Uncertainty Evaluation in Two-Dimensional Indirect Measurement by Evidence and Probability Theories

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***Abstract*—An improved method for 2-D uncertainty expression and propagation based on the theory of evidence and 2-D random-fuzzy variables (RFVs) is described. A previous 2-D RFV approach and two probability-based approaches are also introduced. The improved RFV approach exploits a new algorithm for the combination of random and systematic effects, trying to overcome a drawback of a 2-D RFV method already disclosed in a previous work. One of the two probability-based methods does not take into account any correlation among uncertainty sources and among different time instants of each source, whereas the other probability method exploits time correlation to take into account the repetitive nature of systematic uncertainty sources. All described methods are applied to the 2-D case of a vehicle position measurement on a plane. The obtained results are compared and show the compatibility of all approaches. The improved random-fuzzy method yields better uncertainty evaluation in case of narrow and elongated confidence regions than the previous method. The new 2-D RFV approach also exhibits a better behavior from a theoretical point of view. Main differences between the two probability-based methods are also presented.**

***Index Terms*—Random-fuzzy variable (RFV), theory of evidence, 2-D uncertainty.**

I. Introduction

THE RESULT of a direct or indirect measurement of 2-D quantities should comprise two numerical values, at least a confidence region evaluated for a corresponding level of confidence, and a measurement unit [1], [2]. Thus, to perform a measurement, a suitable method that is able to represent the uncertainty associated with all uncertainty sources must be chosen. In case of indirect measurement, a method for the propagation of selected contributions must be defined to obtain an evaluation of the uncertainty associated with measurement output quantities.

Known and usually accepted procedures for uncertainty propagation are based on the probability theory. These approaches are suited to deal with random contributions to uncertainty but could lead to some questionable results in the presence of uncompensated systematic effects and complete ignorance situations, as explained in [3]–[5]. Conversely, systematic effects and complete ignorance situations can be correctly represented by intervals and the mathematics of the intervals, as described in [5] and [6]. In some recent works [5], [7], [8], an effective and interesting approach to represent and propagate uncertainty has been proposed. This approach is based on the theory of evidence, joining the most useful features of the probabilistic method and the mathematics of the intervals.

In [3], a first procedure based on 2-D RFVs for uncertainty expression and propagation is described. This first 2-D RFV approach separately propagates random and systematic contributions to obtain both propagated confidence 2-D regions associated with random effects and propagated regions associated with systematic contributions. Lastly, the two types of propagated regions are combined together by a simple radial sum. In [3], this method is also applied to the indirect measurement of the Cartesian position of a vehicle, moving along a circular and a roughly rectilinear trajectory. The confidence regions obtained for the circular trajectory have roughly the same shape for both this old 2-D RFV approach and a probabilistic Monte Carlo method. However, this consistency of results does not happen for the rectilinear trajectory. Indeed, in the latter case, systematic effects yield a very narrow inner region elongated along one direction, whereas random effects produce confidence regions that are only marginally wider along that same direction. This way, the radial sum of the two contributions leads to a central protrusion of the external regions, which is inconsistent with the results obtained by the Monte Carlo simulation and does not have a clear explanation.

This paper wishes to improve the 2-D RFV method described in [3], also allowing its application to narrow and elongated confidence regions, without obtaining strange protrusions. The new approach is also based on the innovative RFV method developed for 1-D measurements in [5] and [7]–[12], which explicitly takes into account random effects, systematic effects, and situations of complete ignorance. The described approach tries to extend the RFV approach to the 2-D case and exhibits the further advantage of being a nonparametric estimation method. This particular characteristic can be very useful, as the application of this method does requires very little modeling or assumptions. The introduced RFV method can also be successfully applied when the indirect measurement involves a functional relation (mathematical model) between the output measurement quantities and their contributing input quantities, which comprises numerical models (finite-element analyses, dynamic systems simulations, and identification algorithms, such as linear/nonlinear regression, optimization algorithms, or other complex relations).

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Results are compared with those obtained by the old 2-D RFV method of [3], which is briefly described in Section II, and with those yielded by two methods based on the probability theory and the Monte Carlo propagation, which are already known [13], [14]. Differently from the RFV approach, in the probability approach, there are numerous works that deal with uncertainty evaluation for two- or multidimensional measurement, as it can be seen in [15]–[18].

The particular application described in this work consists of indirect measurement of the Cartesian position of a vehicle moving on plain ground by means of encoders mounted on the vehicle wheels. Position measurement of a vehicle can be performed by many known methods, and it is necessary to properly control an autonomous guided vehicle (AGV). For AGVs, the evaluation of the uncertainty of each measurement is also very important to allow sensor fusion among all vehicle sensors. In [19], the new 2-D RFV approach was applied to the vehicle and was presented a comparison among the results obtained by the probabilistic approach and the two (old and new) 2-D RFV approaches. In this paper, a different probabilistic approach is introduced and applied to the vehicle application. The added probabilistic approach propagates systematic contributions, assuming a complete time correlation for this type of uncertainty sources. This assumption may greatly modify the propagated uncertainty when the output quantities are affected by past values of one or more input time-varying quantities, as in the kinematic model introduced in this work or in all dynamical systems. If all input quantities instantly yield the output quantities, i.e., each output at time t_k only depends on the inputs at the same time t_k, the time correlation of uncertainty sources does not affect the propagated uncertainty. However, if one output at time t_k depends on previous inputs at time $t_1, \ldots, t_{k-1}$, the time correlation of the uncertainty contributions to those inputs becomes as important as the correlation among input quantities. Particularly, when all measured values, from an initial time instant t_1 to a final instant t_N, of one time-varying input quantity are used in the indirect measurement, the time correlation hypothesis implies that the uncertainty contribution to those input values will be roughly similar for all time instants. Conversely, if there is no time correlation, the uncertainty contribution to the considered input quantity may vary from time t_k to time t_{k+1} according to the associated probability density function (pdf). Thus, the effect of this behavior depends on the propagation model. Two simple examples are given as follows: if the propagation model calculates an average of values corresponding to different time instants, the propagated uncertainty does not decrease in the time correlation case, whereas it is reduced when there is no correlation; on the contrary, if the propagation model calculates differences of values corresponding to consecutive time instants, the propagated uncertainty is smaller in the time correlation case since a compensation occurs. Thus, since the time correlation of one or more uncertainty contributions may yield very different results, its presence or absence should explicitly be analyzed during the propagation procedure using the probabilistic approach, as described in this work and not in [19]. Results obtained in the presence of systematic contributions by the new RFV approach are compared with those obtained by the probabilistic approach with time correlation of some uncertainty sources.

In the next section, the two probabilistic methods are introduced, and then, the old and the new 2-D RFV approaches are described. In Section III, the application to the vehicle Cartesian position measurement is described, and the kinematics model used to perform indirect measurements is introduced. In Section IV, results obtained are presented and discussed.

II. Uncertainty Analysis

A. *Method by Means of PDFs and Monte Carlo Simulations*

The Guide to the Expression of Uncertainty in Measurement (GUM) [1] is the internationally accepted reference document for uncertainty appraisal. A basic concept expressed in the GUM and even more strengthened in the supplement [14] is that a distribution of probability is the optimal mean to express the available information on possible values of a quantity. Thus, in this approach, a 1-D pdf is associated to every identified uncertainty source. The aim is to evaluate the uncertainty to be associated with two generic output quantities (variables) Y_1 and Y_2, which are obtainable as indirect measurements of n input quantities (variables) $X_1, \ldots, X_n$. In this section and in the following sections, employed symbols refer to a generic functional relationship between the two output variables Y_1 and Y_2 and input variables $X_1, \ldots, X_n$, i.e.,

$$Y_1 = f_1(X_1, \ldots, X_n) \quad Y_2 = f_2(X_1, \ldots, X_n). \tag{1}$$

Capital letters $(Y_1, Y_2, X_1, \ldots, X_n)$ will be used for all variables, whereas possible values will be expressed by Greek letters (η_i for Y_i and ξ_i for X_i).

As explained in [2], a good approach to obtain a correct confidence region for the output quantities Y_1 and Y_2 requires the propagation of the pdfs of all input quantities X_i to arrive to the evaluation of the joint pdf associated to Y_1 and Y_2. After the joint pdf of Y_1 and Y_2 has been determined, if the marginal pdfs of both Y_1 and Y_2 are unimodal, a unique smallest convex confidence region for a desired coverage probability (e.g., 0,997) can be found out. The joint pdf of the outputs Y_1 and Y_2 can be estimated by means of the Monte Carlo method, which is described in [2], [13], and [14]. After N_{iter} iterations, the Monte Carlo simulation allows evaluating a 2-D frequency distribution dividing the whole 2-D region in a matrix of small bins. This joint frequency can be normalized and employed as an approximation of the Y_1 and Y_2 joint pdf, from which the expectation values of the two output quantities Y_1 and Y_2, and one or more closed regions associated with desired levels of confidence (probability) can be numerically calculated, as explained in [2].

To try and take into account the presence of systematic contributions in the uncertainty sources, a slight modification to the Monte Carlo simulation used to propagate the uncertainty can be introduced. The evaluation of the joint pdf of Y_1 and Y_2 is usually performed using a repeated sampling from each 1-D pdf of the input quantities $X_1, \ldots, X_n$. The sampling phase from a 1-D pdf employs a random number generator, with rectangular distribution in the interval (0,1), and then, the value of the corresponding input quantity is calculated using the inverse cumulative distribution function (cdf) [see [2] and [4] for details]. When the correlation among input quantities is considered, there are two opposite but very simple cases: each input quantity is completely independent or completely correlated with the others; in the first case, at each Monte Carlo iteration, a different random number may be generated for each input quantity, whereas, in the complete correlation case, the same generated random number may be used (in the same iteration) for all input quantities to calculate the input values from the inverse cdfs. For nonscalar input quantities, the correlation or independence of all scalar components should be considered.

To calculate the two output quantities Y_1 and Y_2, the propagation model (functional relationship) may use one or more input quantities (uncertainty sources) that varies over time. In this case, a pdf and a corresponding inverse cdf should be evaluated for each time instant. If the value that a considered uncertainty source may have at a time t_k is completely independent of the values of the same source at all other time instants, a new random number is generated at each time step to calculate the source values from the inverse cdfs in the sampling phase. Conversely, if the value of a considered uncertainty source at a time t_k is completely correlated with the values of the same quantity at all other time instants, the same random number is used for all measured time instants in the sampling phase of the Monte Carlo simulation. In the case of complete time correlation of an uncertainty source, the random number used to calculate values of the source from inverse cdfs (one cdf for each time instant) should be the same for all time instants, but it is randomly changed at each Monte Carlo iteration. Generally, assigning the same random number to all time instants does not mean assigning the same value over time to the uncertainty source since the pdf (and, thus, the inverse cdf) may vary over time, e.g., the expected value may be different over time.

The method previously outlined to treat uncertainty sources over time suggests a simple way to propagate both random and systematic effects using the probabilistic approach. To this aim, the functional relationship to calculate the two output quantities has to explicitly express the uncertainty sources as vector input quantities, with each component corresponding to a different time instant. Since a systematic effect should be repeatable and should yield a similar contribution every time a measurement is acquired, a complete time correlation of the corresponding uncertainty source seems more correct than neglecting the time correlation. Moreover, as explained in [20], for systematic contributions, the operation of averaging of time series of data should not reduce the uncertainty. This behavior can be obtained by explicitly introducing time correlation for this type of contributions in the probability approach or exploiting the Zadeh extension principle in the RFV approach. Thus, for systematic contributions, the same random number is used for all time instants in one iteration of the Monte Carlo simulation, and it is changed every iteration. Conversely, a complete independence over time is assumed for random contributions since they are supposed to vary every time a measurement is performed. In this case, a different random number is used for each time instant at each Monte Carlo iteration.

In this paper, the probabilistic approach is applied to the vehicle position measurement in two ways: without taking into account the time correlation of systematic contributions and taking into account time correlation, as previously described, for the uncertainty sources associated with systematic contributions.

B. Old Method by Means of the Theory of Evidence

In practical applications, there are some situations that could not be properly represented by a probabilistic model and for which the use of distributions of probability could lead to uncertainty undervaluation. A typical situation of this type can be found when there are systematic effects unknown and not explicitly compensated for. Another example is the case of a quantity whose variation range is only known, and nothing else can be specified within this range. This situation of complete ignorance within the range is often represented by a uniform pdf, even if, in such a way, unowned information could be arbitrarily introduced in the model. In fact, the variable could exhibit a pdf that is very different from the uniform pdf within the variation range, only that this pdf is unknown. To cope with these situations, in some recent works [5], [7]–[12], an innovative approach to represent and propagate uncertainty has been proposed. This approach, introducing the concept of random-fuzzy variables (RFVs), tries to join the most useful features of the probabilistic method and the mathematics of the intervals to be able to properly deal with both random and systematic effects. In [3], a first method to employ 2-D RFVs is described, and it is briefly recalled in this Section II-B.

In the case of 2-D problems, e.g., for the two quantities Y_1 and Y_2, an RFV comprises a set of internal regions $A_{\mathrm{int},\alpha}$ associated with complete ignorance and/or noncompensated systematic contributions and a set of external regions $A_{\mathrm{ext},\alpha}$, each of which comprises the corresponding internal region $A_{\mathrm{int},\alpha}$ and defines a confidence region of the quantities Y_1 and Y_2 associated with a predetermined level of confidence $1-\alpha$. This way, the subtraction $A_{\mathrm{ext},\alpha} - A_{\mathrm{int},\alpha}$ defines a lateral region associated with random effect contributions for each level of confidence $1-\alpha$. As for 1-D RFVs, internal regions $A_{\mathrm{int},\alpha}$ of a 2-D RFV may be the same for all levels of confidence $1-\alpha$ or may be smaller for higher values of α. To compute the 2-D RFV of the outputs Y_1 and Y_2, it is assumed that each input X_i is affected by uncertainty systematic contributions properly expressed by a set of internal intervals $[\xi_{2,\alpha}, \xi_{3,\alpha}]_i$ and by uncertainty random contributions expressed by a pdf.

The propagation of systematic and complete ignorance contributions from 1-D internal intervals to internal regions $A_{\mathrm{int},\alpha}$ is performed in accordance with the Zadeh extension principle for possibility distributions [6], [20]–[22]. In practice, for each level of confidence $1-\alpha$, the internal intervals $[\xi_{2,\alpha}, \xi_{3,\alpha}]_i$ of all input quantities X_i yield the internal region $A_{\mathrm{int},\alpha}$ of Y_1 and Y_2 according to the following rule:

$$A_{\mathrm{int},\alpha} = \{(f_1(\xi_1,\ldots,\xi_n), f_2(\xi_1,\ldots,\xi_n)) \,|\xi_i \in [\xi_{2,\alpha}, \xi_{3,\alpha}]_i\} \tag{2}$$

where ξ_i is a possible value of X_i, and $[\xi_{2,\alpha}, \xi_{3,\alpha}]_i$ is the internal interval of the RFV of X_i with level of confidence $1 - \alpha$.

Uncertainty random contributions, which are expressed as a pdf for each input X_i, can be numerically propagated to a joint pdf of the outputs Y_1 and Y_2 by the Monte Carlo simulation. To combine the systematic contributions in the internal regions $A_{\mathrm{int},\alpha}$ with the random contributions and obtain the external regions $A_{\mathrm{ext},\alpha}$, the joint pdf of Y_1 and Y_2 must be transformed into a possibility distribution. This transformation can be performed by the same procedure used in Section II-A for the calculation of confidence regions starting from a numerically evaluated joint pdf. As explained in [2], with the unimodal hypothesis and for each desired level of confidence $1 - \alpha$, a region R_α, whose inner area should comprise the random contributions with a probability of $1 - \alpha$, can be calculated.

Thus, first, the 1-D possibility distributions associated with systematic contributions are propagated according to the Zadeh extension principle. Second, the 1-D pdfs associated with random contributions are propagated with the Monte Carlo simulation, and the 2-D possibility distribution defined by regions R_α is obtained. Lastly, for each level of confidence $1 - \alpha$, the inner region $A_{\mathrm{int},\alpha}$ is merged with the corresponding R_α to find the external region $A_{\mathrm{ext},\alpha}$. This last step may be seen as the propagation of two possibility distributions defined by the regions $A_{\mathrm{int},\alpha}$ and R_α, respectively, through the sum function. Thus, this last step should also be performed according the Zadeh extension principle. In this paper, the old RFV approach, which is described in the following paragraph, does not comply with the Zadeh extension principle, but it is introduced since it is very simple to implement, the computational time required for the propagation is very short, and the obtained results may be similar to those obtained by the new RFV approach in some application cases [19]. Conversely, the new RFV approach described in Section II-C complies with the extension principle and can perform considerably better than the old RFV approach, as presented in Section IV, but it is more difficult to be implemented and requires much longer computational time.

In the old 2-D RFV method, for each $1 - \alpha$, the inner region $A_{\mathrm{int},\alpha}$ and the region R_α are radially summed, starting from the expected values of Y_1 and Y_2. It means that both regions $A_{\mathrm{int},\alpha}$ and R_α have to be preliminarily expressed by their edges with reference to a 2-D polar coordinate system (r, θ) having the expected values of Y_1 and Y_2 as origin. Then, for each $1 - \alpha$, the external region $A_{\mathrm{ext},\alpha}$ can always be calculated by summing, for each angle θ, the radial value of $A_{\mathrm{int},\alpha}$ with that of R_α. Fig. 1 shows a schematic and simplified representation of this radial sum only for a few angles θ.

Thus, the 2-D RFV of Y_1 and Y_2 is defined by the obtained sets of internal regions $A_{\mathrm{int},\alpha}$ associated with systematic effects and external regions $A_{\mathrm{ext},\alpha}$, which express the confidence regions of Y_1 and Y_2 for the desired levels of confidence $1 - \alpha$.

C. New Method by Means of the Theory of Evidence

In the old 2-D RFV method, the radial sum of the internal region $A_{\mathrm{int},\alpha}$ and of the corresponding confidence region R_α is a very simple way to build the external region $A_{\mathrm{ext},\alpha}$, but,

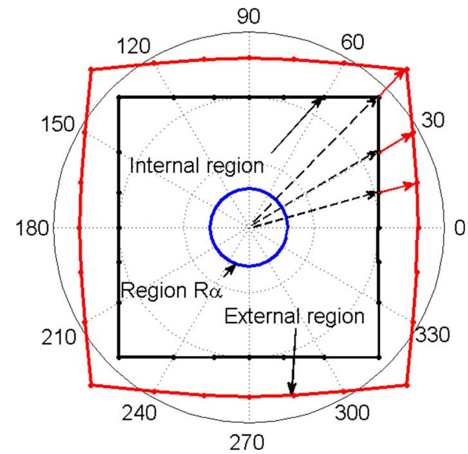

Fig. 1. Schematic of the radial sum of the old 2-D RFV method: internal and external regions.

as already said, it does not comply with the Zadeh extension principle. Moreover, there is no theoretical reason to limit the summing direction, as shown in Fig. 1: the event that the propagated random contribution is aligned and has the same direction as the propagated systematic contribution, even though possible, is deemed not more probable than the case in which the two contributions have no alignment. Thus, the best solution seems to be the following: propagated random contributions, which are expressed by the confidence regions R_α for the output quantities Y_1 and Y_2, are added to each point on the internal region edge along all possible directions.

The new 2-D RFV method is the same as the old method until the sum of the internal region $A_{\mathrm{int},\alpha}$ with the corresponding confidence region R_α; thus, the initial common procedure phases are not repeated here. The difference between the old and the new method lies in the merging of $A_{\mathrm{int},\alpha}$ and the corresponding R_α with the same level of confidence. This merging is performed in this way: each point on the edge of the internal region $A_{\mathrm{int},\alpha}$ is considered as a starting point from which propagated random contributions can be summed as vectors along all directions and not only along the same radial direction of the starting point itself. Fig. 2 shows a schematic and simplified representation of these sums. After these additions are carried out for all points on the edge of $A_{\mathrm{int},\alpha}$, the edge of the region $A_{\mathrm{ext},\alpha}$ can be found as the external envelope of all the outer obtained points, as it can be seen in Fig. 2.

For both the old and the new RFV approach, and particularly for the new method, the number of angularly spaced points that are used to express the edges of the regions may influence uncertainty results. Thus, the number of points should properly be selected as a tradeoff between inaccurate results and too long computational time. The tradeoff number depends on the application characteristics and requirements.

III. Application to Vehicle Position

The described methods for uncertainty expression and propagation are applied to the 2-D position indirect measurement of a vehicle. Estimation of the pose of a mobile robot like that considered is a typical case where the correlation of

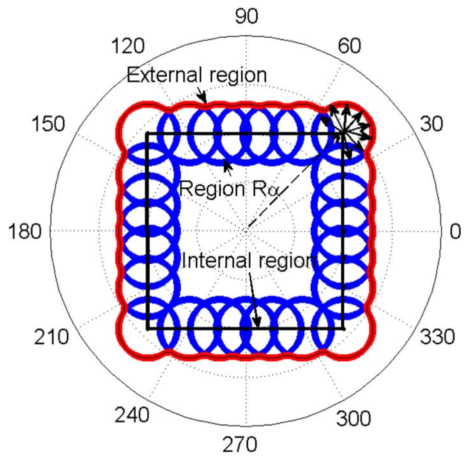

Fig. 2. Schematic of the sums along several directions in the new 2-D RFV method: internal region and external envelope defining the external region.

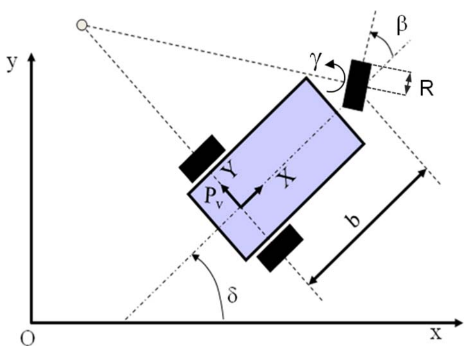

Fig. 3. Kinematics scheme of a three-wheeled AGV.

uncertainty sources shall always be taken into account [23]. The steering wheel angle is known with an uncertainty whose random contribution can be due to floor irregularities or vehicle vibrations, whereas its systematic effect can be due to mechanical tolerances, transmission play, or temperature, and if a self-calibration procedure is not available online, it can be a significant source of uncertainty.

A three-wheeled vehicle, as shown in Fig. 3, is considered. It has two encoders for position measurement: a first encoder measures the steering angle β of the driving wheel, and the second encoder measures the angular rotation γ of the same wheel.

The attitude δ is the angle between the absolute reference system xOy and the mobile reference system $\mathrm{XP}_v Y$. The position P_v is defined by the vector (x, y, δ), which also takes the attitude of the mobile robot into account. The discrete form of the inertial–odometric navigation equations is given as follows:

$$\begin{cases} x_{k+1} = x_k + \Delta\gamma_{k+1} \cdot R \cdot \cos(\beta_k + \beta_0) \cdot \cos(\delta_k) \\ y_{k+1} = y_k + \Delta\gamma_{k+1} \cdot R \cdot \cos(\beta_k + \beta_0) \cdot \sin(\delta_k) \\ \delta_{k+1} = \delta_k + \Delta\gamma_{k+1} \cdot R \cdot \sin(\beta_k + \beta_0) \cdot 1/b \end{cases} \tag{3}$$

where k indicates the time iteration, γ_{k+1} represents the driving wheel rotation angle (in radians), $\Delta\gamma_{k+1}$ is the variation of the rotation angle $\gamma_{k+1} - \gamma_k$, and β_0 is the zero position of the steering encoder.

This kinematic model can be used to calculate the final Cartesian position x, y of the vehicle if the encoder angles are continuously acquired along the trajectory. The vehicle is guided along a predetermined path, and the Cartesian position (x, y) is considered the 2-D output (the generic outputs Y_1 and Y_2 of Section II) of an indirect measurement using the encoder acquisitions β_k and γ_k; the geometrical parameters R (driving wheel radius), b (longitudinal distance between the driving wheel and the rear ones), and β_0 (zero position of the steering encoder); and an initial position (x_0, y_0, δ_0) as inputs. (They are the generic inputs $X_1, \ldots, X_n$ described in Section II.) In detail, the final position x_N and y_N obtained at the end of a trajectory, which are the indirect measurands, can be evaluated by

$$\begin{cases} x_N = f_{1'}(x_{N-1}, \Delta\gamma_N, \delta_{N-1}, \beta_{N-1}, R, \beta_0) \\ \quad\;\; = f_1(\beta_1, \ldots, \beta_{N-1}, \gamma_1, \ldots, \gamma_N, R, b, \beta_0, x_0, \delta_0) \\ y_N = f_{2'}(y_{N-1}, \Delta\gamma_N, \delta_{N-1}, \beta_{N-1}, R, \beta_0) \\ \quad\;\; = f_2(\beta_1, \ldots, \beta_{N-1}, \gamma_1, \ldots, \gamma_N, R, b, \beta_0, y_0, \delta_0). \end{cases} \tag{4}$$

The uncertainty contributions to each input variable $(\beta_1, \ldots, \beta_{N-1}, \gamma_1, \ldots, \gamma_N, R, b, \beta_0, x_0, y_0, \delta_0)$ are divided into two groups, i.e., those associated with random effects and those associated with complete ignorance and systematic effects. In this paper, each uncertainty source of random type is represented by a normal pdf, and each uncertainty source of systematic type, whose only available information is supposed to be a limited interval, is represented by a rectangular pdf in the probability approach and by a rectangular possibility distribution in the RFV approach. In detail, from previous experimental test, random effects are considered to be nonnegligible for the input quantities β_k, γ_k, R, x_0, y_0, and δ_0, whereas systematic effects are taken into account for the quantities β_k, R, b, β_0, x_0, y_0, and δ_0. This way, some of the input quantities are affected by both random and systematic contributions, which have to be summed for the Monte Carlo simulations while separately treated by the RFV approach. The uncertainty sources and their amount are the same for both probability and RFV approaches; only the representation and propagation procedures are different.

In detail, the time-varying steering angle $\beta_1, \ldots, \beta_k, \ldots, \beta_{N-1}$ is affected by both random and systematic uncertainty sources. Thus, each scalar component β_k is affected by a random source, which is expressed by a normal pdf, in all applied approaches and by a systematic source, which is expressed by a uniform pdf in the two probabilistic approaches and by a uniform possibility distribution in the two RFV approaches. As described in Section II-A, in the first probability approach that does not take into account the time correlation, the sampling phase from the pdfs is performed in this way: a different random number is generated for each pdf (normal and uniform) of each component β_k every Monte Carlo iteration. In the second probability approach that *does* take into account the time correlation, the sampling phase becomes the following: every Monte Carlo iteration, a random number is generated for each normal pdf (random source) of each component β_k, whereas, for the uniform pdfs (systematic sources) of all components $\beta_1, \ldots, \beta_{N-1}$, only one random number is generated every Monte Carlo iteration.

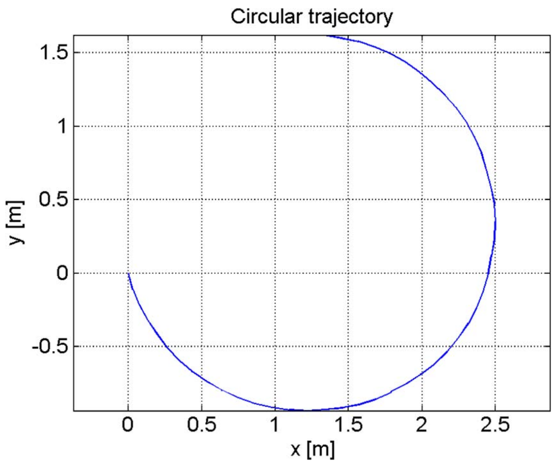

Fig. 4. Circular trajectory.

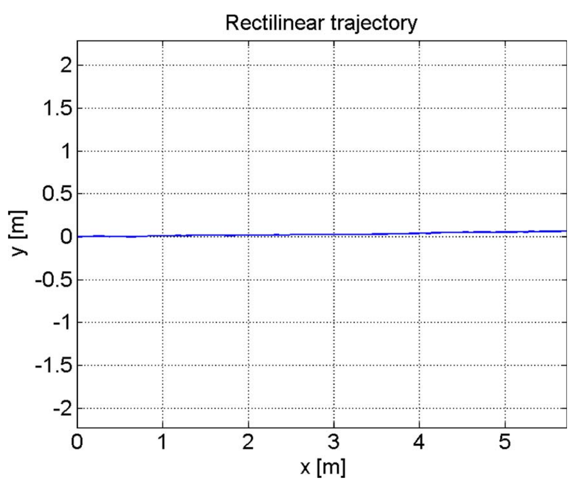

Fig. 5. Rectilinear trajectory.

A correct uncertainty evaluation is critical for the vehicle positioning since uncertainty could be used in many ways: by the navigation unit to fuse measurements from one sensor with those obtained by other sensors in an optimal way and by the trajectory planner and control to optimize a trajectory and follow it, and to verify the practicability of a trajectory. Thus, if uncertainty is underestimated, safety problems may arise; conversely, if it is overestimated, a feasible and safe trajectory could be rejected.

IV. Results and Discussion

The two probabilistic and two RFV methods previously described are applied to evaluate the uncertainty of the end position (x_N, y_N) when the vehicle follows a circular trajectory, as shown in Fig. 4, and a roughly rectilinear trajectory shown in Fig. 5.

For all methods, the obtained results can be summarized by three confidence regions calculated for the levels of confidence 99.73%, 95.45%, and 68.27%.

The results for the circular trajectory are shown in Fig. 6 for the probability approach *without* time correlation of systematic uncertainty sources, in Fig. 7 for the probability approach *with* time correlation of systematic uncertainty sources, in Fig. 8

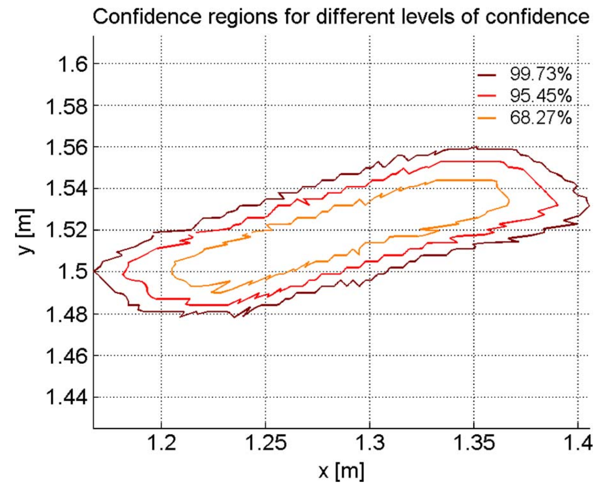

Fig. 6. Method by means of pdfs and Monte Carlo simulations *without* time correlation of systematic uncertainty sources applied to the CIRCULAR trajectory: confidence regions with levels of confidence 99.73% (the darkest), 95.45%, and 68.27% (the lightest).

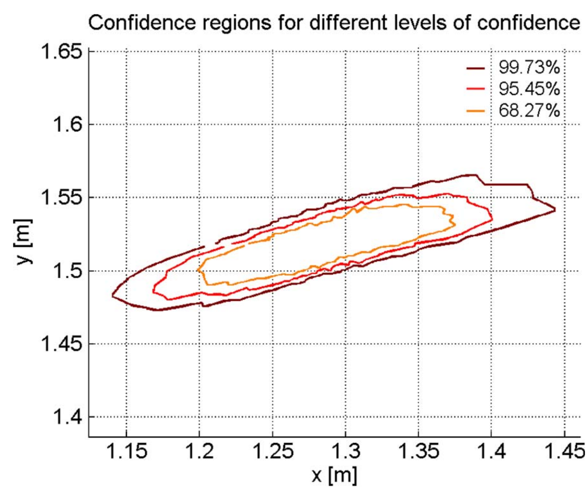

Fig. 7. Method by means of pdfs and Monte Carlo simulations *with* time correlation of systematic uncertainty sources applied to the CIRCULAR trajectory: confidence regions with levels of confidence 99.73% (the darkest), 95.45%, and 68.27% (the lightest).

for the old 2-D RFV method, and in Fig. 9 for the new 2-D RFV method; the same three methods applied to the rectilinear trajectory yield the confidence regions shown in Figs. 10–13, respectively. The figures that refer to the old and new 2-D RFV methods depict the three external regions associated with the desired levels of confidence and only one inner region associated with systematic effects since the latter is the same for all levels of confidence. The reason of this result is that all systematic uncertainty sources are modeled by possibility distributions having a rectangular shape, which means the same interval for all levels of confidence.

Figs. 6–13 show that the applied methods yield compatible results, with the two RFV approaches evaluating slightly wider regions, particularly for medium to low levels of confidence, as expected and reported by many works, e.g., [3] and [5]. Comparing Figs. 6, 7, and 9 for the circular trajectory and Figs. 10, 11, and 13 for the rectilinear trajectory, the propagation of the systematic contributions exploiting their time

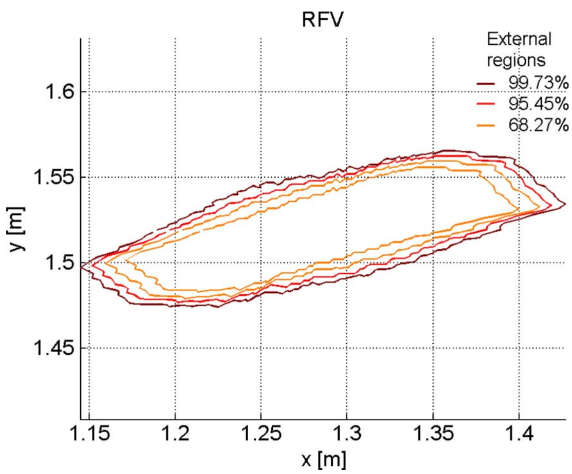

Fig. 8. Old method by means of 2-D RFV applied to the CIRCULAR trajectory: external regions with levels of confidence 99.73% (the darkest), 95.45%, and 68.27%, and the inner region associated with systematic effects (the lightest).

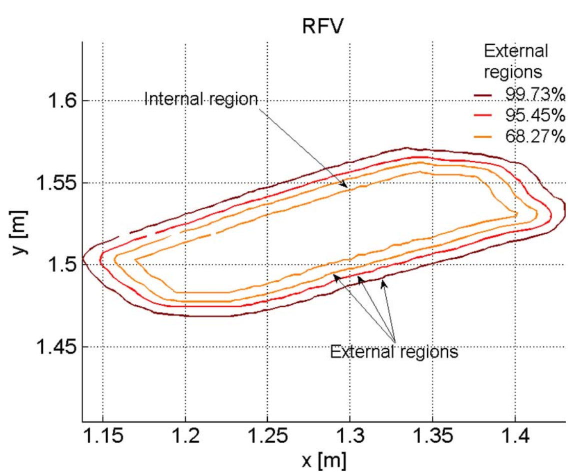

Fig. 9. New method by means of 2-D RFV applied to the CIRCULAR trajectory: external regions with levels of confidence 99.73% (the darkest), 95.45%, and 68.27%, and the inner region associated with systematic effects (the lightest).

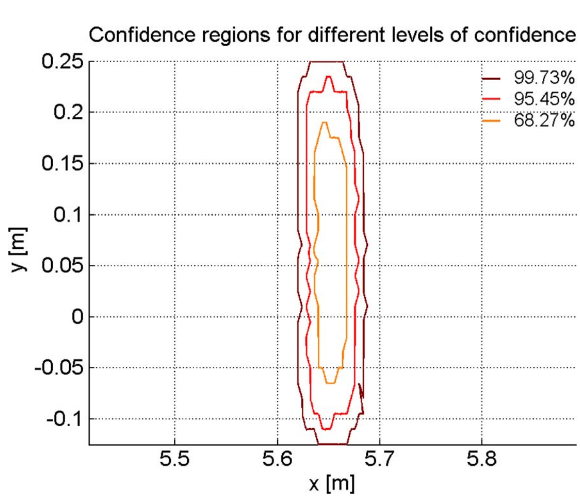

Fig. 10. Method by means of pdfs and Monte Carlo simulations *without* time correlation of systematic uncertainty sources applied to the RECTILINEAR trajectory: confidence regions with levels of confidence 99.73% (the darkest), 95.45%, and 68.27% (the lightest).

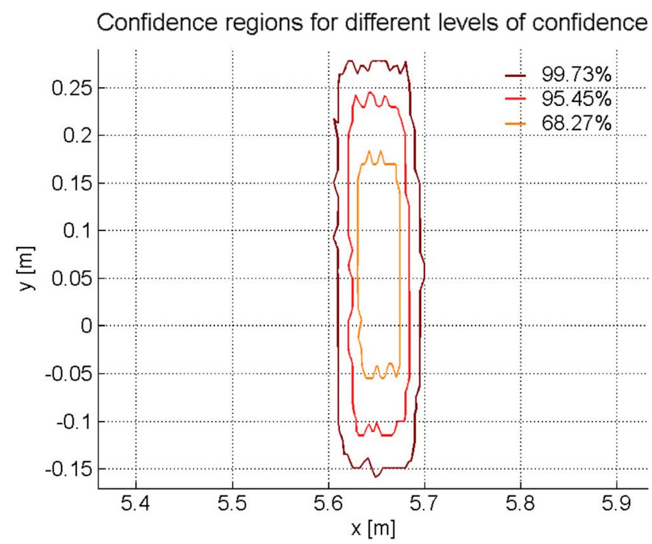

Fig. 11. Method by means of pdfs and Monte Carlo simulations *with* time correlation of systematic uncertainty sources applied to the RECTILINEAR trajectory: confidence regions with levels of confidence 99.73% (the darkest), 95.45%, and 68.27% (the lightest).

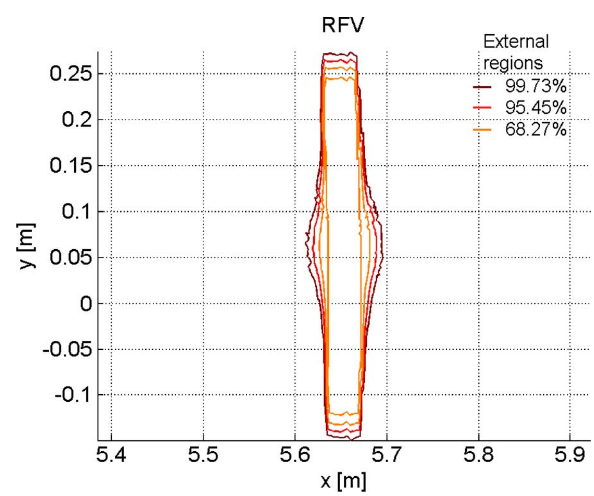

Fig. 12. Old method by means of 2-D RFV applied to the RECTILINEAR trajectory: external regions with levels of confidence 99.73% (the darkest), 95.45%, and 68.27%, and the inner region associated with systematic effects (the lightest).

correlation in the probabilistic approach yields wider confidence regions than the probability approach without time correlation; thus, the obtained regions are more similar to those obtained by the new 2-D RFV approach. Particularly, the confidence regions associated with a level of 99.73% obtained by the probability approach with time correlation and by the new 2-D RFV method have similar dimensions, whereas the confidence regions for the levels of 95.45% and 68.27% are quite different and those from the probability approach remain smaller than the corresponding regions of the RFV approach. The obtained results can be explained by the following reasoning: the time correlation, which is used for systematic uncertainty sources affecting the steering angle, prevents partial compensation of these effects over time, and thus, the obtained uncertainty of the final vehicle position is wider. However, the applied probability approach with time correlation does not allow avoiding the

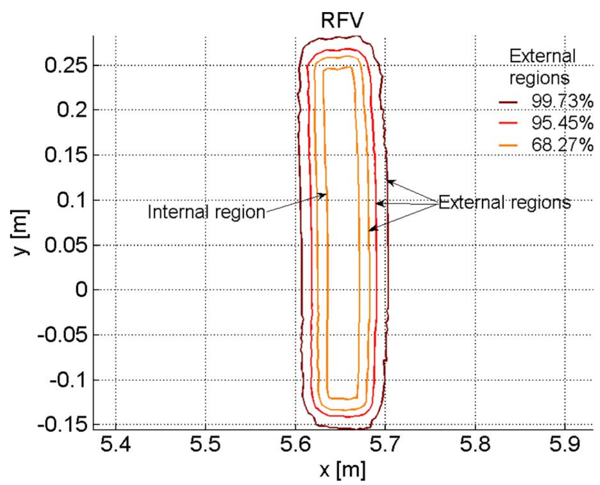

Fig. 13. New method by means of 2-D RFV applied to the RECTILINEAR trajectory: external regions with levels of confidence 99.73% (the darkest), 95.45%, and 68.27%, and the inner region associated with systematic effects (the lightest).

partial compensation that takes place *among* all the systematic and complete ignorance uncertainty sources. If the correlation coefficients among systematic uncertainty sources were known, they could be introduced in the uncertainty propagation, and the probability approach results could improve, but their evaluation is generally difficult and time consuming. Thus, if the only available information for the systematic uncertainty sources is an interval of variation, the RFV approach seems to be a more correct method for avoiding an unjustified partial compensation among systematic and complete ignorance uncertainty sources.

In this application, the only time-varying input quantity that is analyzed in a different way between the two probabilistic methods is the steering angle $\beta_1, \ldots, \beta_k, \ldots, \beta_{N-1}$. Since the presence of a systematic contribution on this quantity causes an increasing drift of measured position in experimental tests, the partial compensation among different time instants that takes place and reduces the estimated uncertainty in the probabilistic approach without time correlation seems not acceptable. Thus, taking into account also the danger of any underestimation in the vehicle positioning uncertainty, the probabilistic method with time correlation and the new RFV approach clearly yield the best results, and the former behaves better than the probabilistic method without time correlation.

With references to Figs. 10–13, the results show that, in case of internal regions $A_{\mathrm{int},\alpha}$, particularly narrow and elongated along one special direction, the radial sum of the old 2-D RFV method proves not to behave in a proper way: when a point on the edge of $A_{\mathrm{int},\alpha}$ is distant from the expected values of Y_1 and Y_2, which define the center of the polar coordinate system (r, θ), the radial direction is roughly parallel to the direction of special elongation and to the local tangent of the internal region border; thus, if the old method is used, the radially added random contribution moves very little from the internal region edge. This behavior of the old 2-D RFV method causes the strange shape of the external regions $A_{\mathrm{ext},\alpha}$, which collapse to the internal regions $A_{\mathrm{int},\alpha}$ when the latter are narrow and elongated. The collapse of $A_{\mathrm{ext},\alpha}$ to the corresponding $A_{\mathrm{int},\alpha}$ makes lateral region $A_{\mathrm{ext},\alpha} - A_{\mathrm{int},\alpha}$, which is associated with random contributions, become negligible in some areas of the 2-D plane. This behavior is very strange and shows an application case where the lack of complying with the Zadeh extension principle yields unacceptable results. Instead, e.g., from Figs. 2 and 13, the sum along different directions employed in the new method avoids this drawback and also allows preserving lateral regions in case of narrow and elongated confidence regions. Clearly, regions $A_{\mathrm{ext},\alpha}$ obtained by the new method have shapes that are more similar to those yielded by the probability approach.

V. Conclusion

In this paper, a new method for 2-D uncertainty expression and propagation based on the theory of evidence has been described. A new algorithm for the combination of propagated random and systematic effects has been introduced to improve the behavior of the method, particularly when the propagated confidence regions have a narrow and elongated shape. A brief theoretical discussion explains the possible reasons of the better behavior of the new 2-D RFV method in comparison with the old method.

Two different implementations of the probabilistic approach have been presented: the first implementation does not take into account any correlation among uncertainty sources and among different time instants of each sources, whereas the second implementation tries to take into account the repetitive nature of systematic uncertainty sources exploiting the time correlation. Results are presented for both implementations.

The described new RFV method is applied to the 2-D case of vehicle position measurement on a plane. The obtained results are compared with those obtained by the probabilistic approach with Monte Carlo simulations and by the old 2-D RFV method. The results show that the confidence regions obtained by all three approaches are compatible and that the improved random-fuzzy method yields a better uncertainty evaluation in case of narrow and elongated confidence regions than the previous region.

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