An overview of matched field methods in ocean acoustics

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Arthur Baggeroer
Massachusetts Institute of Technology

Peter N. Mikhalevsky
Leidos, Inc., Arlington, United States

An Overview of Matched Field Methods in Ocean Acoustics

Arthur B. Baggeroer, *Fellow, IEEE*, William A. Kuperman, and Peter N. Mikhalevsky

(Invited Paper)

***Abstract*—Recent array processing methods for ocean acoustics have utilized the physics of wave propagation as an integral part of their design. The physics of the propagation leads to both improved performance and to algorithms where the complexity of the ocean environment can be exploited in ways not possible with traditional plane wave based methods. Matched field processing (MFP) is a generalized beamforming method which uses the spatial complexities of acoustic fields in an ocean waveguide to localize sources in *range, depth* and *azimuth* or to infer parameters of the waveguide itself. It has experimentally localized sources with accuracies exceeding the Rayleigh limit for depth and the Fresnel limit for range by two orders of magnitude. MFP exploits the coherence of the mode/multipath structure and it is especially effective at low frequencies where the ocean supports coherent propagation over very long ranges. This contrasts with plane wave based models which are degraded by modal and multipath phenomena and are generally ineffective when waveguide phenomena are important. MFP can have either conventional or adaptive formulations and it has been implemented with an assortment of both narrowband and wideband signal models. All involve some form of correlation between the replicas derived from the wave equation and the data measured at an array of sensors. One can view MFP as an inverse problem where one attempts to "invert" the wave equation for these dependencies over the parameter space of the source and the environment. There is currently a large literature discussing many theoretical aspects of MFP including numerous simulations; several experiments acquiring data for MFP now have been conducted in several ocean environments and these have demonstrated both its capabilities and some of its limitations. Consequently, there is a modest understanding of both the theory and the experimental capabilities of MFP. This article provides an overview of both.**

I. Introduction

Recent research on array processing for ocean acoustics is concerned with exploiting to maximum advantage the combination of the complexities of the ocean environment, sophisticated acoustic models, array design and both narrow and broadband coherent processing techniques. This new approach is similar in some respects to what geophysicists and others refer to as inverse methods. It has much in common with matched filtering theory for the problem of detecting a signal with unknown parameters in correlated noise. In underwater acoustics the area is now termed matched field processing (MFP) as a consequence of this. Searching over a parameter space for the unknown parameters can be considered to be an inverse problem; however, the calculation of the matching, or replica, signal is far more difficult because of the effects of the propagating and it involves solving the wave equation. In this article we review recent developments in MFP and their relationship to previously developed classical array processing methods. At the edge of this research area processing algorithms now are emerging that either are tolerant of our uncertain knowledge of the environment or go one step further and simultaneously invert for both the environment and the localization parameters of a target.

Matched field processing (MFP) refers to array processing algorithms which exploit the full field structure of signals propagating in an ocean waveguide. The objectives of the algorithms may include source detection and localization and/or the estimation of parameters of the ocean waveguide. Fundamentally, it is a generalization of plane wave beamforming wherein the "steering vector," or replica, is derived from the Green's function, or spatial point source response, of the medium. The question then is: Why is MFP attractive in a number of ocean acoustic applications? There are several responses to this. First, most of classical beamforming is based upon plane wave signals with homogeneous signal fields and this is often not an appropriate model for signals in the ocean waveguide, especially at low frequencies. The multipaths and/or modes are coherent so this coherence can be exploited to enhance performance well beyond the Rayleigh and Fresnel limits for an array.[1] Since MFP uses replicas which are appropriate for both the coherence and the waveguide, it yields better performance when the environmental model used to design them is accurate. There is an additional benefit *vis à vis* reducing noise. Ocean noise is largely dominated by ambient processes which are subject to the same constraints

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A. Baggeroer is with the Department of Ocean Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139.

W. Kuperman was with the Naval Research Laboratory, Washington, DC 20375. He is now with the Marine Physical Laboratory, Scripps Institution of Oceanography, University of California, San Diego, La Jolla, CA 92093.

P. N. Mikhalevsky is with Science Applications International Corporation, MST 3-5, 1710 Goodrich Dr., McLean, VA 22102.

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[1] The Rayleigh limit for the cross range resolution of an array is given by $\Delta z \sim \lambda(R/L)$ and the Fresnel limit for the in range resolution of an array is given by $\Delta R \sim \lambda(R/L)^2$. See Appendix B of [4] for a more complete discussion.

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of the waveguide; for example, distant shipping is a dominant noise contributor at low frequencies. Adaptive suppression of this directional noise is often done by null placement and "estimator-subtractor" processing. This is implemented more accurately with properly designed replicas and leads to lower residual noise levels. Next, MFP localizes sources far more accurately than plane wave methods; in particular, ranges and depths can be estimated surprisingly well. While simulations predicted this, it is fair to assert that the first experimental verifications surprised most since it was expected that the ocean would not support the high degree of coherence among the multipaths or modes. This concept is not limited to source localization; there have been a number of studies and a few experiments where MFP has been used tomographically to infer environmental parameters such as sound speed profiles and elastic parameters of the seafloor or the ice cover. Finally, MFP establishes upper bounds on performance since it incorporates the full wave physics of the ocean acoustics including both signal and noise processes into the array processing. In this sense it fully exploits the medium.

There are, nevertheless, some liabilities with MFP. Some are unique to it, while others are present in most array processing applications; the MFP context simply magnifies them. The most important liability is the sensitivity to mismatch. Since MFP exploits the environment, the model of it must be accurate, especially when one seeks high performance. There are several forms of mismatch: i) environmental—where one has an incorrect model for the waveguide and/or source, e.g., sound speed profile errors, source doppler; ii) statistical—where one implements adaptive algorithms using estimated covariances for the processes, the so called "snapshot" or "finite degrees of freedom" problem; and iii) system—where arrays and/or sensors are not calibrated or positioned accurately. Initially, the potential of MFP was downplayed by many because they perceived these mismatches to be too great. While the mismatch issue is indeed very real and is discussed more extensively later, several experiments have demonstrated that it can be overcome. Indeed, by accurately incorporating environmental effects using MFP the adaptive mismatch losses are less severe than those introduced by plane wave or wavefront curvature methods which do not include the waveguide propagation effects.

Several factors in ocean acoustics came together at the same time and these stimulated a lot of the work on MFP. First, there were the recognitions that waveguide effects are important and that they can be modeled well. As understanding of the environment, e.g., internal waves, ocean fronts, bathymetry, and the geo-acoustics of the seabed, improved, the accuracy of propagation models in ocean acoustics increased dramatically. Simultaneously, better numerical models could incorporate this environmental knowledge and computational power increased. Results indicating the effects of various environmental models could be obtained within reasonable run times. Next, the exceptional coherence of the propagation at low frequencies became evident in a number of experiments. This led to processors which are phase coherent and formed the basis of MFP. Finally, experiments with large vertical arrays were implemented with considerable care for calibration and navigation. The net result was that much of the signal previously regarded as not usable or, even worse, as unwanted reverberation noise was now useful. In effect, noise became signal as we understood the ocean better and performed our experiments with greater precision.

We first provide a short historical overview of MFP. We then provide background material in both ocean acoustics and array processing needed for MFP. Next, specific algorithms involving both quadratic and adaptive methods are introduced. Since MFP exploits the constraints of the acoustic waveguide, it is sensitive to mismatch. The results on mismatch studies and several algorithms designed to be relatively robust against mismatch are discussed. We next examine simulated MFP for range, depth and bearing localization by using data from a towed array which has been tilted to produce an effective vertical aperture. Several experiments using MFP are then reviewed. One successfully demonstrated MFP at megameter ranges and this has important consequences for recent experiments in global tomography. We finally consider some unique applications of MFP including how it can exploit ocean inhomogeneities and make tomographic measurements of environmental parameters.

II. Matched Field Processing—Historical Overview

The development of MFP parallels the evolution of modeling acoustic propagation in the ocean. Here we review the early contributions to MFP. Clay was the first to recognize the close connection of waveguide models, arrays and signal processing when he examined modal propagation [25]. While he did not address source localization or tomography, he clearly developed the close interaction between modal representations, propagation and array processing. Hinich was the first to examine source localization with a vertical array [57]. He derived the maximum likelihood equations and the Cramer–Rao bounds for the mode amplitude coefficients and the source depth [125]. (These were "true" maximum likelihood equations as used in the statistical literature.) Since zero mean, Gaussian noise models were used for the noise, the estimators were identical to linearized, minimum variance estimators. He later generalized his bounds to include range [58]. While Hinich introduced several important concepts, the lack of credible environmental models relegated the papers to a role of interesting, but not realistic, theory. These issues were also addressed by Carter who developed bounds on the variance for parameter localization with an array; however, he used free space models where only wavefront curvature could be exploited [22].

Bucker is generally credited to be the first to formulate MFP as now used [16]. His paper was significant in two respects. First, Hinich's results led to a linear processor which was sensitive to Doppler mismatch, especially if long integration intervals were used. Bucker recognized this and formulated a quadratic detector which reduced this sensitivity. More importantly, he used realistic environmental models, introduced the concept of an ambiguity surface and demonstrated that there was enough complexity of the wave field to permit inversion, hence localization. His detection factor, or statistic, was essentially what is now termed the "conventional MFP." (More precisely, his detection factor can be related to the log likelihood function for parameter estimation, which is usually termed localization in the MFP literature.) He dropped the diagonal terms in the quadratic sum which lowers the overall level on the ambiguity surface. (Bucker used a shallow water environment; it is a bit remarkable that most experiments have attempted to minimize the bottom interaction intrinsic in shallow water.) Next, Klemm introduced the first form of adaptive processing with what he termed "Approximate Orthogonal Projection." It was closely related to the linear prediction and Maximum Entropy Methods (MEM) [68], [76], [82]. He encountered the complicated sidelobe problems at low Signal to Noise Ratios (SNR) often associated with time series analysis using the MEM. The first application of Bucker's concept was presented in October, 1983 by Heitmeyer, Moseley and Fizell at a Naval Ocean Research and Development Agency (NORDA) Workshop [56]. They presented several simulations using the conventional MFP. The magnitude squared of the cross correlation between the data vector for a narrowband signal on an array, which simulated a "snapshot," and a normalized version of the Green's function, or replica vector, for a Pekeris waveguide was plotted on an ambiguity surface as a function of source position. The simulated source was localized and the resolution of the central peak and the level of sidelobes were clearly indicated to be important issues. Several other simulations also soon appeared. Shang proposed using a vertical array and the sifting, or closure, properties of the eigenfunctions for depth estimation [108]. This has had some useful applications in MFP algorithms based upon mode methods. He then proposed a phase unwrapping algorithm which exploited the modal interference pattern to estimate range [109]. These were very idealized algorithms which required high SNR and well calibrated environments. One of the prominent patterns in the MFP ambiguity surface are periodic sidelobes due to convergence zones in deep water or model interference regions in shallow water. Tappert recognized that a range dependent environment alters this periodicity; he used the parabolic equation (PE) to compute the Green's function for the conventional processor [117]. He also introduced the concept of back propagation for calculating the ambiguity surface.

The Soviet literature has long had an emphasis on waveguides and modal representations of acoustic propagation, so it should not be surprising that MFP concepts appeared early in *Soviet Physics Acoustics*. Kravstov *et al.* formulated the array processing problem of resolving rays and modes in a waveguide in which there was a brief discussion of source resolution [69]. Burov *et al.* formulated the spatial equivalent of the "random signal in noise" detection problem where the covariances differ depending upon the hypothesis [15], [126]. This leads to an MFP algorithm since the covariance matrix of the signal has rank one. The degradation of plane wave beamformers due to waveguide effects were clearly of concern [66]. One of the first papers on adaptive processing using Frost's Least Mean Square (LMS) method within a normal mode formulation was presented by Mal'tsev [83], [49]. Borodin recently has formulated source localization as an inverse problem [10]. There was also a keen appreciation for the computational demands of MFP since there was an optical implementation of the conventional MFP algorithm not very long after these initial papers [137]. Recently an overview of MFP appeared in *Soviet Physics Acoustics* [138]. A special issue of the *IEEE J. Oceanic Eng.* devoted to MFP was published in Sept. 1993 [139].

The first experimental demonstration of MFP using field data was reported by Fizell and Wales [43]. They used low frequency data (47 Hz) recorded on a vertical array suspended from the Arctic ice pack at the FRAM IV ice camp in 1982. They were able to localize a source at a second ice camp 260 km distant to a precision consistent with the satellite navigation of the camps. They also introduced the use of Capon's Minimum Variance, Distortionless Filter (MVDF) algorithm (often erroneously termed the Maximum Likelihood Method (MLM)) [20]. This is by far the most extensively used adaptive algorithm in the MFP literature. It has a number of attributes suitable to MFP; these include good sidelobe suppression and a modest tolerance to mismatch when compared to other adaptive methods.

This provides some background to the early literature; since 1985 articles reporting both theory and experiments have increased greatly; we discuss contributions since then in the various sections of this overview. Recently A. Tolstoy has written the first book on the subject of matched field processing [121].

III. Environmental Acoustics Relevant to the MFP Procedure

A. Green's Functions for a Stratified Ocean Environment

Fig. 1 depicts the source and an N element array in a stratified ocean. The ocean is modeled as a waveguide [62] and the signal from the source can be considered to be the point source solution to the wave equation. The point source solution is composed of modal components which propagate long distances at shallow grazing angles and vertical components which decay rapidly because of the strong interaction with the bottom. The modal components have discrete values of the wavenumber spectrum for the Green's function while the vertical components occupy a continuum in the spectrum at lower wavenumbers, i.e., higher phase speeds. For sources at horizontal ranges that are greater than a few water depths from the receiving array, the discrete portion of the field is the major contributor. On the other hand, for noise sources which originate from and are distributed on the ocean surface such as wind generated noise, there is a significant contribution from the continuous spectrum. Discrete sources which are to be considered as interference with regard to the desired source to be localized have acoustic fields comprised of both components depending on their range from the array. It is, therefore, safest to model the acoustic fields from discrete sources and distributed noise with the full wave solutions. Let $\mathbf{G}$ be the velocity potential (or scaled pressure) for a unit normalized source which satisfies the frequency ($\omega = 2\pi f$) domain wave equation; we then have,

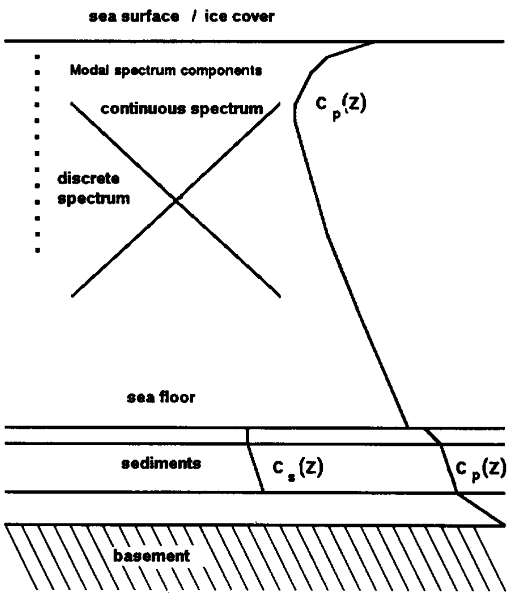


Fig. 1. Array in a stratified ocean: The medium is defined by the sound speed within the water column and the ocean bottom and the bathymetric profile. Other properties, e.g., ice cover and sediment density, may also have an effect upon propagation.

$$\nabla^2 \mathbf{G}(\mathbf{r}, z) + K^2(z)\mathbf{G}(\mathbf{r}, z) = -\delta(\mathbf{r} - \mathbf{r}_s)\delta(z - z_s); \quad K^2(z) = \frac{\omega^2}{c^2(z)} \tag{1}$$

with the subscript s denoting the source coordinates. ($\delta(\mathbf{r})$ denotes a delta function sifting over a two-dimensional vector argument.) The field at any point $(\mathbf{r}, z)$ is clearly a function of frequency f, the source coordinates and the parameters associated with the ocean environment, the latter manifesting themselves as coefficients and boundary conditions of the wave equation. Those parameters other than the frequency we denote by $\mathbf{A}$ so it is understood that $\mathbf{G}(\mathbf{r}, \mathbf{z})$ can alternatively be represented as $\mathbf{G}(f, \mathbf{A})$.

We have used a time dependence of e^{jwt}. In addition, velocity potentials for both compressional and shear waves with approximate coupling at interfaces need to be included for elastic problems. We use the wave equation primarily to introduce the Green's function into the signal processing.

With only horizontal stratification, we may separate the equation by decomposing $\mathbf{G}$ into a vertical dependence $g(\mathbf{k}, z, z_s)$ and set of plane waves, $e^{i\mathbf{k}\cdot r}$, where $\mathbf{k}$ is the horizontal wavenumber of the plane waves.[2]

$$\mathbf{G}(\mathbf{r}, z) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} d^2\mathbf{k}\, g(\mathbf{k}, z, z_s)e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_s)}, \tag{2}$$

[2] We have changed the notation of [72] to be consistent in engineering convention for a Fourier transform. In particular, $\delta(\mathbf{r} - \mathbf{r}_s) = \int\int d\mathbf{k}/(2\pi)^N e^{j\mathbf{k}\cdot(\mathbf{r}-\mathbf{r_s})}$ where N is the dimension of the transform.

and hence, the depth dependent Green's function, $g(\mathbf{k}, z, z_s)$, satisfies

$$\frac{d^2 g}{dz^2} + (K^2(z) - k^2)g = -\delta(z - z_s). \tag{3}$$

The signal solution consists of a discrete and continuous part [74]. For a source not too close to the array, only the discrete portion of the solution contributes at the array.

The noise generated at the ocean surface has near (from above) and far field contributions so that we must describe it by both the discrete and continuous solutions of the wave equation [72], [54]. This situation suggests that the surface generated noise has some overlapping properties with the signal; hence, signal processing based on considerations of plane wave beamforming in white noise is misleading as far as a stratified ocean environment is concerned. In particular, conventional matched field processing corresponds to independent noise at each hydrophone and does not incorporate ocean structured noise. For horizontal arrays with sensor separation less than a wavelength, and vertical arrays more generally, this is not a good model; consequently, the signal like properties of the noise introduce erroneous performance.

B. Signal Models

The Green's function forms the basis for representing the signals used in MFP. We assume that the solutions to the wave equation at the array locations, $\mathbf{G}(\mathbf{r}_i, \mathbf{z}_i)$, are incorporated into a signal vector $\mathbf{G}(\mathbf{f}, \mathbf{A})$ where $\mathbf{A}$ denotes parametric dependencies upon source location and environmental parameters. In addition, we assume that the source level is scaled by a complex Gaussian random variable $\tilde{b}$ whose variance σ_b^2 is equal to the source strength. This models the uncertainty in absolute phase and amplitude in the signal processing. As a result we have

$$\mathbf{G}(\mathbf{f}, \mathbf{A}) = \begin{bmatrix} G(\mathbf{r}_1, z_1, \mathbf{A}) \\ G(\mathbf{r}_2, z_2, \mathbf{A}) \\ \vdots \\ G(\mathbf{r}_N, z_N, \mathbf{A}) \end{bmatrix} \tag{4}$$

for the spatial structure of the signal. The covariance is given by

$$\mathbf{K}_s = \sigma_b^2 \mathbf{G}(\mathbf{f}, \mathbf{A})\mathbf{G}^{\mathbf{H}}(\mathbf{f}, \mathbf{A}). \tag{5}$$

where the notation ^{H} denotes the Hermitian operator, or conjugate transpose, of the matrix.

Almost all the analyses in the MFP literature have used point source signal models; however, more complicated source models, *multipoles* or *distributed sources* can be incorporated with additional complexity. This may be relevant to applications where the source is large compared to a wavelength, such as a large ship or an earthquake, and the radiation pattern becomes important. Most experiments have used point sources.

C. Noise Models

We describe ocean noise in terms of the $\mathbf{K}_n$, cross spectral density matrix or the covariance among the elements at a given frequency. In practice, real ocean noise falls into three categories:

1) Sensor noise: it is spatially uncorrelated and we denote its covariance matrix by $\mathbf{K}_W = \sigma_n^2 \mathbf{I}$ where $\mathbf{I}$ is the identity matrix. This is sometimes called "white noise" which is a misnomer because it does not have a wavenumber spectrum representation, let alone a flat one [3]. Note that the level of the sensor noise may depend upon frequency.
2) Distributed noise, for example surface generated noise: it is correlated and its covariance matrix, denoted $\mathbf{K}_C$ is structured according to the same ocean environment which determines the signal propagation [72]:

$$\mathbf{K}_C(\mathbf{r}_{ij}, z_i, z_j) = q^2 \int d^2\mathbf{k}\, P(\mathbf{k}) g(\mathbf{k}, z_i, z') \cdot g^*(\mathbf{k}, z_j, z') e^{i\mathbf{k}\cdot\mathbf{r}_{ij}}, \quad (6)$$

where

q	the surface noise spectrum level;
$P(\mathbf{k})$	the spatial spectral distribution of homogeneous noise sources;
g	the solutions of (3) with the same ocean environment as the signal and replica fields;
$\mathbf{r}_{ij}$	the horizontal vector between the ith and jth element of an N element receiving array;
z_i and z_j	the corresponding depths and z' is the noise sourced layer depth.

Note that $\mathbf{K}_C$ is actually the "cross spectral density" of the surface generated noise and is a measure of its spatial coherence. For a stratified waveguide, $\mathbf{K}_C$ is spatially homogeneous in the horizontal (it depends only on $\mathbf{r}_i - \mathbf{r}_j$); however, it is inhomogeneous in the vertical because it depends on the absolute depth locations of the array elements. This inhomogeneity is not normally incorporated in standard signal processing where algorithms take advantage of the noise covariance matrix being homogeneous or, more particularly, Toeplitz.

3) Discrete noise sources, which, if they can be resolved, can be treated as point source signals or some other compact source model as mentioned above for signals. Most MFP experiments have used one compact and powerful source so discrete noise interference has not been a major issue. For detection problems at low SNR's the discrete noise sources and their resolution are important issues.

The total covariance of the noise at a receiving array is

$$\mathbf{K_n} = \mathbf{K}_W + \mathbf{K}_C + \mathbf{K}_D \quad (7)$$

where $\mathbf{K}_D$ is the covariance from discrete noise sources. Note, to be precise with the notation this should have include a dependence upon $\mathbf{A}$ if there is an environmental parameter contained in $\mathbf{A}$. We do not include this at the current time, but we return to this issue when we discuss MFP for noise tomography.

D. Normal Mode Representation of the Signal and Noise Fields

Though there exists an assortment of sound propagation models [12], [62], for the purpose of brevity, we use a normal mode description of the signal and noise fields. For stratified media, the normal mode representation of (2) in cylindrical coordinates is [12]

$$\mathbf{G}(r, z) = \frac{i\rho(z_s)}{(8\pi r)^{1/2}} \exp(-i\pi/4) \sum_n \frac{u_n(z_s) u_n(z)}{k_n^{1/2}} \exp(ik_n r). \quad (8)$$

Equation (8) is a far field solution of the wave equation and the quantities u_n are normalized eigenfunctions of the following eigenvalue problem:

$$\frac{d^2 u_n}{dz^2} + [K^2(z) - k_n^2] u_n(z) = 0. \quad (9)$$

The eigenfunctions, u_n, are zero at $z = 0$, satisfy the local boundary conditions descriptive of the ocean bottom properties and satisfy a radiation condition for $z \to \infty$. They form an orthonormal set in a Hilbert space with weighting function $\rho(z)$, the local density. The range of discrete eigenvalues is given by the condition

$$\min_z [K(z)] < k_n < \max_z [K(z)]. \quad (10)$$

This normal mode representation can be used to approximately include the continuous part of the field by introducing a very high-speed deep basement layer. This, in effect, increases the number of modes and makes the left hand side of the above inequality approach zero. The result is that the number of modes is greater than that of the actual discrete spectrum with the lower order modes coinciding with the discrete modes and the higher modes approximating the continuum.

For surface generated noise, we can use the approximate expression

$$\mathbf{K}_C(R, z_1, z_2) \approx \frac{\pi q^2 \rho^2(z')}{2K^2(z')} \cdot \sum_n \frac{u_n^2(z') u_n(z_1) u_n(z_2)}{\alpha_n k_n} J_0(k_n R) \quad (11)$$

where α_n is the attenuation coefficient of the nth mode and R is the horizontal separation between two field points [72]. We again can use the notion of including a high speed deep basement to approximate the continuum (overhead) portion of the noise.

For mildly range dependent problems we can use the adiabatic normal mode representation of the acoustic field,

$$\mathbf{G}(\mathbf{r}, z) = \frac{i\rho(z_s)}{(8\pi r)^{1/2}} \exp(-i\pi/4) \sum_n \frac{u_n(z_s) v_n(z)}{\overline{k_n}_n^{-1/2}} \exp(i\overline{k_n} r). \quad (12)$$

where the range-averaged wave number (eigenvalue) is

$$\overline{k_n} = \frac{1}{r} \int_0^r k_n(r')\, dr' \quad (13)$$

and the $k_n(r')$ are obtained at each range segment from the eigenvalue problem (9) evaluated at the environment at that particular range along the path [12], [62]. The quantities

u_n and v_n are the sets of modes at the source and the field positions, respectively. Note that the mode shapes at the source and receiver are the only ones of direct consequence; the variability of the medium is incorporated only through the average wavenumbers $\overline{k_n}$ which influence the relative phasing among the modes. The corresponding expression for the noise field in a range dependent environment is rather involved and not given here in detail (see [99]).

It is useful later in the paper to express the field $\mathbf{G}(\mathbf{f}, \mathbf{A})$ in matrix/vector notation. This has the form

$$\mathbf{G}(\mathbf{f}, \mathbf{A}) = \gamma \mathbf{W}\mathbf{P}s \tag{14}$$

where

γ the constant, $\gamma = \frac{i\rho(z_s)}{\sqrt{8\pi}} e^{-i\pi/4}$;

s a vector of mode amplitude coefficients at the source,or

$$\mathbf{s} = \begin{bmatrix} u_1(z_s) \\ u_2(z_s) \\ \vdots \\ u_M(z_s) \end{bmatrix};$$

$\mathbf{P}$ an $M \times M$ diagonal propagation matrix,

$$\mathbf{P} = \begin{bmatrix} \ddots & 0 & 0 \\ 0 & \frac{e^{ik_n r}}{\sqrt{k_n r}} & 0 \\ 0 & 0 & \ddots \end{bmatrix};$$

$\mathbf{W}$ an $N \times M$ observation matrix which takes the signal form mode space to sensor space and has the form

$$\mathbf{W} = \begin{bmatrix} u_1(z_1) & u_2(z_1) & \cdots & u_M(z_1) \\ u_1(z_2) & u_2(z_2) & \cdots & u_M(z_2) \\ \vdots & \vdots & & \vdots \\ u_1(z_N) & u_2(z_N) & \cdots & u_M(z_N) \end{bmatrix}.$$

(A slight modification to $\mathbf{W}$ is needed if the sensors are not at the same range, i.e., the array is not vertical, to account for the differential phase across the array.)

IV. Matched Field Processing Algorithms

Preliminaries

Since MFP is a generalization of plane wave beamforming, many of the algorithms developed in the array processing literature have been tested in the MFP context. There are, however, some important differences. Non-adaptive plane wave beamforming has an extensive literature concerning pattern design especially for linear arrays. Theories for calculating weightings to control mainbeam width, sidelobes, etc. are well-honed. There is not a similar literature for MFP; the only non-adaptive weighting is the "conventional" one which uses a normalized version of the Green's function and this is equivalent to equal element weighting with phase shifting for steering. In plane wave beamforming this leads to a beampattern with -13 dB sidelobes. This is not acceptable in most applications because of the potential for false alarms and/or jamming. This sidelobe can be avoided in plane wave beamforming by using one of several pattern design approaches, similar theories for MFP are not available.

The adaptive algorithms are particularly important in MFP. While the complexity of the acoustic field usually permits localization and environmental parameter estimation, the inversion process has many sidelobes which lead to ambiguities in the presence of noise and/or other sources. As a result, adaptive algorithms for control of sidelobes or sidelobe levels are especially useful; this use of adaptive algorithms for control of sidelobes or sidelobe levels are especially useful; this use of adaptive algorithms contrasts to applications where the objective is "high resolution," i.e., very precise localization. In both, the penalty for adaptation is an added sensitivity to mismatch since incorrect adaptation may be worse than no adaptation at all.

In the rest of this section we summarize the various conventional and adaptive algorithms which have been used for MFP. Almost all originated elsewhere in the adaptive array literature; we attempt to indicate who introduced them to MFP and to critique their success on both simulated and/or field data. First, we need to introduce several quantities which are common to many of the algorithms.

There are generally three components to MFP:

1) Signal generation: The environmental model is used to compute the Green's function, $\mathbf{G}(\mathrm{f}, \hat{\mathbf{A}})$, for the spatial response of a source with location/environment parameter $\hat{\mathbf{A}}$. The notation $\hat{\mathbf{A}}$ emphasizes that the computation is done versus a scanning parameter, i.e., a sequence of forward, or trial, solutions.
2) Sample covariance estimation: The data snapshots are used to form an estimate, $\hat{\mathbf{K}}$, of the covariance matrix, $\mathbf{K}$, of the ambient field. Usually, this is done by a sum of outer products on the snapshots.
3) MFP Algorithm: An algorithm which estimates the field distribution versus the location/environment parameter $\hat{\mathbf{A}}$. This algorithm is a function of both $\mathbf{G}(\hat{\mathbf{A}})$ and the sample covariance $\hat{\mathbf{K}}$.

This is illustrated in Fig. 2.

The signal generation model was discussed in the previous section. For the sample covariance estimation we assume we have an array with N sensors located at $\mathbf{z}_i$, $i = 1, N$ and a narrowband model as illustrated in Fig. 3. The field has covariance $\mathbf{K}$ which has uncorrelated signal and noise components with covariances $\mathbf{K}_s$ and $\mathbf{K}_n$ with

$$\mathbf{K} = \mathbf{K}_s + \mathbf{K}_n. \tag{15}$$

While we assume in much of our discussion that these covariances are known, generally, they must be estimated from the data or predicted using some propagation model. For this estimation the signal is segmented into "snapshots" and harmonically decomposed using an FFT.

The snapshots may be windowed and overlapped as often done in Fourier transform based methods of spectrum esti-

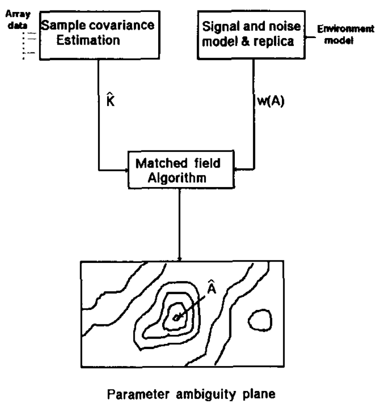


Fig. 2. Matched field processing signal flow algorithm.

mation. These snapshots form a data vector $\mathbf{R}^l(f)$ whose components are

$$R_i^l(f) = \int_{T_l}^{T_l+T_w} r_i(t)a(t-T_l)e^{-i2\pi ft}\,dt \tag{16}$$

where

- $r_i(t)$ the signal from the ith channel,
- $a(t)$ a taper, or window, function applied to the data to control sidelobes,
- T_l the start of the lth segment of data,
- T_w the duration of the taper function.

We assume that window lengths and resolution bandwidths have been chosen so that bias errors are not an issue.

In most MFP algorithms the data vectors are averaged to form the sample covariance matrix

$$\hat{\mathbf{K}}(f) = \frac{1}{L}\sum_{l=1}^{L}\mathbf{R}^l(f)\mathbf{R}^l(f)^H, \tag{17}$$

where L is the number of snapshots. We assume that there are L_{eff} "degrees of freedom" in the sample covariance matrix. (One can also average across a frequency band; however, the bandwidth is limited by the need to preserve approximately the same relative phase shift across, i.e., the separation scaled to a wavelength must be approximately equal across the band.) We assume that the data vectors have zero mean and that the expected value of the sample covariance is the ensemble covariance $\mathbf{K}(\mathbf{f})$. We further assume that the statistics of the field are Gaussian. Then the data vectors are Gaussian and the sample covariance matrix has a complex Wishart distribution of order N with L_{eff} degrees of freedom [1]. The statistical issues of the sample covariance and its impact upon the MFP algorithms are aspects of "snapshot" problem for statistical mismatch.

In several of the MFP algorithms a singular value decomposition (SVD) of the covariance $\mathbf{K}$ is used. This is given by

$$\mathbf{K} = \mathbf{V}\Sigma\mathbf{V}^H. \tag{18}$$

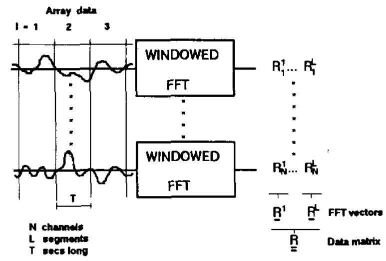


Fig. 3. Array, narrowband model, sample matrix estimation, and data preconditioning for Matched Field Processing.

where $\mathbf{V}$ is a matrix whose columns, $\mathbf{v}_i$, are the eigenvectors and Σ is a diagonal matrix whose entries are the eigenvalues, σ_i^2 of the homogeneous matrix equation

$$\mathbf{K}\mathbf{v}_i = \sigma_i^2\mathbf{v}_i. \tag{19}$$

If L_{eff} exceeds N, then the eigenvalues of the SVD are all positive; often this is not the case and several problems result. These eigenvectors and eigenvalues are not the same as the solution to the wave equation. In fact, one of the important issues in MFP, especially for mode based methods, is the transformation between the bases of these two representations. It is sometimes convenient to use the summation notation directly and the expansion for $\mathbf{K}$ becomes

$$\mathbf{K} = \sum_{i=1}^{N}\sigma_i^2\mathbf{v}_i\mathbf{v}_i^H. \tag{20}$$

Note that if one uses the sample covariance matrix, $\hat{\mathbf{K}}$, instead of the ensemble covariance, $\mathbf{K}$, as done in many adaptive algorithms, then there is an issue with the rank of $\mathbf{K}$. In particular, the rank is bounded by the number of snapshots; if the number of array elements, N, exceeds the number of snapshots, then there will be eigenvalues equal to zero in an SVD of the sample covariance matrix. Even if the number of snapshots exceeds N, there can be practical issues with poorly conditioned matrices. This is discussed in more detail later.

B. Two Parameter Ambiguity Functions

For most of this article we assume that the signal is coherent. This implies that a source with location/environment parameter $\mathbf{A}$ generates a signal $\tilde{b}\mathbf{G}(\mathbf{f}, \mathbf{A})$ across the array as described in the previous section. One of the important issues is how spatially unique the signal is when compared to signals for other locations/environments. This is quantified by the "two-parameter correlation" function, or

$$\phi(f, \mathbf{A}_1, \mathbf{A}_2) = \left(\frac{\mathbf{G}(\mathbf{f}, \mathbf{A}_1)}{|\mathbf{G}(\mathbf{f}, \mathbf{A}_1)|}\right)^{\mathbf{H}}\left(\frac{\mathbf{G}(\mathbf{f}, \mathbf{A}_2)}{|\mathbf{G}(\mathbf{f}, \mathbf{A}_2)|}\right). \tag{21}$$

A second quantity is the ambiguity function which is the magnitude squared of the correlation function, or

$$\Theta(f, \mathbf{A}_1, \mathbf{A}_2) = |\phi(\mathbf{f}, \mathbf{A}_1, \mathbf{A}_2)|^2. \tag{22}$$

These are fundamental measures of the invertibility of the field for MFP. The normalization implies that ϕ and Θ have maxima of unity when $\mathbf{A}_1 = \mathbf{A}_2$. If there are parameter sets $\mathbf{A}_1$, $\mathbf{A}_2$ where ϕ approaches unity, then the inversion is ambiguous and there will be high sidelobes in the MFP parameter plane. High SNR's may permit adaptive processing to suppress these ambiguities, but they still remain. In this situation one needs to be alert for sensitivity to mismatch problems.

From now on it is convenient to suppress the frequency dependence in the notation since in almost all contexts narrowband signals are used, i. e., $\mathbf{G}(f, \hat{\mathbf{A}}) \Rightarrow \mathbf{G}(\hat{\mathbf{A}})$. When there is a need to be explicit about the frequency dependence, it will be included in the notation.

C. Conventional MFP

The Conventional MFP was essentially proposed by Bucker and then implemented by Fizell [16], [43]. Since then it has been used in virtually every article on MFP. It simply forms an average of the projection of the data vectors on the normalized replica vectors, or

$$B_c(\hat{\mathbf{A}}) = \frac{1}{L}\sum_{l=1}^{L} |\mathbf{w_c^H}(\hat{\mathbf{A}})\mathbf{R}^l|^2. \quad (23)$$

where

$$\mathbf{w_c}(\hat{\mathbf{A}}) = \frac{\mathbf{G}(\hat{\mathbf{A}})}{|\mathbf{G}(\hat{\mathbf{A}})|}. \quad (24)$$

This can be expressed in a quadratic form in terms of the sample covariance matrix, $\hat{\mathbf{K}}$, or

$$B_c(\hat{\mathbf{A}}) = \mathbf{w_c^H}(\hat{\mathbf{A}})\hat{\mathbf{K}}\mathbf{w_c}(\hat{\mathbf{A}}). \quad (25)$$

It is useful to interpret this quadratic form when the field consists of a single source with parameter $\mathbf{A}$ and level σ_b^2 plus spatially white noise. The mean value of $(\hat{\mathbf{A}})$ is then given by

$$\overline{B}_c(\hat{\mathbf{A}}) = \sigma_b^2 \overline{TL}(\mathbf{A})\mathbf{N}\boldsymbol{\Theta}(\mathbf{A}, \hat{\mathbf{A}}) + \sigma_n^2 \quad (26)$$

where

$$\overline{TL}(\mathbf{A}) = \frac{|\mathbf{G}(\mathbf{A})|^2}{\mathbf{N}}. \quad (27)$$

The conventional MFP output versus the parameter $\hat{\mathbf{A}}$ consists of a constant due to white noise plus a scaled version of the ambiguity function, $\Theta(\mathbf{A}, \hat{\mathbf{A}})$, positioned at the true source parameter $\mathbf{A}$. The scaling is the product of the source level, σ_b^2, a mean-square transmission loss from the source to the receiver array, $\overline{TL}(\mathbf{A})$, and an array gain which is equal to the number of hydrophones, N. If this peak is unique and there are no significant sidelobes, then the localization is unique. The accuracy of the localization at high SNR's can be determined by the Cramer–Rao bounds [4]. If more than one source is present and they are uncorrelated, then the MFP output is a superposition of these scaled ambiguity functions.

Several algorithms which extend the conventional MFP processor are based upon an SVD of the field. In this case we can express $B_c(\hat{\mathbf{A}})$ in terms of eigenvectors and eigenvalues of the field, or

$$B_c(\hat{\mathbf{A}}) = \sum_{i=1}^{N} \sigma_i^2 |\rho_i(\hat{\mathbf{A}})|^2 \quad (28)$$

where

$$\rho_i(\hat{\mathbf{A}}) = \mathbf{w}_c^{\mathbf{H}}(\hat{\mathbf{A}})\mathbf{v_i}. \quad (29)$$

In practice one would need to use a SVD based upon the data snapshots and a sample covariance as discussed above.

Note that conventional MFP is a "power" estimator and not a "spectral density" estimator. It gives an output level appropriate for the power of a point source just as one obtains the power of a directional source in plane wave beamforming. If one wants to convert to a spectral level, i.e., power per unit wavenumber, or per steradian, then an array directivity factor must be applied. While directivities have been tabulated in plane wave theory for most standard weightings, similar results are not available for MFP.

D. Minimum Variance, Distortionless Filter MFP

The minimum variance, distortionless filter (MVDF), or Capon's Maximum Likelihood Method (MLM), has proven to be one of the more robust of the adaptive array algorithms for MFP [20]. Numerous authors have discussed it using both simulated and field data [100], [44], [4], [34], [40], [118], [79], [98], [41], [123], [120]. (Others have also discussed the MVDF in the context of comparing it to alternative MFP algorithms; we cite these when we review the individual algorithms.)

The MVDF algorithm was originally introduced by Capon in the context of frequency wavenumber analysis of teleseismic events on the Large Aperture Seismic Array (LASA) [20]. The algorithm minimized the variance at the output of a linear weighting, $\mathbf{w_{MV}}(\hat{\mathbf{A}})$ of the sensors subject to the distortionless constraint that signals in the "look direction" have unity gain. Generally, this is done on a narrowband basis although one can formulate broadband versions quite easily. The formulation then is to minimize the variance, or spectral level, given by

$$B(\hat{\mathbf{A}}) = \mathbf{w^H}(\hat{\mathbf{A}})\mathbf{K}\mathbf{w}(\hat{\mathbf{A}}) \quad (30)$$

versus the weighting $\mathbf{w}(\hat{\mathbf{A}})$ and subject to the unit gain constraint that

$$\mathbf{w^H}(\hat{\mathbf{A}})\mathbf{w_c}(\hat{\mathbf{A}}) = \mathbf{1}. \quad (31)$$

The minimum variance weight vector, $\mathbf{w_{MV}}(\hat{\mathbf{A}})$ is

$$\mathbf{w_{MV}}(\hat{\mathbf{A}}) = \mathbf{B_{MV}}(\hat{\mathbf{A}})\mathbf{K}^{-1}\mathbf{w_c}(\hat{\mathbf{A}}) \quad (32)$$

where B_{MV} is the minimum variance and is given by

$$B_{MV}(\hat{\mathbf{A}}) = [\mathbf{w_c^H}(\hat{\mathbf{A}})\mathbf{K}^{-1}\mathbf{w_c}(\hat{\mathbf{A}})]^{-1}. \quad (33)$$

The MVDF algorithm is now defined as $B_{MV}(\hat{\mathbf{A}})$ when the sample covariance is used instead of the ensemble covariance, or

$$B_{MV}(\hat{\mathbf{A}}) = [\mathbf{w_c^H}(\hat{\mathbf{A}})\hat{\mathbf{K}}^{-1}\mathbf{w_c}(\hat{\mathbf{A}})]^{-1}. \quad (34)$$

There are several points here. First, is the assumption that sample covariance can be used in place of an ensemble one. This introduces a bias in the results even when the sample covariance has full rank [21]. If the sample covariance does not have full rank because of too few snapshots, then alternative approaches such as diagonal loading must be used; this is also a source of bias [104]. Finally, like the conventional processor, the output gives the level for a point source; a directivity factor must again be applied to obtain a spectral level. Since the weighting is adaptive the directivity factor is not only a function of the array geometry as in the conventional processor but it is also a function of the relative levels of the noise processes.

Since the MVDCF processor is often used in MFP it is useful to examine it in a some detail. Consider the signal plus noise model with

$$\mathbf{K} = \sigma_b^2 \mathbf{G}(\mathbf{A})\mathbf{G}^{\mathbf{H}}(\mathbf{A}) + \mathbf{K_n}. \tag{35}$$

In this case, the inverse matrix can be expressed using the Woodbury identity in the form [114]

$$B_{MV}(\hat{\mathbf{A}}) = B_{MV-n}(\hat{\mathbf{A}}) \cdot \left[\frac{1 + [\sigma_b^2 \overline{TL}(\mathbf{A})\mathbf{N}/\mathbf{B_{MV-n}}(\mathbf{A})]}{1 + [\sigma_b^2 \overline{TL}(\mathbf{A})\mathbf{N}/\mathbf{B_{MV-n}}(\mathbf{A})][1 - \Theta_n(\mathbf{A}, \hat{\mathbf{A}})]}\right], \tag{36}$$

where $B_{MV-n}(\hat{\mathbf{A}})$ is the MVDF output for noise only, or

$$B_{MV-n}(\mathbf{A}) = [\mathbf{w_c^H}(\mathbf{A})\mathbf{K_n^{-1}}\mathbf{w_c}(\mathbf{A})]^{-1}, \tag{37}$$

and $\Theta_n(\mathbf{A}, \hat{\mathbf{A}})$ is a generalized ambiguity function normed to the Hilbert space of the noise, or

$$\Theta_n(\mathbf{A}, \hat{\mathbf{A}}) = [\mathbf{B_{MV-n}}(\mathbf{A})\mathbf{B_{MV-n}}(\hat{\mathbf{A}})]|\mathbf{w_c^H}(\mathbf{A})\mathbf{K_n^{-1}}\mathbf{w_c}(\hat{\mathbf{A}})|^2. \tag{38}$$

This can be interpreted as follows: $B_{MV-n}(\hat{\mathbf{A}})$ is variance of the MVDF output at parameter $\hat{\mathbf{A}}$ when only the noise with covariance $\mathbf{K}_n$ is present. Since it is the solution to a minimum variance problem, it is always less than or equal to variance of the conventional output $B_{c-n}(\hat{\mathbf{A}})$. The term $\sigma_b^2 \overline{TL}(\mathbf{A})\mathbf{N}/\mathbf{B_{MV-n}}(\mathbf{A})$ is an SNR for the signal at the output of the MVDF. The term $\Theta_n(\mathbf{A}, \hat{\mathbf{A}})$ is a measure of the similarity of the signals at $\mathbf{A}$ and $\hat{\mathbf{A}}$ in the Hilbert space of the noise. Note that the second factor in (36) above is always greater than unity, so in dB measures the presence of the signal always adds to the ambiguity plane of the MFP.

The MVDF is essentially the same as the detection of a signal in "colored noise" which has a very extensive literature [125]. One of the important aspects of this problem is the estimator-subtractor formulation wherein the processor has a structure which estimates the colored noise and then subtracts it from the data. In a spatial formulation the colored noise often consists of discrete components in either the plane wave or MFP context. This leads to "null placement" techniques which were developed by Anderson for his DICANNE (Digital Cancellation of Noise) receiver [2]. Null placement techniques are very important in adaptive arrays especially when one wants to obtain high array gains. The MFP situation does not alter this, and more probably enhances the importance of null placement issues since environmental uncertainties enter as well as the other issues normally present for plane wave beamforming.

There is one final formulation of the MVDF processor which is used in some extensions. If one uses an SVD representation for the covariance, then B_{MV} can be expressed as

$$B_{MV} = \left[\sum_{i=1}^{N} \frac{|\rho_i(\hat{\mathbf{A}})|^2}{\sigma_i^2}\right]^{-1}. \tag{39}$$

where $\rho_i(\hat{\mathbf{A}})$ is given by (29). There is a critical balance here in terms of the accuracy of the eigenvalues and which terms dominate the sum. The dominant terms usually occur when $\rho_i(\hat{\mathbf{A}})$ is close to unity, i.e., the weight vector has a large projection on an eigenvector. The adaption process optimally scales this with the eigenvalue σ_i^2. One needs to consider the small eigenvalues carefully to understand the sensitivity upon them. Since they enter in a reciprocal value, the small eigenvalue terms dominate the sum unless the projections $\rho_i(\hat{\mathbf{A}})$ are low. With accurate SVD's which are based upon accurate estimates of the covariance, this is usually the case; however, if the SVD is not accurate because of inaccurate covariances, then these low order eigenvalues introduce very high sensitivity in the MVDF.

The SVD expansion suggests two approaches which are used when there are small eigenvalues present, i.e., the condition number of $\mathbf{K}$, more often $\hat{\mathbf{K}}$. The first is diagonal loading whereby

$$\mathbf{K} \rightarrow \mathbf{K} + \sigma^2 \mathbf{I}. \tag{40}$$

This assures full rank and is often used to control numerical sensitivity. This essentially increases all the eigenvalues by σ^2, i.e.,

$$\sigma_i^2 \rightarrow \sigma_i^2 + \sigma^2. \tag{41}$$

A second approach is to truncate the sum in the SVD representation for the MVDF above at some threshold level for the low eigenvalues. Both approaches are often used; in particular. Ozard introduces a "reduced minimum variance beamformer (RMVB) [96] and Byrne discusses it in terms of modal decompositions [17].

Power Law MFP's

One can generalize upon the SVD approaches for MVDF in an MFP. There are two versions of this. The first is an extension of the adapted angular response (AAR) spectrum of Borgiotti and Kaplan [11]. It applies a correction to the MVDF to account for the directivity of the array. When extended to MFP, it has the form

$$B_{AAR}(\hat{\mathbf{A}}) = \left[\frac{\mathbf{w_c^H}(\hat{\mathbf{A}})\mathbf{K}^{-2}\mathbf{w_c}(\hat{\mathbf{A}})}{\mathbf{w_c^H}(\hat{\mathbf{A}})\mathbf{K}^{-1}\mathbf{w_c}(\hat{\mathbf{A}})}\right]. \tag{42}$$

This MFP involves ratios of the eigenvalues of the SVD of $\mathbf{K}$, or

$$B_{AAR}(\hat{\mathbf{A}}) = \left[\frac{\sum_{i=1}^{N} \frac{|\rho_i(\hat{\mathbf{A}})|^2}{(\sigma_i^2)^2}}{\sum_{i=1}^{N} \frac{|\rho_i(\hat{\mathbf{A}})|^2}{\sigma_i^2}}\right]. \tag{43}$$

Most adaptive algorithms are not "calibrated" for measuring spectral densities; this is one of the few adaptive algorithms where there has been explicit consideration of the directivity factor as is normally done in conventional plane wave beamforming. Nevertheless, it has not been applied to MFP directly. In practice, it is somewhat less robust than the MVDF algorithm.

A second "power law" MFP algorithm was introduced by Daugherty and Lynch [33]. It was introduced as an extension to the MVDF which was less sensitive to mismatch. It has the form

$$B_{MV-\alpha}(\hat{\mathbf{A}}) = [\mathbf{w}_c^{\mathbf{H}}(\hat{\mathbf{A}})\mathbf{K}^{-\alpha}\mathbf{w}_c(\hat{\mathbf{A}})]^{-1/\alpha} \tag{44}$$

where the fractional power of a matrix is defined as

$$\mathbf{K}^{\alpha} = \sum_{\mathrm{i}=1}^{\mathbf{N}} (\sigma_{\mathrm{i}}^2)^{\alpha} \mathbf{v}_{\mathrm{i}} \mathbf{v}_{\mathrm{i}}^{\mathbf{H}}. \tag{45}$$

They considered three cases in detail: $\alpha = 1., 1.5$, and $2.$, and compared it to the conventional, MVDF and a multiple constraint processor which we discuss later. In their simulations they included the effects of surface waves, internal waves and source motion; they optimized the exponent to maximize the difference from the MVDF output and suggested that $B_{MV-\alpha}(\hat{\mathbf{A}})$ was more robust to these environmental effects. The eigenvalue expression for $B_{MV-\alpha}(\hat{\mathbf{A}})$ has the form

$$B_{MV-\alpha}(\hat{\mathbf{A}}) = \left[\sum_{i=1}^{N} \frac{|\rho_i(\hat{\mathbf{A}})|^2}{(\sigma_i^2)^{\alpha}}\right]^{-1/\alpha}. \tag{46}$$

The exponent on the exterior is of no consequence since it is a monotonic scaling which becomes a scale factor in a dB plot. The exponentiation of the eigenvalues is significant since it determines the weighting of the terms which contribute to the sum.

F. Controlled Sensor Noise MVDF

The most important advantage of the MVDF processor is its adaptive control of sidelobes. The level of the sensor noise, σ_W^2, is a critical term in this. If it is too high, no adaption takes place and one reverts to the conventional processor; if it is too low, the MVDF is very sensitive to mismatch. The concept of the "controlled sensor noise MVDF" is to dynamically set the level of σ_W^2 according to the gain the processor has against the sensor noise. This is a concept advanced by Cox for plane wave beamforming and extended by Velardo for MFP [30], [31], [127]. The response of the MVDF processor to sensor noise is given by

$$B_{\sigma_W^2}(\hat{\mathbf{A}}) = \sigma_W^2 \left[\frac{\mathbf{w}_c^{\mathbf{H}}(\hat{\mathbf{A}})\mathbf{K}^{-2}\mathbf{w}_c(\hat{\mathbf{A}})}{(\mathbf{w}_c^{\mathbf{H}}(\hat{\mathbf{A}})\mathbf{K}^{-1}\mathbf{w}_c(\hat{\mathbf{A}}))^2}\right]. \tag{47}$$

The weight vector of the MVDF can be decomposed into orthogonal components parallel and perpendicular to the conventional weight vector, $\mathbf{w}_c(\hat{\mathbf{A}})$. The use of projection matrices yields

$$\mathbf{w}_{\mathbf{MV}}(\hat{\mathbf{A}}) = \mathbf{w}_c(\hat{\mathbf{A}}) + \mathbf{w}_c^{\perp}(\hat{\mathbf{A}}), \tag{48}$$

where the magnitude squared of the orthogonal component is given by

$$|\mathbf{w}_{\mathbf{c}}^{\perp}(\hat{\mathbf{A}})|^2 = \left[\frac{\mathbf{B}_{\sigma_{\mathbf{W}}^2}(\hat{\mathbf{A}})}{\sigma_{\mathbf{W}}^2}\right] - 1 \tag{49}$$

i.e., the projection along the conventional weighting, $\mathbf{w}_c(\hat{\mathbf{A}})$ is always unity and only the orthogonal component grows. If this second component becomes too large, then the MFP becomes very sensitive to mismatch. The concept of controlled sensor noise gain is to set the sensor noise level such that a constraint on the magnitude of the weight vector is not exceeded. This is done by determining the length of the orthogonal projection since it is the only one which changes. While this requires some extra computation of the magnitude of the weight vector, this is one of the more robust MFP algorithms. In particular, it allows for good sidelobe nulling while suppressing the tendency of the MVDF beamformer to generate very narrow beams using high amplitude weights, to called superdirective effects, and mainlobe nulling which is often present in adaptive arrays. These amplitude weights are dominated by the orthogonal projection which the white noise level cosntrains.

G. Multiple Constraint Processors

Adaptive beamformers require accurate replica signals for their algorithms; this is especially true for MFP since it is the detailed spatial structure of the replica which determines the invertibility in the parameter ambiguity plane. All the algorithms above used $\mathbf{w}_c(\hat{\mathbf{A}})$. In practical context the information needed to compute $\mathbf{w}_c(\hat{\mathbf{A}})$ may be "mismatched" because of environmental, statistical or system errors. Using an adaptive algorithm with "mismatched" information often degrades performance. The issue then becomes how does one achieve the benefits of the adaptive algorithms while tolerating this "mismatch." This has been a problem in the plane wave beamforming literature for some time [128], [115], [38]. There are two issues: i) constraining the shape of the "main beam" and ii) constraining the shape of nulls.

The MVDF applies a "point" constraint in the look direction $\hat{\mathbf{A}}$, i.e., the response pattern is constrained to be unity in but one direction, $\hat{\mathbf{A}}$. If there were some form of mismatch, the constraint would not be satisfied and one encounters a main lobe or self nulling problem.[3] Intuitively, we do not want a point constraint, but a constraints band if we are unsure of the precise shape of $\mathbf{w}_c(\hat{\mathbf{A}})$. In time series analysis one would argue to have a passband consistent with the uncertainty in the signal frequency. In plane wave beamforming this is accomplished by requiring either an angular sector or constraining the derivatives for the beam pattern so it cannot change too rapidly in the presence of the uncertainty. This is easily done for plane wave beamforming since one usually knows the uncertainty to be in the form of an angular error; the situation for MFP is more complicated since

[3] In the adaptive array processing literature the "self nulling" problem occurs when there is high correlation between the desired signal and a nearby noise. The processing requires an accurate representation of the signal since it treats errors in it as the nearby noise and attempts to null them.

the coupling between the environment and the replica vectors is more complicated and the "shape" of the "passband" is not intuitively evident. There have been two approaches to signal uncertainty in the MFP literature, one by Schmidt *et al.* [106] and the other by Krolik [70]. They differ in how the signal uncertainty is modeled.

The formulation for multiple constraint (MCM) processors is done by requiring the optimal weight vector, $\mathbf{w}(\hat{\mathbf{A}})$, to satisfy a set of M constraints instead of a single one, or

$$\mathbf{w}^{\mathbf{H}}(\hat{\mathbf{A}})[\mathbf{e}_1(\hat{\mathbf{A}}) \mid \mathbf{e}_2(\hat{\mathbf{A}})| \cdots |\mathbf{e}_{\mathbf{M}}(\hat{\mathbf{A}})] = [\mathbf{d}_1, \mathbf{d}_2, \cdots, \mathbf{d}_{\mathbf{M}}] \tag{50}$$

or in matrix form

$$\mathbf{w}^{\mathbf{H}}(\hat{\mathbf{A}}) = \mathbf{E}(\hat{\mathbf{A}})\,\mathbf{d}(\hat{\mathbf{A}}) \tag{51}$$

where $\mathbf{E}(\hat{\mathbf{A}})$ is a matrix whose columns are the constraint vectors and $\mathbf{d}(\hat{\mathbf{A}})$ is a column vector. Note that the constraints are dependent upon the look, or scanning, parameter $\hat{\mathbf{A}}$. The output variance is now minimized subject to this set of constraints. The solution is

$$\mathbf{w}_{\mathbf{MCM}}(\hat{\mathbf{A}}) = \mathbf{K}^{-1}\mathbf{E}(\hat{\mathbf{A}})[\mathbf{E}^{\mathbf{H}}(\hat{\mathbf{A}})\mathbf{K}^{-1}\mathbf{E}(\hat{\mathbf{A}})]^{-1}\,\mathbf{d}(\hat{\mathbf{A}}), \tag{52}$$

and

$$B_{MCM}(\hat{\mathbf{A}}) = \mathbf{d}(\hat{\mathbf{A}})[\mathbf{E}^{H}(\hat{\mathbf{A}})\mathbf{K}^{-1}\mathbf{E}(\hat{\mathbf{A}})]^{-1}\,\mathbf{d}^{H}(\hat{\mathbf{A}}). \tag{53}$$

There are several issues here relating to how $\mathbf{E}(\hat{\mathbf{A}})$ and $\mathbf{d}(\hat{\mathbf{A}})$ are chosen. First, columns of $\mathbf{E}(\hat{\mathbf{A}})$ must form a set of linearly independent vectors, or practically, the condition number of the matrix $\mathbf{E}^{H}(\hat{\mathbf{A}})\mathbf{E}(\hat{\mathbf{A}})$ must not be too large; otherwise, instability in the matrix inversion is encountered. This requires a bit of care since one often picks the columns of $\mathbf{E}(\hat{\mathbf{A}})$ to be a set of vectors "near" the conventional weight $\mathbf{w}_c(\hat{\mathbf{A}})$. Next, the constraint magnitudes, d_i must be consistent since large changes lead to very unstable results.

Finally, there is the rationale by which one selects the constraints. For this it is useful to distinguish the parameters in $\mathbf{A}$ to be either source or environmental parameters, i.e., $\mathbf{A} \rightarrow (\mathbf{A}_{\mathbf{s}}, \mathbf{A}_{\mathbf{e}})$. In the approach of Schmidt *et al.* the columns of $\mathbf{E}(\hat{\mathbf{A}})$ are chosen to be a set of conventional weight vectors, $\mathbf{w}_c(\mathbf{A}_{\mathbf{s}_i})$, bracketing a source parameter region, $\mathbf{A}_s$, and the constraint magnitudes aer the values of the conventional beampattern. The motivation is that one wants an algorithm which performs adaption in the sidelobes but acts as a conventional processor in the main lobe. In the approach of Krolik, the columns are chosen to be a set of conventional weight vectors $\mathbf{w}_c(\mathbf{A}_{\mathbf{e}_i})$ derived by perturbing the environment about the nominal one and the constraint magnitudes are the phases of the conventional beampattern. Krolik actually performed a simulation of the adjacent environment and performed an SVD both to select M, the number of constraints and to assure the conditioning required for the matrix inversions. Both approaches have performed well in simulations and have proven to be more robust than direct MVDF; nevertheless, but a lot of care is required to establish a consistent set of constraints.

Adaptive arrays achieve much of the performance by null placement methods; in fact, the optimal processor is a generalized placement of nulls if the ambient field consists of white noise plus directional signals [107]. The theoretical solutions have nulls at precise directions determined by the signal model; the practical approach is to design "broad" nulls which reject a band about the expected region of the interference. "Sector nulling" where there is a reject band about the interfering noise or "derivative nulling" where the first and higher order derivatives are again useful approaches. Intuitively, we want to constrain the shape of the null. The constraint of the shapes of the nulls has not been considered in MFP. This is probably because of the strong interest in localization in the literature and the relatively small concern about jamming and strong signal interference issues. Nevertheless, robust nulling is an important problem in the detection of low SNR signals and adaptive sidelobe control.

H. Matched Mode Processing (MMP)

Normal modes provide a complete representation of the field of signal at long ranges, so one can process modal data to estimate the parameter $\mathbf{A}$. Since the data are measured at a discrete set of sensors, the problem separates into extracting the mode amplitudes from the sensor data and then parameter estimation for $\hat{\mathbf{A}}$ in mode space. The matrix/vector representation of the pressure field in terms of the modes is given

$$\mathbf{r} = \tilde{b}\mathbf{L}\mathbf{s} + \mathbf{n} \tag{54}$$

where (see (14))

$$\mathbf{L} = \gamma\mathbf{W}\mathbf{P}$$

represents the product of the scaling, observation, and propagation matrices in the modal representation of the signal. The quantity $\mathbf{r}_M = \tilde{b}\mathbf{s}$ can be estimated from the data using least squares theory. If we assume N sensors and M modes with $M \leq N$, then we have

$$\mathbf{r}_M = [\mathbf{L}^H\mathbf{L}]^{-1}\mathbf{L}^H\mathbf{r} \tag{55}$$

where $\mathbf{r}_M$ is the least squares estimate of the modes. (If the modes are uniquely invertible ($M = N$), then $\mathbf{r}_M = \tilde{b}\mathbf{s}$. This is the formulation used by Shang [108], [109] and Yang in a series of articles [131], [132], [133], [134]. This does not incorporate any statistical information about the modes coherence or the noise covariance. In practice the matrix $\mathbf{L}^H\mathbf{L}$ is often very close to singular since the array elements do not span the water column; consequently, one either restricts the number of modes, M, used or uses SVD methods to invert the matrix.

Alternatively, one can apply linear estimation theory [50], [125] to obtain

$$\mathbf{r}_M = \sigma_b^2[\sigma_b^2\mathbf{L}\mathbf{K}_s\mathbf{L}^H + \mathbf{K}_n]^{-1}\mathbf{L}^H\mathbf{r}. \tag{56}$$

This formulation takes care of the inversion issues by forcing one to include explicitly the effects of noise; nevertheless, this estimate has not been applied to date although it seems to be a straightforward application of linear estimation theory.

There are several ways of proceeding using the mode coefficients. Following Shang's initial formulation Yang first used the closure properties of the normal modes and the sifting property of Fourier representations to obtain range and depth estimates [131]. This leads to the following matched mode processing (MMP), algorithm

$$B_{MMP}(\hat{r}, \hat{z}) = \frac{1}{L}\sum_{l=1} L\mathbf{s}^H(\hat{r}, \hat{z})\mathbf{r}_M^l, \tag{57}$$

where s is a column vector of modal amplitudes with

$$s_i(\hat{r}, \hat{z}) = e^{-ik_i\hat{r}}u_i(\hat{z}), \tag{58}$$

and $\mathbf{r}_M^l$ are modal "snapshots." This can also be expressed in terms of the sample covariance of the modes, $\hat{\mathbf{K}}_M$, or

$$B_{MMP}(\hat{r}, \hat{z}) = \mathbf{s}^H(\hat{r}, \hat{z})\hat{\mathbf{K}}_M\mathbf{s}(\hat{r}, \hat{z}). \tag{59}$$

Much of Yang's algorithm concerned efficient methods of obtaining an approximate solution to (59) for the source location parameters $\hat{r}$, $\hat{z}$.

Hinich and Sullivan proposed an alternative approach to MFP using modal decomposition based upon Hinich's earlier maximum likelihood formulation [59], [57]. Narrowband data are represented by a sum of modes and then the minimum mean square error is found over the complex modal amplitudes and the source parameters. This leads to much the same formulation of Yang's described above for the modal amplitudes; however, they then continue to find explicit necessary conditions for the maximum likelihood solution and the Fisher information matrix of the Cramer–Rao bounds. Several successful experiments were done on simulated data, but application to field data led to two peaks of approximately the same magnitude with the ambiguity attributed to array tilt mismatch.

The most important advantage of MMP is that it allows one to control the "matching environment." One of the larger uncertainties in acoustic modeling and as a result replica generation is boundary interaction. The higher order modes generally couple to the boundary and are less predictable. In a direct implementation of MFP this leads to replica mismatch whereas in MMP the replica can be confined to a subspace where one has greater confidence about modeling the signal propagation accurately. MMP does have, however, the limitation that the modes must be estimated from the data (e.g. the sound speed profile must be monitored and the bottom properties measured); the SNR's and array geometries to do this are often formidable. Recent progress employing MMP methods are in [24], [8], [19], [135].

I. Subarray Methods

Most of the array processing algorithms in the literature are framed in terms of ray theory and plane wave beamforming. They make the implicit assumptions that: i) the signals are a superposition of plane waves propagating along raypaths and ii) the rays are uncorrelated. Both of these assumptions fail when waveguide phenomena become important. Matched field algorithms succeed when plane wave beamformers fail because the arrays become too large for plane wave representations. There is too much "wavefront curvature" expressed in terms of ray theory. Rays at low frequencies are correlated. Nevertheless, plane wave beamformers have several attractions. They often can be implemented very efficiently and they are robust. Often sets of "preformed beams" are generated using special purpose, high speed hardware. These span the field and reduce the dimensionality of the array processing by transforming the data from element, or sensor, space to beam space. This is also important when one needs to characterize the ambient environment statistically and thus wants to keep the dimensionality low.

The subarray approach to MFP was introduced by Cox and attempts to exploit the advantages of plane wave beamforming while addressing the issues of MFP [32]. The concept is simple and elegant. The arrays are divided into subarrays whose length is established by a criterion on the allowable deviation of the actual phase from a plane wave approximation. The allowable length is

$$L_{\max} \leq c_o\left[2f^2\left|\frac{dc}{dz}\right|\right]^{-1/3} \tag{60}$$

so maximum lengths decrease with frequency and the sound speed gradient. The array length determines the resolution of each subaperture and the angular extent required to cover the propagating waves can be found using ray-angle diagrams based upon sound speed profile; consequently, the number of preformed beams required can be easily determined. Since a beamformer is a linear transformation on the data, the output of the preformed beam network can generally be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{r} \tag{61}$$

where $\mathbf{H}$ is a block diagonal matrix of submatrices representing each subaperture beamforming network. One simply operates on each of the subapertures with the set preformed beams steered in the same way to generate the components of the transformed vector $\mathbf{y}$. After this almost all the various processing algorithms can be reformulated using input/output second moment properties for linear transformations; consequently, there is a large set of algorithms which can be specified with this subarray formulation. Note that not only are the individual beam outputs within each subaperture correlated because of the coherency of the field, but the beams are also correlated across subapertures.

The subarray approach has several advantages. It reduces the dimensionality significantly which aides in adaptation algorithms where the number of snapshots is an issue. It breaks the processing flow up into sectors where decisions about coupling can be addressed directly. Finally, it has an intuitive physical coupling to rays and arrival angles which also has hardware implications.

J. Likelihood Ratio Formulation of MFP

The source/environment parameter estimation for $\mathbf{A}$ can be formulated from first principles using the likelihood ratio. In most cases a Gaussian model leads to many of the very same beamformers as derived using linear estimation methods. The role of uncertain parameters leading to mismatch was addressed in this framework by Richardson and Nolte [102]. The source localization parameters are denoted by $\mathbf{A}_s$ and th environmental ones by $\mathbf{A}_e$. The environmental parameters, $\mathbf{A}_e$, are random variables with a probability density $p_{\mathbf{A}_e}(\mathbf{A}_e)$

and the object is to estimate the source parameters, $\mathbf{A_s}$. The signal model is

$$\mathbf{r} = \tilde{b}\mathbf{s}(\mathbf{A_s}, \mathbf{A_e}) + \mathbf{n} \tag{62}$$

where $\tilde{b}$ is a complex Gaussian noise vector for the noise. The likelihood formulation maximizes the *a posteriori* probability density $p_{\mathbf{A_s}|\mathbf{r}}(\mathbf{A_s} \mid \mathbf{r})$, i.e., the probability density of the source location given the observed data $\mathbf{r}$ [125]. This can be written in terms of the *a priori* densities using Bayes' Law. The uncertainty in environment parameters is included by conditioning the observation on them and then integrating with respect to their probability density, or

$$p_{\mathbf{A_s}|\mathbf{r}}(\mathbf{A_s} \mid \mathbf{R}) = \frac{p_{\mathbf{A_s}}(\mathbf{A_s})}{\mathbf{p_r}(\mathbf{R})} \cdot \int p_{r|\mathbf{A_s},\mathbf{A_e}}(\mathbf{R} \mid \mathbf{A_s}, \mathbf{A_e}) \mathbf{p_{A_e}}(\mathbf{A_e})\, \mathbf{dA_e}. \tag{63}$$

When the random phase and amplitude of $\tilde{b}$ are integrated out, the posterior density which is plotted on an ambiguity surface as a function of the source parameters, $\mathbf{A_s}$, is given by

$$p_{\mathbf{A_s}|\mathbf{r}}(\mathbf{A_s} \mid \mathbf{R}) = C p_{\mathbf{A_s}}(\mathbf{A_s}) \cdot \int (\mathbf{E}+1)^{-1} \mathbf{e}^{\frac{\sum_{\mathbf{l}=1}^{\mathbf{L}} |\mathbf{R}(\mathbf{l})|^2}{\mathbf{E}+1}} \mathbf{p_{A_e}}(\mathbf{A_e})\, \mathbf{dA_e} \tag{64}$$

where C is scaling constant and

$$E = \sigma_b^2 \mathbf{w_c^H}(\hat{\mathbf{A}})\mathbf{K}^{-1}\mathbf{w_c}(\hat{\mathbf{A}})$$

and

$$\mathbf{R}(l) = \sigma_b^2 \mathbf{w}_c^{\mathbf{H}}(\hat{\mathbf{A}})\mathbf{K}^{-1}\mathbf{R}^{\mathbf{l}}$$

$\mathbf{R}(l)$ is a scaled version of the projection of the data sample $\mathbf{R}^l$ upon the weight vector, so the term in the sum is essentially the conventional MFP. The noise is assumed to be uncorrelated. One can interpret (64) as an exponential average of the beamformer output with respect to the environmental parameters, $\mathbf{A_e}$.

Richardson and Nolte [102] used a perturbation of a Munk profile [91] for their environmental model. They used simulation studies and concluded that their algorithm based upon maximizing the above *a posteriori* density was less sensitive to these environmental uncertainties. These techniques have also been applied to MFP of array tilt [103].

K. Other MFP Algorithms

Most of the algorithms in the plane wave beamforming literature have been applied to MFP; however, those algorithms which are based on data from equally spaced arrays operating in homogeneous media are not applicable and cannot be used. This includes all the autoregressive and maximum entropy formulation. Klemm did use the prediction error form in his formulation [68]. If one has an equally spaced array at constant depth, then autoregressive methods can be used. Several have used this to identify modes; in particular, Shang used Prony's method in the first step of a range localization algorithm [110].

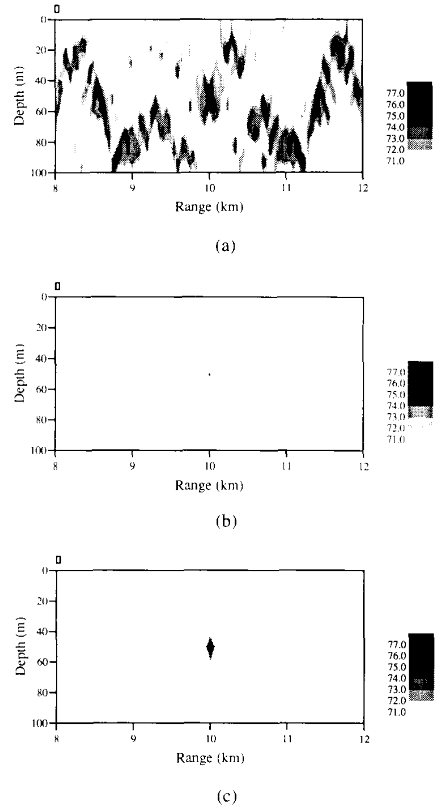


Fig. 4. Ambiguity surface of array output levels for a 130 dB source. (a) Conventional Beamformer; (b) MVDF Beamformer; (c) MCM Beamformer.

The MUSIC algorithm is based upon an SVD of the sampled covariance matrix and this can be used if one substitutes the conventional weight vector for the steering vector. Ozard used this both theoretically and experimentally with a small array [94].

Minmax approaches in plane wave beamforming lead to Tchebycheff array design which control maximum sidelobe levels. These approaches are restricted to equally spaced line arrays and emphasize constraining the sidelobes. Preisig has used a generalized minmax approach to formulate a very elegant theory for minmax MFP [101]. He minimizes the maximum value of an estimate of the field over a range of environmental uncertainties in the environment. In one sense his approach can be interpreted as a "joint replica estimate for MFP with MVDF" which attempts to couple MCM methods and focalization. He has studied it in a number of mismatch context and it appears to be very robust. Note, however, that these techniques do not lead to a minmax ambguity function such as Tchebycheff beam pattern. Gingras and Geer [52] has also applied minmax procedures to develop robust, broadband MFP techniques.

Recently efforts to apply neural network methods to MFP have commenced. Very rudimentary algorithms have been discussed for simple point source, free field geometries in terms of training networks [116]. The issue of extracting weak signals from noise by training a network using high SNR data has also been approached [95].

It has long been recognized that the acoustic propagation in the ocean is a stochastic process; nevertheless, MFP algorithms formulated to data have not incorporated this directly into their design. One can do this using either normal modes or rays to formulate a "stochastic matched field processor" [6]. Recently the theory of using a reference to overcome ocean uncertainty has been explored [90].

V. Examples of MFP Simulations

In this section we give some examples for the purpose of displaying the essentials of MFP ambiguity surfaces. Fig. 4 indicates three ambiguity surfaces for different MFP algorithms: conventional; MVDF (33); and MCM (53). The axes of each plot are the search parameters: source range and depth for a two dimensional model. This is essentially the shallow water environment of Fig. 1 with a 15 element vertical array spanning the 100 m water column and the source frequency is 100 Hz. The most prominent feature of Fig. 4(a) is that though the source is correctly localized at 10 km, the ambiguities are significant and they coincide with the cyclic nature of the propagation. The MVDF example in Fig. 4(b) suppresses these ambiguities as discussed in Section IV.D. This perfect result requires perfect knowledge of the environment and subsequently, perfect propagation modeling, both of which are highly unlikely. The MCM processor also correctly locates the source but the larger spot size, and hence lower position resolution results in a processor which is lower in resolution(and hence more tolerant) of the environmental parameters such as sound speed. The plot is for perfect knowledge of the environment, but it has been shown [106], within certain limits, that the MCM maintains correct localization while the MVDF performance degrades with uncertainty in the environment. An important goal of research in MFP techniques is the development of robust algorithms.

VI. Matched Field Processing with Towed Arrays

Almost all matched field studies and experiments have used some type of moored vertical array. Towed horizontal arrays have many advantages in terms of mobility, ease of deployment and number of sensors, so there is significant motivation to be able to apply MFP to towed arrays. The 1987 Single Vertical Line Array (SVLA) experiment (see Section XI) successfully demonstrated that 3-D MFP worked with a relatively small effective horizontal aperture and this led to considering the possibility of introducing both a vertical "tilt" and a horizontal "bow" in a towed array to provide a vertical aperture and to break the right/left ambiguity of a towed array. This concept is termed array symmetry breaking (ASB) and was introduced by Mikhalevsky *et al.* (1988), [88]. They demonstrated that range and depth as well as bearing parameters could be estimated with towed horizontal arrays if they were tilted and kinked. Figs. 5(a)–(d) shows the results of simulations with a tilted ($\approx 20°$) 1.8 km "horizontal" array. These simulation results include a uniformly distributed phase error of $\pm\lambda/10$. The target is at 500 km range, 200 m depth and a 85° bearing. Note the much improved sidelobe rejection of the adaptive MFP (referred to as MVDR on the plots and the same as MVDF in the text.

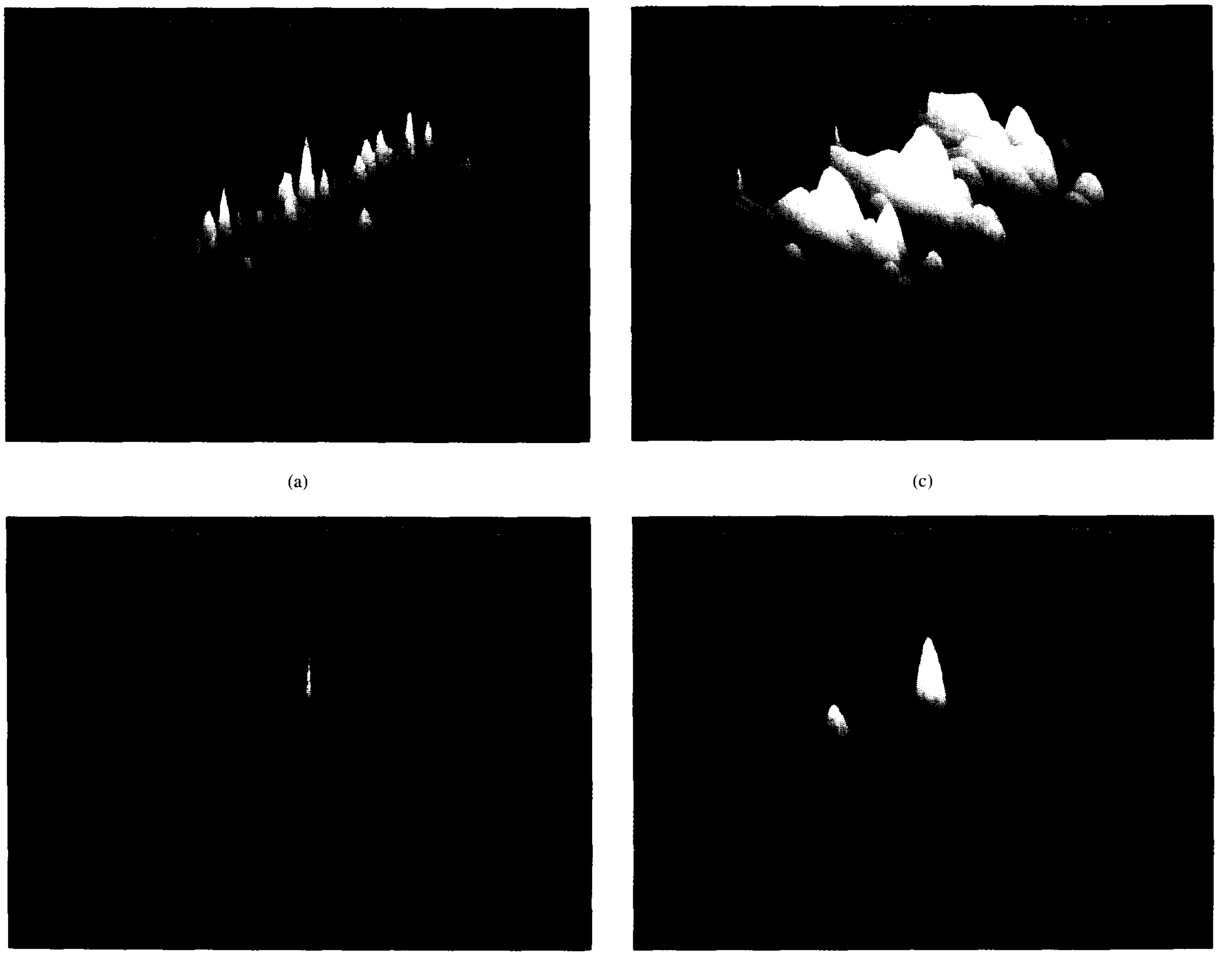
In the canonical deep sound channel environment (one without in-range and azimuthal sound speed changes that are comparable to the sound speed changes in the vertical) it was shown that the bearing resolution of a tilted array is very close to a horizontal array with a length equal to the projection of the tilted array on the horizontal. Likewise the depth and range resolution of a tilted array is closely approximated by that of a vertical array with length equal to the projection of the tilted array on the vertical. Note in Fig. 5(b) that the range resolution produced by the vertical projection of the array with MFP is a few km's while the Fresnel range resolution provided by the horizontal projection of the array is essentially infinite.

Breaking the symmetry of the horizontally stratified ocean acoustic waveguide allows three dimensional localization with a linear (but tilted!) array. At the same time and independently Perkins and Kuperman, [98], showed via simulation that in a highly range dependent environment near the Gulf Stream even tilting the array was not required (see Section VIII). By exploiting the asymmetry of the three dimensional sound speed structure it was shown shown that a linear vertical array could localize in three dimensions which was dubbed environmental symmetry breaking (ESB). Essentially, ASB and ESB (different expressions for the same phenomena) show the power of matched field processing which makes the propagation medium via the signal processing part of the array itself.

VII. Mismatch Studies

As mentioned above, there are essentially three sources of mismatch in MFP: environmental, statistical, and system. Environmental mismatch has been studied early in great detail, [110], [118], [34], [51], [101], [18], [113]. Environmental mismatch refers to uncertainty in the propagation model, e.g., sound speed profile errors, bottom composition uncertainty. There have been numerous perturbation studies some satisfying realistic oceanographic constraints, others quite arbitrary.

Statistical mismatch refers to the need for covariance matrices in the design of adaptive filters. The fundamental problem is that the number of degrees of freedom in an arbitrary covariance matrix can grow as N^2 if *a priori* constraints are not employed; as a result, the number of degrees of freedom often exceeds the number of available snapshots from the data. For adaptive arrays one is left with a paradox that larger arrays may be more difficult to use than smaller ones because one does not have the data to constrain the sample covariance matrix, $\hat{\mathbf{K}}$. This matrix is often modeled probabilistically using the Wishart density which is essentially a matrix generalization of the χ^2 density and analytic results can be derived using this density [1], [9]. Certainly, one needs a number of snapshots which exceed the number of sensors to a have full rank in the sample covariance matrices. Even when the rank is full, Capon and Goodman demonstrated that there is a bias for a finite number of snapshots. [21] Brennan *et al.* suggest that



(a) (c)

(b) (d)

Fig. 5. (a) Conventional MFP ambiguity surface of bearing versus range for a tilted "horizontal" array. This is a slice of the 3-D ambiguity volume at 200 m depth. (b) Adaptive MFP (MVDF algorithm) ambiguity surface of bearing versus range for the tilted array. Note the much improved sidelobe rejection evident with the adaption. (c) Conventional MFP ambiguity surface of depth versus range for the tilted array. This is a slice of the 3-D ambiguity volume on a bearing of 85°. (d) Adaptive MFP (MVDF algorithm) ambiguity surface of depth versus range for the tilted array. Note the much improved sidelobe rejection with the adaption.

at least $3N$ snapshots are required for MVDF beamformers [13]. It is clear that for large arrays some form of *a priori* knowledge which exploits both the statistical structure and the physical constraints must be applied [14], [85], [5]. This is an important issue in beamforming in general and is not confined to MFP.

System mismatch refers to errors in the receiving system, e.g., array tilt, hydrophone sensitivities, phase shifts. Adaptive processing can be quite sensitive to these since cancellation is often part of the adaption process There has been a fair amount of work done on this since it is a common problem in plane wave beamformers. Many times the sensitivity is characterized in terms of an effective sensor noise level and artificially controlling this is one way to reduce the mismatch [31]. MFP introduces its own issues because of the complexities of the field and array distortions. All successful experiments with MFP have had some form of array shape calibration since MFP appears to be quite sensitive to tilt [122]. While there have been several simulations, there has not been a good analysis to date on the effects of tilt. Doppler is another form of system mismatch. In principle it can be modeled and included; however, when it is not known, it expands the parameter search space significantly. Most experiments have used fixed sources and receivers and there has not been much work on the doppler compensation.

To summarize, the results of these studies have had a twofold impact. First, they led to a deeper understanding of the matched field processing technique, in general. Second, the emerging realization of the sensitivity of MFP to the environment directly led to the possibility of matched field tomography. The statistical issues for MFP are similar to those for plane wave beamforming. System issues, such as array tilt, had been addressed somewhat. On the other hand, most system/engineering issues, particularly for larger arrays, still remain to be explored.

VIII. Three-Dimensional MFP (or Environmental Signal Processing)

The complexity of the real ocean environment can, in principle, be used to enhance MFP [98], [136]. The stringent and unrealistic requirement, though, is that the environment must be known to great accuracy over a wide area. Assuming that to be the case, the azimuthal asymmetry of the three-dimensional environment adds additional spatial structure to multipath complexity of the acoustic field.

As an example, consider a 21 element vertical array in the center Gulf Stream environment [98] of Fig. 6 with 75 m spacing whose top hydrophone is at a depth of 100 m. For such an environment, the propagation is different along each radial outward from the array. Therefore, the acoustic field as expressed by (12) should be considered a function of all three spatial coordinates,

$$\mathbf{G}(x, y, z) = \frac{i\rho(z_s)}{(8\pi)^{1/2}} \exp(-i\pi/4) \cdot \sum_{n=1}^{m} u_n(x_s, y_s, z_s) v_n(x, y, z) \frac{e^{i\overline{k_n}(x, y)r}}{\sqrt{\overline{k_n}(x, y)r}} \quad (65)$$

where $\overline{k_n}(x, y)$ is the horizontal wavenumber (eigenvalue) of the nth normal mode averaged over the radial direction from the source position (x_s, y_s) to the receiver position (x, y). This expression, which is difficult enough to calculate over large regions [74] does not include full horizontal refraction; however, it is sufficient to illustrate the concept. We use this expression as the replica generator for the MVDF ambiguity surface displayed in Fig. 6. The environment is such that the region north of the Gulf Stream is shallower with colder ("slope") water while the region south of the Gulf Stream consists of warmer, Sargasso Sea, deep water. The source, in the eastern kink of Gulf Stream is localized, not only in range, but in azimuth with a vertical array. The excellent localization arises from the fact that propagation between the source and array is a unique function of position and that perfect knowledge of the environment combined with the MVDF processor suppresses the ambiguities. The only contributor to horizontal aperture in the problem is the environment. Hence, we use the terminology Environmental Signal Processing.

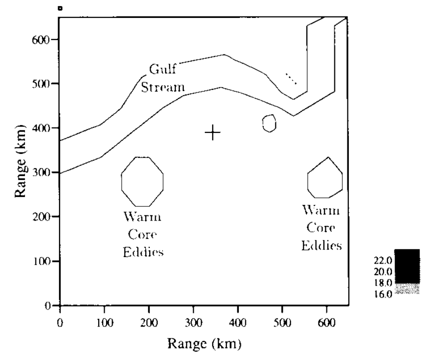


Fig. 6. MVDF ambiguity surface at the known source depth with range and cross range as the search parameters. Gulf Stream and eddies are superimposed. The array is at the + and the top 6 dB of the ambiguity are the several dots in the upper right corner just north of the Gulf Stream [98].

IX. Focalization

Recently, there have been attempts to go beyond tolerance of environmental uncertainty beamforming to alleviate the effect of uncertainty in knowledge of the environment (or the mismatch problem). Focalization [28] is a generalization of MFP in which both the source parameters and the environmental parameters are unknown or partially unknown. The environment is treated as a complicated acoustic lens and is focused (i.e., the ocean-acoustic parameters are adjusted) in an attempt to localize the acoustic sources with a high-resolution cost function. Focalization has the primary goal of determining source location and perhaps the secondary goal of determining effective ocean-acoustic parameters. The ocean-acoustic parameters are determined at most along the propagation paths between the sources and the array (a two-dimensional equivalent of the three-dimensional environment) to the appropriate resolution for the frequency, source distribution, and array configuration involved. In principle, focalization can perform high-resolution tomography if a large number of configurations of the source and array are used.

Focalization requires a parameter optimization method for focusing the acoustic lens (i.e., searching for the minimum of the cost function). Simulated annealing, recently applied to time-domain beamforming problems [73], is efficient for finding the global minimum of a cost function that depends on many parameters. This Monte Carlo method is analogous to slowly cooling a pure liquid substance to form a perfect crystal (the lowest energy state of the system). A random perturbation is chosen (using a random number generator) at each iteration of the simulated annealing Markov process. The perturbation is accepted automatically as the new parameter state if the cost, or energy, is reduced. To allow escape from local minima, the perturbation is accepted according to a Boltzmann probability distribution if the energy is increased. The annealing process is controlled by the distribution of the perturbations and by an artificial parameter, the temperature, which is decreased slightly after each iteration. Eventually, the temperature becomes so low that the probability of accepting an energy increase is low. In practice, the temperature annealing schedule is a trial and error procedure to obtain the best results. For many optimization problems, the sensitivity of the cost to parameter perturbations is not the same for all of the parameters. In focalization, for example, the ocean-acoustic parameters tend to be more important near the source and the array than in the region between the source and the array (this fact can be seen explicitly in the adiabatic normal mode solution). For these types of problems, methods of improving efficiency can be found by using the analogy of a mixture of liquid substances with different freezing temperatures.

First simulation attempts at focalization were made using ray and mode based algorithms. For exmaple, Collins [28] used a cost function based on modal phases; from (11) define replica adiabatic modal phases $\psi_n(r) = r\overline{k_n}$ which are actually parameterized using empirical orthogonal functions (EOF's) to characterize the ocean environment. The search parameters, which are the coefficients of the EOF expansion were adjusted to match the measured modal phases ξ_n. The high-resolution cost function used in the simulated annealing search/optimization procedure was

$$E = \min_r [F(r)], \tag{66}$$

$$F(r) = \left\{ \frac{1}{N-1} \sum_{n=2}^{N} \sin^2 [\xi_n - \xi_{n-1} - \psi_n(r) + \psi_{n-1}(r)] \right\}^{1/2} \tag{67}$$

where N is the number of modes used for focalization. Since the difference between eigenvalues is relatively small, the differential modal phases appearing in (67) vary much slower than the modal phases. Thus, F has a relatively wide peak at the source range. This is an attractive property because it allows a sparse sampling of F in range. Differential modal phases also have the advantage of being independent of source phase. Simulated annealing was used to search for the environmental parameters that minimize E; the estimate at each iteration for the source range is the range at which the minimum of F occurs.

The results (given in detail in [28]) indicate that for a "simple" mismatch or uncertainty in the environment, both the source location and an effective environment can be found. However, since acoustic fields tend to be more sensitive to variations in source location than to variations in the ocean-acoustic parameters, the focalization search process usually locks in to the source location before the ocean-acoustic parameters. In some cases, this parameter hierarchy is so extreme that the algorithm locks in to the correct source location but the wrong ocean-acoustic parameters. This ambiguity is acceptable if the primary goal is to determine source location. The ambiguity is most likely to occur in gradually range-dependent environments in which there is little mode coupling.

X. Experimental Demonstrations of MFP

Over the last few years, experiments have been reported in the literature which demonstrated the feasibility of MFP with an assortment of the above mentioned processors. Experiments were performed in deep water, shallow water, and Arctic environments. Here we summarize some of these results.

MFP using mode matching was first successfully tested by Yang [131] in a 4000 m deep water Arctic environment; the data was from the 1982 FRAM IV experiment. A 27-element array approximately spanned the upper quarter of the water column and a source was located at 90 m depth and a range of 260 km. The frequency was 23.5 Hz and the average signal-to-noise ratio was greater than 9 dB. The mode matching method resulted in lower sidelobes than the conventional MFP processor.

Experimental results with a 120-element vertical array for MFP in deep water at a frequency of 200 Hz were recently reported on by Tran and Hodgkiss [122], [123]. Their 900 m vertical array had sensors spaced at 7.5 m deployed from a depth of 400 m in the North-East Pacific in a water depth of about 5000 m. The 200 Hz source, at a depth and range of 400 m and 165 km, respectively, was correctly localized (range more successfully than depth) for high (10 dB) and low (−10 dB) SNR's. An attempt to localize the source at a range of 300 km resulted in a maximum peak one convergence off the true position. Array position measurements suggest that tilt can be the cause of significant mismatch leading to depth localization error and to a lesser extent range error. Including array tilt in replicas improves the accuracy of the MVDF processor. Fluctuations with a period of the order of 8 minutes correlated with predicted internal wave periods associated with a Brunt Vaisala frequency derived from CTD measurements at the array. Furthermore, propagation modeling through a modeled monochromatic internal wave field resulted in amplitude fluctuations consistent with the MFP measurements.

Pacific Echo was a recent experiment in deep water [23]. Its objectives were to determine the effects upon MFP of the geoacoustic properties of the seafloor. Vertical and horizontal arrays acquired data from which the effects of both compressional and shear wave propagation can be determined.

The work of Ferris [39] and Ingenito [61], though not aimed at MFP, experimentally demonstrated mode separation in a shallow water environment with a vertical array. Years later, Feuillade *et al.* [41] reported the results of an MFP experiment with full field replicas in the same Gulf of Mexico region where they localized a source at a range of 2.2 km from a 16-element vertical array in 33 m of water. Both conventional and MVDF processors were used, the latter displaying the great ambiguity of a repetitive sidelobe structure. Similar, but longer range results were obtained by Hamson and Heitmeyer [55] in shallow water in the Mediterranean. Localization out to a range of 19 km was obtained. In both of the above studies, it was determined that water depth and sound speed were the critical environmental parameters. Precise knowledge of the density and attenuation of the bottom sediment was less important.

Other MFP techniques were also demonstrated in shallow water. Ozard [94] used an eigenvector method with a sparse horizontal array of four sensors on the bottom. Reasonable bearing estimation was obtained through range localization was ambiguous; however, further analysis indicated that a 15-element array should unambiguously localize in range, depth and azimuth, assuming a 10 dB SNR. Jesus [64] used a mode-matching method based on Yang's work [131] to analyze a Mediterranean shallow water MFP experiment. The results of his analysis indicated that mode-matching was superior to conventional MFP and performed extremely well, even with apertures as small as half the water column. For smaller aperture, the sensitivity of the processor to the environmental parameters increases and this sensitivity is a function of aperture size. This work was extended to broadband processing [65].

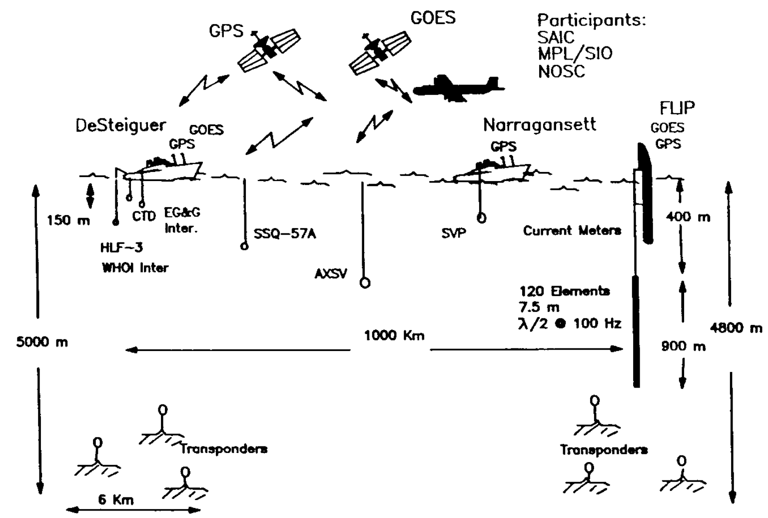


Fig. 7. SVLA experimental geometry (SVLA Experiment, Fall 1987).

Broadband information enhances MFP performance as was demonstrated in [4]. One of the early experiments by Parvulescu and Clay and took this to the limit using a single sensor where they reversed the response of the acoustic channel and played it back so it would in effect match filter itself [97]. Clay as well as several others [42] have demonstrated that the broadband impulse response of the channel is sufficient for localization in range; however, success in at sea experiments has been limited by the stochastic components of the propagation [26], [78], [27]. This has recently been pursued in a MFP context by Hodgkiss and Brienzo [60] and Frazier and Pecholcs [47] where they both demonstrated that both range and depth could be estimated simultaneously under not too restrictive conditions. Experimentally, Westwood developed a ray based broadband MFP algorithm and applied it to data in the Gulf of Mexico. He used 55–95 Hz pseudorandom signals from a moving 5 m/s moving source at a depth of 100 m in water 4500 m deep. A bottom mounted six-element vertical array localized the source at ranges 42 km.

Except for the 120-element vertical array experiment the generally successful MFP results were for large SNR's. Furthermore, a conventional $10 \log n$ argument for the 120 element array gives a 20 dB array gain so that one should not be too impressed with the MFP results for a signal-to-noise of -10 dB as far as array gain is concerned. However, conventional plane wave beamformers do not locate sources in space. Finally, the MFP procedure suggests additional noise gain possibilities vis-à-vis the spatial compartmentalization of correlated noise Such gains have yet to be conclusively demonstrated. With respect to the coherence of signal and noise, the optimal sampling procedure for estimating the cross-spectral density matrix with regard to MFP has at best been briefly addressed in the experimental literature. Finally, the design of the array geometry and sensor placement for MFP in a 3-D environment, which is essentially a spatial sampling strategy, is poorly understood.

XI. MFP at Megameter Ranges

Prior to 1987 the application of matched field processing at very long ranges (1000 km's) in the temperature oceans was speculative at best. The MFP requirement for predictable phase coherent propagation to ranges well beyond the 260 km reported by Fizell was debated [43]. The 1985 matched field processing of the FRAM IV data was the first application of MFP to field data. The Arctic sound channel, however is somewhat special *vis-à-vis* MFP. First, the Arctic Ocean is well known for its extreme stability in phase [86], which was not generally accepted for the temperate oceans. Second, the ocean dynamic activity, particularly internal waves and mesoscale phenomena, are much less energetic in the Arctic [77]. Thus the favorable Arctic results would not necessarily translate to the temperate seas especially at 1000 km's. The theory by Flatte *et al.* [45], and work by Dozier and Tappert [35], [36] had led many to believe that internal waves would destroy the coherence of the acoustic field even at low frequencies (where the theory had not been advertised to be necessarily correct [46]) especially when pushed to ranges of 1000 km's or more. There was further debate whether PE, adiabatic modes or other "approximate" solutions to the wave equation would be of sufficient accuracy to generate accurate full field replicas. The amount of sound speed information that was required, including the in-range spacing of samples and measurement accuracy (direct SVP, CTD, or derived from XBT) was an issue. If fully coupled normal modes or finite difference methods were required then it was argued that a prohibitive amount of computer time would be needed for any useful applications.

In 1987 the Single Vertical Line Array (SVLA) experiment was conducted in the Pacific Ocean [87]. This was the first

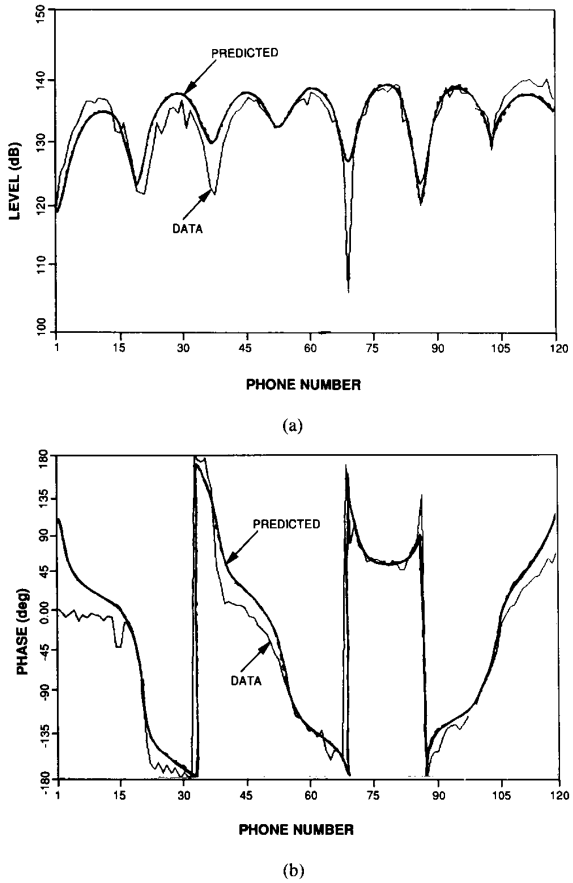


Fig. 8(a) A comparison of the predicted and measured amplitude of the acoustic pressure field on the SVLA for a 1000 km, 27 Hz CW transmission. (b) A comparison of the predicted and measured phase of the acoustic pressure field on the SVLA for a 1000 km, 27 Hz CW transmission.

in a series of three experiments that were conducted over the next five years that began to explore the above issues. Fig. 7 shows the experimental geometry. The SVLA experiment successfully demonstrated that MFP would be applied to localize in depth and range at very long ranges in the ocean. Fig. 8 shows the amplitude and phase of a 27 Hz CW signal measured on the VLA and the computed amplitude and phase with a normal mode code using only one measured CTD based SVP. The close agreement, especially the phase permitted successful MFP. Adaptive matched field (MVDF) and the weight norm constrained version were applied consistently and successfully, significantly improving the sidelobe rejection performance over conventional MFP.

The use of adaptive array processing in sonar never received the same attention or as wide an application as it did in radar. This was due to both the more complicated propagation media, making modeling errors the limiting factor for adaptive processing in sonar, and the more diffuse ambient noise field. The SVLA experiment showed that modeling errors could be reduced to the point that well known signal suppression of adaptive processing would be less than the additional gain against noise providing a net SNR improvement over conventional processing. The use of MFP in lieu of standard plane-wave modeling, and spherical wavefront curvature modeling (and of course for any array with any vertical aperture) permits the exploitation of the well-known sidelobe and noise rejection properties of adaptive processing in the ocean. For any practical applications of adaptive MFP, errors in the estimation of the covariance matrix also degrade performance. In estimating the sample covariance the loss due to finite degrees of freedom that were predicted theoretically by Brennan, Mallet, and Reed [13] lead to the use of a factor of three in the ratio of the number of independent snapshots of the sample covariance to the number of array elements in the data processing. This result was borne out in the SVLA data processing and in simulations by Mikhalevsky and Baggeroer [89].

As indicated above the issue of coherence in long range acoustic propagation is central to the successful application of MFP in the ocean. Without predictable phase coherence MFP and especially adaptive versions and would not be possible. Most certainly the fundamental frequency and range limits will be dictated by internal waves that have time and space scales that are far too small for any realistic ocean measurement program that might be used to accommodate these fluctuations in the replica formation. While the SVLA experiment pushed the envelope to 1000 km at frequencies less than 100 Hz, contrary to theoretical calculations, it is not at all certain that this represents the limit. In fact the Heard Island Feasibility Test [6] provides strong evidence that at frequencies less than 100 Hz the range limits may be very much greater than 5000 or even 10 000 kms. In this experiment acoustic CW and phase encoded M (Maximum length shift register) sequences centered at 57 Hz were transmitted from Heard Island in the southern Indian Ocean (54 S, 74 E) to receiving sites around the world to distances as far as 18 000 km. One of the more remarkable demonstrations of phase stability from this experiment is shown in Fig. 9. This figure illustrates the detected phase of the CW signal at Ascension Island, 9000 km away, after removal of a linear phase ramp for the mean Doppler shift. Also illustrated is the phase predicted by projecting the GPS navigation along the geodesic to Ascension. The agreement is to within 10 m. The implications for successful MFP and matched field tomography (MFT) at long ranges in the ocean is positive.

XII. Matched Field Tomography

Tomography generally refers to applying some form of inverse theory to observations in order to infer properties of the propagation medium. Matched field tomography (MFT) is a field of emerging interest. MFT is a natural extension of MFP since the latter is simply a method to perform inversion by solving the forward problem a very large number of times in order to match the data. Ocean acoustic tomography largely relies upon travel time measurements of the identifiable ray paths whereas MFT exploits the full field across an array with spatially phase coherent signal processing.

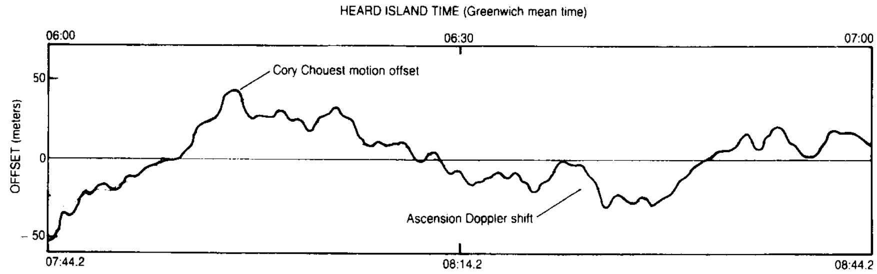


Fig. 9. Comparison of the phase of a signal after 9000 km and the Doppler induced phase by ship translation during the Heard Island Feasibility Test (by Dzieciuch, Munk and Forbes [37], reprinted from [7]).

Acoustics has long been used to infer ocean environmental parameters so it is difficult to separate cleanly what is and what is not tomography. We consider some recent experiment where the MFP concept of matching the spatial structure of observations directly, instead of some derived parameter, as the distinguishing feature of MFT. Some of the concepts of MFT have appeared in the matched field literature. For example, both the application of the Prony method to MFP by Shang *et al.* [110] and the focalization technique [28] reduce to MFT if the source positions are known. For the purpose of tomography part of the motivation is to apply wave theoretic frequency domain solutions to the inverse problem with the intent of utilizing both phase and amplitude. Efforts along these lines were stimulated by Shang [111], [112] who suggested a method of applying adiabatic mode theory to perform tomographic inversion. Tran and Hodgkiss [124] used MFT in a simulation to obtain range averaged sound speed profile with a vertical array and adiabatic mode replicas. Tolstoy *et al.* [119], [120] have proposed a specific MFT scheme of ocean volume tomography as an alternative to the more "classical tomographic methods" introduced by Munk and Wunsch [92], [93]. In Tolstoy's approach explosives (or an equivalent broadband source) would be dropped from an aircraft along the perimeter of the volume of interest and inversion would be performed with the Fourier components being matched against solutions of the wave equation. The equivalent to the averaging procedure used in MFP for estimating the cross-spectral density of the measured field is hypothesized to be accomplished by the multiplicity of paths through each cell.

While several MFT algorithms have been proposed, experiments to test them have just begun. A forerunner of this method was the mode separation experiment reported on by Ingenito [61] in which a vertical array was used to separate and identify normal modes in shallow water. Together with a theory which related modal attenuation to the attenuation properties of the bottom sediment, he was able through simple fitting of the data to determine these absorption coefficient properties. One of the first experiments for MFT has been performed by Goncharov *et al.* [53]. He recently reported their results of a tomography experiment in the Norwegian Sea. The element locations of a 560 m array were determined using 14 kHz acoustic pulses. Empirical orthogonal functions were used to describe the ocean volume and up to a 6 parameter fit was made using MFP, including a matched mode version. Path lengths of the order of 100 km were studied. Five profiles along the path were estimated and agreements with measured profiles were accurate to 1 m/s with the least accuracy near the surface. Karangelen and Diachok [67] used explosive sources to invert for a range dependent profile in the Pacific.

Geo-acoustic inversion using matched field tomography was also recently reported by Collins *et al.* [29] using data taken in the Gulf of Mexico by Lynch *et al.* [81]. Full field MFP was employed using a combination of parabolic equation (PE) modeling and simulated annealing as a nonlinear search algorithm. The range dependence of the bottom properties incorporated in the PE method proved the key to matching the data. MFP using wavenumber was also investigated indicating that, for range independent environments, such an approach may be the most efficient for determining the geo-acoustic properties of stratified ocean bottoms.

The properties of the ice in Arctic waters have been studied with MFT. Livingston and Diachok [79] applied MFT for the purpose of investigating under-ice reflectivity to the same data set from FRAM IV that Yang [131] used for the early MFP demonstrations. Recently, Miller and Schmidt [84] applied full wave solutions to the elastic wave equation [105] as replicas to perform MFT to determine the bulk properties of Arctic ice. Seismo-acoustic data from a 1987 experient (PRUDEX) was inverted to obtain estimates of the compressional and shear wave bulk velocities and the attenuation of the ice, the latter tending to be less reliable. Furthermore, this analysis identified the presence of unexpected horizontally polarized transverse (SH) waves, thereby providing impetus for further research into the complex field of coupling elastic waves in plates and, in particular, ice sheets.

XIII. Conclusions

Matched field methods are a generalized form of beamforming which incorporates the propagation physics directly into the array processing. As such it involves solving a forward problem a very large number of times to match the source and/or environmental parameters. Source localization or determination of medium properties requires a careful consideration of the close coupling between the propagation physics and the estimation algorithms. One must consider both aspects especially in experiments where various forms of mismatch are inevitably encountered. An extensive literature has evolved since 1985 when the first experiments demonstrated that MFP could work. Both theory and experiments have advanced rapidly. Clearly, this new field presents great challenges and will be an important part of research in underwater acoustics for ocean exploration.

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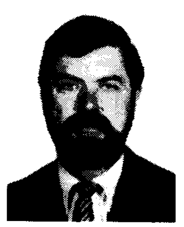
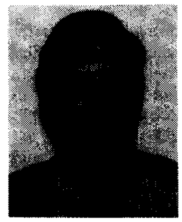


Arthur B. Baggeroer received the B.S.E.E. degree from Purdue University in 1963 and the Sc.D. degree from MIT in 1968. He has been on the faculty at MIT since 1968. He is the Ford Professor of Engineering in the Departments of Ocean and Electrical & Computer Science at the Massachusetts Institute of Technology. He has had a long involvement with the Woods Hole oceanographic Institution through the MIT-Woods Hole Joint Program in Oceanography and Oceanographic Engineering, of which he was the MIT Director from 1983 to 1988. He was consultant to the Director of the NATO SACLANT Ctr. during a sabbatical leave in 1977 and a Green Scholar at the Scripps Institution of Oceanography during a sabbatical leave in 1990. His research has focused upon the application of advanced signal processing methods to underwater acoustics and geophysics. He has been the chief scientist at six ice stations in the high Arctic, where large aperture arrays for acquiring acoustical and geophysical data have been deployed. These data have been used for long range propagation, reverberation, ambient noise and tectonic measurements in the Arctic. He has developed several algorithms for matched field processing. He led the first efforts for applying advanced digital communication methods for high data rate acoustic telemetry systems in reverberant ocean environments. He is now involved in the ATOC (Acoustics Thermometry of Ocean Climate) and studies on low frequency reverberation.

Dr. Baggeroer has authored many articles concerning signal processing and underwater acoustics. He is now writing a text on array processing. He is a Fellow of the Acoustical Society of America and was Chairman of the Technical Committee on Underwater Acoustics from 1985–1988. He is a Fellow of the Institute of Electrical and Electronic Engineers. He was an elected member of the School Committee for the Town of Westwood, MA from 1977–1989.

William A. Kuperman received the B.S., M.S., and Ph.D. degrees in physics from the Polytechnic Institute of Brooklyn in 1965, the University of Chicago in 1966 and the University of Maryland in 1972, respectively.

In 1967, he joined the Acoustics Division of the Naval Research Laboratory (NRL) and then in 1976 he went to the SACLANT Undersea Research Centre in La Spezia, Italy for 5 years where he founded the Environmental Modeling Group. Following his Italian sojourn, he became the head of the Numerical Modeling Division at what is now the Stennis Space Center Detachment of NRL. In 1985 he returned to Washington, DC and until late 1993, was the Senior Scientist of NRL's Acoustics Division. He was elected a Fellow of the Acoustical Society of America in 1980 and was chairman of its Technical Committee on Underwater Acoustics from 1982–1985. From 1986–1993 he was an Associate Editor of the Journal of the Acoustical Society of America. He has just co-authored a book entitled *Computational Ocean Acoustics* (F. Jensen, W. A. Kuperman, M. Porter and H. Schmidt, AIP Press, 1994). Presently, he is a Professor and Director of the Marine Physical Laboratory of the Scripps Institution of Oceanography, University of California San Diego.



Peter N. Mikhalevsky received the B.A. and M.S. degrees from Harvard University in 1972 and the Ph.D. degree from M.I.T. in 1979.

Upon graduation from college Dr. Mikhalevsky was commissioned an officer in the U.S. Navy and served on active duty for 12 years. His specialty was in Anti-Submarine Warfare and he was instrumental in the introduction of towed arrays in the Navy in the mid 1970's. Upon leaving the Navy in late 1983 he spent 2 years on the faculty of M.I.T. where he was an Associate Professor in the Department of Ocean Engineering teaching acoustics and array processing. He joined SAIC in 1985 and is currently a Vice President and Operations Manager of the Ocean Sciences and Technology Operation in McLean, VA.

For his research in underwater acoustics Dr. Mikhalevsky is the recipient of the Biennial Award of the Acoustical Society of America, the A.B. Wood Medal and Prize from the Institute of Acoustics U.K., and the Decibel Award from the Naval Undersea Warfare Center, New London, CT.