A Comparative Analysis of the Statistical and Random-Fuzzy Approaches in the Expression of Uncertainty in Measurement

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***Abstract*—The present practice for uncertainty expression and estimation in measurement, endorsed in the IEC-ISO Guide to the Expression of Uncertainty in Measurement, is based on a statistical approach, which is also the basis for the Monte Carlo method generally employed to overcome the problems met in the strict application of the guide. More recently, methods based on the fuzzy theory have been proposed too, with encouraging results. This paper compares the results obtained, in the expression of uncertainty, by the use of the Monte Carlo method and the random-fuzzy variable method. Both methods are applied to a real, digital signal processing-based instrument for electric power quality measurement, and the obtained results are compared and discussed.**

***Index Terms*—Measurement uncertainty, Monte Carlo method, power quality measurement, random-fuzzy method.**

I. Introduction

IN THE present measurement practice, uncertainty in measurement is supposed to be originated only by random contributions. In fact, the reference standard, the IEC-ISO Guide to the Expression of Uncertainty in Measurement [1], recommends applying corrections for all possible and recognized systematic effects, so that the remaining sources of deviation of the measured values from the expected measurement result can be attributed to random effects only.

Under these assumptions, the statistical approach to the expression and estimation of uncertainty in measurement appears to be the most suitable one, since the distribution of the possible measurement results can be fully described by a probability distribution function.

If the hypotheses of the central limit theorem are also supposed to be satisfied, this distribution can be fully represented by its first two moments, that is, its mean value and its variance. These considerations are the theoretical basis of the recommendations of the guide [1] for expressing and combining uncertainty in measurement.

Although these recommendations are very effective in a large number of practical situations, still a number of situations remain where their application is sometimes troublesome (when a large number of individual measurement results must be combined), sometimes not cost-effective (when the correction for all systematic effects is too expensive), and sometimes even not possible (when the relationship between each single measurement result and the final result is not derivable) [2]–[5].

In order to overcome these problems, different approaches to the expression of uncertainty in measurement than the one proposed by the guide [1] have been recently proposed in the literature. To the authors' knowledge, the most significant ones are those based on the Monte Carlo method [5], [6], which has been recently considered in a draft supplement to the guide [7], and those based on the fuzzy variables [8]–[11] and the random-fuzzy variables (RFVs) [12], [13].

On the other side, the approach based on the RFVs is framed in the more general theory of evidence and allows one to take correctly into account both the random and nonrandom contributions to uncertainty [12]. Moreover, its implementation does not require a derivable relationship linking the final measurement result to the single measurement results when the combined uncertainty has to be expressed, so that this approach can be used also when decision rules are embedded in the measurement algorithm [14].

The comparison of the results obtained by means of the Monte Carlo method and the RFV method is hence quite interesting in assessing the validity of these methods and their optimal application fields.

This paper is aimed at comparing the two methods on a real measurement system for electric power quality measurement, since the need for such measurement systems is increasing more and more and they represent a significant example of complex measurement systems; moreover, they show all of the problems listed above when their measurement uncertainty has to be estimated.

The following sections recall briefly the measurement application chosen for the comparison, give the procedure for the uncertainty estimation by means of the Monte Carlo and RFV methods on a real instrument, and show the results of the comparison.

II. Measurement Application

One of the most challenging measurement applications in the field of electric power systems is concerned with the estimation of the supply and loading quality in the presence of harmonic and nonharmonic disturbances on voltages and currents.

The metrological characterization of the instrumentation devoted to such measurements is even more challenging: in fact, the employed measurement systems are generally digital signal-processing (DSP)-based instruments, executing complex algorithms, with embedded decisions, on a large number of

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input samples [15]. Their metrological characterization seems to catalog almost all problems that may be encountered in the application of the guide [1]. For this reason, such a measurement application is here considered as a meaningful test bench for comparing the different methods for uncertainty estimation.

As far as electric power quality measurements in a three-phase three-wire system are concerned, the following quantities [16] are considered as the most interesting.

A. Collective RMS Values of Voltages and Currents

These quantities are defined, for a three-wire system, as [16]

$$U_\Sigma = \sqrt{\sum_{j=1}^{3} U_{\mathrm{L}_j}^2} \quad \text{and} \quad I_\Sigma = \sqrt{\sum_{j=1}^{3} I_{\mathrm{L}_j}^2} \tag{1}$$

with U_{L_j} and I_{L_j} being the root mean square (rms) values of the zero-sum line voltages and the line currents, respectively.

B. Total Active Power

This quantity is defined as [16]

$$P_\Sigma = \frac{1}{T} \cdot \int_{t-T}^{t} (u_{\mathrm{L}_1}(\tau)\cdot i_{\mathrm{L}_1}(\tau) + u_{\mathrm{L}_2}(\tau)\cdot i_{\mathrm{L}_2}(\tau) + u_{\mathrm{L}_3}(\tau)\cdot i_{\mathrm{L}_3}(\tau))d\tau \tag{2}$$

where T is the period of voltages and currents.

C. The Global Total Harmonic Distortion Factors

These quantities are defined as [15]

$$\mathrm{GTHD}_{\mathrm{U}}^{+} = \sqrt{\frac{U_\Sigma^2}{U_{\Sigma_{+1}}^2} - 1} \quad \text{and} \quad \mathrm{GTHD}_{\mathrm{I}}^{+} = \sqrt{\frac{I_\Sigma^2}{I_{\Sigma_{+1}}^2} - 1} \tag{3}$$

where $U_{\Sigma_{+1}}$ and $I_{\Sigma_{+1}}$ are the collective rms values of the fundamental frequency, positive sequence components of the line voltages and currents, respectively. These factors quantify the conformity of the considered signals to balanced, positive sequence sinewaves.

D. The Supply and Loading Quality Index

This index is defined as [15]

$$\xi_{\mathrm{slq}} = \frac{P_\Sigma}{P_{\Sigma_{+1}}} \tag{4}$$

where $P_{\Sigma_{+1}}$ is the total active power associated with the fundamental frequency, positive sequence components of voltages and currents.

It can be proved [15] that, when the distortion and/or unbalance of the supply prevail over the load distorting and unbalancing effects, $\xi_{\mathrm{slq}} > 1$. On the contrary, when the load distorting and/or unbalancing effects prevail over the supply voltage distortion and/or unbalance, $\xi_{\mathrm{slq}} < 1$.

E. Harmonic Global Index

This index is defined as [17]

$$\xi_{\mathrm{HGI}} = \frac{\|\mathbf{I}_{\Sigma_{\mathrm{L}}}\|^2}{\|\mathbf{I}_{\Sigma_{\mathrm{S}}}\|^2} \tag{5}$$

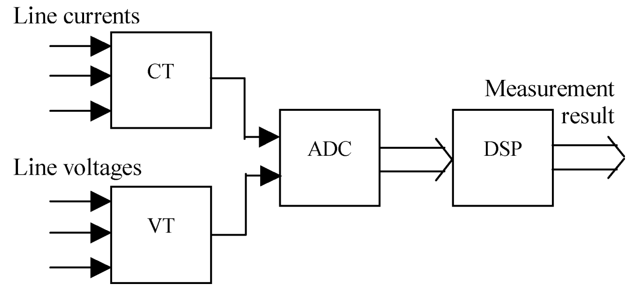

Fig. 1. Basic structure of a three-phase power quality meter based on DSP techniques.

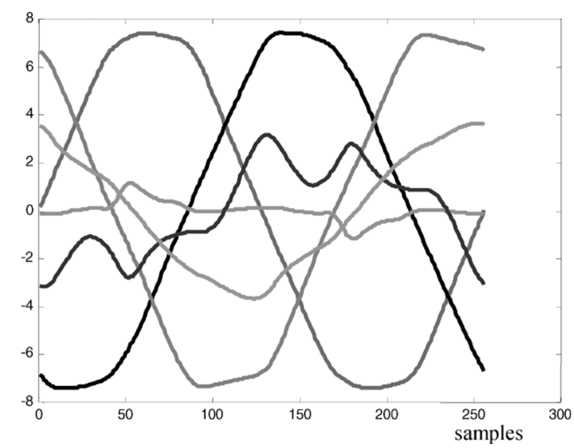

Fig. 2. Acquired signals.

where $\mathbf{I}_{\Sigma_{\mathrm{L}}}$ is the vector of the collective rms values of the harmonic and sequence components associated with active powers reflected backward from the load to the source and $\mathbf{I}_{\Sigma_{\mathrm{S}}}$ is the vector of the collective rms values of the harmonic and sequence components associated with active powers flowing from the source toward the load. The higher the value assumed by (5), the higher the load contribution to distortion.

It is worth while noting that the estimation of the measurement uncertainty for this index is quite troublesome, because the assignment of the current harmonic components to vectors $\mathbf{I}_{\Sigma_{\mathrm{L}}}$ and $\mathbf{I}_{\Sigma_{\mathrm{S}}}$ depends on the uncertainty associated to each measured value of the harmonic active powers. The different ways with which the Monte Carlo method and the RFV method take into account this effect will represent a meaningful index of their effectiveness in correctly estimating the uncertainty associated with the measured value of ξ_{HGI}.

III. Uncertainty Estimation

The quantities and indexes listed in the previous section have been measured by a dedicated DSP-based instrument [15]. The block diagram of this instrument is reported in Fig. 1.

In order to estimate the uncertainty associated to each measured value, the signals shown in Fig. 2 have been considered. These signals have been acquired by the system shown in Fig. 1.

The measurement algorithm has been executed in order to apply both the Monte Carlo method and the RFV method. With both methods, the same contributions to the measurement uncertainty due to the analog-to-digital conversion (ADC) gain, offset, and quantization errors have been considered, while the voltage (VT) and current (CT) transducers have been supposed to be ideal, for the sake of simplicity.

A. Monte Carlo Method

As far as the Monte Carlo method is concerned, each measurement algorithm is processed 1000 times, each one with slightly different signals, obtained by corrupting each ideal sample of the acquired signals with a different extraction of the ADC gain, offset, and quantization errors from their probability distributions. These probability distributions have been obtained experimentally, after having compensated the ADC offset and gain errors and confined them in the $\pm 1/2$ LSB range [5]. Therefore, a type A evaluation of the uncertainty due to the ADC offset and gain errors has been accomplished. This evaluation showed that, after compensation, offset and gain errors affect each sample in a different way [5]. Therefore, all extractions of these values from the associated probability distributions will be considered as noncorrelated in the following. As for the quantization error, nothing can be said except that it falls inside the $\pm 1/2$ LSB interval. This situation is the typical situation of total ignorance which, in the probability theory, is represented by a uniform probability distribution. As far as the transducers are concerned, they are supposed to be ideal, and therefore their gain is supposed to be constant and equal to the nominal value, and their time delay between the input and output signals is supposed to be null. In order to obtain the required measurement results, each ideal sample is reported at the input of the measurement chain.

Therefore, each single acquired sample s_n is changed into a distribution y_n of samples, whose single element $y_{i,n}$ is obtained as

$$y_{i,n} = \frac{1}{g_{tr}} \cdot \left[\frac{(s_n - q_{i,n})}{g_{i,n}} - o_{i,n} \right] \tag{6}$$

where $g_{i,n}$, $o_{i,n}$, and $q_{i,n}$ are random extractions of gain, offset, and quantization of the ADC from their corresponding probability distributions and g_{tr} is the nominal gain of the employed transducers.

This procedure provides 1000 different results for each considered quantity or index, so that a relative frequency histogram can be drawn, approximating the probability distribution of the measurement result [5].

B. Random Fuzzy Variable (RFV) Method

The RFV method is based on the representation of a measurement result and its associated uncertainty in terms of an RFV, as shown in [12], to which the reader is addressed for the definition and all mathematical details.

As far as the method application is concerned, each sample s_n of the acquired signals is converted into an RFV according to the following relationship:

$$S_n = \frac{1}{g_{tr}} \cdot \left[\frac{(s_n - Q)}{G} - O \right] \tag{7}$$

where G, O, and Q are the RFVs representing the ADC gain, offset, and quantization, respectively. G and O have been obtained from the same experimental characterization as that from which the probability distributions employed with the Monte Carlo method have been obtained [13]. The same situation of total ignorance is still considered for the quantization error. In the possibility theory, total ignorance is represented by a uniform possibility distribution that is a pure fuzzy variable with a rectangular membership function [12]. As in the previous case, g_{tr} is the nominal gain of the employed transducers. Therefore S_n represents each actual sample of the signals at the input of the measurement chain. With this representation, each sample can be interpreted as the result of a measurement together with its associated uncertainty due to the ADC gain, offset, and quantization errors.

With this method, each measurement algorithm from (1) to (5) is executed only once on the obtained RFVs, and the procedure provides each considered quantity or index in terms of an RFV. In this way, according to the definition of RFV [12], the distribution of all possible measurement results is obtained as a set of nested confidence intervals, together with the associated level of confidence.

IV. Comparison

The following figures show the comparison between the distributions of the measured values in terms of relative frequency histograms and RFVs. In all figures, the upper plot shows the histogram of the relative frequencies for the 1000 measurement results obtained by the application of the Monte Carlo method; the lower plot shows the RFV representing the measurement result obtained executing the measurement algorithm on the samples expressed in terms of RFVs. In the same figures, the expected value, evaluated in absence of uncertainty contributions, is also reported as a black vertical line in the lower plot.

Figs. 3 and 4 show the collective rms value of voltages and currents, respectively. Fig. 5 shows the total active power of the three phase system. Figs. 6 and 7 show the global total harmonic distortion factors of voltages and currents respectively. Fig. 8 shows the supply and loading quality index. Fig. 9 shows the harmonic global index.

Figs. 3–8 clearly show the good agreement between the two considered approaches. The expected value is always included in both the relative frequency distribution and the RFV.

The width of the relative frequency distributions is always narrower than that of the RFVs. This is compatible with the theories on which the two different approaches are based. In fact, the Monte Carlo method is a typical statistical approach, and all contributions to the measurement uncertainty are treated as random contributions. This leads to a compensation of the uncorrelated uncertainty contributions. Therefore, when, as in this case, the prevailing contributions to uncertainty are supposed to be uncorrelated, the greater the number of computations in the measurement algorithm is, the narrower the relative width of the corresponding distribution is.

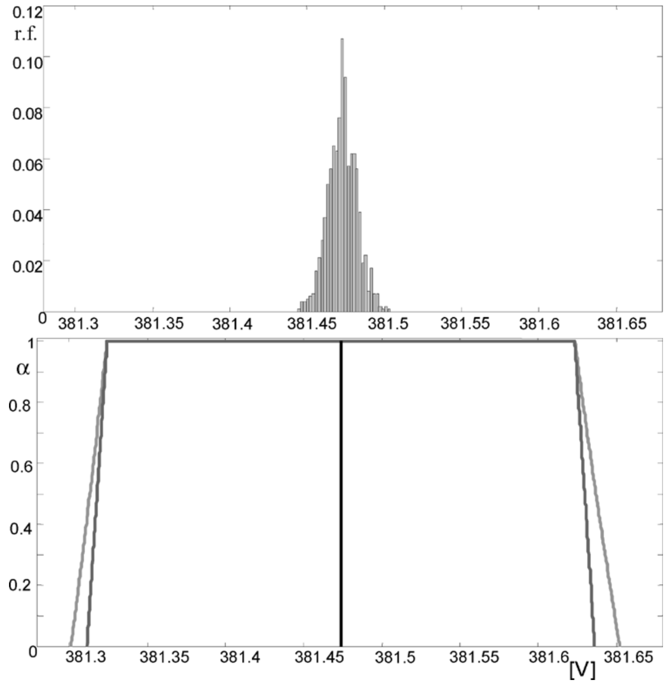

Fig. 3. Collective rms value of voltages.

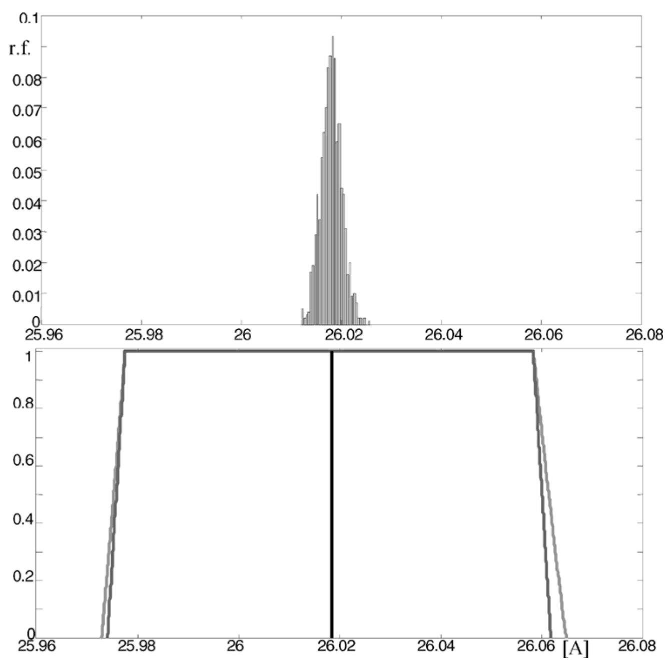

Fig. 4. Collective rms value of currents.

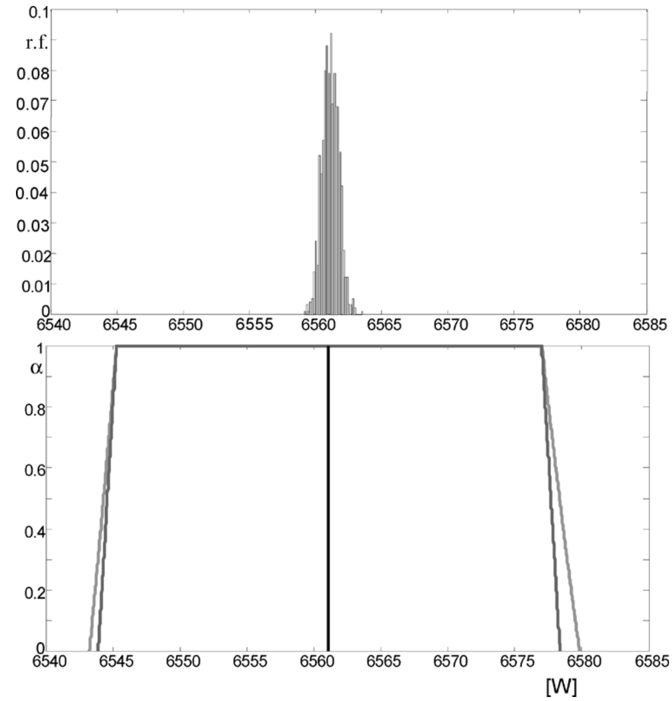

Fig. 5. Total active power.

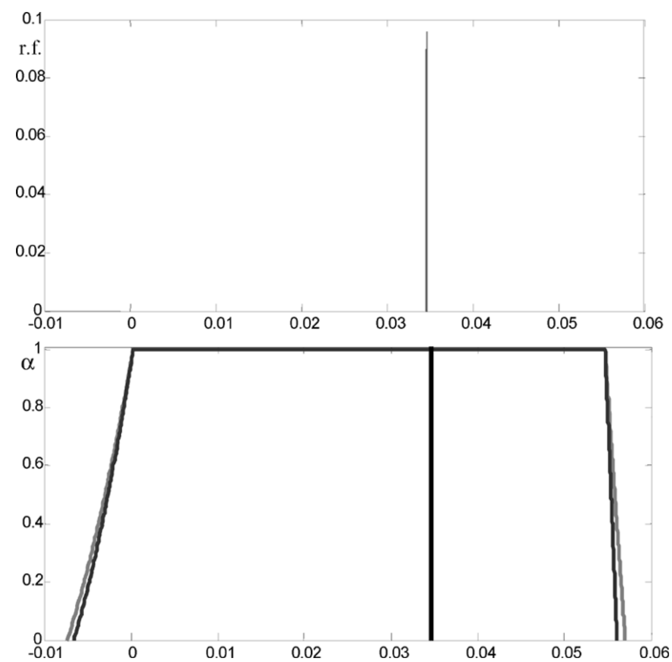

Fig. 6. Global total harmonic distortion factor of voltages.

The plot in Fig. 9, representing the harmonic global index, deserves more consideration. This index is in fact the most critical one, since its definition contains a series of *if…then…else* structures: the current harmonic components are assigned to the two different sets $\mathbf{I}_{\Sigma_L}$ and $\mathbf{I}_{\Sigma_S}$ depending on the value (positive or negative) of the correspondent active power harmonic component. Since, when the RFV approach is followed, each intermediate measurement result is expressed by an RFV, the evaluation of index (5) requires the definition of a suitable decision rule [14]. Of course, the decision is immediate when the whole RFV is above the zero value, or below it. On the contrary, the decision is troublesome when the RFV lies across the zero value, as shown in Fig. 10. Since the RFV subtends an area, it can be said [14] that the RFV is greater than zero if the relative area from zero to the right extreme of the RFV is greater than the relative area from the left extreme to zero. Of course, this last decision is not as certain as the one taken in the two previous cases; therefore, it is useful to associate a credibility factor λ to the decision taken. When the decision is taken with full certainty, the credibility factor is one; on the contrary, the credibility factor is, numerically, the relative area subtended by

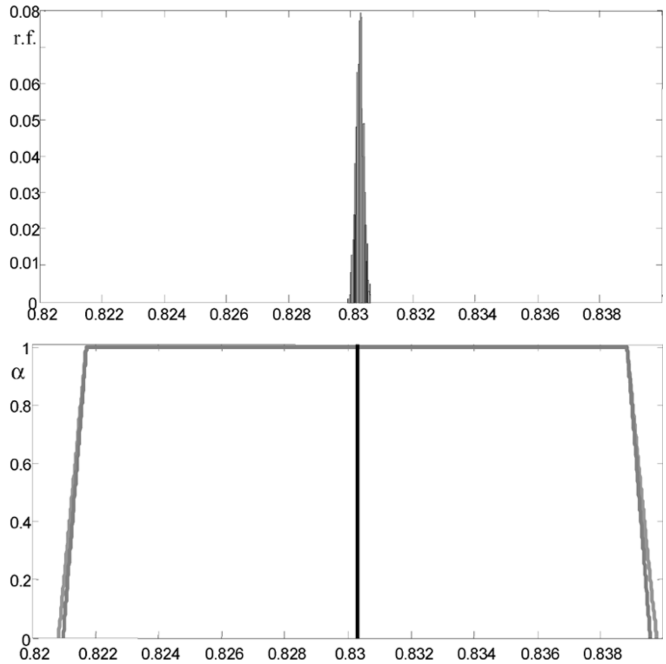

Fig. 7. Global total harmonic distortion factor of currents.

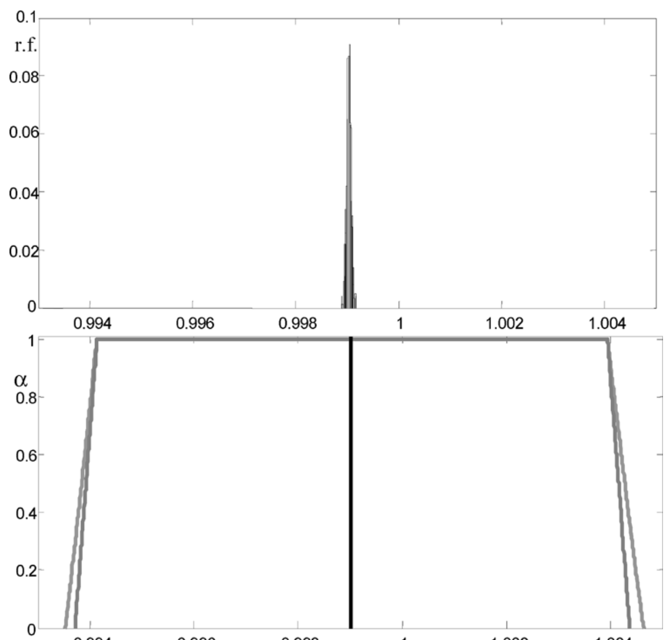

Fig. 8. Supply and loading quality index.

that part of the RFV above, or below, the zero value. Of course, in this last case, the sum of the two credibility factors related to the two opposite decisions is one. Finally, the decision that must be taken is the one to which corresponds the greater credibility factor ($\lambda > 0.5$) [14].

Moreover, when the measurement algorithm requires many decisions to be taken, and the final result depends on all of them, a credibility factor is associated to the final result, given by a suitable combination of the single credibility factors of each decision [14]. In this particular case, the final credibility factor is the mean value of the credibility factors associated with each single decision taken [14]. Applying this procedure, the RFV in

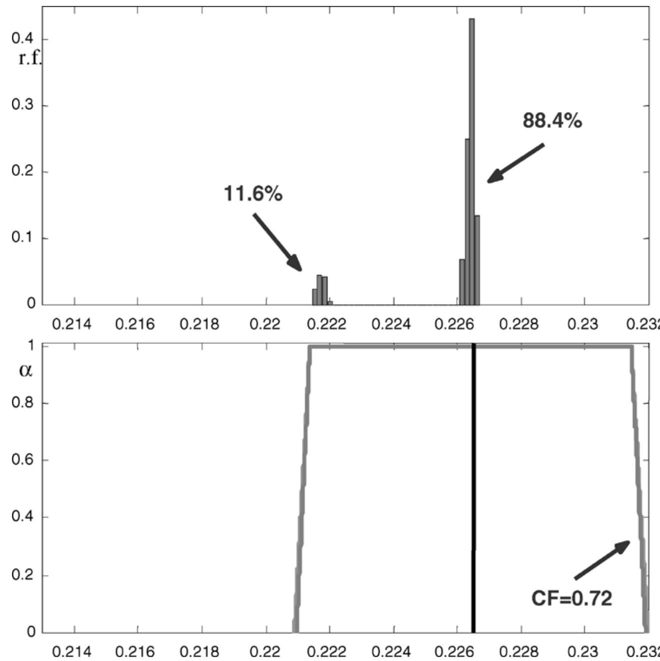

Fig. 9. Harmonic global index.

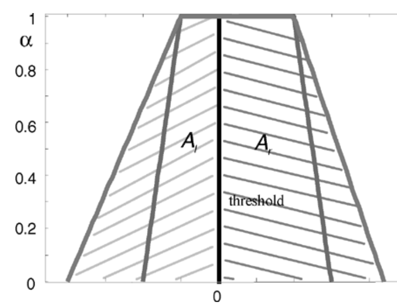

Fig. 10. Example of implementation of the decision rules on an RFV. The RFV is lower than the threshold with a credibility factor A_l/A, and it is greater with a credibility factor equal to A_r/A; A is the total area.

the lower part of Fig. 9 is obtained, to which a credibility factor equal to 0.72 is associated.

On the contrary, when the Monte Carlo method is followed, the relative frequency histogram in the upper part of Fig. 9 is obtained. The histogram is actually divided into two separate subhistograms that are due to the way the Monte Carlo method works. In fact, each new extraction of the input samples by means of (6) provides new values for the harmonic active powers. According to their sign, each current harmonic component is assigned to one set or the other. If the distribution of a given harmonic active power (obtained from the 1000 values of the same harmonic active power, provided by the 1000 executions of the measurement process) falls across the zero value, the same value of the current harmonic component is assigned sometimes to one set and sometimes to the other, thus giving rise to the double peak of the distribution of index (5), shown in the upper plot of Fig. 9.

Of course, the probability that the measured value of index (5) falls into one of the two subhistograms can be computed from the histograms themselves, as reported in Fig. 9. Obviously, the expected value of (5) falls within only one subhistogram, that is, the one with the higher probability value. Moreover, this subhistogram is well centered in the obtained RFV, while the second subhistogram is almost outside it.

V. Conclusion

This paper presents a comparative analysis of the Monte Carlo and the RFV approaches in the expression of the measurement uncertainty and its propagation through the measurement algorithm. The two approaches have been shown to be in perfect agreement, even if the second one is sometimes preferable, for at least three reasons.

1) The Monte Carlo method provides the measurement results with an underestimation of the measurement uncertainty when nonnegligible unknown systematic contributions are present.

   In fact, when a statistical approach is followed, as in the case of the Monte Carlo method, all uncertainty contributions are treated as random ones, so that they tend to compensate each other during their propagation through the measurement algorithm. However, it is known that this is the typical behavior of uncorrelated random contributions to the measurement uncertainty, while uncompensated systematic contributions act in the opposite way. This may lead to an underestimation of the measurement uncertainty.

2) The Monte Carlo method takes a much longer time to be implemented, despite the relatively high computational burden of the RFV approach.

   In fact, the computational burden for the RFV approach is quite high because all measurement algorithms must be implemented on RFVs that are represented, in mathematical terms, by matrices [12], [13]. On the other hand, the Monte Carlo method works on the simple samples, and therefore each iteration surely requires a shorter time. However, the Monte Carlo method also requires a great number of iterations, thus resulting in a much longer execution time. Let us also consider that the value chosen for the number of iterations in the Monte Carlo method in this paper (1000) represents the minimum number of iterations that ensures meaningful results. For the simulations described in the paper, which have been implemented in MatLab, a 1 GHz Pentium III PC has been used. The Monte Carlo approach took about 6 h, while the random-fuzzy approach took about 30 min.

3) The results provided by the Monte Carlo approach might be misleading when the measurement process contains *if…then…else* structures.

   In fact, when the RFV approach is followed, each decision is taken by considering the whole RFV of the intermediate result involved in the decision itself. On the contrary, when the Monte Carlo method is followed, the decision must be taken a number of times equal to the number of iterations, each time by considering the actual value of the intermediate result, obtained by processing the measurement algorithm over extractions of the input samples. In case the considered intermediate result takes values that are near the discriminating threshold [values near zero in the case of the evaluation of index (5)], the decisions can be not coherent with each other. Therefore, a certain number of times the measurement algorithm is executed considering a positive decision, and a certain number of times the measurement algorithm is executed considering a negative decision. This leads to two separate distributions for the final measurement result, as happened in the evaluation of index (5), as shown in Fig. 9. The separate distributions are more difficult to interpret than a single RFV.

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