A t-Norm-Based Fuzzy Approach to the Estimation of Measurement Uncertainty

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Abstract—**From a metrological point of view, a measurement process rarely consists of a direct measurement but, rather, of digital signal processing (DSP) performed by one or more instruments. The measurement algorithm makes the numerical results available as functions of acquired samples from input signals. Moreover, when repeated direct measurements are performed, one may speak about interactions in subsequent results (and it may be dependent on the type of instrument being used). With mathematical formalism, the complex relations involved can be described, although again, an indirect measurement result would be obtained. Regardless, no matter what kind of process is being examined, the distribution of the uncertainty associated with the measurement needs to be known. To express a measurement result with its associated uncertainty, the recommendations of the ISO Guide need to be met. Many published papers have proposed the use of fuzzy intervals to describe both the systematic and statistical effects of repeated measurements on the distribution of their results. In this paper, we use a random–fuzzy model, the single measure is represented as a fuzzy set, and the propagation of the possibility distribution through the DSP stage (which simply consists of an average operation) is performed using the extension principle of Zadeh based on a particular triangular norm: the so-called Dombi's.**

Index Terms—**Distribution of measurement results, measurement uncertainty, random–fuzzy model.**

I. Introduction

THE DEMANDS of statistical investigations in measurements have inspired the remarkable development of probabilistic methods [1]. However, the probability theory did not prove to be fully adequate for all types of uncertainty. The probability theory is excellent if the ambiguity is to be modeled, but its attempts to describe vagueness are quite inconsistent with common sense [2].

In the framework of the standard approach given in the Guide [1], the result of a measurement is a random variable, and the term *uncertainty* is defined as "a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand." Thus, uncertainty is a way to describe the vagueness of the measurement process, whose estimation depends on the model of measured quantities.

In [3], the authors described extended uncertainty as α-cut and used fuzzy arithmetic based on the extension principle of Zadeh to evaluate the uncertainty propagation. Other authors have also noted that this is a good method to describe systematic effects but not random effects. In fact, in [4], a more effective way to estimate measurement uncertainty in terms of random–fuzzy variables (RFVs) was proposed, but as was noticed in [5], there is no unique statistical theory on the topic, and there is no complete theory to describe and represent both the *random and systematic effects* on the distribution of measurement results. In this paper, we agree with the authors of [5] and apply a random–fuzzy model in which each measurement result is expressed in terms of fuzzy intervals, whereas the measuring function is a random variable (banally, the average operator). The problem with such an approach is how to propagate the fuzziness of single measures to the measurand estimation. A natural extension of the principle of fuzziness is to consider intersection connectives; accordingly, the use of multivalued t-norms is a simple step in this direction. In our proposed approach, the correlation or interaction of measurements is described by Dombi's triangular norm, and the use of the "t-*norm*"-based arithmetic on fuzzy intervals describes the propagation of both the systematic and statistical effects of the measurements.

II. Background on Uncertainty Representation and Propagation Based on the Fuzzy Theory

If Y is the space of the measurement results, then the measurement of a quantity x is just a mapping of the space of events Ω into Y, i.e.,

$$f : \Omega \rightarrow Y. \tag{1}$$

Then, as one can see in [5], it is possible to define a random–fuzzy model where the space Y of the measurement results is a set of fuzzy intervals, the measuring function is random, and the extended uncertainty for a given significance level and degree of possibility is defined as half-width of α-cut.

This way, the results of measurements are random–fuzzy intervals with a level of confidence of $1 - \alpha$ (our so-called "α-cut sets"). One should note that if α is a number, i.e., $0 \leq \alpha \leq 1$, the α-cut of fuzzy variable A (Fig. 1) is defined as

$$A_\alpha = \{z : \mu_A(z) \geq \alpha\}. \tag{2}$$

From a statistical point of view, once measurement results are obtained, the ISO Guide requires one to choose the estimator

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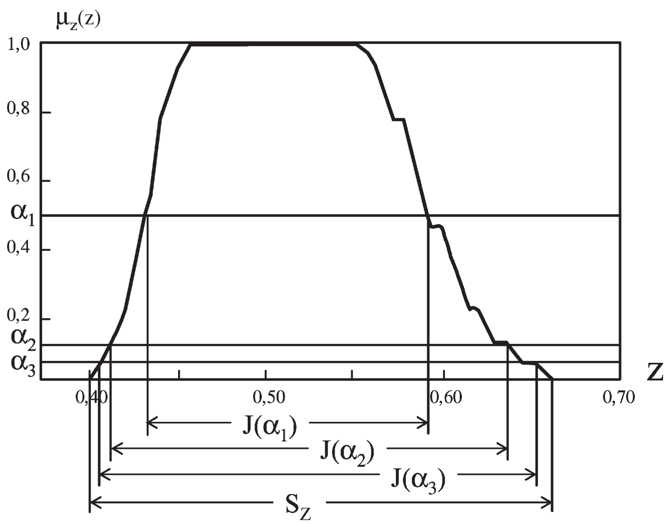

Fig. 1. Example of a membership function with α-cuts marked.

of the measurand; for this paper, we opted for the average operator. In a probabilistic model, one would average some random variables to get another random variable. Its density of probability function depends on the density functions of single input variables (measurement results) and on the correlation between measurements. For the random–fuzzy model, if A_1 and A_2 are two measurement fuzzy results (i.e., that their own associated uncertainties are expressed in terms of α-cuts), and if the indirect measurement algorithm performs the output that is a function g of A_1 and A_2, i.e., $Z = g(A_1, A_2)$, then the evaluation of uncertainty of Z requires a joint membership function (taking into account correlations between random contributions to uncertainty). One should note that a generic t-norm may indicate the principle of joining variables, i.e.,

$$\mu_{A_1,A_2}(a_1, a_2) = T\left[A_1(a_1), A_2(a_2)\right]. \tag{3}$$

Then, the membership function $\mu_Z(z)$ is simply (extension principle)

$$\mu_Z(z) = \sup_{g(a_1,a_2)=z} T\left[A_1(a_1), A_2(a_2)\right]. \tag{4}$$

A. "t-Norm"-Based Arithmetic

A fuzzy interval $\tilde{a}$ is a fuzzy set of real numbers $\Re$ with a continuous, compactly supported, unimodal, and normalized membership function. It is well known that any fuzzy interval $\tilde{a}$ can be described with the following membership function [6]:

$$A(t) = \begin{cases} L\left(\frac{a-t}{\alpha}\right), & \text{if } t \in [a-\alpha, a] \\ 1, & \text{if } t \in [a, b] \\ R\left(\frac{t-b}{\beta}\right), & \text{if } t \in [b, b+\beta] \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where $[a, b]$ is the peak of $\tilde{a}$; a and b are the lower and upper modal values; and L and R are shape functions $[0,1] \to [0,1]$, with $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$, which are nonincreasing continuous mappings.

We shall call these fuzzy intervals LR-type and use the notation $\tilde{a} = (a, b, \alpha, \beta)LR$. The support of $\tilde{a}$ is exactly $[a - \alpha, b + \beta]$.

In fuzzy logic systems, the basic aggregation operations are performed by the logical connectives AND and OR, which provide pointwise implementations of the intersection and union operations. It has been well established in the literature that the appropriate characterizations of these operators in the multivalued logic environment are the triangular norm operators. The *t-norm operator* provides the characterization of the AND operator. It is a mapping defined as $\otimes : [0,1]^2 \to [0,1]$, with the following properties:

$$\begin{array}{ll} 1)\ a \otimes b = b \otimes a & \text{(commutative)} \\ 2)\ a \otimes b \geq c \otimes d \text{ for } a \geq c \text{ and } b \geq d & \text{(monotonic)} \\ 3)\ a \otimes (b \otimes c) = (a \otimes b) \otimes c & \text{(associative)} \\ 4)\ a \otimes 1 = 1 \otimes a = a & \text{(with 1 as a neutral element).} \end{array} \tag{6}$$

If $\otimes$ is continuous, and $a \otimes a < a\ \forall a \in]0,1[$, then $\otimes$ is called an Archimedean t-norm; if, further, $\otimes$ is strictly monotonic (i.e., strictly satisfying (6) or, at the second point, with $\geq$ replaced by $>$), then $\otimes$ is a strict Archimedean t-norm.

A decreasing generator is a continuous strictly decreasing function $f : [0,1] \to \Re$ such that $f(1) = 0$. The pseudoinverse of a decreasing generator f is the function $f^{(-1)} : \Re \to [0,1]$, which is defined by

$$f^{[-1]}(y) = \begin{cases} 1, & \text{if } y < 0 \\ f^{-1}(y), & \text{if } y \in [0, f(0)] \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where f^{-1} is the ordinary inverse of f. If $\widetilde{a}_1$ and $\widetilde{a}_2$ are fuzzy sets of the real line (i.e., fuzzy quantities), then their t-sum (triangular norm-based sum) $\widetilde{A} = \widetilde{a}_1 + \widetilde{a}_2$ is defined by (4). Thus

$$\widetilde{A}(z) = \sup_{x_1+x_2=z} T\left[\tilde{a}_1(x_1), \tilde{a}_2(x_2)\right], \qquad z \in \Re.$$

This can be written in the form

$$\widetilde{\widetilde{A}}(z) = f^{(-1)}\left(\inf_{x_1+x_2=z}\left(f\left(\widetilde{a}_1(x_1)\right)\right) + \left(f\left(\widetilde{a}_2(x_2)\right)\right)\right). \tag{8}$$

Let T be an Archimedean t-norm with an additive generator f, and let $\widetilde{a}_i = (a_i, b_i, \alpha, \beta)LR$, $i = 1, \ldots, n$ be fuzzy numbers of LR-type. If L and R are twice-differentiable concave functions, and f is a twice-differentiable strictly convex function, then the membership function of the T-sum $\widetilde{A}_n = \widetilde{a}_1 + \cdots + \widetilde{a}_n$ is (see [6])

$$A_n(z) = \begin{cases} 1, & \text{if } A_n \leq z \leq B_n \\ f^{-1}\left\{n \cdot f\left[L\left(\frac{A_n - z}{n\alpha}\right)\right]\right\}, & \text{if } A_n - n\alpha \leq z \leq A_n \\ f^{-1}\left\{n \cdot f\left[R\left(\frac{z - B_n}{n\beta}\right)\right]\right\}, & \text{if } B_n \leq z \leq B_n + n\beta \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

where $A_n = a_1 + \cdots + a_n$, and $B_n = b_1 + \cdots + b_n$.

Hence, we interpret the t-norm as a basis for the definition of the arithmetic of interactive fuzzy measurement results.

TABLE I
PARAMETERS OF THE t.p.d. APPLIED TO THE TRIANGULAR, UNIFORM, AND GAUSSIAN LAWS

	t.p.d. for the triangular law	t.p.d. for the uniform law	t.p.d. for the Gaussian law
x_n	2.54σ	1.73σ	2.58σ
x_ε	1.63σ	x_n	1.54σ
ε	0.11	0	0.12

III. UNCERTAINTY OF A MEASUREMENT RESULT

As shown in [5], measured data $\{x_i\}^N$ are expressed in terms of a vector of fuzzy intervals. To obtain our result, N measurements are performed, which furnish a vector of pure real numbers. Each "pure" result is affected by uncertainty. Accordingly, one can define an interval built around the "one-point" estimation, in which there is a given probability for this interval to contain the "real" value.

In [3], we have seen that the general shape of the resulting possibility distributions is the optimal possibility distribution, because a probability/possibility transformation has been applied to symmetric probability distributions (i.e., Gaussian law, Laplace law, triangular law, and uniform law). The authors suggest that this shape is not easy to handle, and a parameterized distribution—the pseudo triangular possibility distribution (t.p.d.)—would be more interesting.

A truncated distribution expression has been applied to the four most encountered unimodal and symmetric probability laws, with mean values and standard deviations that are well known. For the uniform and triangular laws, the fuzzy t.p.d. is such that $\mu A_i(x_i) = 1$ (in the sense that A_i is built in such way that x_i belongs to it with the maximum possibility), where x_i is chosen equal to the mean value of the interval (the measured pure value). As these are two bounded laws, no further approximations are required. Now, what can one say about the measurement results when data at our disposal only consist of a vector of "pure" numbers generally distributed with the Gaussian law? For unbounded laws, an approximation is needed to correctly choose the required parameters [3], as shown in Table I; the pseudo t.p.d. is a piecewise linear approximation of the optimal possibility distribution (Fig. 2), which itself is completely determined when x_c, x_n, x_ε, and ε are known [3].

In our model, every result of measurements x_i is transformed into a fuzzy set A_i. Thus, in terms of membership functions, we take an x from the real interval $[x_{\min}, x_{\max}]$ (where $x_{\min}$ is the minimum measured datum, and $x_{\max}$ is the maximum datum), and $\mu A_i(x) = 0$. Nevertheless, one must correctly describe the systematic effects on the measurement results, as it is intrinsic to the concept of systematic effect, and it always appears with the same "impact" during the execution of all the tests. The ith result is such that $x_i - x_i^* = s + a$ (s and a, respectively, are related to the systematic and random effects on the measurement results, whereas x_i^* represents the true value and is therefore unknown). Unfortunately, no information may be given to us about the sign of component s, but it is considered to be contained in the interval $[-\delta, \delta]$, i.e., that δ is the maximum absolute value of s, as authors have expressed in [3]. Accordingly, the true measurand value should

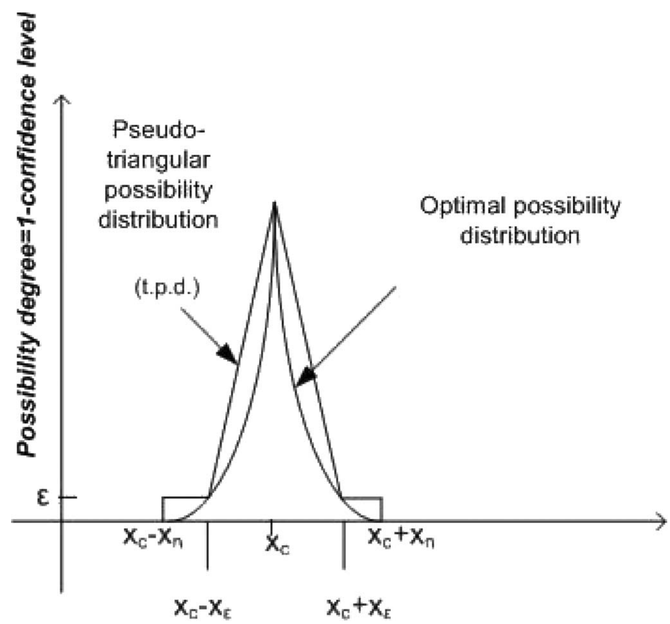

Fig. 2. General shape of the t.p.d.

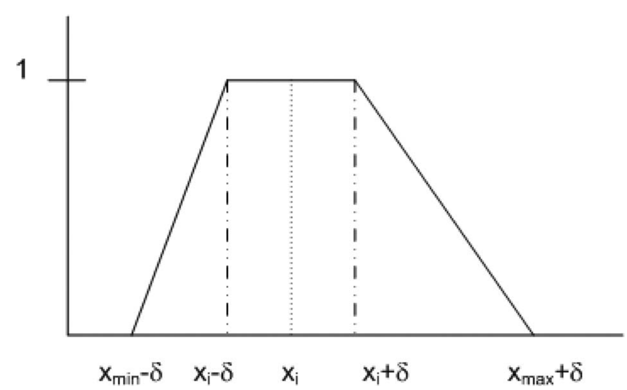

Fig. 3. Membership function of the ith results of a single measurement.

stay between $x_i - \delta$ and $x_i + \delta$ with the maximum possibility (δ is an unsigned real number). Now, let us define the ith fuzzy set as follows:

$$\mathrm{Supp}(A_i) = [x_{\min} - \delta, x_{\max} + \delta] \tag{10}$$

$$\mathrm{Ker}(A_i) = [x_i - \delta, x_i + \delta] \tag{11}$$

where A_i is a trapezoidal interval (Fig. 3).

We have chosen the averaging operation E_T for the fuzzy interval series $\{A_i\}^N$ as an estimator of measurand, which is in agreement with the Guide. If we say that a relation (or

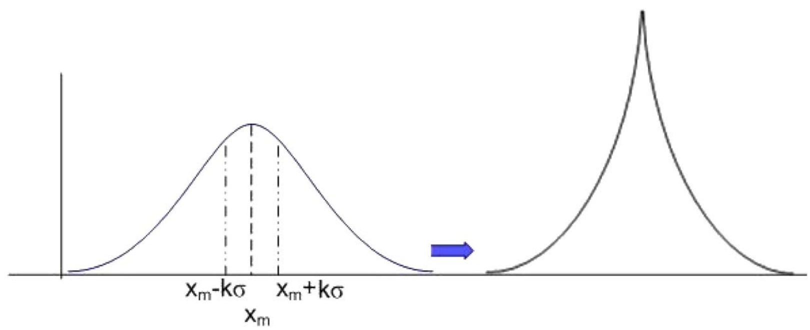

Fig. 4. (Right) Optimal possibility distribution resulting from (left) transformation of the Gaussian law.

more) is established among the single measurement results (crisp numbers), a fuzzy conditional rule is needed to express this connection. Some authors have mentioned two examples where fuzzy sets can be fruitfully applied: 1) classes with vaguely defined boundaries and 2) numbers that are only known to lie within an interval (which is the case of a measurement result, as well known). The aforementioned examples, as well as many similar examples appearing throughout the literature, involve uncertainty about the degree to which objects belong to sets; on the other hand, the manner in which fuzzy sets are *combined* (e.g., by unions and intersections) *does not involve any uncertainty*. For example, given two fuzzy sets A and B and an element x, the degree to which x belongs to both A and B is given by $A(x)\hat{}B(x)$; no uncertainty is involved in the application of the $\hat{}$ connective. A natural extension of the principle of fuzziness is to consider the use of multivalued t-norms.

The triangular norms are the operations that are considered to fit the notion of conjunction as much as possible, and this is why we want to apply extended addition and multiplication based on the t-norm.

IV. Parameterized Archimedean Dombi's t-Norm

Earlier in this paper, we showed our appreciation of the work of Urbanski and Wasowsky [5]. The treatment of the subject is quite complete and simply expressed, and the final result is a trapezoidal fuzzy set that is simple to handle. This treatment was a more natural and simple approach, but it does not really consist of a probability/possibility transformation, because *a priori* knowledge of the range or dispersion of measurements is excluded. The reason why we propose a more natural expression of the ith fuzzy set in such a way is that when given a vector of data, the conventional partition of the universe of discourse implies that each element belongs to the data set with a certain degree of belonging. Hence, if the vector is a group of repeated measurement results, it is reasonable to think that the extreme results have the minimum possibility degree to be considered part of the measure sets. We assume this minimum value as a zero level of possibility (excepted for the fuzzy set centered on the extreme measures). However, most of all, the real shape of the membership function should not be linear. In fact, if we can be sure that N measures are normally distributed, then

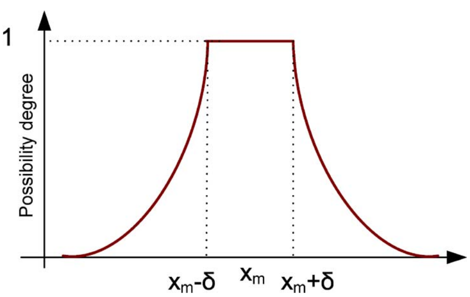

Fig. 5. Membership function of the "expected" fuzzy set result.

the probability/possibility transformation of Fig. 4 is executed according to the following formula:

$$\begin{aligned}\int_{x_m-k\sigma}^{x_m+k\sigma} \mathrm{pdf}_X(x)dx &= 1-\alpha \\ &= 1-\mu_X(x_m-k\sigma) = 1-\mu_X(x_m+k\sigma). \quad (12)\end{aligned}$$

Thus, if one hypothesizes that each of the N measures is affected by B-type uncertainty (i.e., systematic effects of repeated measurements on the final result), as represented in Fig. 3, the final set should have a membership function similar to that of the amaranthine in Fig. 5. As a first trial solution, we look for a parameterized triangular norm, which may be used instead of Yager's, to obtain a meaningful approximation of the shape of Fig. 5.

The Dombi operator ${}_p\otimes$ is defined as [7]

$$a_p \otimes b = \begin{cases} 0, & a=0 \text{ or } b=0 \\ \frac{1}{\left(\left(\frac{1}{a}-1\right)^p+\left(\frac{1}{b}-1\right)^p\right)^{1/p}+1}, & \text{otherwise.} \end{cases} \quad (13)$$

For fuzzy numbers with identical spreads like $\widetilde{a}_i = (a_i, b_i, \beta_L, \beta_R)LR, i=1,\ldots,n$, one gets (14) and (15), shown at the bottom of the next page, for the membership function of Dombi's t-norm-based sum $\widetilde{A}_N = \widetilde{a}_1 + \cdots + \widetilde{a}_N$, where $A_N = \sum_i a_i$, $B_N = \sum_i b_i$, and L (which is the same for R) obviously has to be replaced with the concave shape of the membership function for LR numbers or intervals ($L(z) = R(z) = 1 - z$ for trapezoidal intervals).

According to (14), the left and right spreads for Dombi's t-norm-based sum of LR-fuzzy intervals are $N\beta_L$ and $N\beta_R$, respectively. Fig. 6 shows the membership function shape of the inferred set with Dombi's t-norm. Thus, to approximate in

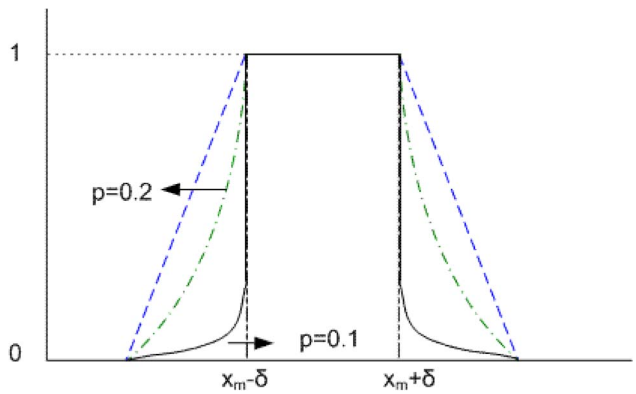

Fig. 6. Inferred fuzzy set with Dombi's adaptive t-norm.

a better way the Gaussian distribution of repeated measures, we have proposed for the first time the use of Dombi's t-norm to build the average set A^{AV}.

V. Comparative Analysis

In [4], the authors suggested the use of RFVs to describe all the possible effects of repeated measurement acts on a quantity (Fig. 7).

According to Fig. 7, a possible value x belonging to the intervals $[x_1^\alpha, x_2^\alpha]$ or $[x_3^\alpha, x_4^\alpha]$ is supposed to be randomly distributed, according to a given probability density function, whereas the interval $[x_3^\alpha, x_4^\alpha]$ represents a confidence interval with confidence level $1-\alpha$ describing the systematic effects of the repeated measurement acts. Despite the fact that this is a good idea, a weak point of such an approach is that the RFV describing both the systematic and random effects is built by roughly summing the intervals related to them, and then, two different algebraic tools are used to compose data belonging to internal and external intervals. Our opinion is that it is lacking in sense, because once the RFV is created using the two membership functions, the probability/possibility transformation has been made, and there is no longer a distinction in the way to treat internal and external intervals but only the significance that we may attribute to them. The formula used to reach the final result (the uncertainty interval for a given confidence level) seems to

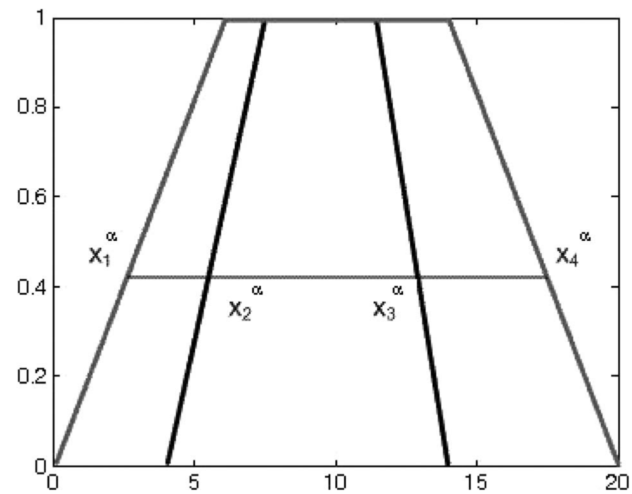

Fig. 7. Example of an RFV [4].

be a method that patches up the inconvenience of the impossibility of having a good interpretation of random and systematic effects through the simple observation of the obtained RFV. The formula, moreover, does not exhaustively explain how to obtain the uncertainty interval for a given confidence level in the event of any distribution but the Gaussian distribution.

An alternative approach, as shown in [5], is the t-norm distribution. First, let us see some useful theoretical hints.

In [8], we see some well-known continuous t-norms, such as the minimum operator T_M, the algebraic product T_P, and the Lukasiewicz t-norm T_L.

T_M is the strongest (greatest) t-norm, which is defined as

$$T_M(x,y) = \min(x,y). \tag{16}$$

T_W is the weakest (smallest) t-norm, i.e.,

$$T_W(x,y) = \begin{cases} \min(x,y), & \text{if } \max(x,y) = 1 \\ 0, & \text{elsewhere.} \end{cases} \tag{17}$$

The following inequality holds:

$$\begin{aligned} T_W(a,b) \le T_L(a,b) &= \max(a+b-1, 0) \\ &\le ab \le \min(a,b), \qquad \forall a,b \in [0 \quad 1]. \end{aligned} \tag{18}$$

$$\tilde{A}_N(z) = \begin{cases} 1, & \text{for } A_N \le z < B_N \\ \left[1 + N^{1/p}\left(\frac{1}{L\left(\frac{A_N - z}{N\beta_L}\right)} - 1\right)\right]^{-1}, & \text{for } A_N - N\beta_L \le z < A_N \\ \left[1 + N^{1/p}\left(\frac{1}{R\left(\frac{z - B_N}{N\beta_R}\right)} - 1\right)\right]^{-1}, & \text{for } B_N \le z < B_N + N\beta_R \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

$$E_T(\{\tilde{a}_i\}^N) = \begin{cases} 1, & \text{for } \frac{A_N}{N} \le z < \frac{B_N}{N} \\ \left[1 + N^{1/p}\left(\frac{1}{L\left(\frac{A_N/N - z}{\beta_L}\right)} - 1\right)\right]^{-1}, & \text{for } \frac{A_N}{N} - \beta_L \le z < \frac{A_N}{N} \\ \left[1 + N^{1/p}\left(\frac{1}{R\left(\frac{z - B_N/N}{\beta_R}\right)} - 1\right)\right]^{-1}, & \text{for } \frac{B_N}{N} \le z < \frac{B_N}{N} + \beta_R \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

Moreover, for any triangular norm T

$$T_W(a,b) \leq T(a,b) \leq T_M(a,b). \tag{19}$$

The corollary described in [8] suggests that for the addition based on the minimum operator, the resulting spreads are the sum of the incoming spreads, whereas for the addition based on the weakest t-norm, the resulting spreads are the greatest of the incoming spreads.

Letting T_1 and T_2 be two t-norms such that $T_1(a,b) \leq T_2(a,b)\ \forall a,b \in [0\ \ 1]$, then for any fuzzy quantities A and B, the T-sum of these fuzzy intervals is $A \oplus_{T_1} B \leq A \oplus_{T_2} B$ (see [9]).

One can readily take note of a first difference between the result of the average of fuzzy sets, as shown in Fig. 3, based on Yager's t-norm and that based on Dombi's: in the former case, the addition of trapezoidal fuzzy sets is just a trapezoidal fuzzy set (see [5]), whereas in the latter case, one may find what is described in Fig. 6, which is a more realistic case, if the probability/possibility transformation is considered. From a numerical application of Yager's t-norm, we have noted that the interval of possibility for the fuzzy model and the extended uncertainty for the probabilistic model are very similar for $p \in (2, 3)$ but are not identical. We have noted that the α-cut interval proves to be rather narrow when p is less than 2 and when a large body of data is unavailable. We propose the use of Dombi's t-norm with $p \in (0.2, 0.3)$ to build the fuzzy variable A^{AV}, as follows:

$$a_L^{\mathrm{AV}} - \beta_L^{\mathrm{AV}} = x_{\min} - \delta, \quad a_R^{\mathrm{AV}} + \beta_R^{\mathrm{AV}} = x_{\max} + \delta$$
$$a_L^{\mathrm{AV}} = \frac{\sum_i a_L^i}{N}, \quad a_R^{\mathrm{AV}} = \frac{\sum_i a_R^i}{N}.$$

By varying the parameter p of Dombi's, one can obviously modify the curve shape of the left and right spread profiles. Thus, when averaging $E_T(\{A_i\}^N)$, one obtains a fuzzy number that is fully determined by the membership function of (14). Then, one can express the measure of uncertainty (with confidence level of $1-\alpha$) as half-width of α-cut, and to obtain the α-cut interval, it is sufficient to find z_1 and z_2 such that $z_1 < z_2$ and $\mu_{A^{\mathrm{AV}}}(z_1) = \mu_{A^{\mathrm{AV}}}(z_2) = \alpha$. Then

$$\mu_\alpha(A^{\mathrm{AV}}) = \frac{1}{2}(z_2 - z_1). \tag{20}$$

The fuzzy set representation of measurement uncertainty is preferable in all the cases in which measures acquired have to be further processed, and in these events, a conversion into a symbolic format may also be necessary to supply reasoning systems.

VI. Conclusion

In this paper, we have proposed a novel use of a particular triangular norm to define the arithmetic on fuzzy intervals to describe both the systematic and random effects on the distribution of measurement results. The purpose of this approach was to obtain measurement results that can be considered compatible with the ISO Guide method and that may be considered of great advantage when ensuring that correlation terms are considered when a long measurement chain occurs.

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