IMPLICATION OF SHALLOW WATERS FOR SOURCE LOCALIZATION: TIME-DELAYS ESTIMATION VERSUS MATCHED-FIELD PROCESSING.

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ABSTRACT

Since matched-field processing (MFP) has been widely studied, and due to the increasing interest about shallow water conditions, this paper addresses the rightful problem of comparing such a promising technique with the classical multipath time-delay estimation (TDE) and least-squares fitting, for a source localization purpose. At first the response function of both methods (MFP and TDE) are established for a single broadband source, considering an equispaced horizontal linear array and taking into account the assignment ambiguity between the set of time-delay measurements and all the possible model replicas. Then their Cramer-Rao bounds are concurrently processed and compared in the case where the acoustic medium is perfectly known. Finally, biases in range and depth estimation are studied for TDE and a robust version of MFP (ie. adaptive with multiple constraints) in the presence of environmental mismatch as celerity gradient errors . TDE reveals itself as being more robust and less sensitive than MFP.

1. INTRODUCTION

Source localization via passive means is a problem of major interest in the field of ocean acoustics. A great number of techniques have been proposed dealing with such an estimation problem. We distinguish two classes of methods: the first ones that take simply advantage of the relative geometry between the source and the receiver, they are based on bearing and/or Doppler shift measurements [1, 2], and the other ones that make rather the most of the specific propagation through the acoustic medium. The latter has received much interest in the recent years and specially concerning the so-called coherent methods (based on matched-field processing) versus the incoherent ones (based on Time-Delay estimation between multipath) [3]. Actually, the problem is to know at which level a propagation model has to be entered into the signal processing chains. If one considers that the global signal processing is two-folded: a first stage dedicated to conventional array processing (e.g. the energy beams determine only the source direction) and a second one to time-delay post-processing (the beam autocorrelation provides time-delay estimates), then one have an incoherent localization of the source position (range and depth) by computing a least-squares fit between the time-delay measurements and a model including propagation effects. On the contrary, one can initiate the beamforming with a realistic medium modeling: the signal phase contributes to estimate simultaneously the source direction but its range and depth. The question is: which of the two techniques is the most efficient to address the case of shallow waters, and especially when the relevant propagation parameters are not accurately monitored?

The following discussion is structured into four sections. Sec. 2 establishes the two different source location estimators: multipath time-delay (TDE) and robust matched-field processing (MFP). Plots such as the ambiguity function or the asymptotic response enable to compare them in terms of spatial contrast and sidelobe level. Furthermore, the multipath uncertainty in TDE (which couple of ray paths corresponds to which time-delay) is taken into account by means of an assignment solution. Sec. 3 presents computation of the Cramer-Rao bounds for both TDE and MFP and Sec. 4 considers effects of mismodeling sound celerity gradients on the source detection and localization biases. Finally, Sec. 5 concludes that TDE is not sensitive to celerity mismatch from a detection point of view: localization bias does exist but remains reasonable; whereas MFP proves to be dramatically sensitive: computing its location bias is no more relevant since the source might be even not detected.

2. PROBLEM STATEMENT

2.1. Hydrophonic signals and spatial filters

Considering shallow waters and low frequencies, induces naturally the resort to a modal theory of acoustic propagation in a waveguide [4]. Thus in a MFP context, avoiding computation of the eigenrays, the transfer function from the source to the receiver is based on a modal description of the acoustic pressure or equivalently the acoustic signal delivered by the hydrophones of the receiving antenna. Given an acoustic source located at range R and depth z in the direction θ from a K-hydrophones equispaced linear array, the received noiseless signal on the k-th sensor at depth z_k

can be expressed in the frequency domain as

$$x_k(f) = \gamma(f) \exp 2\jmath\pi f t \sum_{i=1}^{N} \frac{\phi_i(z_k)\phi_i(z)}{\sqrt{\kappa_i R_k}} \exp \jmath\kappa_i R_k \tag{1}$$

where $\gamma^2(f)$ is the power spectral density of the source, N is the number of modal functions ϕ_i which are the depth solutions of the wave equations, with their associated horizontal wave number κ_i. R_k is the effective range between the k-th sensor and the source. Collecting the K signals into a column vector and adding a process noise $n(f)$ yields the following measurement model

$$X(f) = \gamma(f) M \Phi + n(f) \tag{2}$$

where M is a $K \times N$ matrix whose general term is $M_{ki} = \phi_i(z_k)/\sqrt{\kappa_i R_k} \exp \jmath\kappa_i R_k$ and $\Phi = \mathrm{col}_{i=1}^{N}\{\phi_i(z)\}$. Here, the wave front shape is represented by the vector $w = M\Phi$.

On the contrary, in a TDE context, the wave front shape w is generally taken as a single plane wave so

$$x_k(f) = \gamma(f) \exp 2\jmath\pi f (t - \tau_k) \tag{3}$$

where the delay between sensors for a horizontal array $\tau_k = (k-1)d/c \cos(\theta)$ classically depends only on the source bearing; d being the distance between sensors, and c the mean sound speed.

Given a wave front shape w defining the medium response enables to calculate the beamforming filter h according to the well known various forms: Bartlett, Capon or adaptive with multiple constraints (Baggeroer [5]) filter, as

$$h_{Bar} = \frac{w}{\|w\|^2} \tag{4}$$

$$h_{Cap} = \frac{\Gamma^{-1} w}{w^+ \Gamma^{-1} w} \tag{5}$$

$$h_{Bag} = \Gamma^{-1} W (W^+ \Gamma^{-1} W)^{-1} e \tag{6}$$

where Γ is the covariance spectral density matrix, and W a matrix stacking in line the wave front vector w for the beam of interest, the other w's being chosen in its immediate vicinity; constraint vector e being equal to the classical gains (ie. $e_1 = 1$ and $e_l = w_1^+ w_l$).

The MFP consists simply in a specific spatial filter mapping directly the 3D-space range $\times$ depth $\times$ bearing. So applying this filter to the signals X, followed by a detection procedure, yields the source location estimate.

2.2. Time-delay estimation

For TDE, a plane-wave classical filter is generally sufficient: it determines the source bearing, then autocorrelation of the beam will yield the time-delay measurements; estimates of source range and depth are given by a least-squares fitting between the TD measurements and TD delivered by a model. Since MFP and TDE must be compared in the same realistic case (eg. a 2-layered sound speed profile), one have to consider the same model of propagation for time-delays computation. There exists no explicit expressions but we can detail the propagation travel time T (of which the TD measurement τ is made with variance σ^2) of each eigenray in terms of the source depth z and the elevation angle φ at the receiver, as [6]

$$T = H(\varphi, z) , \tag{7}$$

$$\varphi = G^{-1}(R, z) , \tag{8}$$

the function G and H have been primarily introduced in [7].

3. AMBIGUITY ANALYSIS

As suggested by [8], a common form can be used as an ambiguity function for both coherent MFP and incoherent TDE. In a gaussian noise context, it consists in writing the log-likelihood ratio of the two hypotheses: H_0 the source position is located in (θ_0, R_0, z_0) or in (θ, R, z) (hypothesis H_1). H_0 can be viewed as the true source location and H_1 as the guessed one; plotting the ambiguity versus the H_1 parameters (eg. range and depth, rather than the bearing) will demonstrate the performance of localization methods. Designating by (μ_0, Σ_0) (resp. (μ, Σ)) the mean and the covariance of the observation in H_0 (resp. in H_1) the ambiguity is defined as

$$\mathcal{A} = \frac{1}{2} \ln \frac{|\Sigma_0|}{|\Sigma|} + \frac{1}{2} \mathrm{tr} \Sigma (\Sigma_0^{-1} - \Sigma^{-1}) + \frac{1}{2} \mathrm{tr} \Sigma_0^{-1} (\mu - \mu_0)(\mu - \mu_0)' \tag{9}$$

For coherent MFP the information is contained in the variance of the signals, $\mu = E(X) = 0$, $\Sigma = \Gamma$, so the ambiguity reduces to

$$\mathcal{A}_{MFP} = \frac{1}{2} \ln \frac{|\Sigma_0|}{|\Sigma|} + \frac{1}{2} \mathrm{tr} \Sigma (\Sigma_0^{-1} - \Sigma^{-1}) \tag{10}$$

Notice that this definition does not take into account the specific spatial MFP filter h. So considering rather the MFP output $\mu = E(h^+ X)$ and $\Sigma = h^+ \Gamma h$ yields

$$\mathcal{A}_{Bar} = \frac{1}{2} \ln \frac{w_0^+ \Gamma_0 w_0}{w^+ \Gamma_0 w} + \frac{1}{2} \left(\frac{w^+ \Gamma_0 w}{w_0^+ \Gamma_0 w_0} - 1 \right) \tag{11}$$

$$\mathcal{A}_{Cap} = \frac{1}{2} \ln \frac{w^+ \Gamma_0^{-1} w}{w_0^+ \Gamma_0^{-1} w_0} + \frac{1}{2} \left(\frac{w_0^+ \Gamma_0^{-1} w_0}{w^+ \Gamma_0^{-1} w} - 1 \right) \tag{12}$$

$$\mathcal{A}_{Bag} = \frac{1}{2} \ln \frac{e_0^+ (W_0^+ \Gamma_0^{-1} W_0)^{-1} e_0}{e^+ (W^+ \Gamma_0^{-1} W)^{-1} e} + \frac{1}{2} \left(\frac{e^+ (W^+ \Gamma_0^{-1} W)^{-1} e}{e_0^+ (W_0^+ \Gamma_0^{-1} W_0)^{-1} e_0} - 1 \right) \tag{13}$$

these last expressions are not so far from the classical MFP response simply $h^+ \Gamma_0 h$.

For the TDE, when the variances are assumed equal, $\mu = [\tau_1, \ldots, \tau_N]'$, $\Sigma = \mathrm{diag}_{i=1}^{N}\{\sigma_i^2\}$

$$\mathcal{A}_{TDE} = \frac{1}{2} (\mu - \mu_0)' \Sigma^{-1} (\mu - \mu_0) \tag{14}$$

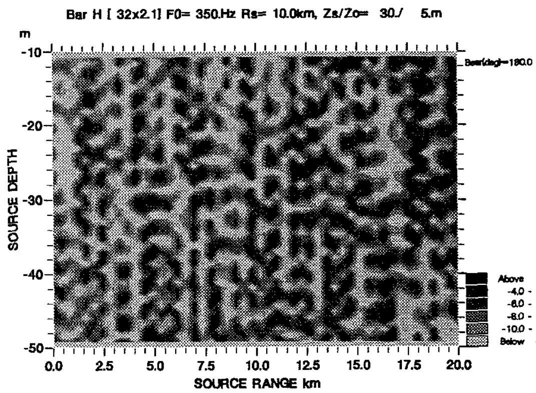

Figure 1. MFP Bartlett ambiguity

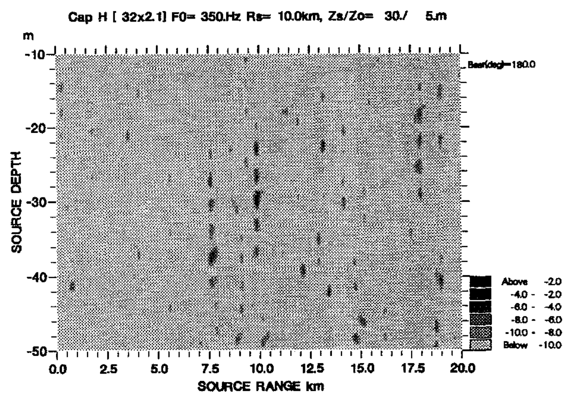

Figure 2. MFP Capon ambiguity

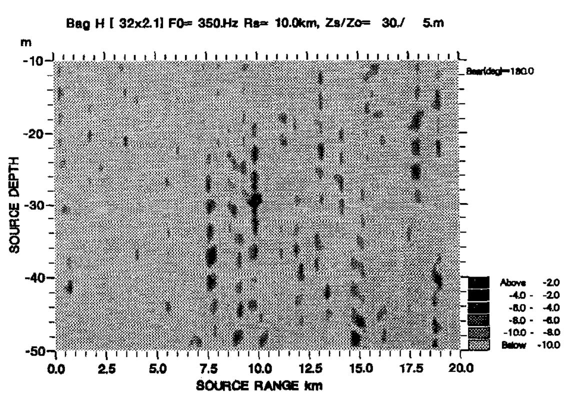

Figure 3. MFP Baggeroer ambiguity

which is equivalent to the Mahalanobis distance (except a factor $2\dim(\mu)$) between the two hypotheses (this distance being minimized to estimate the source location when μ is the noisy measurement). However, the unknown type of rays is implicitly included into the mean μ. So the modified TDE ambiguity is

$$\mathcal{A}_{TDE} = \min_{\mu} \frac{1}{2}(\mu - \mu_0)' \Sigma^{-1}(\mu - \mu_0) \qquad (15)$$

where the minimization over μ is the optimal assignment of the true TD vector μ_0 and the assumed one μ [9]. Let us apply these previous definitions to the following shallow water case (bottom depth: 100 m): a source range $R = 10$ km, depth $z = 30$ m, and bearing endfire; the antenna being horizontal at 5 m, with 32 equispaced sensors. Signal to noise ratio at the sensors output is 0 dB for the MFP, and standard deviations for the TDE are $\sigma_\tau = 1$ ms corresponding to a 0.5 kHz bandwidth. The acoustic waveguide is characterized by an upper celerity gradient $g_1 = 0.01$ ms^{-1}/m in the 55 first meters, then a lower gradient $g_2 = 0.2$ ms^{-1}/m and a rigid bottom. Figs. 1-3 shows the ambiguity function for the three MFP's: adaptive MFP's presents a much better contrast than Barlett MFP. The TDE ambiguity is displayed in Fig. 4 for only 1 TD measurement, and in Fig. 5 for 4 TD measurements. Due to the unknown type of the rays, total ambiguities appear and are reduced by adding several measurements.

4. CRAMER-RAO BOUNDS

4.1. MFP case

The ambiguity is linked with the 1st order statistics of the observations, whereas the Cramer-Rao bounds (CRB) are related to the 2nd order as being a lower bound for the covariance of the source position estimation. Given the statistic of the observation $p(X)$, the CRB are the inverse of the Fisher information matrix which general term F_{ij} is expressed as

$$F_{ij} = -E\{\partial^2_{ij} \ln(X)\} \qquad (16)$$

Considering gaussian signals

$$p(X) = (2\pi)^{-\frac{K}{2}} |\Gamma|^{-\frac{1}{2}} \exp -\frac{1}{2}\mathrm{tr}(\Gamma^{-1} X X^{+}) \qquad (17)$$

yields (Whittle's formula)

$$F_{ij} = \frac{1}{2}\mathrm{tr}(\Gamma^{-1}\partial_i\Gamma.\Gamma^{-1}\partial_j\Gamma) \qquad (18)$$

Recalling the expression of matrix $\Gamma = \gamma^2 I + \sigma^2 w w^{+}$ allows to compute quite easily each term of the Fisher matrix, being understood that indexes i and j designates the source parameter R and z respectively (θ is assumed known). Accordingly, Fig. 6 shows the range CRB for any source in a vertical plane [0,20] km and [10,50] m.

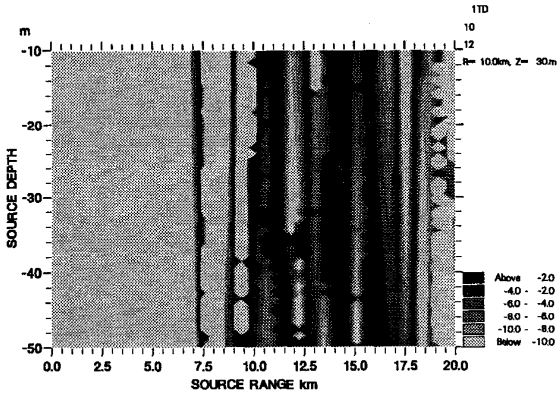

Figure 4. TDE ambiguity (1 τ)

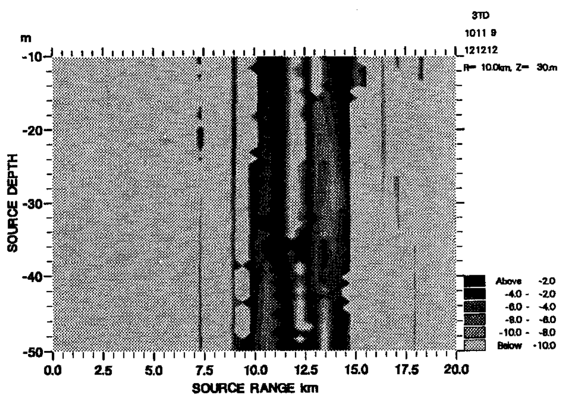

Figure 5. TDE ambiguity (3 τ)

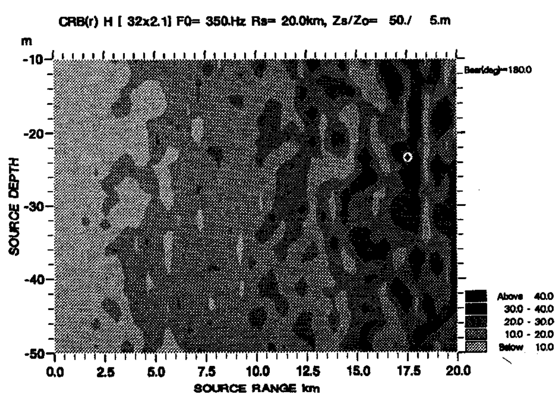

Figure 6. MFP range CRB (%)

4.2. TDE case

In the same manner, the CRB for TDE is given by the Fisher matrix as

$$F(X) = \partial_X \underline{\tau}' \Sigma_\tau^{-1} \partial_X \underline{\tau} + \partial_X \underline{\varphi}' \Sigma_\varphi^{-1} \partial_X \underline{\varphi} \qquad (19)$$

where $\underline{\tau}$ and $\underline{\varphi}$ are the vector of measurements. The partial derivatives are rather tedious to calculate [6] specially for a 2-layered medium. Fig. 7 shows the range CRB in the same conditions as previously for the ambiguity.

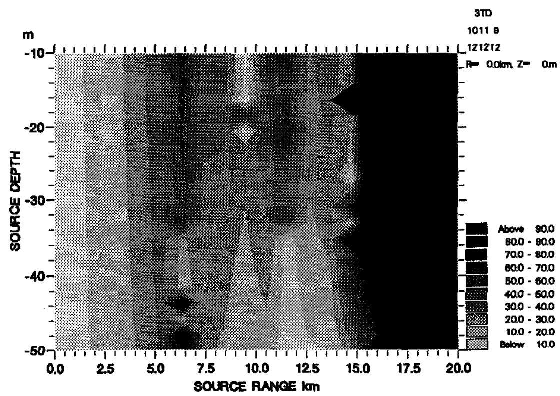

Figure 7. TDE range CRB (%) (3 τ)

5. ROBUSTNESS AGAINST MEDIUM MISMATCH

A rather similar behaviour has been observed for MFP and TDE in a CRB sense. Now consider the case of mismatches in the celerity gradients. For TDE, the robustness consists in studying the range and depth biases implied by a medium mismatch. For MFP, the problem is essentially to detect the source in the presence of a mismatch and after to estimate the localization biases. All the parameters of the scenario are the same as for the previous study of ambiguity. The mismatch consists in an upper gradient ranging from 0.008 to 0.012 ms^{-1}/m and a lower gradient ranging from 0.19 to 0.21 ms^{-1}/m. Concerning MFP, Figs. 8-10 show the resulting contrast for source detection (ie. the ratio between the peak of maximum energy and the highest sidelobe expressed in dB): Bartlett MFP is clearly robust but initially (i.e. without mismatch) the contrast is poor; Capon MFP is too much sensitive and Baggeroer MFP is an interesting compromise between the two.

Notice that this is especially true for a vertical array (Figs. 11-13 in the same conditions as Figs 8-10): the +/- 10% variation of the surface gradient and +/- 5 % for the bottom gradient severely reduce the performance of the MFP with a horizontal array, but are quite acceptable for the vertical array.

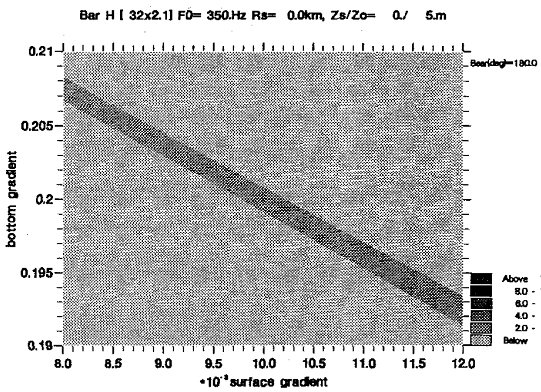

Figure 8. Bartlett MFP detection index (Hor. array)

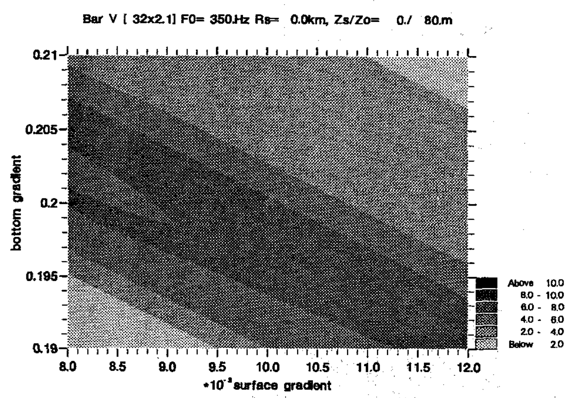

Figure 11. Bartlett MFP detection index (Vert. array)

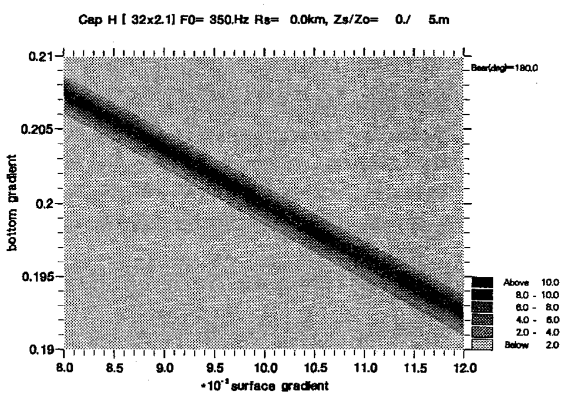

Figure 9. Capon MFP detection index (Hor. array)

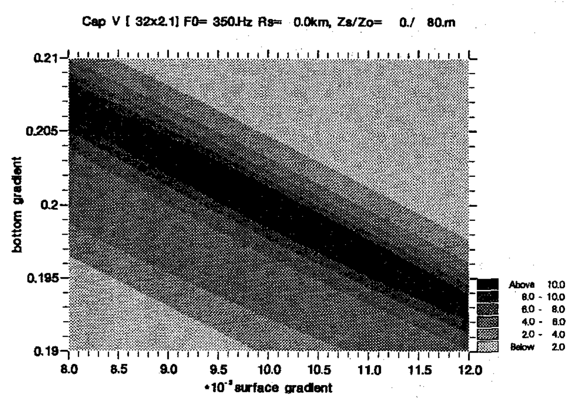

Figure 12. Capon MFP detection index (Vert. array)

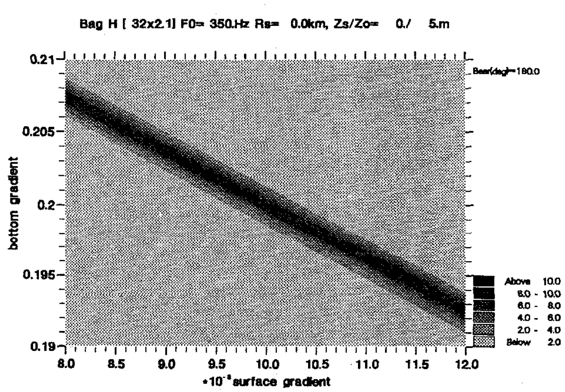

Figure 10. Baggeroer MFP detection index (Hor. array)

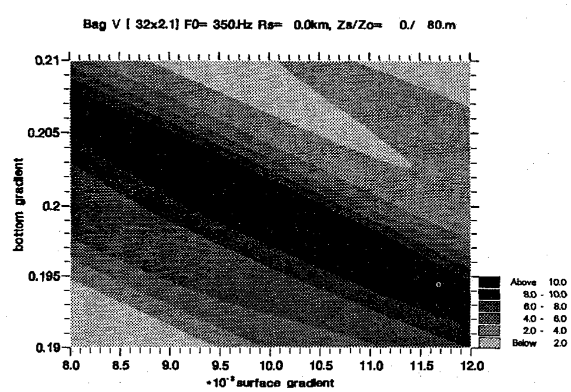

Figure 13. Baggeroer MFP detection index (Vert. array)

However, when these rather small variations of gradient are overpassed, the MFP delivers no detection and consequently, localization biases are irrelevant.

On the contrary, TDE delivers solutions even if the celerity gradients are moderately distorted. Fig. 14 shows the bias for TDE: it is limited to a few % in range.

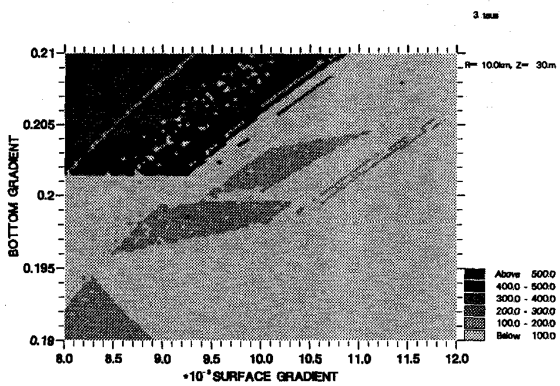

Figure 14. TDE range bias (m)

6. CONCLUSION

Sensitivity to the acoustic waveguide errors is a major limitation to operate matched-field methods specially when only a poor calibration of the medium is available. On the contrary time-delay estimation and least-square fitting enables to locate a source even in the presence of a few % of celerity gradient mismatch. In that condition, TDE solution exhibits reasonable biases whereas MFP prevents even from detecting the source itself. Further developments will necessary consider the way of estimating partly the medium, with respect to the observability of the corresponding parameters [10, 11].

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