values of β

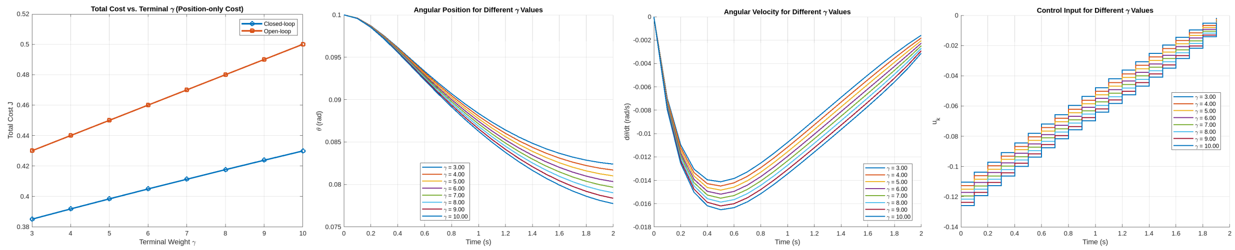
β	**Closed-Loop Cost**	**Open-Loop Cost**	**Cost Improvement**
0.25	0.39837	0.45	0.051635
0.50	0.39837	0.45	0.051635
0.75	0.39837	0.45	0.051635
1.00	0.39837	0.45	0.051635
2.00	0.39837	0.45	0.051635
3.00	0.39837	0.45	0.051635
4.00	0.39837	0.45	0.051635
5.00	0.39837	0.45	0.051635



Changing γ

Table 13: Total Costs for Varying Values of γ

γ	**Closed-Loop Cost**	**Open-Loop Cost**	**Cost Improvement**
3	0.38501	0.43	0.044986
4	0.39175	0.44	0.048254
5	0.39837	0.45	0.051635
6	0.40487	0.46	0.055126
7	0.41127	0.47	0.058725
8	0.41757	0.48	0.062429
9	0.42376	0.49	0.066236
10	0.42986	0.50	0.070142



Changing δ

Table 14: Total Costs for Varying Values of δ

δ	Closed-Loop Cost	Open-Loop Cost	Cost Improvement
0.25	0.39837	0.45	0.051634
0.50	0.39837	0.45	0.051633
0.75	0.39837	0.45	0.051632
1.00	0.39837	0.45	0.051631
2.00	0.39837	0.45	0.051627
3.00	0.39838	0.45	0.051623
4.00	0.39838	0.45	0.051619
5.00	0.39838	0.45	0.051615