Bayesian Neural Networks

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Deep Learning course

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Project goal

Goal

Estimate posterior distributions of the model parameters from data

Probabilistic Programming:

- Uncertainty in predictions;
- Uncertainty in representations;
- Regularizations with priors;
- Transfer learning;
- Hierarchical Neural Networks.

Problem

Monte Carlo sampling is very slow for high-dimensional data

Related work

1. Salvatier J, Wiecki T. V., Fonnesbeck C. Probabilistic programming in Python using PyMC3. // *PeerJ Computer Science*. 2016.

2. Blundell C. et al. Weight Uncertainty in Neural Network // *Proceedings of The 32nd International Conference on Machine Learning*. 2015.

3. Kucukelbir A. et al. Automatic Differentiation Variational Inference // *arXiv preprint arXiv:1603.00788*. – 2017.

Problem Statement

Inference problem

Bayes' theorem states: $\mathbb{P}(\boldsymbol{\theta} \mid \mathbf{X}) = \frac{\mathbb{P}(\mathbf{X} \mid \boldsymbol{\theta})\mathbb{P}(\boldsymbol{\theta})}{\mathbb{P}(\mathbf{X})}$

Maximum A Posteriori (MAP) estimation

$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta}} \left[\ln \mathbb{P}(\boldsymbol{\theta} \mid \mathbf{X})\right] = \arg\max_{\boldsymbol{\theta}} \left[\ln \mathbb{P}(\mathbf{X} \mid \boldsymbol{\theta}) + \ln \mathbb{P}(\boldsymbol{\theta})\right]$$

Monte Carlo approach:

- Metropolis-Hastings sampling;
- Gibbs sampling;
- No-U-Turn Sampling (NUTS).

Variational Inference

Goal

Approximate posterior distribution $p(\boldsymbol{\theta}|\boldsymbol{X})$ by function $q(\boldsymbol{\theta})$ from parametric family.

$$\begin{array}{ccc}
\ln p(\mathbf{X}) = & \mathrm{KL}(q||p) & + \ \mathrm{ELBO}(q) \\
 & \Updownarrow & \Updownarrow \\
\int q(\boldsymbol{\theta}) \ln \frac{q(\boldsymbol{\theta})}{p(\boldsymbol{\theta}|\mathbf{X})} \mathrm{d}\boldsymbol{\theta} & & \int q(\boldsymbol{\theta}) \ln \frac{p(\mathbf{X}, \boldsymbol{\theta})}{q(\boldsymbol{\theta})} \mathrm{d}\boldsymbol{\theta}
\end{array}$$

Minimization of $\mathbf{KL(q||p)}$ $\Leftrightarrow$ Maximization of $\mathbf{ELBO(q)}$

Automatic Differentiation Variational Inference (ADVI)

- Automatic transformation of constrained variables $\boldsymbol{\zeta} = T(\boldsymbol{\theta})$;
  Example: $\theta \in \mathbb{R}_+ \Rightarrow \zeta = T(\theta) = \log\theta$, then $\zeta \in \mathbb{R}$.
- $q(\boldsymbol{\zeta}) = \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\Sigma}$ is diagonal;

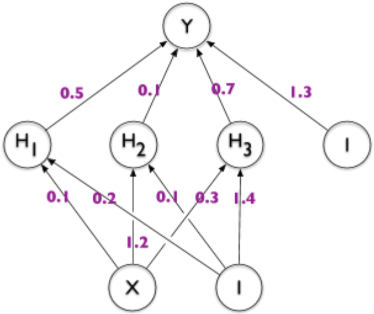
$$\boldsymbol{\mu}^*, \boldsymbol{\Sigma}^* = \arg\max_{\boldsymbol{\mu},\boldsymbol{\Sigma}} \mathrm{ELBO}(q)$$

- Stochastic optimization;
- Reparametrization trick to apply automatic differentiation;
- Adaptive step-size.

Deep Learning

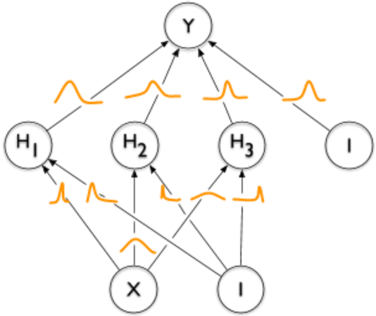
Neural Networks

Predict values of parameters by fitting complex model on the huge dataset



Bayesian Neural Networks

Predict the parameters of the weights distributions from the dataset



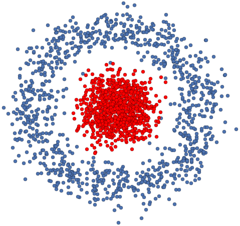
http://bit.ly/2rMQuDq

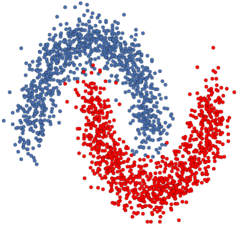
Experiments

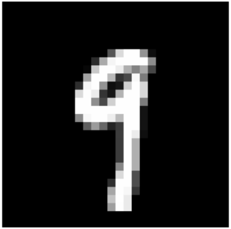
Goals:

- investigate influence of different priors on the predictions
- visualize uncertainties in predictions
- analyze the model behaviour

Datasets:







Course of work

Normal **Laplace** **Cauchy** **Flat**

Fix

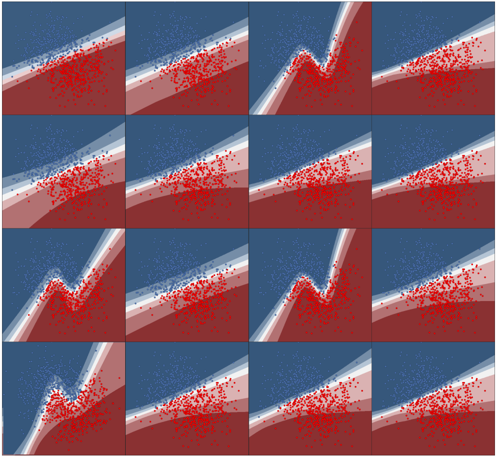
Fixed values

Hierarchical modeling

Half-Normal

Half-Cauchy

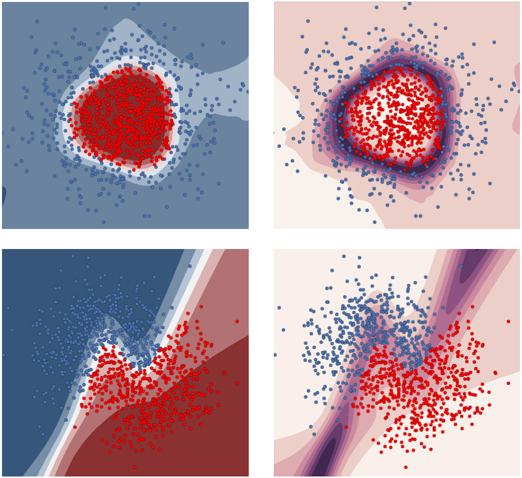
Inverse-Gamma



Synthetic data

Posterior Probability

Uncertainty



Prior: Cauchy
Hyperprior:
Inverse-Gamma
Accuracy: 0.735

Prior: Normal
Hyperprior:
Inverse-Gamma
Accuracy: 0.851

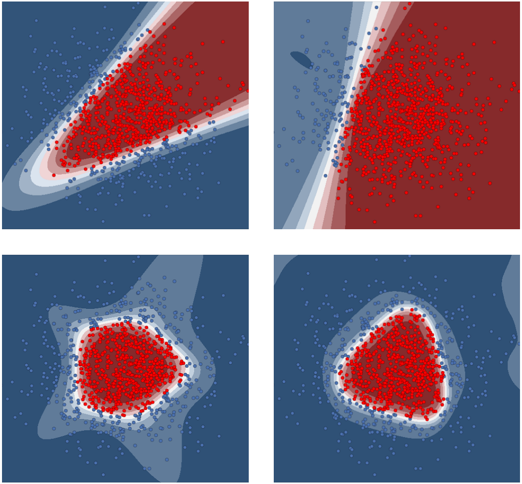
Hierarchical modelling

Prior: Normal

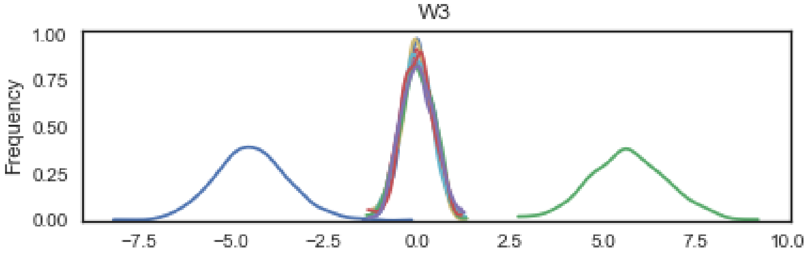
Prior: Laplace

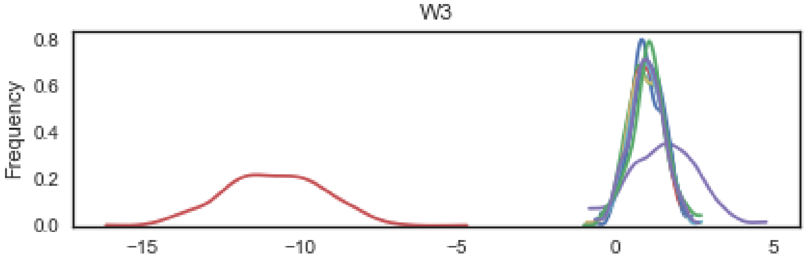
Hyperprior:
Fixed values

Hyperprior:
Inverse-Gamma

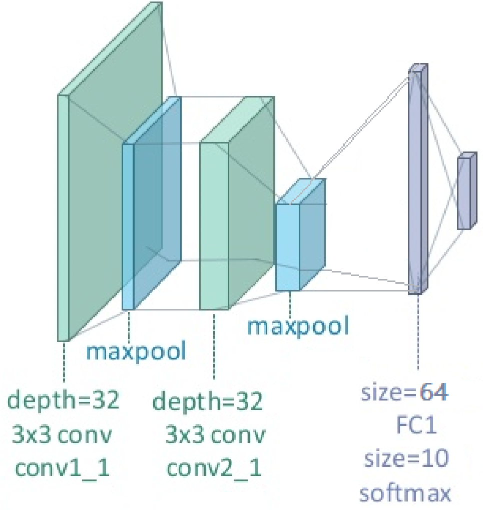


Laplace sparsity





MNIST


maxpool
maxpool
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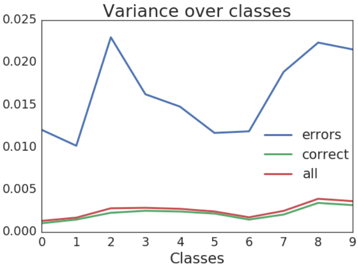
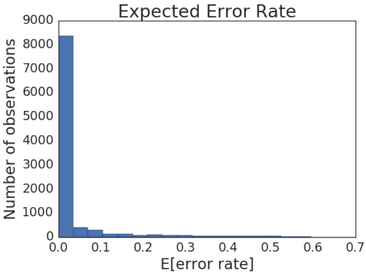
3x3 conv
conv1_1
depth=32
3x3 conv
conv2_1
size=64
FC1
size=10
softmax

MNIST





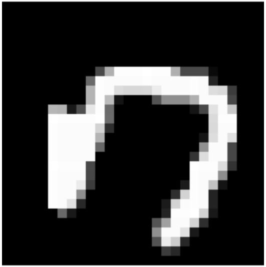
Conclusions:

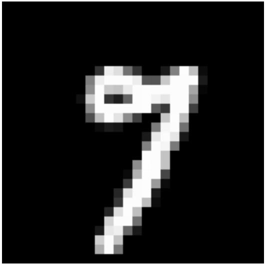
- Accuracy score: 97.7%;
- Variance is much higher for misclassified pictures;
- Model is not always confident.

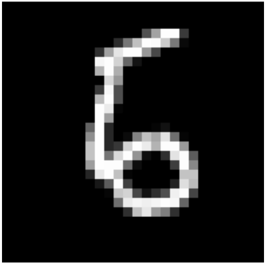
MNIST

Misclassified pictures with **zero expected error rate**:

True	Prediction	True	Prediction	True	Prediction
7	0	9	7	5	6



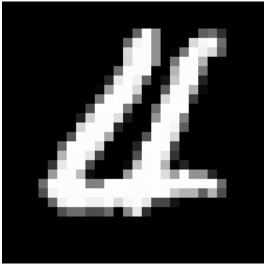


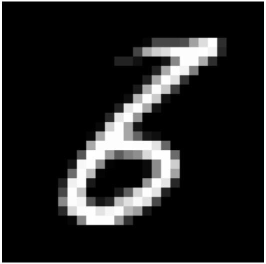


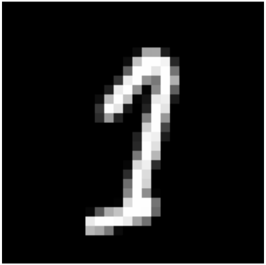
MNIST

Pictures with the **lowest confidence**:

True	Prediction	True	Prediction	True	Prediction
4	0	6	8	1	1







Conclusion

- Posterior distribution helps to make conclusions about uncertainties
- Variational inference allows to approximate posterior distribution for high-dimensional data
- Hierarchical models have more degrees of freedom