Machine Learning in Complex Domains: Assignment 3

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4 Blocked Gibbs Sampler

4.1 Analysis Questions

We have the following equation

$$p(z_{d,i} = k, x_{d,i} = c|\mathbf{z} - z_{d,i}, \mathbf{x} - x_{d,i}, \mathbf{c}, \mathbf{w}; \alpha, \beta, \lambda) = \frac{p(\mathbf{z}, \mathbf{x}, \mathbf{c}, \mathbf{w}|\alpha, \beta, \lambda)}{p(\mathbf{z} - z_{d,i}, \mathbf{x} - x_{d,i}, \mathbf{c}, \mathbf{w}; \alpha, \beta, \lambda)}$$

The full join (numerator) is given to us in the assignment. We can factor this according to the joint and every term that does not involve $z_{d,i}$ or $x_{d,i}$ will cancel. This is basically a union of (15) and (18). This yields the following:

$$p(z_{d,i} = k, x_{d,i} = 0|\mathbf{z} - z_{d,i}, \mathbf{x} - x_{d,i}, \mathbf{c}, \mathbf{w}; \alpha, \beta, \lambda) = \frac{p(x=0|\lambda)\frac{\Gamma(n^k_{w_{d,i}}+1+\beta)}{\Gamma(1+\sum_w n^k_w+\beta)}\frac{\Gamma(n^d_k+1+\alpha)}{\Gamma(1+\sum_{k'} n^d_{k'}+\alpha)}}{\frac{\Gamma(n^k_{w_{d,i}}+\beta)}{\Gamma(\sum_w n^k_w+\beta)}\frac{\Gamma(n^d_k+\alpha)}{\Gamma(\sum_{k'} n^d_{k'}+\alpha)}}$$

$$p(z_{d,i} = k, x_{d,i} = 0|\mathbf{z} - z_{d,i}, \mathbf{x} - x_{d,i}, \mathbf{c}, \mathbf{w}; \alpha, \beta, \lambda) \propto \frac{(1-\lambda)(n^k_{w_{d,i}} + \beta)(n^d_k + \alpha)}{(n^k_* + K\beta)(n^d_* + V\alpha)}$$

By analogy, we get the followoing for the case that $p(x = 1)$.

$$p(z_{d,i} = k, x_{d,i} = 1|\mathbf{z} - z_{d,i}, \mathbf{x} - x_{d,i}, \mathbf{c}, \mathbf{w}; \alpha, \beta, \lambda) \propto \frac{\lambda(n^k_{w_{d,i}} + \beta)(n^d_k + \alpha)}{(n^k_* + K\beta)(n^d_* + V\alpha)}$$

Most of the heavy lifting for this question was done in the appendix so there isn't really any more we can write.

5 Text Analysis with MCLDA

5.1 Empirical Questions

1. On all three chains, the test likelihood is lower than the training likelihood by approximately the same amount. By around iteration 200, the likelihoods have more or less converged, indicating that the Markov chain is sufficiently mixed. Because our topic counts are based on the training data, it makes sense that the test likelihood is lower than the training likelihood.

Figure 1: Question 1

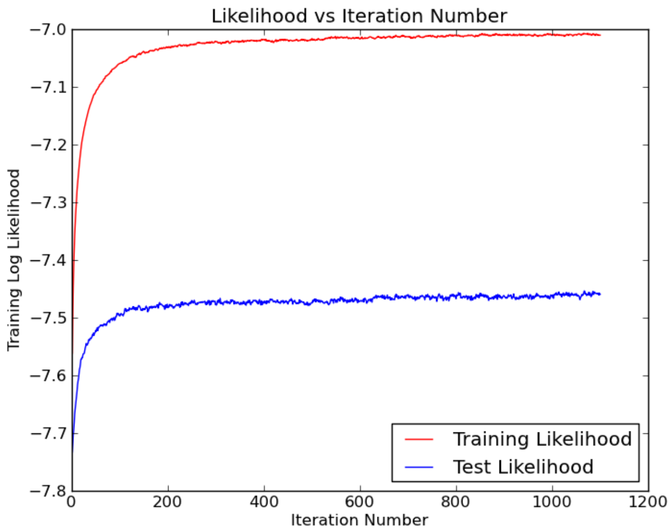


Figure 2: Question 1

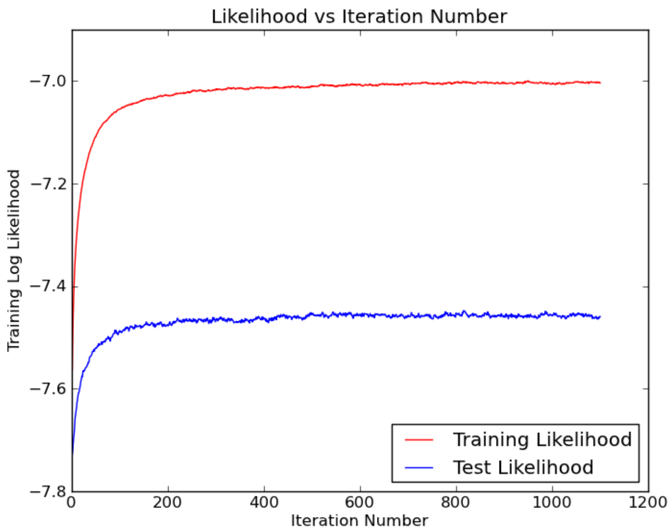
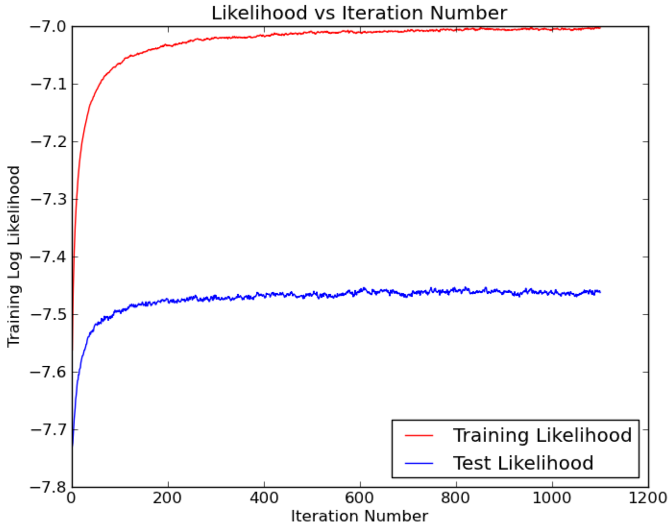
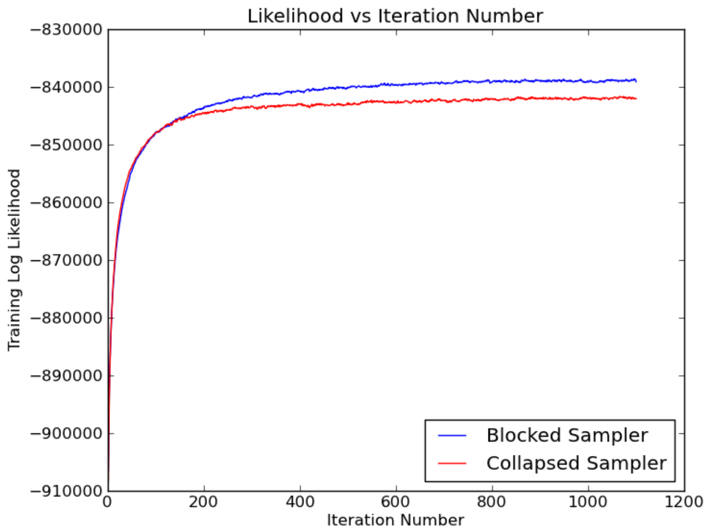


Figure 3: Question 1



2. The collapsed sampler seems to mix about 50-100 iterations faster than the blocked sampler. However, the blocked sampler reaches a higher final likelihood.

Figure 4: Question 2



3. The average runtime for the blocked iterations was 300ms and for the collapsed iteration it was 229ms.

Figure 5: Question 3

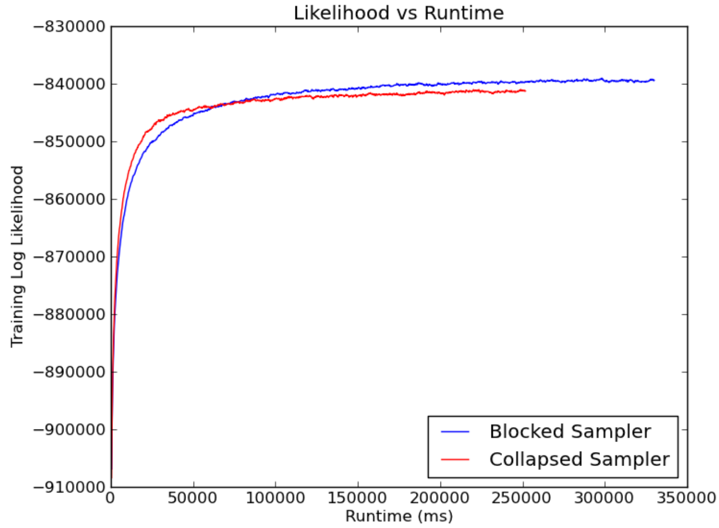


Figure 6: Question 4

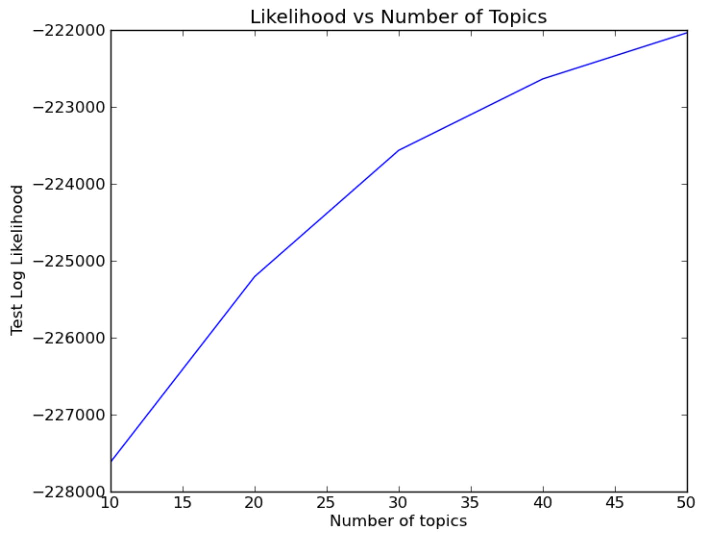
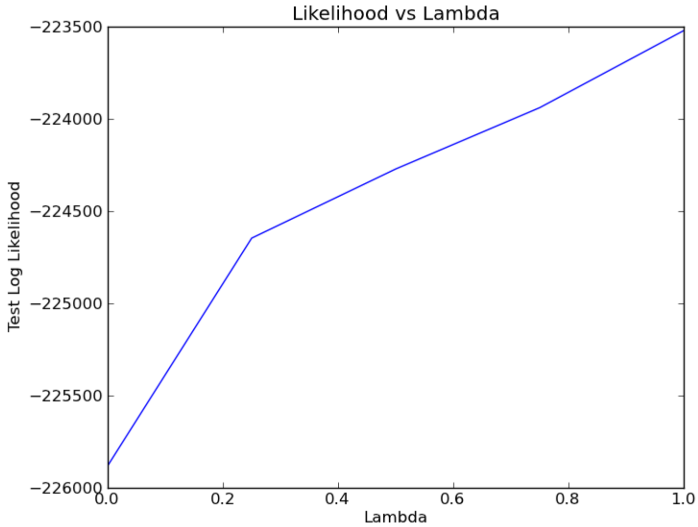
4. 

Figure 7: Question 5



5.

6. (a) Both conferences publish papers exclusively on statistical methods and other machine learning techniques. Naturally, from this there will arise a similarity between the global and local topics across each conference. However, we do see a difference between the specificity between the global and local topics.

   Among the global topics are a few that represent general machine learning concepts such as a "feature" topic which include "feature," "extraction," "classifier," "hyperplane," as well as a "modeling" topic which include words such as "statistical", "probabilistic", "framework," "entropy," "model." In addition, we pick out a few topics that relate with topics that relate to all computer science conference papers. These include results, significant, new important.

   On the local level, we see topics that very clearly belong to one or the other conference. In NIPS we see a reinforcement learning that includes "Markov," "reinforcement," "control," and "decision." We also see a computational neural science and neural network topic with related words. The ACL clearly contains NLP related topics with topics related to parsing, SMT, IR, QA, and Semantics.

   (b) When the value for λ is close to 0, this means that all of the topics come from the global corpus. The global topics are rather diverse as before. You can see topics that come from the NIPS and ACL corpora. These tend to be general machine learning topics. The corpus-specific topics are all the same. This is because none of the words are drawn from the corpus-specific topic, so each of the words is equally likely. Therefore, all of the topics are the same.

   Then when λ goes to 1, all of the global corpus topics are identical. This follows a similar argument as before. The NIPS and ACL topics are very specific to each group. The NIPS has topics about neural networks and imaging; the ACL topics deal with semantics and other NLP related topics. This is expected.

   (c) As α goes to infinity, the Dirichlet from which every θ will be sampled will become a point mass. In that extreme case, we should get every topic to be equally likely in each document, which will make it harder to associate individual topics with each document.

6 Variational Inference

6.1 Analysis Questions

We toiled and troubled on this task writing many equations on the whiteboard and on paper. Below we've provided a simple derivation that skips over many steps. For a more detailed approach see the `variational.tex` file we've attached. We believe this is enough since it is roughly equivalent to the amount of work they showed in the original paper.

Notation: w_n^v is an indicator variable that means the type of the nth word is v. c_d^c means the corpus type of the dth document is c. This is very similar to the notation used in the paper. The subscripts have the following meaning d = document index, n = word token index v = word type index , g = global topic, l = local topic c = corpus id, k = topic id. Also note that $\log(\phi_{d,v,l,k,c})$ is a point estimate for expected value of the natural statistic of a Dirichlet (the difference of the two digammas) according to https://lists.cs.princeton.edu/pipermail/topic-models/2009-June/000560.html. We can now factorize p and q according to the graphical model

The upper bound on the likelihood.

$$\mathcal{L}_{[\gamma,\delta,\epsilon,\psi;\alpha,\beta,\lambda]} = E_q\left[\log(p(\theta|\alpha)p(\mathbf{w}|\mathbf{z},\mathbf{x},\mathbf{c},\phi)p(\mathbf{c})p(\mathbf{x}|\lambda)p(\mathbf{z}|\theta)p(\phi|\beta,\lambda)\right] - H_q\left[\log(q(\theta|\gamma)q(\mathbf{z}|\delta)q(\mathbf{x}|\epsilon)q(\phi|\psi))\right]$$

$$\begin{aligned}\mathcal{L}_{[\gamma,\delta,\epsilon,\psi;\alpha,\beta,\lambda]} &= E_q\left[\log(p(\theta|\alpha)\right] + E_q\left[\log(p(\mathbf{w}|\mathbf{z},\mathbf{x},\mathbf{c},\phi)\right] + E_q\left[\log(p(\mathbf{c})\right] + E_q\left[\log(p(\mathbf{x}|\lambda))\right] \\ &+ E_q\left[\log(p(\mathbf{z}|\theta))\right] + E_q\left[\log(p(\phi|\beta))\right] - E_q\left[\log(q(\theta|\gamma))\right] - E_q\left[\log(q(\mathbf{z}|\epsilon))\right] - E_q\left[\log(q(\mathbf{x}|\delta))\right] - \\ &\qquad E_q\left[\log(q(\phi|\psi))\right]\end{aligned}$$

δ

δ is the variational parameter for x and we need an update rule for it.

$$\mathcal{L}_{[\delta]} = \mathbb{E}_q\left[\log(p(\mathbf{w}|\mathbf{z},x,c,\phi))\right] + \mathbb{E}_q\left[\log(p(x|\lambda))\right] - \mathbb{E}_q\left[\log(q(x|\lambda))\right]$$

We can expand this to (note that T = {g,l}

$$\begin{aligned}&\sum_{n=1}^{N_d}\sum_{k=1}^{K}\sum_{t=1}^{T} w_n^w c_d^c\,\delta_{d,t,n}\epsilon_{d,n,k}\log(\phi_{d,n,v,k,c}) + \sum_{n=1}^{N_d}\sum_{t=1}^{T}\delta_{d,t,n}\log(\lambda_t) \\ &\qquad - \sum_{n=1}^{N_d}\sum_{t=1}^{T}\delta_{d,t,n}\log(\delta_{d,t,n}) + \zeta_n(\sum_{t=1}^{T}\delta_{d,t,n} - 1)\end{aligned}$$

Now we take the partial for the global and local δ and ζ_i is a Lagrance multiplier.

$$\frac{\partial}{\partial\delta_{d,g,n}}\left[\mathcal{L}_{[\delta]}\right] = \sum_{k=1}^{K}\epsilon_{d,n,k}\log(\phi_{d,v,g,k,c}) + \log(\lambda_g) - \log(\delta_{d,g,n}) - \zeta_g$$

Setting this equal to 0 and relocating the $\log(\delta)d,g,n$ to the left hand size and then exponentiating yields.

$$\delta_{d,g,n} \propto w_n^v\lambda_g\exp\left\{\sum_{k=1}^{K}\epsilon_{d,n,k}\log(\phi_{d,v,g,k,c})\right\}$$

By analogy, we achieve the update rule for each local corpus

$$\delta_{d,l,n} \propto w_n^v c_n^c \lambda_l \exp\left\{\sum_{k=1}^{K} \epsilon_{d,n,k} \log(\phi_{d,v,l,k,c})\right\}$$

Note that here we have treated λ as a vector so λ_g (global) $= \lambda$ (old notation) and λ_l (local) $=$ $(1-\lambda)$.

ϵ

ϵ is the variational parameter for θ. We get

$$\mathcal{L}_{[\epsilon]} = \mathbb{E}_q\left[\log(p(\mathbf{w}|\mathbf{z}, x, c, \phi))\right] + \mathbb{E}_q\left[\log(p(\mathbf{z}|\theta))\right] - \mathbb{E}_q\left[\log(q(\mathbf{z}|\epsilon))\right]$$

Now since we are clever, we note that the only difference between this and the $\mathcal{L}_{[\phi]}$ in Blei et al. (2003) is $\mathbb{E}_q\left[\log(p(\mathbf{w}|\mathbf{z}, x, c, \phi))\right]$. Thus we can simply write down adapation of equation (16) to include the δ term.

$$\epsilon_{d,n,k} \propto \exp\left\{\sum_{t}^{T} w_n^v c_n^c\ \delta_{d,t,n} \log(\phi_{d,v,l,k,c}) + \Psi(\gamma_{dk})\right\}$$

ϕ

Finally to get the updates for ϕ we look at A.4.1 in Blei et al. (2003). We notice that only addition will be $\delta_{d,t,n}$ which will be constant with respect to the derivative. So we can write down by inspection the global update

$$\phi_{d,v,g,k} \propto \beta + \sum_{d=1}^{D} \sum_{n=1}^{N_d} w_{d,n}^v\ \delta_{d,g,n} \epsilon_{d,n,k}$$

Now to get the local update, we have

$$\phi_{d,v,l,k,c} \propto \beta + \sum_{d=1}^{D} \sum_{n=1}^{N_d} w_{d,n}^v c_d^c\ \delta_{d,l,n} \epsilon_{d,n,k}$$

The updates for γ are identical. I repeat them for the sake of repeating them but this is extremely straight forward since no part of the model has changed.

$$\gamma_{dk} = \alpha + \sum_{n=1}^{N_d} \epsilon_{d,n,k}$$

6.3 Empirical Questions

1.

2.

3.

4.