

# Fully Comprehensive Mathematical Approach to the Expression of Uncertainty in Measurement

Alessandro Ferrero, *Fellow, IEEE*, and Simona Salicone, *Member, IEEE*

**Abstract**—This paper analyzes the result of a measurement in the mathematical model of incomplete knowledge and shows how it can be treated in the framework of the theory of evidence. The random fuzzy variables are considered in order to express the result of a measurement together with its uncertainty, and a significant application is considered to prove the practical utility of the proposed approach.

**Index Terms**—Theory of evidence, uncertainty.

## I. INTRODUCTION AND HISTORICAL BACKGROUND

ONE OF THE MOST challenging—probably the most challenging—part of the measurement activity is the qualification of the obtained measurement result—that is, the estimate of how different this result is from the actual value of the measurand.

It has been clear, since the early beginning of the scientific experimental activity, that any measurement process is capable of providing only an approximation of the value of the measurand, due to a number of nonideal situations affecting the employed measuring instruments and devices, the adopted measurement method, the measurement conditions, the model with which the measurand is described, or the operator. In general, everything, known or unknown, may interact with the measurement process.

It has also been soon clear that any measurement result would have been absolutely useless without an estimate of how different it was from the actual value of the measurand. In order to provide useful measurement results, the measurement science has, from the start, dealt with the fundamental problem of how the inaccuracy of the measurement result could be expressed and estimated.

This problem shows also a further difficulty, since many quantities cannot be measured in a direct way, but can be known only starting from other measured quantities. Therefore, not only must the measurement result be expressed in such a way that its deviation from the value of the measurand can be estimated, but it is also necessary to find a way to combine these estimates so that the result of indirect measurements can also be qualified.

Until a few decades ago, this problem was dealt with in a deterministic way by means of the measurement-error concept. The error concept was based on the fundamental assumption

that the true value of the measurand could be known so that the measurement error could be expressed as the difference between the result of the measurement and the true value.

The usual way to qualify the result of a measurement was based on the estimate of the maximum error, and then the estimate of an interval, centered about the result of the measurement, and showing a width equal to twice the estimated maximum error. The propagation of the measurement error, when indirect measurements were taken into account, could be obtained by applying the mathematics of the intervals to the intervals defined for each single measurement result.

More recently, this approach has been rejected and, in particular, two points have been considered quite critically. The first one, which is based more on philosophical considerations rather than on technical ones, is concerned with the impossibility of knowing the true value of the measurand. The second one pointed out that the way the error theory handled error propagation was correct only when the errors were due to uncompensated systematic effects, but disregarded the possible mutual compensation effects that could take place when the errors were due to random effects.

This discussion has led to the formulation of the International Organization for Standardization (ISO)–International Electrotechnical Commission (IEC)–International Organization of Legal Metrology (OIML)–The International Bureau of Weights and Measures (BIPM) Guide to the Expression of Uncertainty in Measurement (GUM) [1], which represents the present reference document in metrology and measurement practice.

In that document, the underlying philosophical assumption is the denial of the true-value concept, which implies, from both the theoretical and practical points of view, the impossibility of knowing the true value of the measurand. In fact, the GUM states: “it is now widely recognized that, when all the known or suspected components of error have been evaluated, and the appropriate corrections have been applied, there still remains an uncertainty about the correctness of the stated result, that is, a doubt about how well the result of a measurement represents the value of the quantity being measured” [1].

Therefore, the qualification of the result of a measurement no longer refers to the estimate of how different it is from an unknown and unknowable true value of the measurand but to the estimate of a new quantifiable attribute of the measurement result, called “measurement uncertainty,” which must also be capable of providing “an interval, about the measurement result within which the values that could reasonably be attributed to the quantity subject to measurement may be expected to lie with a high level of confidence” [1].

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The authors are with the Dipartimento di Elettrotecnica, Politecnico di Milano, Milano 20133, Italy.

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Both the “historical” approach, in terms of measurement error, and the GUM approach, in terms of measurement uncertainty, can be seen, from a mathematical point of view, as an attempt to handle, also from a quantitative point of view, incomplete knowledge and incomplete information about the measurand. Always from the mathematical point of view, this means that we need a theory to estimate, from a quantitative point of view, how incomplete the available information (given by the result of the measurement) is and to process this incomplete information in order to estimate how incomplete the knowledge about a measurand is when the measurement result is obtained by means of indirect measurements.

Up to now, the most widely known and assessed mathematical theory that deals with incomplete information is the probability theory: For this reason, it has been considered by the GUM as the main mathematical tool for expressing and estimating uncertainty [1]. However, the probability theory covers only a particular branch of incomplete knowledge: the one in which the reason for the incompleteness of knowledge is the presence of sole random effects. For this reason, the GUM is forced to state that “it is assumed that the result of a measurement has been corrected for all recognized significant systematic effects” [1]. The limitations implied by such a statement have been widely discussed in [2], together with the limitations implied by the assumptions underlying the uncertainty propagation law suggested in [1], and are only partially removed by the Supplement 1 to the GUM [3], which are now in the revision process.

This discussion has led to the conclusion that a more general theory for dealing with incomplete knowledge is needed, which is capable of taking into account all kinds of effects, and not only the random ones. This more general theory can be found in Shafer’s theory of evidence [4], as shown in [2] and [5].

This paper will briefly reconsider the fundamentals of the theory of evidence, show how the probability and possibility theories can be derived as two distinct particular cases of the theory of evidence, define the random fuzzy variables (RFVs) as the mathematical tool for expressing and processing incomplete knowledge, and show how they can be used for expressing the result of a measurement together with its uncertainty.

## II. THEORY OF EVIDENCE AND THE EXPRESSION OF UNCERTAINTY

From a mathematical point of view, the correct representation of the result of a measurement and its associated uncertainty needs a mathematical theory able to handle incomplete information, even when the contributions that are responsible for this incompleteness are not necessarily random. Of course, this theory should include the probability theory as a particular case so that this last one is naturally used when only random contributions affect the measurement process.

According to the considerations reported in the previous section, the theory that seems to be particularly suitable for the representation of incomplete information is the theory of evidence, conceived in the 1970s by Shafer [4] and, as far as the fuzzy logic is concerned, by Zadeh [6].

The theory of evidence is a mathematical theory based upon three fundamental concepts: the basic probability assignment function ( $m$ ), the belief measure (Bel), and the plausibility measure (Pl) [2]. These quantities are defined over set  $A \in P(X)$ , where  $X$  is the universal set, and  $P(X)$  is the power set of  $X$ .

The basic probability assignment function is defined as [2]

$$m : P(X) \rightarrow [0, 1] \quad (1)$$

and satisfies the following relationships:

$$m(\emptyset) = 0 \quad (2)$$

$$\sum_{A \in P(X)} m(A) = 1. \quad (3)$$

Every set  $A$  for which  $m(A) > 0$  is called a focal element of  $X$  [2]. For each set  $A \in P(X)$ ,  $m(A)$  represents the confidence, or belief, in the statement “the element  $x$  belongs to  $A$ ,” supported by all available and relevant evidence. This value  $m(A)$  refers only to set  $A$  and does not imply anything about the belonging of that element to the various subsets of  $A$  [2].

Given the basic probability assignment function  $m(A)$  for each set  $A \in P(X)$ , the belief measure and the plausibility measure are uniquely determined by

$$\text{Bel}(A) = \sum_{B|B \subseteq A} m(B) \quad (4)$$

$$\text{Pl}(A) = \sum_{B|A \cap B \neq \emptyset} m(B). \quad (5)$$

Relations (4) and (5) show that, while  $m(A)$  characterizes the degree of evidence or belief that the considered element belongs only to set  $A$  (exactly to set  $A$ ),  $\text{Bel}(A)$  represents the total evidence or belief that the considered element belongs to  $A$  and to its various subsets, and  $\text{Pl}(A)$  represents not only the total evidence or belief that the considered element belongs to set  $A$  and all its subsets, but also the additional evidence or belief that the considered element belongs to the sets that overlap with  $A$ . In other words,  $m(A)$  is the degree of evidence, or belief, in the information we have that the considered element belongs to set  $A$ ;  $\text{Bel}(A)$  is the degree of evidence that the considered element belongs to set  $A$ , based on the information that directly support this statement, and  $\text{Pl}(A)$  is the degree of evidence that the considered element belongs to set  $A$ , based on the information that does not directly contradict this statement.

The probability theory can be framed within the theory of evidence as a particular case, where the focal elements degenerate into singletons. In such a case, the belief measure and the plausibility measure assume the same value, called probability measure [2]:

$$\text{Bel}(A) = \text{Pl}(A) = \text{Pro}(A). \quad (6)$$

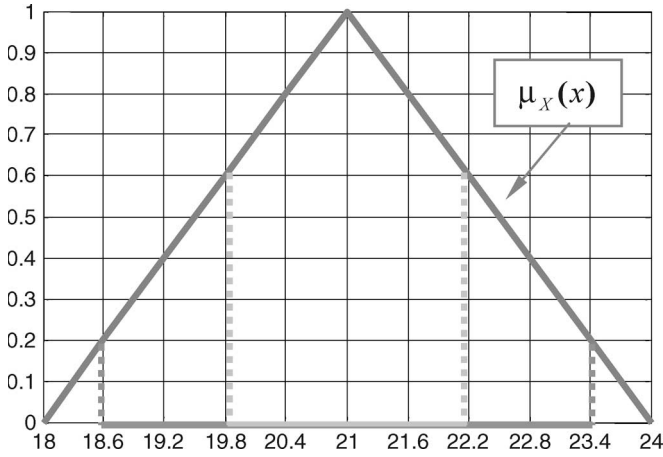


Fig. 1. Example of a triangular fuzzy variable and two  $\alpha$ -cuts.

The well-known random variables are defined within this theory.

Another particular case of the theory of evidence is the possibility theory. In this case, the focal elements satisfy the following relationship [2]:

$$A_1 \subset A_2 \subset \dots \subset A_n \equiv X \quad (7)$$

that is, the focal elements are all nested. Within the possibility theory, the belief and plausibility measures are called necessity measure  $\text{Nec}(A)$  and possibility measure  $\text{Pos}(A)$ , respectively, and are obtained by (3)–(5) as

$$\text{Nec}(A_j) = \sum_{k=1}^j m(A_k) \quad (8)$$

$$\text{Pos}(A_j) = \sum_{k=1}^n m(A_k) \equiv 1. \quad (9)$$

The most interesting application of the possibility theory, at least for the metrology purpose, is the monodimensional case, where the focal elements become nested intervals. In this frame, the fuzzy variables can be defined.

It is known that a fuzzy variable is defined by a membership function  $\mu_X(x)$  over  $X$ .  $\mu_X(x)$  takes all values between 0 and 1 and is convex [2].

A fuzzy variable can be alternatively described by a set of nested intervals, called  $\alpha$ -cuts, in such a way that, given a value  $\alpha$ , between 0 and 1, the correspondent  $\alpha$ -cut is an interval defined as

$$X_\alpha = \{x | \mu_X(x) \geq \alpha\}. \quad (10)$$

Fig. 1 gives an example of a fuzzy variable with a triangular membership function; two  $\alpha$ -cuts are also reported, at levels  $\alpha = 0.2$  and  $\alpha = 0.6$ , in order to show how  $\alpha$ -cuts satisfy (7).

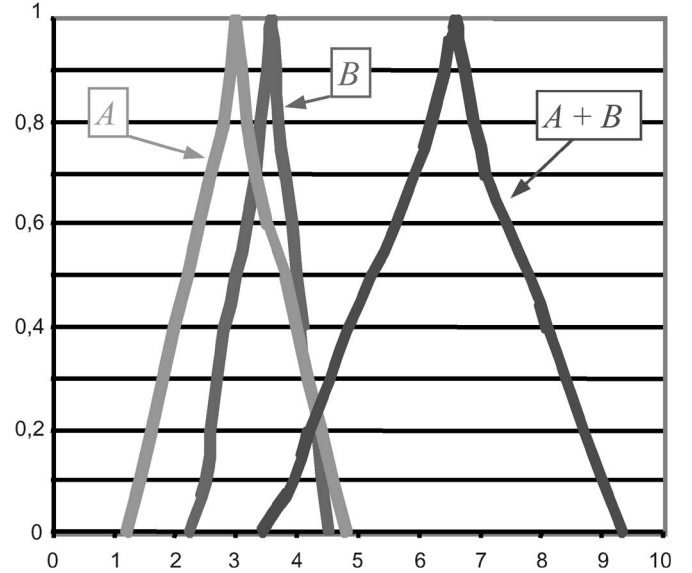


Fig. 2. Sum of two fuzzy variables.

Therefore, the  $\alpha$ -cuts are nested focal elements; this means that the possibility and the necessity measures can be defined on each  $\alpha$ -cut.

In particular, it is

$$\text{Nec}(X_\alpha) = 1 - \alpha \quad (11)$$

$$\text{Pos}(X_\alpha) = 1.$$

Moreover, it is possible to prove [2] that each  $\alpha$ -cut  $X_\alpha$  can be interpreted as a confidence interval and that the associated level of confidence is given by  $\text{Nec}(X_\alpha)$ , which is the necessity measure for the considered  $\alpha$ -cut.

According to (8), (9), and (11), a fuzzy variable can be suitably employed to represent incomplete knowledge in every case where the available information shows that a value falls within a given interval but does not reveal whereabouts and with which probability.

This situation is called “total ignorance” and is typical when the only available information is the interval provided by the instrument’s manufacturer or by the calibration certificate or when, in the measurement process, an unknown systematic contribution is present.

In all these cases, the associated fuzzy variable has a rectangular membership function with a width equal to the given interval [2].

The mathematics of the fuzzy variables is very simple. In fact, fuzzy variables can be combined together by employing the well-known mathematics of the intervals applied to each  $\alpha$ -cut at the same level  $\alpha$  [2]. This means that, for example, when two fuzzy variables are summed up (Fig. 2), for each level  $\alpha$ , the width of the  $\alpha$ -cut of the sum  $A + B$  is equal to the sum of the widths of the  $\alpha$ -cuts of  $A$  and  $B$ .

According to the above considerations, it can be concluded that the fuzzy variables can also represent, alternatively to random variables, the distributions of values associated to the result of a measurement. Moreover, because of the different

way the focal elements of the two different theories are defined (nested intervals in one case and singletons in the other case) and the different way the two kinds of variables are combined (mathematics of the intervals in one case and statistics in the other one), they represent two distinct and complementary ways for representing incomplete information [2].

In particular, a random variable can be suitably and correctly employed to take into account random effects on the result of a measurement. On the other hand, a fuzzy variable can be suitably and correctly employed to take into account unknown effects (total ignorance) and uncompensated systematic effects on the result of a measurement.

In order to clarify this point, let us consider that, if an unknown systematic effect is present in the measurement process, the only way to consider it in the uncertainty evaluation is to say that it falls within a given interval (total ignorance), since this is the only available information. It is therefore obvious that if two measurement results affected by the same unknown systematic contribution are averaged, then the average will be represented by a rectangular fuzzy variable whose width is the same as that of the original ones. On the other hand, if the considered effect behaves in a strict systematic way and the measurement procedure is supposed to fully compensate for it (as in the case of the double-weighing method, when the results of the two weighings on a weighbridge are averaged), additional information is available supporting the evidence that the considered systematic effect no longer affects the final measurement result. In this case, using the mathematics of the intervals (as in the previous example) would not take into account this additional available information, thus leading to an overestimate of the measurement uncertainty. In this particular case, the available information leads to conclude that the systematic effect does not contribute to the final measurement uncertainty, and hence, the measurement result is expressed as a scalar value.

In general, expressing uncertainty in measurement requires taking into account all kinds of effects, both random and unknown. Although it is possible to consider them separately, with a random variable or a fuzzy variable, this is definitely impractical, and a unique way to handle all effects should be found.

Since the possibility theory (within which the fuzzy variables are defined) and the probability theory (within which the random variables are defined) have a common root in the theory of evidence, it is natural to define a new more general variable within this more general theory. This new variable should synthesize the behavior of both the fuzzy and the random variables and therefore represent the different kinds of contributions to the measurement uncertainty. Due to the conceptual synthesis implied by this new variable, it has been called an RFV.

Its definition requires a few careful considerations about the actual meaning of the different considered distributions. A fuzzy variable is defined by a membership function, whose values vary between 0 and 1. On the other hand, a random variable is defined by a probability density function, with a unitary integral. This difference prevents from considering both distributions in a unique mathematical object. It is therefore necessary to transform the probability density function into

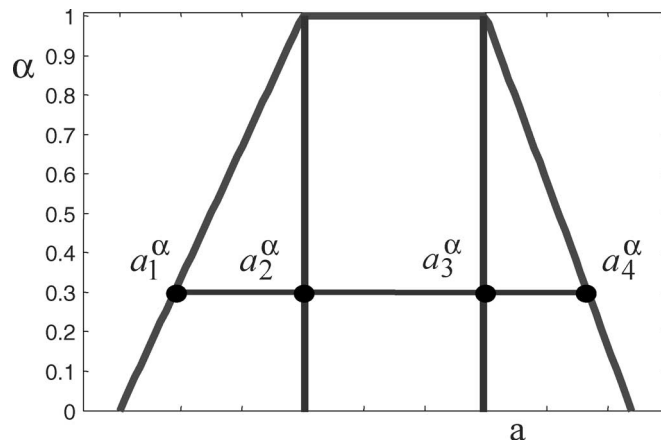


Fig. 3. Example of an RFV.

a membership function or vice versa. Since the GUM suggests that “the ideal method for evaluating and expressing measurement uncertainty should be capable of readily providing ... a confidence interval,” [1] and, as already stated, a fuzzy variable is just defined as a set of confidence intervals, the transformation of the probability density function into a membership function appears the more natural one [7].

Several probability-possibility transformations have been proposed in the literature, ranging from simple ratio scaling to more sophisticated transformations based upon various principles [8]–[10].

To the authors’ knowledge, the simpler ones do not provide correct results in all possible situations, while the most complex ones seem to be difficult to apply in practice.

For this reason, an original implementation has been proposed by the authors [2], [7], which seems to be quite simple and deal with the concepts of level of confidence and confidence interval reported in the GUM [1] in an effective way. The aim of the method is the construction of the RFV associated to the available information [2], [7].

The RFV is defined by two membership functions, as shown in the example of Fig. 3.

When the result of a measurement is represented by an RFV, its internal membership function must take into account all contributions to the measurement uncertainty and its external membership function must take into account all kinds of contributions.

In particular, the construction of an RFV considers two separate steps: The first step takes into account the different kinds of contributions separately, giving rise to two different membership functions; the second step combines them [2], [7].

As for the first step, the result of a measurement affected by unknown contributions is generally given in terms of a confidence interval, within which it is known that the measurement result will lie and no other information is available. As already stated, this situation is referred to, within the theory of evidence, as “total ignorance,” and it is efficiently described by a rectangular fuzzy variable [2]; therefore, the internal membership function of the final RFV is immediately given.

On the other hand, the result of a measurement affected by random contributions is generally given in terms of a

probability density function. In this case, a membership function must be obtained from this probability distribution. The probability-possibility transformation is immediate if the following is considered.

- 1) Given a probability density function  $p(x)$ , the level of confidence  $\lambda$  (where  $0 \leq \lambda \leq 1$ ) associated to the confidence interval  $[x_a, x_b]$  is given by  $\lambda = \int_{x_a}^{x_b} p(x)$ .
- 2) Given a fuzzy variable  $X$ , the level of confidence  $\text{Nec}(X_\alpha)$  associated to the generic  $\alpha$ -cut  $X_\alpha$  is given by  $\text{Nec}(X_\alpha) = 1 - \alpha$ .

Therefore, each interval  $[x_{a_k}, x_{b_k}]$ , which is suitably chosen within the support of  $p(x)$  [2], [7], is the  $\alpha$ -cut of the corresponding fuzzy variable at level  $\alpha_k = 1 - \lambda_k$  [2].

As for the second step, the first membership function (unknown effects) is simply “inserted” into the second one (random effects) [2]. Therefore, the first membership function becomes the internal one of the RFV, while the second one, which is suitably widened in order to accommodate the first one, becomes the external membership function of the RFV [2].

Similar to the fuzzy variable, also, the RFV can be defined by a set of nested  $\alpha$ -cuts [2] identified by four numbers:

$$X_\alpha = \{x_1^\alpha, x_2^\alpha, x_3^\alpha, x_4^\alpha\}. \quad (12)$$

as shown in Fig. 3 for  $\alpha = 0.3$ .

Each  $\alpha$ -cut  $X_\alpha = [x_1^\alpha, x_4^\alpha]$  is the confidence interval associated to the considered level of confidence  $\text{Nec}(X_\alpha) = 1 - \alpha$ . In detail, the  $\alpha$ -cut also identifies three subintervals  $[x_1^\alpha, x_2^\alpha]$ ,  $[x_2^\alpha, x_3^\alpha]$ , and  $[x_3^\alpha, x_4^\alpha]$ , which provide further information.

- 1) Inner interval  $[x_2^\alpha, x_3^\alpha]$  shows the unknown contributions to uncertainty affecting the measurement result.
- 2) External intervals  $[x_1^\alpha, x_2^\alpha]$  and  $[x_3^\alpha, x_4^\alpha]$  show the random contributions to uncertainty affecting the measurement result.
- 3) While for the inner interval nothing can be said about the way the generic value  $x$  is distributed, suitable probability distributions are associated to the external intervals.

These considerations lead to the conclusion that the result of a measurement can be correctly expressed in terms of an RFV [2], and this variable contains full information about the possible intervals “within which the values that could reasonably be attributed to the quantity subject to measurement may be expected to lie” [1]. In fact, a level of confidence can be associated to each of these intervals, and unknown and random contributions can be recognized too.

In order also to process uncertainty, when the result of an indirect measurement must be qualified, a suitable mathematics must be defined for the RFVs.

This mathematics is applied to each different  $\alpha$ -cut and is able to treat the internal and external intervals in which the  $\alpha$ -cut is divided in the appropriate way. In fact, since different interpretations are given for the central and side intervals, it is natural to define a different mathematics for them.

As far as the internal intervals are concerned, by definition, the internal membership function of the RFV is a pure fuzzy variable; therefore, the same mathematics of fuzzy variables is applied. On the other hand, as far as the external intervals are concerned, more information is available: the probability

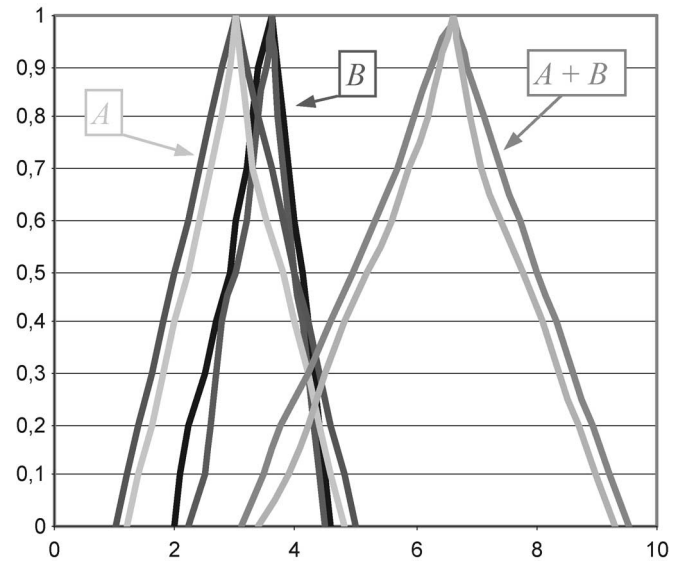


Fig. 4. Sum of two RFVs.

distribution of the possible values within these intervals. If the external membership functions were combined with the mathematics of the intervals, this further information would not be considered, and neither the natural compensation that occurs when random contributions are processed. In some way, the mathematics of the external intervals must take into account the associated probability density function. A very simple way to obtain this is the following, which also encompasses the suggestions of the GUM [1].

Let us consider that since the side intervals of each  $\alpha$ -cut of an RFV have been defined in order to take into account purely random contributions, the values falling within these same intervals are randomly distributed according to a Gaussian distribution by definition [5]. In particular, the right-half side of a Gaussian distribution is defined over interval  $[x_3^\alpha, x_4^\alpha]$ , and the left-half side of a Gaussian distribution is defined over interval  $[x_1^\alpha, x_2^\alpha]$ . By definition, the normal distribution whose right part is defined over interval  $[x_3^\alpha, x_4^\alpha]$  has mean value on  $x_3^\alpha$  and standard deviation equal to one third the width of  $[x_3^\alpha, x_4^\alpha]$ . Since the probability that a value falls within an interval  $\pm 3\sigma$  around the mean value is 0.9973, it is reasonable to suppose that the probability is zero outside this interval [5]. The same applies for  $[x_1^\alpha, x_2^\alpha]$ .

Since the composition of normal distributions is still a normal distribution, it is possible to determine its standard deviation by applying the well-known standard uncertainty propagation law, defined by the GUM [1], which also allows taking into account the possible correlations. Finally, from the knowledge of the standard deviations of the two half-Gaussian distributions, we can immediately determine the widths of the two side intervals of the final RFV by multiplying it by three [5].

Fig. 4 shows an example of the sum of two RFVs, with uncorrelated random contributions.

Thanks to this mathematics, the measurement results and their associated uncertainties can be processed in a quite immediate way, in order to estimate, as required by the GUM [1], the measurement uncertainty of an indirect measurement,

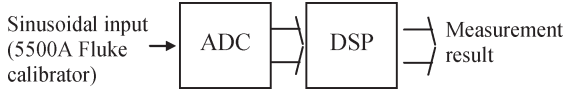


Fig. 5. Single-channel instrument for digital signal processing.

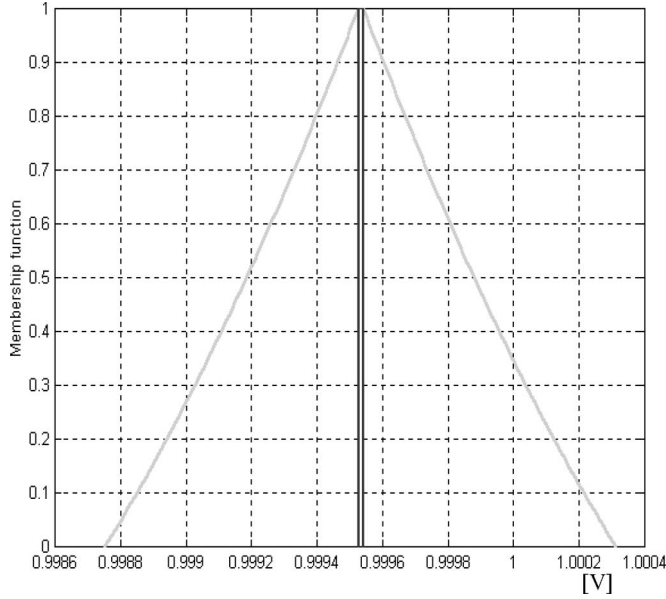


Fig. 6. RFV representing the ADC gain.

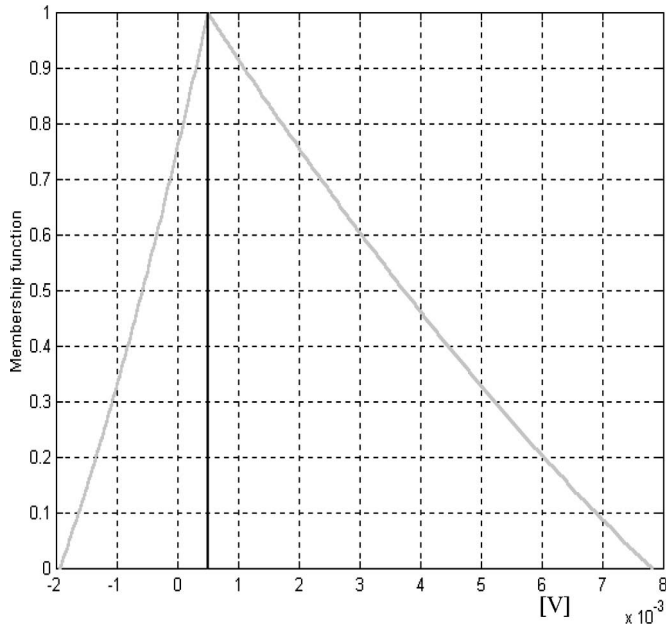


Fig. 7. RFV representing the ADC offset.

taking correctly into account the different kinds of uncertainty contributions.

### III. PRACTICAL APPLICATION

In order to prove the validity of the proposed approach, a simple practical example is reported: the evaluation of the rms value of a sinusoidal voltage with DSP techniques [7]. The digital instrument is reported in Fig. 5, while Figs. 6–8 represent, respectively, the ADC gain  $G$ , offset  $O$ , and quantiza-

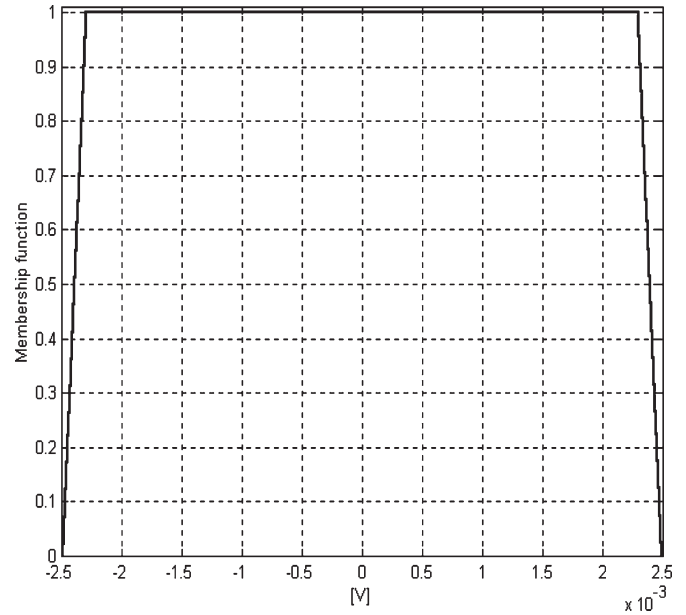


Fig. 8. RFV representing the ADC quantization.

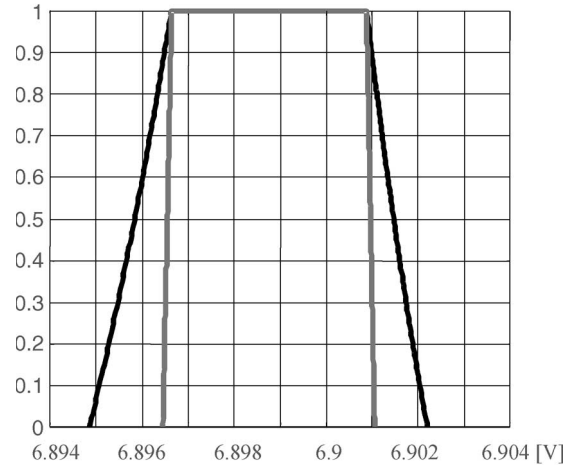


Fig. 9. Measured rms value.

tion  $Q$  contributions to uncertainty, obtained after a calibration process, expressed in terms of RFVs [7].

Due to these uncertainty contributions, each input sample  $s_n$  to the DSP can be converted into the RF sample  $S_n$  by

$$S_n = \frac{(s_n - Q)}{G} - O. \quad (13)$$

Therefore, the rms value of the input voltage is given by the mathematical expression  $V_{\text{rms}} = (1/N)(\sum_{n=1}^N S_n^2)^{1/2}$ , involving RFVs, and the measurement result is directly provided in terms of the RFV, as shown in Fig. 9 for a 6.9- $V_{\text{rms}}$  input sinewave.

It can be noted that the value (6.9  $V_{\text{rms}}$ ) of the input voltage provided by the calibrator falls within the output RFV, thus proving that all considered uncertainty sources have been correctly described in terms of RFVs. Moreover, according to the theoretical considerations reported in the previous section, the

different uncertainty contributions due to random and systematic and unknown effects are clearly identified.

#### IV. CONCLUSION

This paper has shown that the result of a measurement can be seen, from a philosophical point of view, as incomplete information about the measurand and, consequently, that estimating measurement uncertainty means quantifying how incomplete this information is. According to this formulation, this paper has presented a new approach to the expression of uncertainty in measurement, framed within the theory of evidence, and fully compatible with the theoretical background of the GUM.

The theory of evidence, which includes the probability theory and the possibility theory as two different particular cases, allows handling incomplete information originated by all kinds of effects and is therefore particularly suitable for expressing uncertainty in measurement.

#### REFERENCES

- [1] *ISO-IEC-OIML-BIPM: Guide to the Expression of Uncertainty in Measurement*, 1992.
- [2] S. Salicone, *The Mathematical Theory of Evidence and Uncertainty in Measurement*. New York: Springer, 2006.
- [3] *ISO-IEC-OIML-BIPM: Guide to the Expression of Uncertainty in Measurement. Supplement 1. Numerical Methods for the Propagation of Distributions*, 2004.
- [4] G. Shafer, *A Mathematical Theory of Evidence*. Princeton, NJ: Princeton Univ. Press, 1976.
- [5] A. Ferrero and S. Salicone, "The random-fuzzy variables: A new approach for the expression of uncertainty in measurement," *IEEE Trans. Instrum. Meas.*, vol. 53, no. 5, pp. 1370–1377, Oct. 2004.
- [6] L. A. Zadeh, "Fuzzy logic and approximate reasoning," *Synthese*, vol. 30, no. 1, pp. 407–428, 1975.
- [7] A. Ferrero, R. Gamba, and S. Salicone, "A method based on random fuzzy variables for on-line estimation of the measurement uncertainty of DSP-based instruments," *IEEE Trans. Instrum. Meas.*, vol. 53, no. 5, pp. 1362–1369, Oct. 2004.
- [8] G. J. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic. Theory and Applications*. Englewood Cliffs, NJ: Prentice-Hall, 1995.
- [9] G. Mauris, V. Lasserre, and L. Foulloy, "A fuzzy approach for the expression of uncertainty in measurement," *Measurement*, vol. 29, no. 3, pp. 165–177, 2001.
- [10] —, "A simple probability-possibility transformation for measurement error representation: A truncated triangular transformation," in *Proc. World Congr. IFSA*, Prague, Czech Republic, 1997, pp. 476–481.



**Alessandro Ferrero** (M'88–SM'96–F'99) was born in Milan, Italy, in 1954. He received the M.Sc. degree in electrical engineering from the Politecnico di Milano, Milan, in 1978.

In 1983, he joined the Dipartimento di Elettrotecnica, Politecnico di Milano, as an Assistant Professor of electrical measurements. From 1987 to 1991, he was an Associate Professor of measurements on electrical machines and plants at the University of Catania, Catania, Italy. From 1991 to 1994, he was an Associate Professor of electrical measurements at the Dipartimento di Elettrotecnica, Politecnico di Milano. Since 1994, he has been a Full Professor of electrical and electronic measurements at the same department. His current research interests are concerned with the application of digital methods to electrical measurements and measurements on electric power systems under nonsinusoidal conditions.

Prof. Ferrero is a member of the Italian Association of Electrical and Electronic Engineers (AEIT) and the Italian Association for Industrial Automation (ANIPLA) and is the Chair of the Italian Association for Electrical and Electronic Measurements (GMEE). He is also a member of the AdCom of the IEEE IM Society.



**Simona Salicone** (S'01–M'05) was born in Milan, Italy. She received the M.Sc. and Ph.D. degrees in electrical engineering from the Politecnico di Milano, Milan, in 2000 and 2004, respectively.

In 2000, she joined the Dipartimento di Elettrotecnica of the Politecnico di Milano as a Part-Time Researcher on a research project aimed at the metrological characterization of complex distributed measurement systems. Since 2005, she has been an Assistant Professor of electrical and electronic measurements at the same university.

Dr. Salicone is a member of the Italian Association for Electrical and Electronic Measurements (GMEE).