

The Ecological Effects of Trait Variation in a u -Predator, v -Prey System

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Funding

- ▶ National Science Foundation
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Observations

- ▶ Predator/Prey interactions are prevalent in nature
 - ▶ Crab vs. gastropod
 - ▶ Protist vs. bacteria
- ▶ There is trait variation within species
 - ▶ Thickness of plant cuticula
 - ▶ Strength of gastropod shell
- ▶ Incorporating trait variation provides richer dynamics than classical Lotka-Volterra models

$$\frac{dN}{dt} = N(b - aM)$$
$$\frac{dM}{dt} = M(eaN - d)$$

Variables

- ▶ $N \equiv$ Prey Density
- ▶ $M \equiv$ Predator Density

Parameters

- ▶ $a \equiv$ Attack rate
- ▶ $b \equiv$ Prey birth rate
- ▶ $e \equiv$ Efficiency
- ▶ $d \equiv$ Predator death rate

$$\frac{dN}{dt} = N(b - aM)$$
$$\frac{dM}{dt} = M(eaN - d)$$

Variables

- ▶ $N \equiv$ Prey Density
- ▶ $M \equiv$ Predator Density

Parameters

- ▶ $a \equiv$ Attack rate \leftarrow *No variation!*
- ▶ $b \equiv$ Prey birth rate
- ▶ $e \equiv$ Efficiency
- ▶ $d \equiv$ Predator death rate

$$a(m) = \alpha \exp \left[-\frac{(m - \theta)^2}{2\tau^2} \right]$$

Variables

- ▶ $N \equiv$ Prey Density
- ▶ $M \equiv$ Predator Density
- ▶ $m \equiv$ Predator Character (Trait Value)

Parameters

- ▶ $\alpha \equiv$ Maximum attack rate
- ▶ $\theta \equiv$ Optimal trait value
- ▶ $\tau \equiv$ Specialization Constant

$$a(m) = \alpha \exp \left[-\frac{(m - \theta)^2}{2\tau^2} \right]$$

Variables

- ▶ $N \equiv$ Prey Density
- ▶ $M \equiv$ Predator Density
- ▶ $m \equiv$ Predator Character (Trait Value)

Parameters

- ▶ $\alpha \equiv$ Maximum attack rate
- ▶ $\theta \equiv$ Optimal trait value \longleftarrow *No variation!*
- ▶ $\tau \equiv$ Specialization Constant

$$a(m, n) = \alpha \exp \left[-\frac{(m - n - \theta)^2}{2\tau^2} \right]$$

Variables

- ▶ $N \equiv$ Prey Density
- ▶ $M \equiv$ Predator Density
- ▶ $n \equiv$ Prey Character (Trait Value)
- ▶ $m \equiv$ Predator Character (Trait Value)

Parameters

- ▶ $\alpha \equiv$ Maximum attack rate
- ▶ $\theta \equiv$ Optimal trait *difference*
- ▶ $\tau \equiv$ Specialization Constant

Distribution Assumptions

- ▶ Trait values are **normally distributed** over the populations

$$p(n, \bar{n}) = \frac{1}{\sqrt{2\pi\beta^2}} \exp \left[-\frac{(n - \bar{n})^2}{2\beta^2} \right]$$

$$p(m, \bar{m}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[-\frac{(m - \bar{m})^2}{2\sigma^2} \right]$$

Variables

- ▶ $N \equiv$ Prey Density
- ▶ $\bar{n} \equiv$ Mean Prey Character
- ▶ $M \equiv$ Predator Density
- ▶ $\bar{m} \equiv$ Mean Predator Character

Parameters

- ▶ $\beta^2 \equiv$ Prey Trait Variance
- ▶ $\sigma^2 \equiv$ Predator Trait Variance

Average Attack Rate

$$\begin{aligned}\bar{a}(\bar{m}, \bar{n}) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a(m, n) \cdot p(m, \bar{m}) \cdot p(n, \bar{n}) dm dn \\ &= \frac{\alpha\tau}{\sqrt{\sigma^2 + \beta^2 + \tau^2}} \exp \left[-\frac{(\bar{m} - \bar{n} - \theta)^2}{2(\sigma^2 + \beta^2 + \tau^2)} \right]\end{aligned}$$

Variables

- ▶ $N \equiv$ Prey Density
- ▶ $\bar{n} \equiv$ Mean Prey Character
- ▶ $M \equiv$ Predator Density
- ▶ $\bar{m} \equiv$ Mean Predator Character

Parameters

- ▶ $\beta^2 \equiv$ Prey Trait Variance
- ▶ $\sigma^2 \equiv$ Predator Trait Variance
- ▶ $\alpha \equiv$ Maximum attack rate
- ▶ $\theta \equiv$ *Optimal trait difference*
- ▶ $\tau \equiv$ Specialization Constant

Fitness Assumptions

- ▶ Prey experiences logistic growth in absence of predator
- ▶ Predator experiences exponential decay in absence of prey

$$Y(m, n, M, N) = r \left(1 - \frac{N}{K} \right) - Ma(m, n)$$

$$W(m, n, N) = eNa(m, n) - d$$

Variables

- ▶ $N \equiv$ Prey Density
- ▶ $n \equiv$ Prey Character
- ▶ $M \equiv$ Predator Density
- ▶ $m \equiv$ Predator Character

Parameters

- ▶ $r \equiv$ Intrinsic Prey Growth Rate
- ▶ $K \equiv$ Prey Carrying Capacity
- ▶ $d \equiv$ Predator Death Rate
- ▶ $e \equiv$ Efficiency

Average Fitness

$$\begin{aligned}\overline{Y}(\overline{m}, \overline{n}, M, N) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Y(m, n, M, N) \cdot p(m, \overline{m}) \cdot p(n, \overline{n}) dm dn \\ &= r \left(1 - \frac{N}{K} \right) - M \overline{a}(\overline{m}, \overline{n})\end{aligned}$$

$$\begin{aligned}\overline{W}(\overline{m}, \overline{n}, N) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(m, n, N) \cdot p(m, \overline{m}) \cdot p(n, \overline{n}) dm dn \\ &= e N \overline{a}(\overline{m}, \overline{n}) - d\end{aligned}$$

Variables

- ▶ $N \equiv$ Prey Density
- ▶ $\overline{n} \equiv$ Mean Prey Character
- ▶ $M \equiv$ Predator Density
- ▶ $\overline{m} \equiv$ Mean Predator Character

Parameters

- ▶ $r \equiv$ Intrinsic Prey Growth Rate
- ▶ $K \equiv$ Prey Carrying Capacity
- ▶ $d \equiv$ Predator Death Rate
- ▶ $e \equiv$ Efficiency

Ecological Components

$$\frac{dN}{dt} = N \cdot \bar{Y}(\bar{m}, \bar{n}, M, N)$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}, N)$$

Variables

- ▶ $N \equiv$ Prey Density
- ▶ $\bar{n} \equiv$ Mean Prey Character
- ▶ $M \equiv$ Predator Density
- ▶ $\bar{m} \equiv$ Mean Predator Character

Parameters

- ▶ $r \equiv$ Intrinsic Prey Growth Rate
- ▶ $K \equiv$ Prey Carrying Capacity
- ▶ $d \equiv$ Predator Death Rate
- ▶ $e \equiv$ Efficiency

Evolutionary Components

- ▶ The evolution of the mean character is always in the direction which increases the mean fitness in the population.

$$\frac{d\bar{n}}{dt} = \beta_G^2 \frac{\partial \bar{Y}}{\partial \bar{n}}$$
$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Variables

- ▶ $N \equiv$ Prey Density
- ▶ $\bar{n} \equiv$ Mean Prey Character
- ▶ $M \equiv$ Predator Density
- ▶ $\bar{m} \equiv$ Mean Predator Character

Parameters

- ▶ $\beta_G^2 \equiv$ Prey genetic variance
- ▶ $\sigma_G^2 \equiv$ Predator genetic variance

The Complete 1×1 Model (One Predator Species, One Prey Species)

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Ecological Components

$$\frac{dN}{dt} = N \cdot \bar{Y}(\bar{m}, \bar{n}, M, N) = N \left[r \left(1 - \frac{N}{K} \right) - M \bar{a}(\bar{m}, \bar{n}) \right]$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}, N) = M [eN \bar{a}(\bar{m}, \bar{n}) - d]$$

Evolutionary Components

$$\frac{d\bar{n}}{dt} = \beta_G^2 \frac{\partial \bar{Y}}{\partial \bar{n}} = \beta_G^2 \frac{M(\theta + \bar{n} - \bar{m})}{\sigma^2 + \beta^2 + \tau^2} \bar{a}(\bar{m}, \bar{n})$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}} = \sigma_G^2 \frac{eN(\theta + \bar{n} - \bar{m})}{\sigma^2 + \beta^2 + \tau^2} \bar{a}(\bar{m}, \bar{n})$$

Prey Fitness

$$Y(m, n, M, N) = r \left(1 - \frac{N}{K} \right) - Ma(m, n)$$

Predator Fitness

$$W(m, n, N) = eNa(m, n) - d$$

Prey Fitness

$$Y(m, n, M, N) = r \left(1 - \frac{N}{K} \right) - Ma(m, n)$$

↓

$$Y_j([m_i]_{i=1}^u, n_j, [M_i]_{i=1}^u, N_j) = r_j \left(1 - \frac{N_j}{K_j} \right) - \sum_{i=1}^u M_i a_{ij}(m_i, n_j)$$

Predator Fitness

$$W(m, n, N) = eNa(m, n) - d$$

Notation

$$[x_i]_{i=1}^u = x_1, \dots, x_u$$

Prey Fitness

$$Y(m, n, M, N) = r \left(1 - \frac{N}{K} \right) - Ma(m, n)$$

↓

$$Y_j([m_i]_{i=1}^u, n_j, [M_i]_{i=1}^u, N_j) = r_j \left(1 - \frac{N_j}{K_j}\right) - \sum_{i=1}^u M_i a_{ij}(m_i, n_j)$$

Predator Fitness

$$W(m, n, N) = eNa(m, n) - d$$

↓

$$W_i(m_i, [n_j]_{j=1}^v, [N_j]_{j=1}^v) = \sum_{j=1}^v \left[e_{ij} N_j a_{ij}(m_i, n_j) \right] - d_i$$

Notation

$$[x_i]_{i=1}^u = x_1, \dots, x_u$$

Average Fitness

$$\begin{aligned}\bar{Y}_j([\bar{m}_i]_{i=1}^u, \bar{n}_j, [M_i]_{i=1}^u, N_j) \\&= \int_{\mathbb{R}^{u+1}} Y_j \cdot \prod_{i=1}^u \left[p_i(m_i, \bar{m}_i) \right] \cdot p(n, \bar{n}) \prod_{i=1}^u \left[dm_i \right] dn_j \\&= r_j \left(1 - \frac{N_j}{K_j} \right) - \sum_{i=1}^u M_i \bar{a}_{ij}(\bar{m}_i, \bar{n}_j)\end{aligned}$$

$$\begin{aligned}\bar{W}_i(\bar{m}_i, [\bar{n}_j]_{j=1}^v, [N_j]_{j=1}^v) \\&= \int_{\mathbb{R}^{u+1}} W_i \cdot p_i(m_i, \bar{m}_i) \cdot \prod_{j=1}^v \left[p(n_j, \bar{n}_j) \right] dm_i \prod_{j=1}^v \left[dn_j \right] \\&= \sum_{j=1}^v \left[e_{ij} N_j \bar{a}_{ij}(\bar{m}_i, \bar{n}_j) \right] - d_i\end{aligned}$$

The Complete $u \times v$ Model

(u Predator Species, v Prey Species)

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Ecological Components

$$\frac{dN_j}{dt} = N_j \bar{Y}_j = N_j \left[r_j \left(1 - \frac{N_j}{K_j} \right) - \sum_{i=1}^u M_i \bar{a}_{ij}(\bar{m}_i, \bar{n}_j) \right]$$

$$\frac{dM_i}{dt} = M_i \bar{W}_i = M_i \left[\sum_{j=1}^v \left[e_{ij} N_j \bar{a}_{ij}(\bar{m}_i, \bar{n}_j) \right] - d_i \right]$$

Evolutionary Components

$$\frac{d\bar{n}_j}{dt} = \beta_{jG}^2 \frac{\partial \bar{Y}_j}{\partial \bar{n}_j} = \beta_{jG}^2 \sum_{i=1}^u \left[\frac{M_i(\theta_{ij} + \bar{n}_j - \bar{m}_i)}{\sigma_i^2 + \beta_j^2 + \tau_{ij}^2} \bar{a}_{ij}(\bar{m}_i, \bar{n}_j) \right]$$

$$\frac{d\bar{m}_i}{dt} = \sigma_{iG}^2 \frac{\partial \bar{W}_i}{\partial \bar{m}_i} = \sigma_{iG}^2 \sum_{j=1}^v \left[\frac{e_{ij} N_j(\theta_{ij} + \bar{n}_j - \bar{m}_i)}{\sigma_i^2 + \beta_j^2 + \tau_{ij}^2} \bar{a}_{ij}(\bar{m}_i, \bar{n}_j) \right]$$

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Equilibria

$$\frac{dN}{dt} = N \cdot \bar{Y}(\bar{m}, \bar{n}, M, N)$$

$$\frac{d\bar{n}}{dt} = \beta_G^2 \frac{\partial \bar{Y}}{\partial \bar{n}}$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}, N)$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Extinction

$$(N^*, M^*, \bar{n}^*, \bar{m}^*) = (0, 0, _, _)$$

Exclusion

$$(N^*, M^*, \bar{n}^*, \bar{m}^*) = (K, 0, _, _)$$

Equilibria

$$\frac{dN}{dt} = N \cdot \bar{Y}(\bar{m}, \bar{n}, M, N)$$

$$\frac{d\bar{n}}{dt} = \beta_G^2 \frac{\partial \bar{Y}}{\partial \bar{n}}$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}, N)$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Extinction *Unstable*

$$(N^*, M^*, \bar{n}^*, \bar{m}^*) = (0, 0, _, _)$$

Exclusion

$$(N^*, M^*, \bar{n}^*, \bar{m}^*) = (K, 0, _, _)$$

Equilibria

$$\frac{dN}{dt} = N \cdot \bar{Y}(\bar{m}, \bar{n}, M, N)$$

$$\frac{d\bar{n}}{dt} = \beta_G^2 \frac{\partial \bar{Y}}{\partial \bar{n}}$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}, N)$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Extinction *Unstable*

$$(N^*, M^*, \bar{n}^*, \bar{m}^*) = (0, 0, _, _)$$

Exclusion *Stable under certain conditions*

$$(N^*, M^*, \bar{n}^*, \bar{m}^*) = (K, 0, _, _)$$

$$\frac{dN}{dt} = N \cdot \bar{Y}(\bar{m}, \bar{n}, M, N)$$

$$\frac{d\bar{n}}{dt} = \beta_G^2 \frac{\partial \bar{Y}}{\partial \bar{n}}$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}, N)$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Extinction *Unstable*

$$(N^*, M^*, \bar{n}^*, \bar{m}^*) = (0, 0, _, _)$$

Exclusion *Stable under certain conditions*

$$(N^*, M^*, \bar{n}^*, \bar{m}^*) = (K, 0, _, _)$$

Necessary Conditions for Stable Exclusion:

- ▶ $d > e\bar{a}(\bar{m}^*, \bar{n}^*)K$
- ▶ $(\bar{m}^* - \bar{n}^* - \theta)^2 < \sigma^2 + \beta^2 + \tau^2$

$$\frac{dN}{dt} = N \cdot \bar{Y}(\bar{m}, \bar{n}, M, N)$$

$$\frac{d\bar{n}}{dt} = \beta_G^2 \frac{\partial \bar{Y}}{\partial \bar{n}}$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}, N)$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Coexistence

$$(N^*, M^*, \bar{n}^*, \bar{m}^*) = \left(\frac{d\sqrt{A}}{e\alpha\tau}, \frac{r\sqrt{A}}{\alpha\tau} \left(1 - \frac{d\sqrt{A}}{Ke\alpha\tau} \right), \mu^*, \mu^* - \theta \right)$$

where $A = \sigma^2 + \beta^2 + \tau^2$ and μ^* is an arbitrary value.

Equilibria

$$\frac{dN}{dt} = N \cdot \bar{Y}(\bar{m}, \bar{n}, M, N)$$

$$\frac{d\bar{n}}{dt} = \beta_G^2 \frac{\partial \bar{Y}}{\partial \bar{n}}$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}, N)$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Coexistence *Stable under certain conditions*

$$(N^*, M^*, \bar{n}^*, \bar{m}^*) = \left(\frac{d\sqrt{A}}{e\alpha\tau}, \frac{r\sqrt{A}}{\alpha\tau} \left(1 - \frac{d\sqrt{A}}{Ke\alpha\tau} \right), \mu^*, \mu^* - \theta \right)$$

where $A = \sigma^2 + \beta^2 + \tau^2$ and μ^* is an arbitrary value.

$$\frac{dN}{dt} = N \cdot \bar{Y}(\bar{m}, \bar{n}, M, N)$$

$$\frac{d\bar{n}}{dt} = \beta_G^2 \frac{\partial \bar{Y}}{\partial \bar{n}}$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}, N)$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Coexistence *Stable under certain conditions*

$$(N^*, M^*, \bar{n}^*, \bar{m}^*) = \left(\frac{d\sqrt{A}}{e\alpha\tau}, \frac{r\sqrt{A}}{\alpha\tau} \left(1 - \frac{d\sqrt{A}}{Ke\alpha\tau} \right), \mu^*, \mu^* - \theta \right)$$

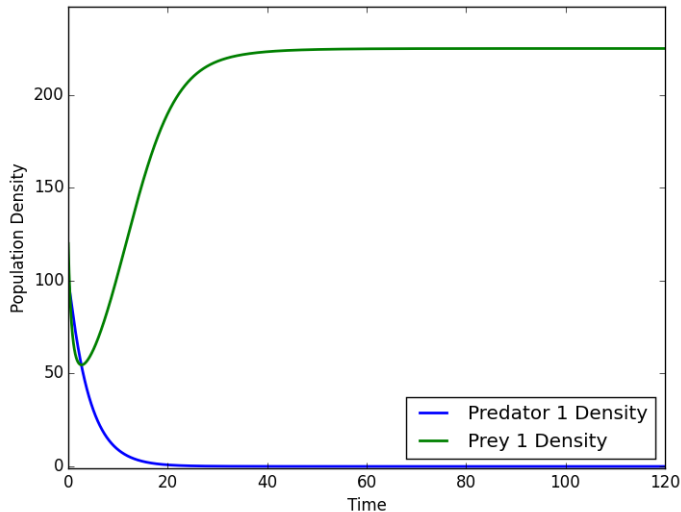
where $A = \sigma^2 + \beta^2 + \tau^2$ and μ^* is an arbitrary value.

Necessary Condition for Stable Coexistence:

$$\blacktriangleright d\sigma_G^2 > r\beta_G^2 \left(1 - \frac{d\sqrt{A}}{Ke\alpha\tau} \right)$$

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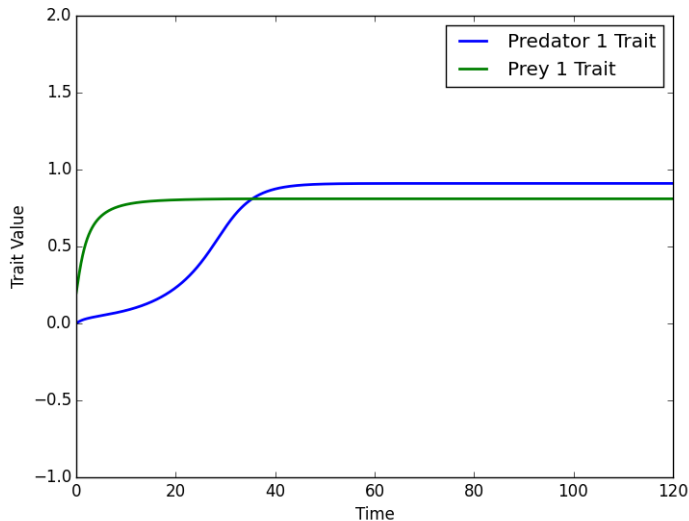
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1×1

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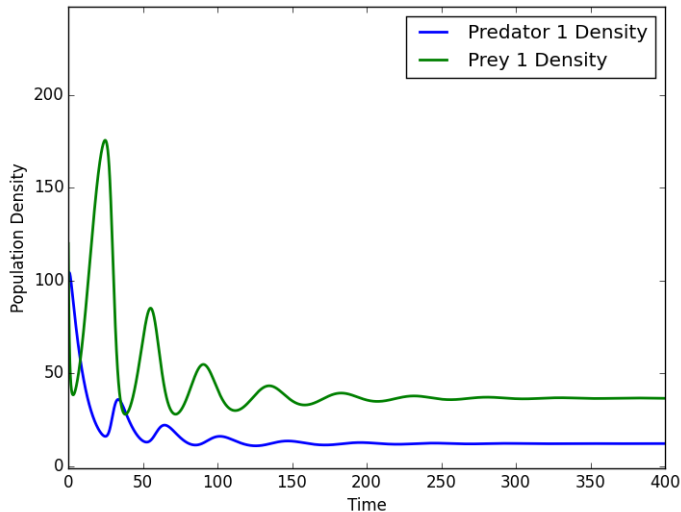
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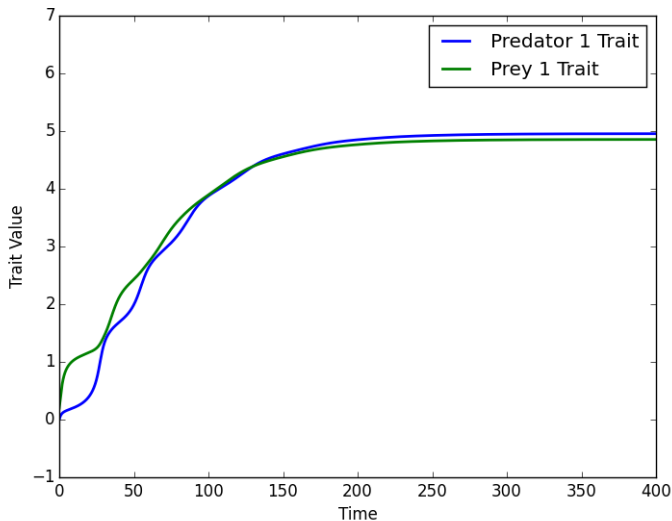
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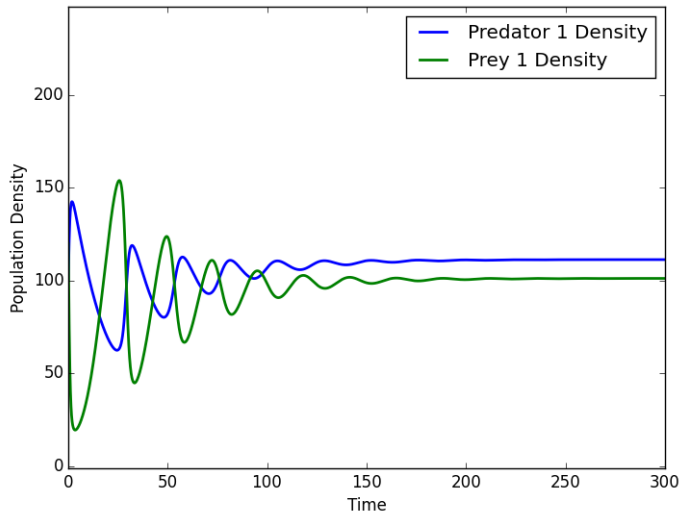
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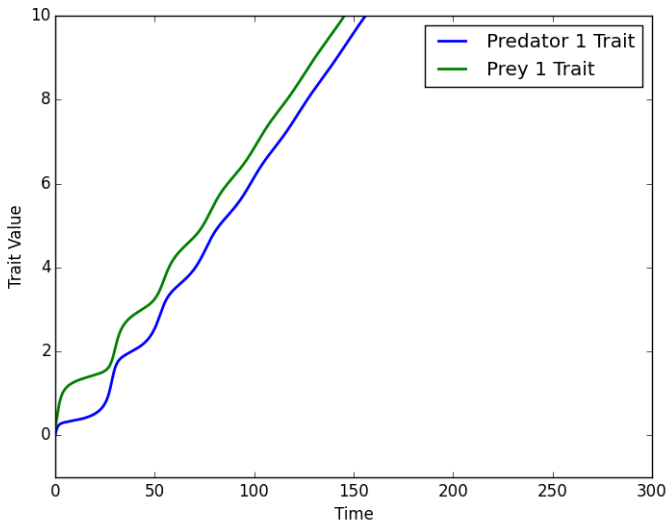
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Unstable Coexistence



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Equilibria

$$\frac{dN_1}{dt} = N_1 \cdot \bar{Y}_1(\bar{m}, \bar{n}_1, M, N_1)$$

$$\frac{dN_2}{dt} = N_2 \cdot \bar{Y}_2(\bar{m}, \bar{n}_2, M, N_2)$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}_1, \bar{n}_2, N_1, N_2)$$

$$\frac{d\bar{n}_1}{dt} = \beta_{1,G}^2 \frac{\partial \bar{Y}_1}{\partial \bar{n}_1}$$

$$\frac{d\bar{n}_2}{dt} = \beta_{2,G}^2 \frac{\partial \bar{Y}_2}{\partial \bar{n}_2}$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Equilibria

$$\frac{dN_1}{dt} = N_1 \cdot \bar{Y}_1(\bar{m}, \bar{n}_1, M, N_1)$$

$$\frac{dN_2}{dt} = N_2 \cdot \bar{Y}_2(\bar{m}, \bar{n}_2, M, N_2)$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}_1, \bar{n}_2, N_1, N_2)$$

$$\frac{d\bar{n}_1}{dt} = \beta_{1,G}^2 \frac{\partial \bar{Y}_1}{\partial \bar{n}_1}$$

$$\frac{d\bar{n}_2}{dt} = \beta_{2,G}^2 \frac{\partial \bar{Y}_2}{\partial \bar{n}_2}$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Extinction

$$(N_1^*, N_2^*, M^*, \bar{n}_1^*, \bar{n}_2^*, \bar{m}^*) = (0, 0, 0, _, _, _)$$

Equilibria

$$\frac{dN_1}{dt} = N_1 \cdot \bar{Y}_1(\bar{m}, \bar{n}_1, M, N_1)$$

$$\frac{dN_2}{dt} = N_2 \cdot \bar{Y}_2(\bar{m}, \bar{n}_2, M, N_2)$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}_1, \bar{n}_2, N_1, N_2)$$

$$\frac{d\bar{n}_1}{dt} = \beta_{1,G}^2 \frac{\partial \bar{Y}_1}{\partial \bar{n}_1}$$

$$\frac{d\bar{n}_2}{dt} = \beta_{2,G}^2 \frac{\partial \bar{Y}_2}{\partial \bar{n}_2}$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Extinction *Unstable*

$$(N_1^*, N_2^*, M^*, \bar{n}_1^*, \bar{n}_2^*, \bar{m}^*) = (0, 0, 0, _, _, _)$$

Equilibria

$$\frac{dN_1}{dt} = N_1 \cdot \bar{Y}_1(\bar{m}, \bar{n}_1, M, N_1)$$

$$\frac{dN_2}{dt} = N_2 \cdot \bar{Y}_2(\bar{m}, \bar{n}_2, M, N_2)$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}_1, \bar{n}_2, N_1, N_2)$$

$$\frac{d\bar{n}_1}{dt} = \beta_{1,G}^2 \frac{\partial \bar{Y}_1}{\partial \bar{n}_1}$$

$$\frac{d\bar{n}_2}{dt} = \beta_{2,G}^2 \frac{\partial \bar{Y}_2}{\partial \bar{n}_2}$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Extinction *Unstable*

$$(N_1^*, N_2^*, M^*, \bar{n}_1^*, \bar{n}_2^*, \bar{m}^*) = (0, 0, 0, _, _, _)$$

Exclusion

$$(N_1^*, N_2^*, M^*, \bar{n}_1^*, \bar{n}_2^*, \bar{m}^*) = (K_1, K_2, 0, _, _, _)$$

Equilibria

$$\frac{dN_1}{dt} = N_1 \cdot \bar{Y}_1(\bar{m}, \bar{n}_1, M, N_1)$$

$$\frac{d\bar{n}_1}{dt} = \beta_{1,G}^2 \frac{\partial \bar{Y}_1}{\partial \bar{n}_1}$$

$$\frac{dN_2}{dt} = N_2 \cdot \bar{Y}_2(\bar{m}, \bar{n}_2, M, N_2)$$

$$\frac{d\bar{n}_2}{dt} = \beta_{2,G}^2 \frac{\partial \bar{Y}_2}{\partial \bar{n}_2}$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}_1, \bar{n}_2, N_1, N_2)$$

$$\frac{d\bar{m}}{dt} = \sigma_G^2 \frac{\partial \bar{W}}{\partial \bar{m}}$$

Extinction *Unstable*

$$(N_1^*, N_2^*, M^*, \bar{n}_1^*, \bar{n}_2^*, \bar{m}^*) = (0, 0, 0, _, _, _)$$

Exclusion *Stable under certain conditions*

$$(N_1^*, N_2^*, M^*, \bar{n}_1^*, \bar{n}_2^*, \bar{m}^*) = (K_1, K_2, 0, _, _, _)$$

Equilibria

$$\frac{dN_1}{dt} = N_1 \cdot \bar{Y}_1(\bar{m}, \bar{n}_1, M, N_1)$$

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Generalist Becomes Specialist

$$\left(\frac{d\sqrt{A_1}}{e_1\alpha_1\tau_1}, K_2, \frac{r_1\sqrt{A_1}}{\alpha_1\tau_1} \left(1 - \frac{d\sqrt{A_1}}{K_1e_1\alpha_1\tau_1} \right), \mu_1^*, \mu_2^*, \mu_1^* - \theta_1 \right)$$

where $A_1 = \sigma^2 + \beta_1^2 + \tau_1^2$, μ_1^* is an arbitrary value, and μ_2^* is sufficiently far from $\mu_1^* - \theta_1$.

Equilibria

$$\frac{dN_1}{dt} = N_1 \cdot \bar{Y}_1(\bar{m}, \bar{n}_1, M, N_1)$$

$$\frac{d\bar{n}_1}{dt} = \beta_{1,G}^2 \frac{\partial \bar{Y}_1}{\partial \bar{n}_1}$$

$$\frac{dN_2}{dt} = N_2 \cdot \bar{Y}_2(\bar{m}, \bar{n}_2, M, N_2)$$

$$\frac{d\bar{n}_2}{dt} = \beta_{2,G}^2 \frac{\partial \bar{Y}_2}{\partial \bar{n}_2}$$

$$\frac{dM}{dt} = M \cdot \bar{W}(\bar{m}, \bar{n}_1, \bar{n}_2, N_1, N_2)$$

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Extinction *Unstable*

$$(N_1^*, N_2^*, M^*, \bar{n}_1^*, \bar{n}_2^*, \bar{m}^*) = (0, 0, 0, _, _, _)$$

Exclusion *Stable under certain conditions*

$$(N_1^*, N_2^*, M^*, \bar{n}_1^*, \bar{n}_2^*, \bar{m}^*) = (K_1, K_2, 0, _, _, _)$$

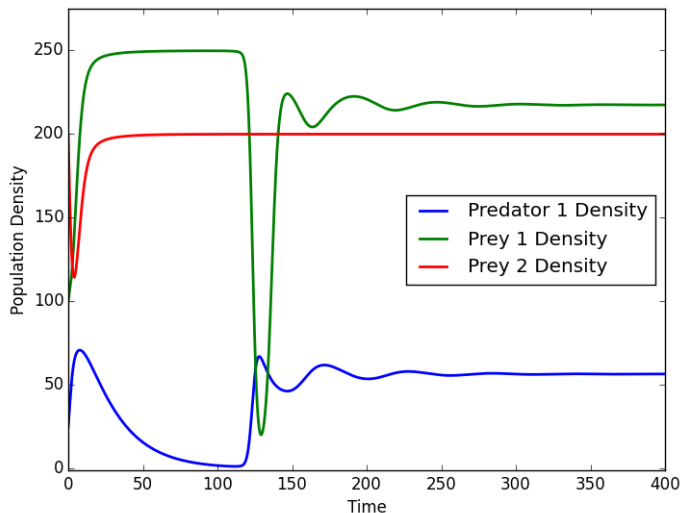
Generalist Becomes Specialist *Stable under certain conditions*

$$\left(\frac{d\sqrt{A_1}}{e_1\alpha_1\tau_1}, K_2, \frac{r_1\sqrt{A_1}}{\alpha_1\tau_1} \left(1 - \frac{d\sqrt{A_1}}{K_1e_1\alpha_1\tau_1} \right), \mu_1^*, \mu_2^*, \mu_1^* - \theta_1 \right)$$

where $A_1 = \sigma^2 + \beta_1^2 + \tau_1^2$, μ_1^* is an arbitrary value, and μ_2^* is sufficiently far from $\mu_1^* - \theta_1$.

Figures

Generalist Becomes Specialist



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of Trait Variation in a
 μ -Predator, ν -Prey
System

Sam Fleischer, Pablo
Chavarria

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Model Formulation

Lotka-Volterra

Schreiber, Bürger,
and Bolnick

Our Extension

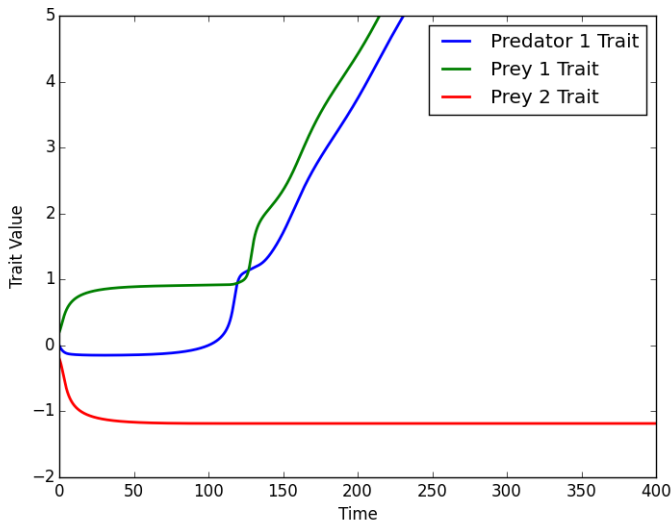
Preliminary Results

1×1

1×2

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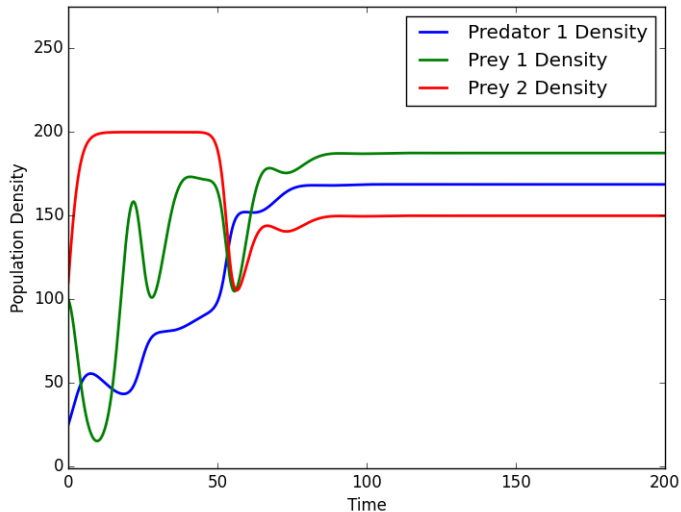
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