Research on Improved TDOA Algorithm for Shallow Water Underwater Positioning

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ABSTRACT

Accurate positioning in shallow water regions is crucial for various underwater applications. TDOA algorithms estimate the signal source location by measuring the time differences of signal arrival at multiple receivers. However, these methods often suffer from Non-Line-of-Sight (NLOS) errors and noise, leading to reduced positioning accuracy in shallow water environments. This research proposes an improved TDOA estimation algorithm that utilizes cross-correlation techniques to mitigate NLOS errors and combines the strengths of the Chan and Taylor algorithms to enhance positioning precision. The proposed algorithm addresses challenges such as time ambiguity and alpha-stable noise, offering advantages in TDOA modeling for underwater positioning. Implementation details are provided, including the design of matching basis functions, time delay matching procedures, algorithm flow, and computational steps. Simulation tests validate the effectiveness of the algorithm, demonstrating significant improvements in positioning accuracy. The computational complexity is also analyzed to assess the algorithm's practicality. Finally, the research findings are summarized, and directions for future work are discussed.

Keywords: TDOA algorithm; underwater positioning; shallow water positioning

1. INTRODUCTION

Time Difference of Arrival (TDOA) is an important wireless positioning technology that determines the position of a signal source by measuring the time difference of signal arrival at different receivers [1]. The Federal Communications Commission (FCC) in the United States has stipulated that location services will be a basic function of future wireless cellular networks [1]. Therefore, improving the accuracy of TDOA estimation is crucial to meet these needs. However, multi-path propagation leads to significant Non-Line-of-Sight (NLOS) errors, which seriously affect the accuracy of TDOA estimation [2]. Similar to multi-path phenomena in underwater positioning, in the field of underwater acoustic communication and positioning, sound waves are easily affected by factors such as seabed terrain, currents, and temperature changes during propagation, resulting in reflection, refraction, and scattering phenomena. This causes signals to arrive at the receiver along multiple paths, forming underwater multi-path effects. These multi-path effects not only cause attenuation of signal strength but also lead to delays and phase changes in signal arrival time, greatly increasing the difficulty and error of positioning. Compared with NLOS errors in wireless cellular networks, underwater multi-path can also be regarded as a "non-direct path" error[3]. Existing TDOA estimation methods mainly include methods based on timing synchronization and methods based on cross-correlation [4]. For example, the traditional TDOA estimation method based on TOA subtraction can reduce NLOS errors but still cannot completely eliminate them [5][6].Due to model assumptions and computational complexity, existing methods also have insufficient noise resistance and estimation accuracy when facing high or low signal-to-noise ratio conditions [7]. For example, the time delay estimation algorithm based on the generalized quadratic cross-correlation sparse Fourier transform performs poorly under low signal-to-noise ratio conditions [8][9].

To overcome the limitations of existing Time Difference of Arrival (TDOA) estimation methods in shallow water environments—where underwater positioning is often hindered by multi-path effects and noise leading to signal distortion—we propose an improved cross-correlation-based TDOA algorithm[10]. By modeling impulsive noise using the heavy-tailed characteristics of alpha-stable distributions and applying cross-correlation functions for signal matching, we enhance positioning accuracy. Additionally, interference from alpha-stable noise is mitigated using the cross-power spectrum function. This improved algorithm significantly advances positioning accuracy in shallow waters, addressing key challenges in underwater positioning technology[10].

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2. PRELIMINARIES OF TDOA

TDOA is a technique that determines the position of a target by utilizing the difference in signal arrival times at different receivers. This technique does not require knowledge of the specific time the signal was sent but calculates the position of the target by measuring the time difference of signal arrival at two or more receivers [10]. The basic principle of the TDOA method is to locate based on signal propagation speed and arrival time difference. When a signal propagates from a source point to multiple receiving points, the different distances from each receiving point to the source point result in different signal reception times. By measuring these time differences, the position of the target can be deduced. For example, in three-dimensional space, if three receivers are used, a hyperboloid equation system can be formed to solve the position of the target [4][6]. (See Figure 1).

The advantage of the TDOA method is that it requires less stringent synchronization between receivers. Compared to the Time of Arrival (TOA) method, which requires strict synchronization, TDOA only requires synchronization between receivers, making it more flexible and convenient in practical applications [11]. In addition, TDOA can also reduce the amount of data transmission because only the time difference data needs to be exchanged [9][11][12].

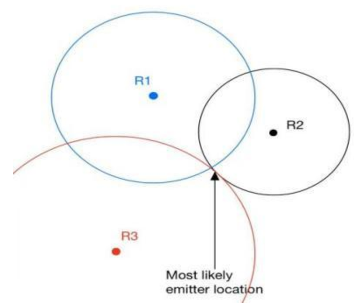

Figure 1. TDOA positioning diagram

3. PROPOSED TDOA ESTIMATION METHOD

3.1 Improved TDOA Modeling

TDOA is a positioning method that determines the location of a signal source by measuring the differences in signal arrival times at multiple monitoring stations. By analyzing these time differences, the relative distances from the signal source to pairs of hydrophones can be calculated. Each pair of monitoring stations defines a hyperbola on which the signal source must lie—the hyperbola is the set of points with a constant difference in distances to the two stations (the foci). By constructing hyperbolas for multiple pairs of monitoring stations, the intersection point of these hyperbolas pinpoints the position of the signal source (see Figure 2).

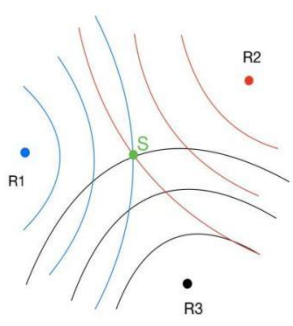

Figure 2. Hyperbolic intersection positioning

TDOA is a multi-station positioning system, so at least 3 or more monitoring stations are required to locate the signal simultaneously. Each monitoring station mainly includes a receiver, antenna, and timing synchronization module. In theory, existing monitoring stations can be upgraded to TDOA monitoring stations as long as they have timing synchronization modules. In actual underwater positioning, there are many interference, such as multi-path effects and various noises. There may be non-thermal noise sources such as ships and marine life. The noise generated by these noise sources often does not conform to the Gaussian distribution. Due to the complexity of the underwater environment, the received signals may be affected by non-Gaussian noise, causing signal distortion. Alpha-stable noise (as shown in Figure 3) is a type of non-Gaussian noise whose probability density function has heavy tails and is suitable for describing environments with pulse noise or impact noise. The multi-path problem is caused by the sound wave propagating from the transmitter to the receiver through multiple paths, including the direct path and the path reflected and refracted by underwater objects. The difference in the length of different paths leads to different arrival times at the receiver. This change in time delay directly affects the positioning algorithm based on time difference, such as the Cramér-Rao lower bound and the maximum likelihood estimate. By combining the multi-path effect and alpha-stable noise, a positioning model that is more in line with the actual underwater acoustic environment can be constructed:

Signal construction:

$$X = s + n + s' \tag{1}$$

Where s is the original signal, n is the non-Gaussian noise, and s' is the noise caused by multi-path effects. Alpha-stable noise is introduced to simulate non-Gaussian noise sources and signal distortion.

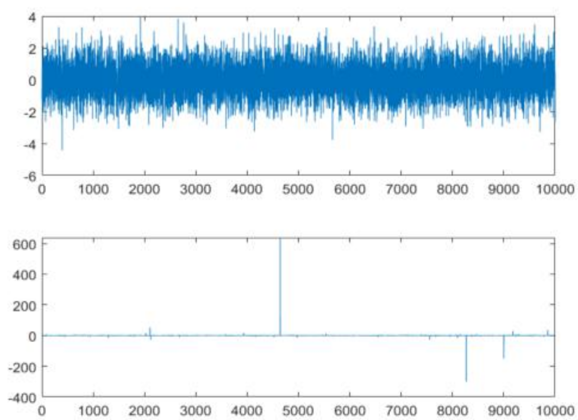

Figure 3. Alpha-stable noise

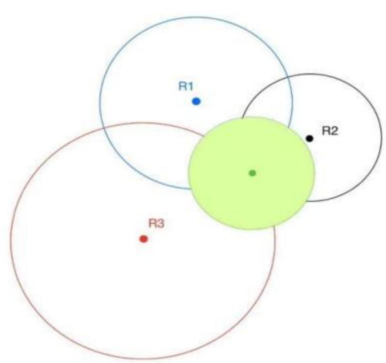

Figure 4. Time ambiguity diagram

3.2 Solving Time Ambiguity and Alpha-Stable Noise Problems

When each hydrophone emits a signal, due to the influence of multi-path effects, time delays will occur, and these changes in time delay will directly affect the positioning algorithm based on time difference, causing different sensors to receive signals at different times, resulting in time ambiguity (as shown in Figure 4 dashed circle area), leading to inaccurate positioning. At this time, cross-correlation functions can be used to match signals pairwise to improve the accuracy of positioning. By convoluting two signals to produce cross-correlation functions, the similarity between the two signals is measured. Analyzing the cross-correlation functions and finding their peaks, and processing the peaks through convex optimization and least squares methods makes the time t more accurate. The use of cross-power spectrum functions can eliminate the interference of alpha-stable noise, making the alpha-stable noise approximately zero in the overall range received.

3.3 Advantages of TDOA Modeling

TDOA does not have the problem of phase ambiguity, so the direction-finding baseline can be unrestricted. Traditional direction-finding methods need to calculate the azimuth angle through phase, and phase measurement has a 2π period of uncertainty. Therefore, the method of using an antenna baseline shorter than the signal wavelength is often used to avoid the 2π period of reverberation. However, the wavelength of high-frequency signals is short, which makes the distance of the test antenna close, easy to produce signal coupling, and cause measurement errors. Each TDOA monitoring station only needs one antenna, which fundamentally solves the problem of signal coupling.

The complexity of the TDOA system is low. For TDOA monitoring stations, only monitoring antennas and receivers need to be configured, and the requirements for antennas are not high. Even different monitoring points can use different antennas.

The positioning accuracy of the TDOA system is high. For TDOA detection stations, its positioning accuracy depends on the accuracy of time measurement. With the optimized algorithm, the calculation error of time difference is at the 100ns level, and the positioning accuracy is about 30m. The error of Class A direction-finding stations is generally 1 degree, and the signal error for 5km away is 87m, and the signal error for 10km away is 174m.

4. ALGORITHM IMPLEMENTATION DETAILS

4.1 Design of Matching Basis Functions

Eliminating multi-path signals in underwater positioning is a complex and important issue because underwater signals have time-varying characteristics, severe multi-path effects, Doppler effects, and complex noise, making underwater positioning technology more complex and difficult than land positioning. In order to effectively eliminate multi-path signals, based on the improved TDOA algorithm, we have combined the Chan algorithm with the Taylor algorithm. The Chan algorithm is suitable for preliminary positioning due to its high efficiency, and the Taylor algorithm provides an effective means to optimize positioning results in complex environments. The combination of these two algorithms provides strong technical support for improving underwater positioning accuracy.

The Chan algorithm is a non-recursive hyperbolic equation group solution method with an analytical expression solution. The idea of the Chan algorithm is: ① Transform the nonlinear TDOA measurement into a linear equation group; ② Then use WLS (weighted least squares method) to get an initial solution;③ Then use the estimated position coordinates and additional variables obtained for the first time and other known constraints to perform a second WLS estimation to obtain an improved estimated position. Define the variable za=[x,y,R]: that is, the position and distance vector to be estimated. And the TDOA noise model is: $\varphi = h - Gz^{\circ}$.

$$h = \frac{1}{2}\begin{bmatrix} {R_{21}}^2 - {x_2}^2 - {y_2}^2 + {x_1}^2 + {y_1}^2 \\ {R_{31}}^2 - {x_3}^2 - {y_3}^2 + {x_1}^2 + {y_1}^2 \\ \vdots \\ {R_{M1}}^2 - {x_M}^2 - {y_M}^2 + {x_1}^2 + {y_1}^2 \end{bmatrix} \quad (2)$$

$$Gz = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & R_{21} \\ x_3 - x_1 & y_3 - y_1 & R_{31} \\ & \vdots & \\ x_M - x_1 & y_M - y_1 & R_{M1} \end{bmatrix} \quad (3)$$

Due to the influence of measurement noise, then

$$\mathrm{R_{i1}} = \mathrm{R_{i1}}^0 + \mathrm{n_{i1}}; \mathrm{R_i}^0 = \mathrm{R_{i1}}^0 + \mathrm{R_1}^0 \quad (4)$$

Therefore, the noise vector error can be expressed as:

$$\varphi = \begin{bmatrix} \mathrm{R_2}^0\mathrm{n_{21}} + \frac{1}{2}\mathrm{n_{21}}^2 \\ \mathrm{R_3}^0\mathrm{n_{31}} + \frac{1}{2}\mathrm{n_{31}}^2 \\ \vdots \\ \mathrm{R_M}^0\mathrm{n_{M1}} + \frac{1}{2}\mathrm{n_{M1}}^2 \end{bmatrix} \quad (5)$$

In fact, the square term of the noise in the above formula can be ignored, and the noise error is a Gaussian random vector with a covariance matrix.

$$\begin{cases} \varphi = \mathrm{E}(\varphi\varphi^{\mathrm{T}}) = \mathrm{BQ} \\ \mathrm{B} = \mathrm{diag}\{\mathrm{R_2}^0, \mathrm{R_3}^0, \cdots \mathrm{R_M}^0\} \\ \mathrm{Q} = \mathrm{diag}\{\sigma_{21}^2, \sigma_{31}^2, \cdots \sigma_{M1}^2\} \end{cases} \quad (6)$$

Because B contains an unknown distance, further similarity is needed. Assuming B is a unit matrix, the WLS is obtained as follows: $Z_a = ({G_a}^T \varphi^T G_a)^{-1} {G_a}^T \varphi^{-1} h$(obtained from Q, obtained from R, obtained from S, obtained from B)When an initial solution for calculating the B matrix is obtained, the first WLS calculation result is as follows:

$$\mathrm{Z_a} = (\mathrm{G_a}^{\mathrm{T}}\varphi^{\mathrm{T}}\mathrm{G_a})^{-1}\mathrm{G_a}^{\mathrm{T}}\varphi^{-1}\mathrm{h} \quad (7)$$

The estimated covariance is equal to$cov(Z_a)$=$({G_a}^T\varphi^{-1}G_a)^{-1}$, and the corresponding estimated errors aree_1,e_2, e_3, then，$Z_{a1} = x^0 + e_1$,$Z_{a2} = y^0 + e_2$,$Z_{a3} = {R_1}^0 + e_3$; construct another equation set as follows:$\varphi = h' - G_a'Z_a'$.

$$\mathrm{h'} = \begin{bmatrix} (\mathrm{Z_{a1}} - \mathrm{x_1})^2 \\ (\mathrm{Z_{a2}} - \mathrm{y_1})^2 \\ \mathrm{Z_{a3}}^2 \end{bmatrix} \quad (8)$$

$$\mathrm{G_a}' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \quad (9)$$

$$\mathrm{Z_a} = \begin{bmatrix} (\mathrm{x}^0 - \mathrm{x_1})^2 \\ (\mathrm{y}^0 - \mathrm{y_1})^2 \end{bmatrix} \quad (10)$$

When an initial solution for calculating the B matrix is obtained, the first WLS calculation result is as follows:

$$\varphi_1' = 2(\mathrm{x}^0 - \mathrm{x_1})\mathrm{e_1} + \mathrm{e_1}^2 \approx 2(\mathrm{x}^0 - \mathrm{x_1})\mathrm{e_1} \quad (11)$$

$$\varphi_2' = 2(\mathrm{y}^0 - \mathrm{y_1})\mathrm{e_2} + \mathrm{e_2}^2 \approx 2(\mathrm{y}^0 - \mathrm{y_1})\mathrm{e_2} \quad (12)$$

$$\varphi_3' = 2\mathrm{R_1}^0\mathrm{e_3} + \mathrm{e_3}^2 \approx 2\mathrm{R_1}^0\mathrm{e_3} \quad (13)$$

The covariance of this estimate is

$$\varphi' = \mathrm{E}\lfloor\varphi'\varphi'^{\mathrm{T}}\rfloor = 4\mathrm{B}'\mathrm{cov}(\mathrm{z_a})\mathrm{B}' \quad (14)$$

$$\mathrm{B}' = \mathrm{diag}\{\mathrm{x}^0 - \mathrm{x_1}, \mathrm{y}^0 - \mathrm{y_1}, \mathrm{R_1}^0\} \quad (15)$$

Since the position of the intermediate element is replaced by the estimated value from the first step (x^0 ,y^0 unknown, obtained by replacing x, y, R1 from the first step), an approximate estimate can be obtained$Z_a'({G_a'}^T{\varphi'}^{-1}G_a')^{-1}{G_a'}^T{\varphi'}^{-1}h'$; So the final MS positioning result is$\begin{bmatrix} x \\ y \end{bmatrix} = \sqrt{Z_a'} + \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$either or$\begin{bmatrix} x \\ y \end{bmatrix} = -\sqrt{Z_a'} + \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$, obtaining (x^0,y^0)and selecting the solution located in the positioning area as the final solution. Further utilizing Taylor series expansion to solve the local least squares (LS) of TDOA measurement error to improve the estimation of position. For a set of TDOA values, perform Taylor expansion on the selected (x^0,y^0)(which is obtained by calling the Chan algorithm to obtain an initial estimated position), ignoring components above second order, and convert it into:

$$\varphi = \mathrm{h_t} - \mathrm{G_t}\delta \quad (16)$$

$$\delta = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \quad (17)$$

$$h_t = \begin{bmatrix} R_{21}-(R_2-R_1) \\ R_{31}-(R_3-R_1) \\ \vdots \\ R_{M1}-(R_M-R_1) \end{bmatrix} \quad (18)$$

$$G_t = \begin{bmatrix} \frac{x_1-x_0}{R_1}-\frac{x_2-x_0}{R_2} & \frac{y_1-y_0}{R_1}-\frac{y_2-y_0}{R_2} \\ \frac{x_1-x_0}{R_1}-\frac{x_3-x_0}{R_3} & \frac{y_1-y_0}{R_1}-\frac{y_3-y_0}{R_3} \\ \vdots & \\ \frac{x_1-x_0}{R_1}-\frac{x_M-x_0}{R_M} & \frac{y_1-y_0}{R_1}-\frac{y_M-y_0}{R_M} \end{bmatrix} \quad (19)$$

The weighted least squares solution is$\delta = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$=$(G_t{}^T Q^T G_t)^{-1} G_t{}^T Q^{-1} h_t$, where Q is the covariance matrix of the TDOA measurement values. In the next recursion, ， $x_0 = x_0 + \Delta x, y_0 = y_0 + \Delta y$， repeat the above process until it is small enough to meet a certain threshold $|\Delta x + \Delta y| < \varepsilon$ or $\sqrt{\Delta x^2 + \Delta y^2} < \varepsilon$, and output the corresponding Δx，Δy .In summary, before applying the Chan algorithm and Taylor algorithm, first perform cross-correlation analysis on the signal to extract signal features, such as peak positions, to estimate the initial TDOA. Use the cross-correlation peak to initialize the TDOA estimation, providing a more accurate initial value for the Chan algorithm. In the Taylor algorithm iteration, use the cross-correlation method to refine the TDOA estimation and improve positioning accuracy.As shown in Figure 5, it is the simulation of the Chan and Taylor algorithm based on cross-correlation calculation.

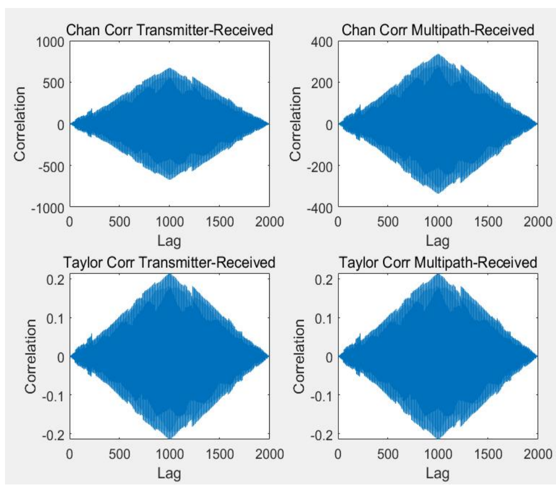

Figure 5. Simulation of Chan and Taylor algorithms based on cross-correlation calculation

4.2 Implementation of Time Delay Matching

The Time Difference of Arrival algorithm is generally divided into two main steps. The first step involves using a hydrophone array to collect sound source signals for time delay estimation. The second step is to determine the position of the sound source based on these time delay values. Figure 6 illustrates the overall flowchart of the algorithm.

Time delay estimation specifically refers to capturing the sound source signals with the hydrophone array and calculating the time delays between the signal arrivals at different microphones. When implemented in the frequency domain, the auto-correlation and cross-correlation functions of the signal are first calculated. These functions are then used to perform cross-correlation operations again, further improving the accuracy of the time delay estimation. The signal X is cross-correlated with$\int_0^{\pi} \mathrm{x}\,\mathrm{x}(\mathrm{t}-\tau)\mathrm{dt}$, then

$\int_0^{\pi}$ （$s + s' + n$）$(s(t-\tau)s'(t-\tau) + n(t-\tau)dt$ （20）

Due to the characteristics of time delay, the signal s itself will superimpose to produce a higher peak value, and the signal produced by multi-path will have inconsistent time and will decline. As shown in Figure 7.

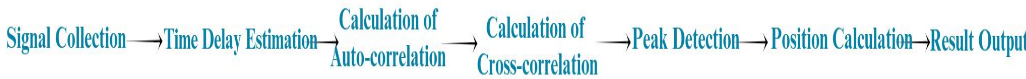

Figure 6. Flowchart

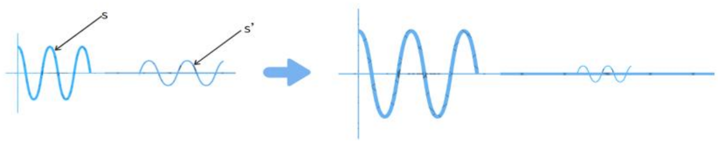

Figure 7. cross-correlation operation result diagram

4.3 Algorithm flow and calculation steps

The TDOA algorithm is generally divided into two steps. The first step is to use the hydrophone array to collect the sound source signal for time delay estimation, and the second step is to determine the position of the sound source based on the time delay value. In the presence of noise, taking the four-element cross hydrophone array as an example, the signal received by the hydrophone i is:

$$x_i(t)=s(t-\tau_i)+n_i(t)+s' \tag{21}$$

Among them, S(t) is the sound source signal received by the hydrophone, $\tau_i$is the time delay difference of the remaining hydrophones relative to the reference signal, and$n_i(t)$ represents the noise signal received by the microphone.Taking two hydrophones M1 and M2 in the four-element cross array as an example, as shown in Figure 1, t1 and t2 represent the time from the sound source to the hydrophone M1 and M2, c represents the speed of sound, generally taken as 340m/s. Then these two receive models are shown in Figure 8:

$$x_1(t)=s(t)+n_1(t) +s' \tag{22}$$

$$x_2(t)=s(t-\tau_{12})+n_2(t) \tag{23}$$

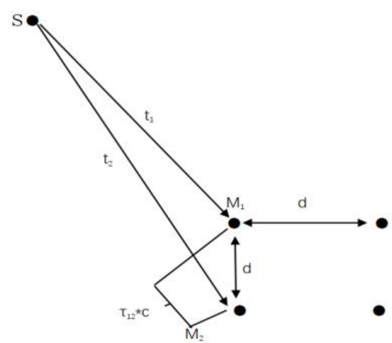

Figure 8. Hydrophone receiving model

4.4 Construction of the optimization function

The traditional cross-correlation algorithm assumes that the signal is stationary. However, in practical applications, many signals are non-stationary and are affected by noise. The generalized cross-correlation algorithm enhances the correlation between signals and suppresses the influence of noise by introducing a weighting function for non-stationary signal processing. The relationship between the cross-correlation function and the cross-power spectrum function can be expressed as:

$$R_{x1x2}(\tau)=\int_0^{\pi} G_{x1x2}(\omega)e^{-j\omega\tau}d\omega \int_0^{\pi} X_1(\omega)X_2^{*}(\omega)\, e^{-j\omega\tau}d\omega \tag{24}$$

In order to avoid the influence of noise on the peak value of the cross-correlation function and to improve the accuracy of time delay estimation, the signal and noise in the frequency domain can be weighted to enhance the peak value of the cross-correlation function:

$$R_{x1x2}(\tau)=\int_0^{\pi}\psi_{12}(\omega)X_1(\omega)X_2^{*}(\omega)\,e^{-j\omega\tau}d\omega \tag{25}$$

Among them,φ12（w） is the weighting function under frequency domain conditions. There are many forms of weighting functions, among which the most commonly used weighting function is the phase transformation (Phase Transformation) weighting function, and its expression is as follows:

$$\psi_{12}(\omega)=\frac{1}{|G_{x1x2}(\omega)|} \tag{26}$$

Under far-field conditions, the inter-element spacing of the hydrophone array is usually much smaller than the distance from the sound source to the hydrophone array, and the TDOA estimation is a very small amount, so a higher sampling frequency is required for processing. Under normal circumstances, directly increasing the sampling frequency will have high requirements for computer hardware, and the operation speed is very slow. Interpolation algorithms can be used to increase the sampling rate in the time domain, which is faster than directly increasing the sampling rate. Under the assumption that signals and noise are not related to each other, the secondary cross-correlation model for the two signals is as follows:

$$R_{x1x2}=E\{x_1(t)x_1（t-\tau）\}=R_{ss}(\tau) \tag{27}$$

$$R_{x1x2}=E\{x_1(t)x_2（t-\tau）\}=R_{ss}(\tau-\tau_{12}) \tag{28}$$

the two formulas are cross-correlated to obtain:

$$R_{R_{x1x2}R_{x1x2}}=\{R_{x1x2}(t)R_{x1x2}(t-\tau)\} \tag{29}$$

Finally, it is derived that:

$$R_{x1x2}=R_{ss}(\tau-\tau_{12}) \tag{30}$$

This study takes cubic spline interpolation as an example for time delay estimation and proposes a secondary cross-correlation algorithm based on cubic spline interpolation . After cross-correlation, the peak value detection can obtain the maximum value of the TDOA estimation value.Figure 9 is Flowchart of the secondary cross-correlation algorithm based on cubic spline interpolation.

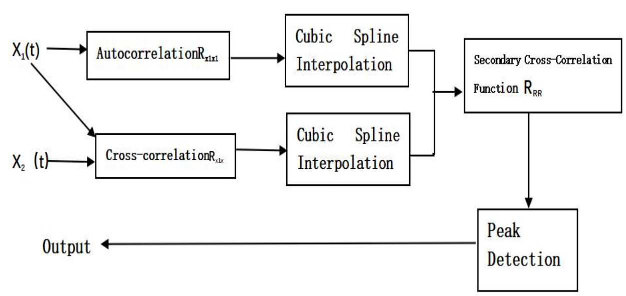

Figure 9. Flowchart of the secondary cross-correlation algorithm based on cubic spline interpolation

However, calculating the cross-correlation of the signal itself and the cross-correlation of two signals cannot eliminate the interference of multi-path signals and alpha-stable noise delays. Therefore, we use the energy accumulation function for convex optimization processing.In the above text, we have defined the target function as: X=s+n+s′,Set S(t) as the sound source signal received by the hydrophone, $\tau_i$as the time delay difference of the remaining hydrophones relative to the reference signal, n_i (t)represents the noise signal received by the microphone. Then the received signal can be expressed as:$x_i(t)=s(t-\tau_i)+n_i(t)+s'$,Define the target function as the energy accumulation function:

$$E=\int|xi(t-\tau)*s(t)|^2dt'' \tag{31}$$

As the time delay τ, * represents the convolution operation. From this, we can find the time delay that maximizes the energy accumulation, thereby estimating the direct signal delay. Set the multi-path signal as m(t) , and add constraints. For example, the energy of the multi-path signal can be limited to be less than a certain threshold M, that is:

$$\int|m(t)|^2\,dt\le M \tag{32}$$

For alpha-stable noise, its statistical characteristics can be used to add constraints. The variance of the noise can be limited to be less than a certain threshold N, that is, the target function and constraints are combined into a convex optimization oblem, and the Lagrange multiplier method or other convex optimization methods can be used to solvepr it. For example, the problem can be formulated as:

$$\mathrm{Min}\tau\text{-}E+\lambda 1\left(\int |m(t)|^2\,dt - M\right)+\lambda 2\left(\mathrm{Var}\big(n(t)\big) - N\right) \qquad (33)$$

Among them, λ1 and λ2 are Lagrange multipliers.

5. SIMULATION TEST

5.1 Signal Simulation

A sine wave is generated as the signal source signal model, with a frequency of 20kHz and a duration of 1 second.

5.2 Signal Reception

Set the positions for four hydrophones and place them in different positions. Each hydrophone will receive the original signal plus multi-path effects and alpha-stable noise.Figures 10 -15 illustrate the results of simulations conducted using Matlab.

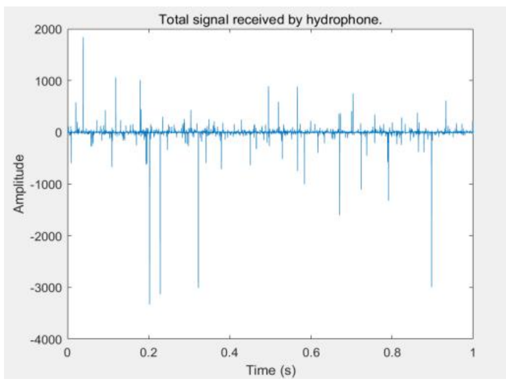

Figure 10. Total received signal of the hydrophone without the algorithm

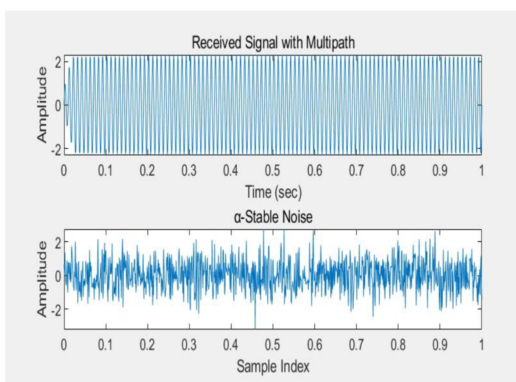

Figure 11. Multi-path signal and alpha-stable noise

5.3 TDOA Estimation

Use the well TDOA cross-correlation technology to estimate the time range of the signal source, reduce the time delay, and eliminate the time ambiguity phenomenon. Calculate the cross-correlation matrix between all hydrophones and use the energy accumulation function to eliminate the influence of time delay.

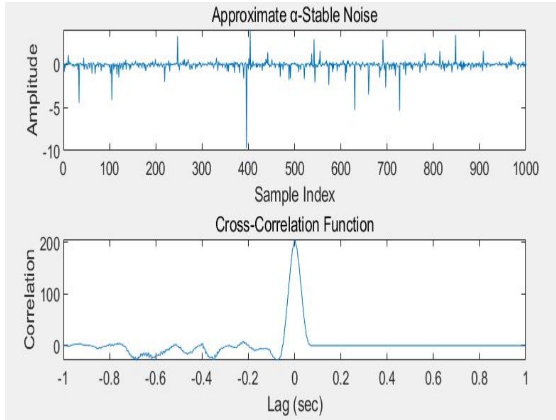

Figure 12. Alpha-stable noise for cross-correlation calculation

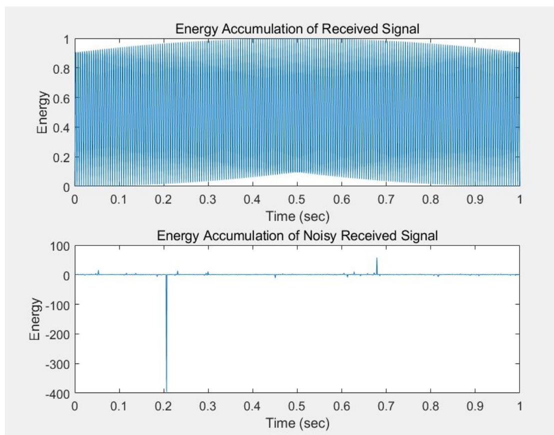

Figure 13. Energy accumulation function calculation

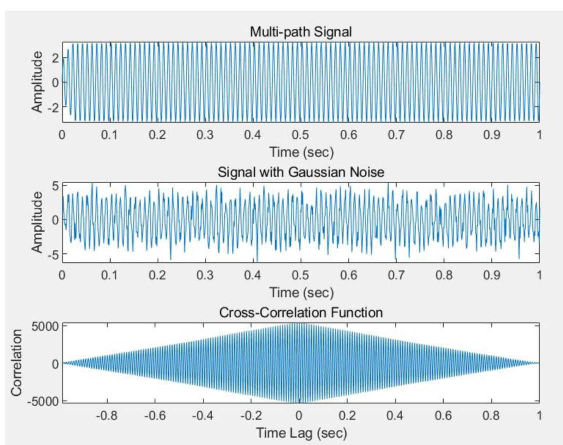

Figure 14. Simple cross-correlation calculation

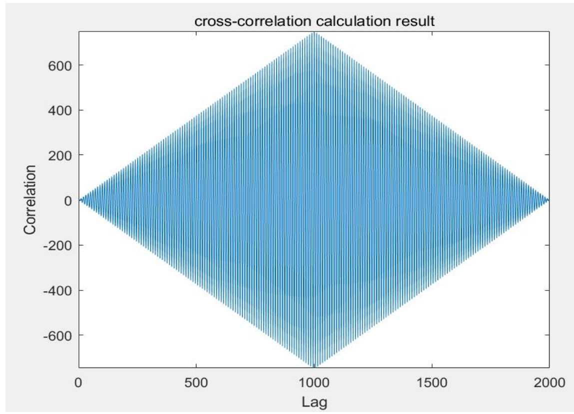

Figure 15. Improved TDOA cross-correlation calculation

In our simulations, we demonstrate that integrating the Chan and Taylor algorithms significantly enhances positioning accuracy in underwater environments. By employing cross-correlation analysis and convex optimization processing, the robustness of the combined algorithm is improved, especially under complex conditions. The high efficiency of the Chan algorithm makes it suitable for real-time systems. While cross-correlation computation, convex optimization, and interpolation algorithms each have advantages and limitations in signal processing and time delay estimation, optimizing the algorithm design and selecting appropriate interpolation methods ensure accuracy while reducing computational complexity. Our simulations show that the cross-correlation method performs well in time delay estimation under low signal-to-noise ratio conditions.In studies of sound source localization and tracking, the generalized cross-correlation weighting function—specifically, the cross-power spectrum phase (CSP) weighting function—is utilized. Results indicate that this method correctly estimates time delays even under low signal-to-noise ratios. During dynamic tracking, the maximum positional deviation does not exceed a few centimeters, and the maximum response time is within a few seconds, indicating rapid response and effective tracking performance. However, errors increase when the sound source is farther from the hydrophone array, primarily due to environmental noise interference, variations in sound propagation speeds across different media, and limitations imposed by experimental conditions.

6. CONCLUSION

TDOA positioning technology is fundamental for precise underwater target localization, yet traditional methods suffer from decreased accuracy in high-noise and NLOS environments common in shallow waters. In this paper, we conducted an in-depth study of TDOA and its derivative algorithms within underwater sensor networks to address these challenges. We proposed an improved TDOA estimation algorithm based on the auto-correlation function , transforming the estimation problem into frequency domain analysis to effectively reduce NLOS errors. Numerical simulations verified the algorithm's stability and superiority under varying signal-to-noise ratios, significantly reducing computational cost while ensuring accurate time delay estimation. While the improved algorithm and positioning system were implemented and their core functions verified through simulations and actual measurements, limitations remain due to mathematical modeling and hardware constraints. These findings highlight areas for future enhancement, including integrating theoretical improvements into practical systems and optimizing hardware performance, to develop a more robust and accurate underwater positioning system capable of effective operation in complex, high-noise environments.

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