Problem 5

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Algorithm and Complexity
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Branch and bound

	1	2	3	4
a	26	31	32	8
b	23	8	10	29
c	13	32	9	27
d	19	27	19	27

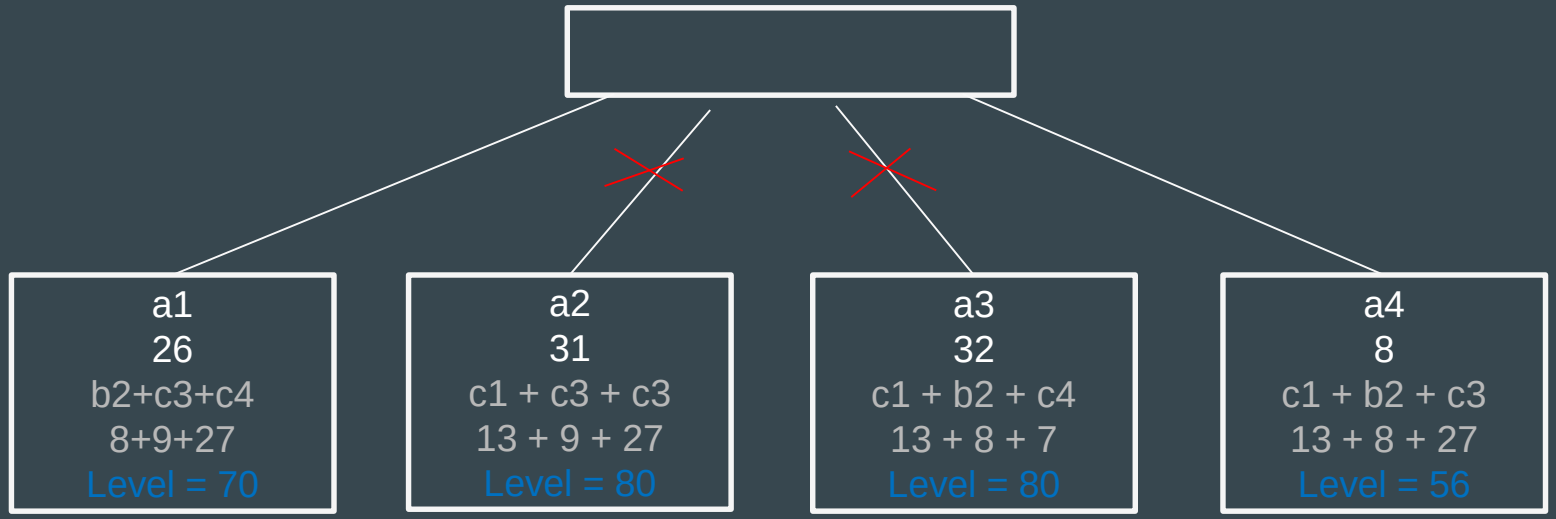
$a_{11} = 7$, $a_{12} = 6$, $a_{13} = 5$, $a_{14} = 5$
$b_{11} = 6$, $b_{12} = 6$, $b_{13} = 6$, $b_{14} = 8$
$c_{11} = 1$, $c_{12} = 8$, $c_{13} = 3$, $c_{14} = 27$
$d_{11}=19$, $d_{12} = 27$, $d_{13} = 19$, $d_{14} = 27$

Boundaries (I)

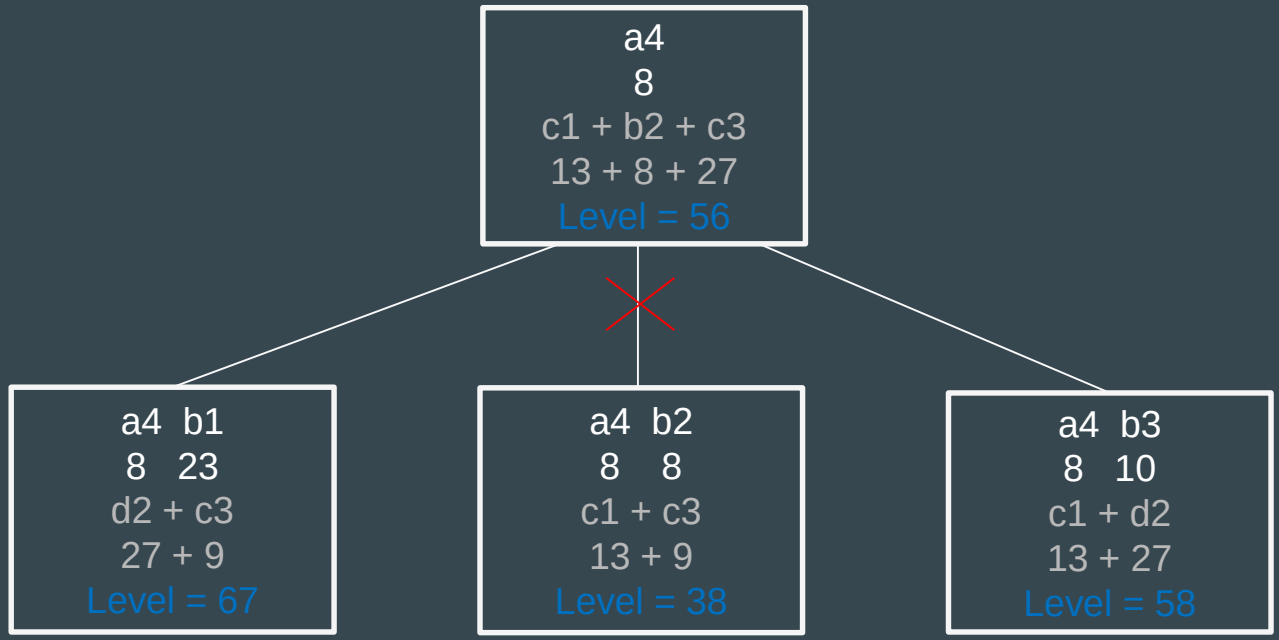
- Real level: based on known solutions that allow us to bound undesirable candidates. I will use as a real value any valid assignment that is easy to find; for example the solution a1, b2, c3, d4
  - Real level = a1 + b2 + c3 + d4 = 26 + 8 + 9 + 27 = 70
  - Any partial candidate that has a cost higher than 70 will be rejected since instead of continuing to generate it we could stay with the complete solution already found.

Boundaries (II)

- Estimated value: serves to quickly end the problem if there are candidates with values close to it. As an estimated value we will consider, for example, the minimum cost that would be on the part of the agents, that is, the minimum of the rows:
    - 8(a4) + 8(b2) + 9(c3) + 19(d1) = 44
- We could also consider the minimum cost to perform the tasks, that is, the minimums of columns
    - 13(c1) + 8(b2) + 9(c3) + 8(a4)=38.
- Considering both estimates, the estimated value will be max{44, 38}=44. Thus, the range of valid values to explore will be [44, 70].



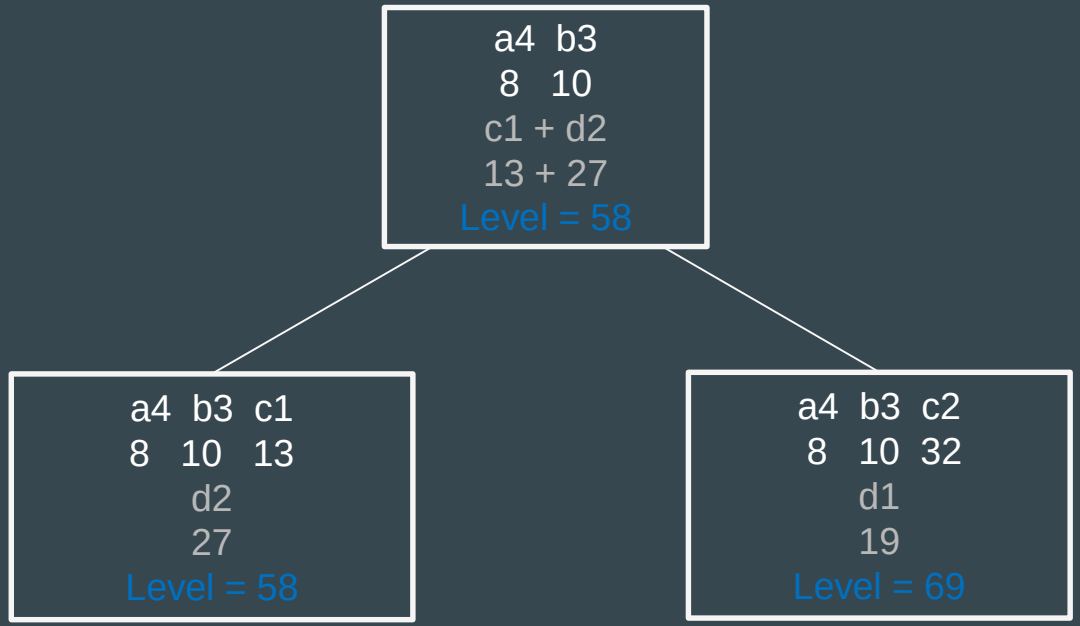
Of these four nodes, we can see that the second and the third nodes, in the best case, has a cost of 80, which is higher than the known upper bound. Therefore, that node does not need to be scanned and bounded. Of the other two nodes, called living nodes, the one with the best available elevation is chosen and the exploration is continued, trying to assign the agent b.
So the best available elevation is a4.



We can see that the second node, in the best case, has a cost of 38, which is lower than the known lower bound. In addition, two tasks have been assigned to agent c. Therefore, that node does not need to be scanned and bounded.

Of the live nodes, the one with the smallest dimension, in this case the third node, is taken and the agent c is added, and since there would only be one task left to perform the estimation, it is really a complete solution.

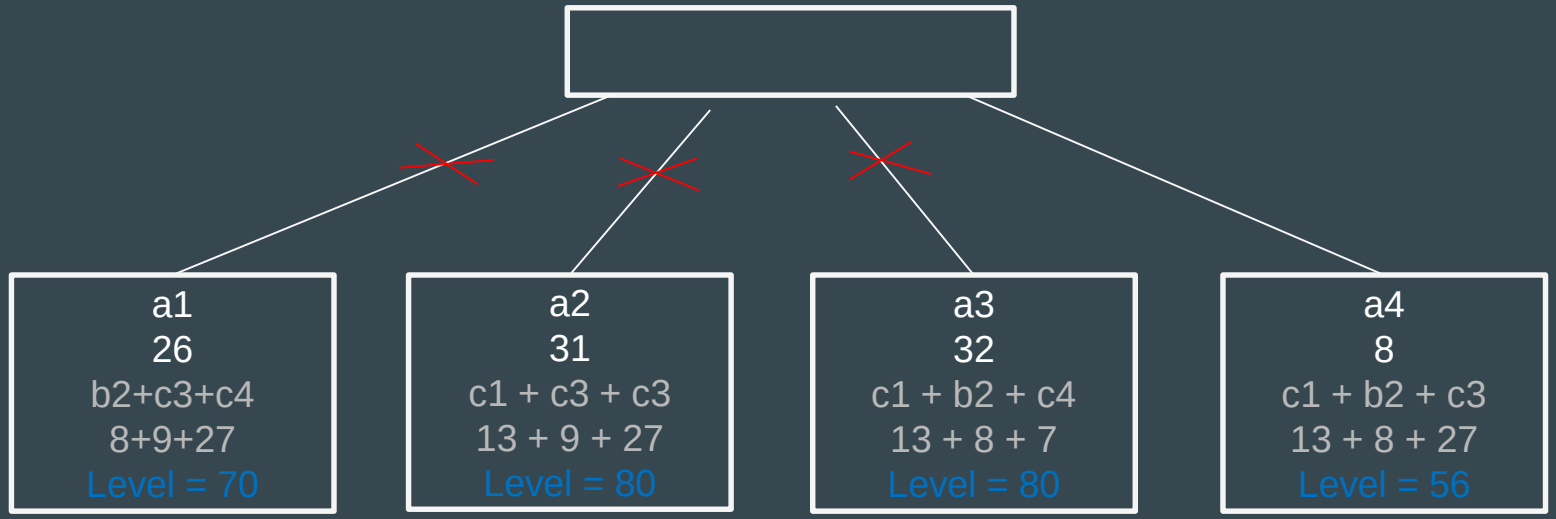
The best available elevation is a4 b3.



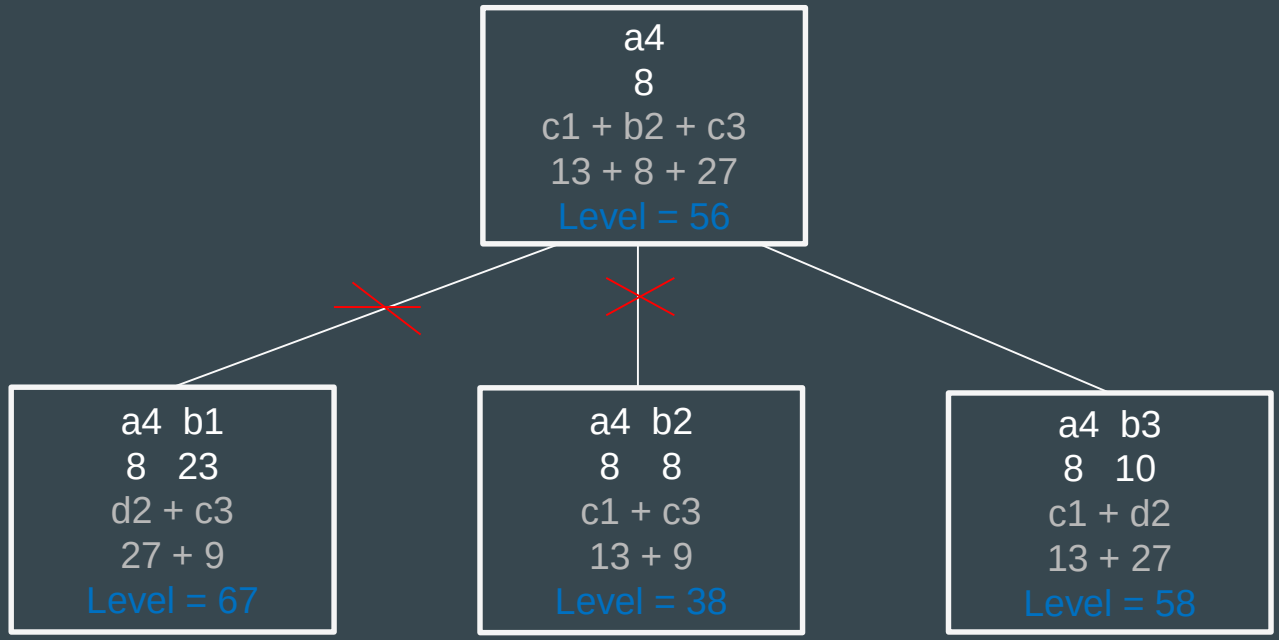
Of these two nodes, the one with the best available elevation is chosen and the exploration ends. The best available elevation is a4. b3 c1 d2 with a optimal level of 58.

As we have a valid assignment, we can update the value of the upper bound of the boundary from 70 to 58.

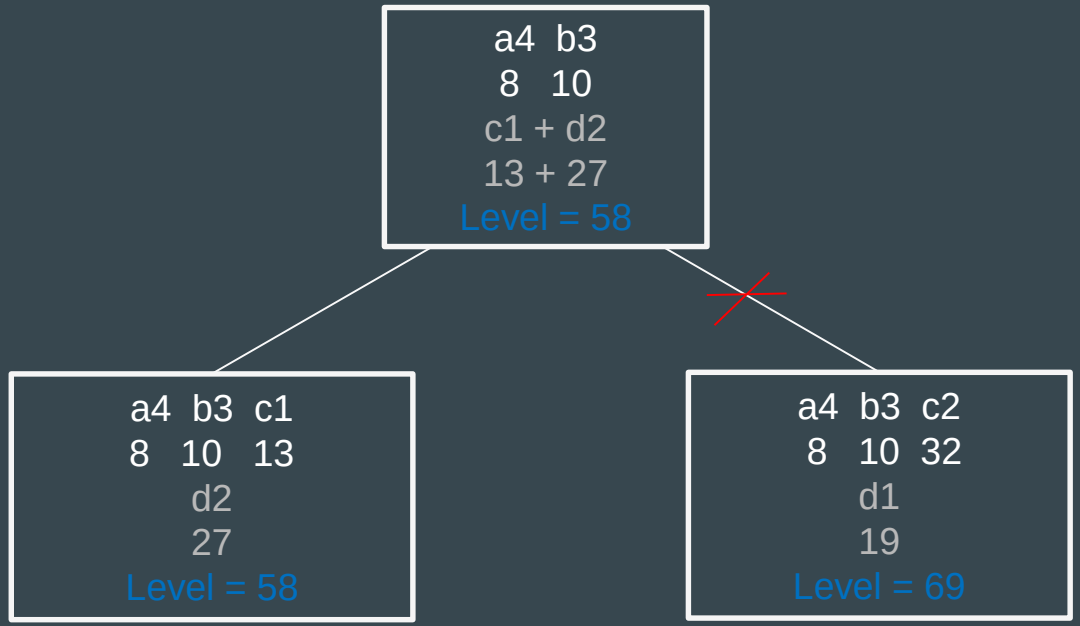
New boundary = [44,58]



As we have a new boundary, we can bound those branches whose level in the best case has a cost of more than 58 that is the new upper bound. Therefore, the first node now does not need to be scanned and bounded.



Here we can bound the first branch because the level in the best case has a cost of more than 58 that is the new upper bound.



Here we can bound the second branch because the level in the best case has a cost of more than 58 that is the new upper bound.

Conclusion

- The optimal solution is the assignation of a4 – b3 – c1 – d2.
- If a complete search had been made it would have been necessary to go through a number of nodes equal to 4+4*3+4*3*2=40 nodes, while with the branch and bounding only 9 nodes had to be created.