Management of Measurement Uncertainty for Effective Statistical Process Control

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***Abstract*—In the context of quality assurance strategies, statistical process control techniques and conformance testing are necessary to perform a correct quality auditing of process outcomes. However, data collection is based on measurements and every measurement is intrinsically affected by uncertainty. Even if adopted instruments are in a condition of metrological confirmation, random and systematic measurement errors can not be completely eliminated. Moreover, the consequence of wrong measurement-based decisions can seriously decrease company profits because of larger repairing and shipping costs, as well as for the loss of reputation due to customers' dissatisfaction. This paper deals with a theoretical analysis aimed at estimating the growth in decisional risks due to both random and systematic errors. Also, it provides some useful guidelines about how to choose the test uncertainty ratio of industry-rated measurement instruments in order to bound the risk of making wrong decisions below a preset maximum value.**

***Index Terms*—Conformance testing, control charts, measurement uncertainty, random error, systematic error, test uncertainty ratio (TUR).**

I. Introduction

THE WORLDWIDE diffusion of the quality management practices described in the ISO 9000 series of standards, has deeply influenced the role of test and measurement for the improvement of primary company processes. However, due to the generality of the concepts explained in the ISO 9000 norms, whose applicability ranges from manufacturing to servicing, only some basic requirements, such as the need for metrological confirmation, are explicitly mentioned [1]. Further details about measurement and instrumentation management techniques are described in other specific documents [2]–[5]. One of the main goal of such standards is to provide methods and criteria aimed at achieving valid results useful to support decisions. However, in spite of these standardization efforts, it is known that the unavoidable presence of uncertainty in every measurement process tends to increase the risks of making wrong decisions [6], [7]. In this paper, after a short introduction about the main sources of uncertainty affecting electronic measurement equipment, the effects of random contributions and systematic errors on control charts are analyzed and compared. Moreover, some design and management directions are provided to achieve a good tradeoff in terms of decisional risks and instrument-related costs. Finally, an analysis of the influence of unknown systematic errors on conformance testing outcomes is also presented.

II. Random Errors, Systematic Errors, and Quality Control

In performing a measurement, the final result is unavoidably affected by a total error that originates from several possible sources such as measuring parameter biases, limited equipment resolution, operator mistakes, environmental factors, aging of devices and random noises [8]. Each error can be modeled as a random variable that follows a statistical distribution with a certain standard deviation, whose estimate is usually referred to as *uncertainty*. Clearly, there is an uncertainty contribution associated with each error source. The total measurement uncertainty results from the composition of all these elementary contributions and it can be estimated using either type A or type B techniques [9]. Also, in order to ensure that a given instrument is in compliance with the requirements for its intended use, a maximum uncertainty value needs to be set and periodic calibration or verification procedures are to be carried out [3], [10]. In fact, when any measurement equipment in employed in operating conditions that are different from those associated with the calibration environment, measurement errors still affect the experimental data. If the various influence factors exhibit nondeterministic variations during repeated measurements, the corresponding errors show a random behavior. Conversely, if the influence factors are constant, the errors introduced by the instrument tend to be systematic over a certain period of time. In most industrial applications, the latter situation usually occurs because the operating conditions are approximately stable and it is excessively expensive to implement systems for the accurate control of environmental quantities. Hence, when assessing the measurement uncertainty, the range of admissible values for the influence factors has to be large enough to keep into account all possible operating conditions. To this purpose, electronic instrumentation manufacturers usually report heuristic models in their specifications aimed at bundling together the effect of various environmental factors such as time, temperature, humidity and power line fluctuations [11]. These estimates hold as long as all parameters lie within a given window of operation and include, as a particular case, the conditions used to run the calibration procedure.

In quality-oriented organizations, the influence of measurement errors becomes particularly critical because it affects the results provided by statistical quality assurance strategies, thus leading to possible additional management costs. Consequently, the effect of measurement errors on process control techniques such as *control charts* and *conformance testing* needs to be carefully investigated. To this aim, suppose that $\mathbf{x}$ is a Gaussian random variable with mean value μ_x and standard deviation σ_x

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modeling the process outcomes. If $\boldsymbol{\varepsilon}$ represents the measurement error, then the new random variable $\mathbf{y} \triangleq \mathbf{x} + \boldsymbol{\varepsilon}$, with mean value μ_y and standard deviation σ_y, models the measurement results. Note that, in case of random contributions, $\boldsymbol{\varepsilon}$ can be assumed to be a zero mean Gaussian random variable with a standard deviation equal to σ_ε [12], [13]. On the other hand, when dealing with systematic errors, $\boldsymbol{\varepsilon}$ takes a constant value which is included in the interval $[-\varepsilon_M, \varepsilon_M]$, depending on the environmental conditions. The upper bound $\varepsilon_M > 0$ is usually reported in the instrument specifications and it represents the worst-case measurement offset. Notice that, because of the central limit theorem, unless other information is available or a 100% data containment inside preset limits has been observed, the normal distribution should be considered as the default error distribution to describe the behavior of systematic errors in the domain of influence factors [14]. Therefore, ε_M can be usually set equal to $3\sigma_\varepsilon$.

III. Control Charts and Measurement Uncertainty

Control charts are usually employed to monitor some meaningful parameters of a process, such as the mean and the standard deviation of a quality-critical characteristic ($\hat{\mu}$—$\hat{\sigma}$ charts). Unfortunately, due to the influence of measurement uncertainty on collected data, the parameters that are actually monitored are μ_y and σ_y. In fact, if N is the number of observations, the central tendency of the process is given by

$$\hat{\boldsymbol{\mu}}_{\mathrm{y}} = \frac{1}{N}\sum_{n=0}^{N-1}\mathbf{y}[n] \quad (1)$$

that is the commonly used unbiased estimator of the mean value μ_y. Similarly

$$\hat{\boldsymbol{\sigma}}_{\mathrm{y}} = \sqrt{\frac{1}{N-1}\sum_{n=0}^{N-1}(\mathbf{y}[n]-\hat{\mu}_y)^2} \quad (2)$$

is a biased estimator of σ_y, as proved by the fact that [12]

$$E\{\hat{\boldsymbol{\sigma}}_{\mathrm{y}}\} = \sigma_y c_4(N), \quad c_4(N) \triangleq \sqrt{\frac{2}{N-1}}\frac{\Gamma(N/2)}{\Gamma((N-1)/2)} \quad (3)$$

where $E\{\cdot\}$ is the expectation operator and $\Gamma(\cdot)$ is the gamma function [15]. Basically, a control chart consists of a center line (CL), which represents the wished value of the parameter of interest, an upper control limit (UCL), and a lower control limit (LCL) [12]. Random contributions and systematic errors tend to modify the experimental process outcomes in two complementary ways. In fact, random contributions increase the variance of the measured parameter but they do not have any effect on its mean value. On the contrary, systematic errors add an offset to the original data without altering the intrinsic variability of the process.

In the next subsections, only the influence of random and systematic measurement errors on $\hat{\mu}$ charts will be described. Indeed, the impact of random contributions on $\hat{\sigma}$ charts has already been discussed in [7], whereas systematic errors do not affect the variance of the experimental outcomes. Shortly, the main goal of the proposed analysis is to quantify the probability of type II errors associated with the use of $\hat{\mu}$ control charts, referred to also as false acceptance risk $\beta_\cdot(\cdot)$, under the influence of different kinds of measurement uncertainty. To this purpose, the two following basic conditions are assumed.

1) The process is centered and the chart CL is either fixed by regulatory requirements or it derives from a long-term correct estimation based on a high-accuracy measuring procedure.[1]
2) Both LCL and UCL are determined by setting the maximum probability of type I errors α equal to a given value.

Generally speaking, the false acceptance risk can be mathematically determined following a hypothesis-based testing approach [12]. In fact, if δ is referred to as the possible deviation of the actual process mean value from the expected CL, the process in analysis can be considered in control when $\delta = 0$ (null hypothesis, H_0), whereas it results to be out of control when $\delta \neq 0$ (alternative hypothesis, H_1).

A. Random Contributions and $\hat{\mu}$ Control Charts

According to the basic definition of mean control chart, the UCL and LCL limits are given by [12]

$$\mathrm{LCL} \triangleq \mathrm{CL} + Z_{\frac{\alpha}{2}}\frac{\sigma_y}{\sqrt{N}} \quad \mathrm{UCL} \triangleq \mathrm{CL} - Z_{\frac{\alpha}{2}}\frac{\sigma_y}{\sqrt{N}} \quad (4)$$

where $\mathrm{CL} = \mu_x$ because of the zero-mean value of $\boldsymbol{\varepsilon}, Z_p$ is the p-level quantile associated with a zero-mean, unity-variance Gaussian random variable and σ_y replaces σ_x to take into account the measurement uncertainty. Notice that, as $\alpha \leq 1, Z_{(\alpha/2)} \leq 0 \ \forall\alpha$. By assuming that $\mathbf{x}$ and $\boldsymbol{\varepsilon}$ are independent random variables, it results that $\sigma_y = \sigma_x\sqrt{1+1/R^2}$, where $R \triangleq \sigma_x/\sigma_\varepsilon$ is referred to as test uncertainty ratio (TUR). Therefore, if $\mu_y = \mathrm{CL} + \delta$ represents the mean value of the out-of-control process, the false acceptance risk $\beta_r(\cdot)$ is given by

$$\begin{aligned}\beta_r(R) &\triangleq \Pr\{\text{Type II err.}\} = \Pr\{\mathrm{LCL} < \hat{\boldsymbol{\mu}}_{\mathrm{y}} \leq \mathrm{UCL} \,|\, H_1\}\\ &= \Pr\left\{\mu_x + Z_{\frac{\alpha}{2}}\frac{\sigma_y}{\sqrt{N}} < \hat{\boldsymbol{\mu}}_y \leq \mu_x - Z_{\frac{\alpha}{2}}\frac{\sigma_y}{\sqrt{N}} \,\middle|\, H_1\right\}\\ &= \Phi\left(-Z_{\frac{\alpha}{2}} - \frac{\delta\sqrt{N}}{\sigma_x\sqrt{1+\frac{1}{R^2}}}\right)\\ &\quad - \Phi\left(Z_{\frac{\alpha}{2}} - \frac{\delta\sqrt{N}}{\sigma_x\sqrt{1+\frac{1}{R^2}}}\right) \end{aligned} \quad (5)$$

where $\Phi(\cdot)$ is the distribution function of a zero-mean, unity-variance normal random variable. In Fig. 1(a), by assuming the meaningful case of one-at-time data $(N = 1)$ and $\alpha = 0.05$, several operating characteristic curves are shown for different

[1] In this context, high accuracy means that both systematic and random contributions associated with the measurements employed to calculate the control charts parameters are negligible with respect to σ_ε. In fact, if, for instance, the chart central line was estimated using measures that are affected by a systematic error, the resulting CL would be shifted from its nominal value, thus hiding the effects of the systematic error itself.

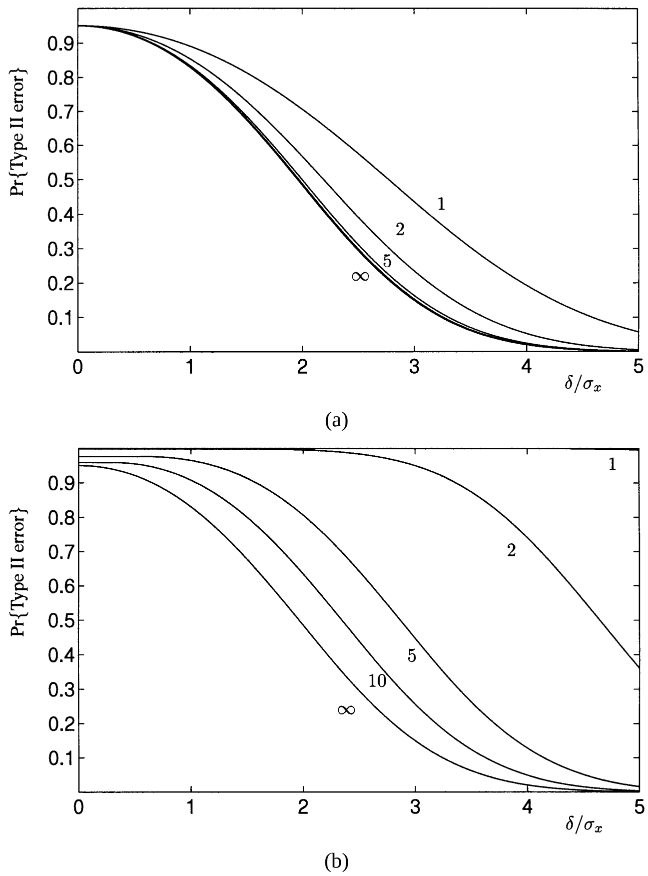

Fig. 1. Probabilities of type II errors related to a generic mean control chart for different values of the TUR. In (a), the effects of random contributions are considered, while in (b), the worst-case operating characteristic curves are shown when an unknown systematic error affects the measurements. In both cases, it results that $\alpha = \alpha_M = 0.05$ and $N = 1$.

values of the TUR. Each operating characteristic curve represents by definition the probability of type II errors $\beta_r(\cdot)$ as a function of the normalized process deviation δ/σ_x after setting both α and N [12]. The hypothesis $N = 1$ is justified by the fact that, due to time-related and economical reasons, many processes are controlled on the basis of a single measurement [16]. Of course, for a given value of δ, the greater is the TUR, the lower is $\beta_r(\cdot)$. Note that (5) holds not only in the domain of time (where N is simply the number of sequential observations), but also in the domain of every other possible influence factor (e.g., temperature) causing a random, normally-distributed behavior of measurement errors.

B. Systematic Contributions and $\hat{\mu}$ Control Charts

As stated in Subsection III.A, the upper and lower limits of a mean control chart should be determined by setting the probability of type I errors α equal to a given value. However, if the measured data are affected by an unknown systematic error whose actual value depends on the instrument operating conditions, the resulting measurement uncertainty will obscure the information about the in–control or out–of–control state of the considered process. As a consequence, in order to establish when the process is out–of–control, the chart limits have to be widened to keep into account the effects of the worst–case systematic error [2]. In particular, such limits have to be chosen so that the probability of type I errors α is lower than or equal to a preset value α_M. Given that the actual value of the systematic error affecting the measurements is not known until a calibration or, at least, a verification is carried out, the real probability of type I errors α is usually strictly lower than α_M. In the following, by assuming that the chart central line $\mathrm{CL} = \mu_x$ is fixed by regulatory or specification requirements, the maximum risk of not detecting an out-of-control condition due to the influence of systematic contributions is estimated in two stages. In the former, the upper and lower chart limits are set on the basis of α_M; in the latter, the maximum probability of type II errors $\beta_s(\cdot)$ is determined. If the process results to be in a condition of statistical control ($\delta = 0$), the values of LCL and UCL, whose general expressions are given by

$$\mathrm{LCL} \triangleq \mathrm{CL} + T\frac{\sigma_x}{\sqrt{N}} \quad \mathrm{UCL} \triangleq \mathrm{CL} - T\frac{\sigma_x}{\sqrt{N}} \tag{6}$$

can be obtained by finding the value $T < 0$, by which $\hat{\mu}_y$ lies outside the interval [LCL, UCL] with probability lower than or equal to α_M. To this purpose, using the definition of probability of type I errors, it results that $\alpha(\cdot,\cdot)$ is given by

$$\begin{aligned}\alpha(T,\varepsilon) &\triangleq \Pr\{\text{Type I err.}\} = \Pr\{\hat{\boldsymbol{\mu}}_{\mathrm{y}} \le \mathrm{LCL}\,|\,H_0\} \\ &\quad + \Pr\{\hat{\boldsymbol{\mu}}_{\mathrm{y}} > \mathrm{UCL}\,|\,H_0\} \\ &= \Pr\left\{\hat{\boldsymbol{\mu}}_{\mathrm{y}} \le \mu_x + T\frac{\sigma_x}{\sqrt{N}}\right\} \\ &\quad + \Pr\left\{\hat{\boldsymbol{\mu}}_{\mathrm{y}} > \mu_x - T\frac{\sigma_x}{\sqrt{N}}\right\} \\ &= \Pr\left\{\hat{\boldsymbol{\mu}}_{\mathrm{x}} + \varepsilon \le \mu_x + T\frac{\sigma_x}{\sqrt{N}}\right\} \\ &\quad + \Pr\left\{\hat{\boldsymbol{\mu}}_{\mathrm{x}} + \varepsilon > \mu_x - T\frac{\sigma_x}{\sqrt{N}}\right\} \\ &= \Phi\left(T - \frac{\varepsilon\sqrt{N}}{\sigma_x}\right) + \Phi\left(T + \frac{\varepsilon\sqrt{N}}{\sigma_x}\right)\end{aligned} \tag{7}$$

where $-\varepsilon_M \le \varepsilon \le \varepsilon_M$ is the systematic error. It can easily be shown that over the interval $[-\varepsilon_M, \varepsilon_M]$, (7) reaches its maximum when $|\varepsilon| = \varepsilon_M$, where ε_M is usually known. Therefore, T can be calculated as the solution $T_{(\alpha/2)}$ of $\alpha(T_{(\alpha/2)}, \varepsilon_M) = \alpha_M$ by reversing numerically the equation:

$$\alpha_M = \Phi\left(T_{\frac{\alpha}{2}} - \frac{\varepsilon_M\sqrt{N}}{\sigma_x}\right) + \Phi\left(T_{\frac{\alpha}{2}} + \frac{\varepsilon_M\sqrt{N}}{\sigma_x}\right) \tag{8}$$

Hence, the corresponding operating characteristic curve $\beta_s(\cdot)$ is given by

$$\begin{aligned}\beta_s(\varepsilon) &\triangleq \Pr\{\text{Type II err.}\} \\ &= \Pr\{\mathrm{LCL} < \hat{\boldsymbol{\mu}}_{\mathrm{y}} \le \mathrm{UCL}\,|\,H_1\} \\ &= \Pr\left\{\mu_x + T_{\frac{\alpha}{2}}\frac{\sigma_x}{\sqrt{N}} < \hat{\boldsymbol{\mu}}_{\mathrm{x}} + \varepsilon \le \mu_x - T_{\frac{\alpha}{2}}\frac{\sigma_x}{\sqrt{N}}\,\middle|\, H_1\right\} \\ &= \Phi\left(-T_{\frac{\alpha}{2}} - \frac{\varepsilon\sqrt{N}}{\sigma_x} - \frac{\delta\sqrt{N}}{\sigma_x}\right) \\ &\quad - \Phi\left(T_{\frac{\alpha}{2}} - \frac{\varepsilon\sqrt{N}}{\sigma_x} - \frac{\delta\sqrt{N}}{\sigma_x}\right),\end{aligned} \tag{9}$$

where both ε and δ are usually unknown. As a consequence, only the maxima of $\beta_s(\cdot)$ over $-\varepsilon_M \leq \varepsilon \leq \varepsilon_M$ can be univocally determined. In particular, if $|\delta| < \varepsilon_M$, the maximum of $\beta_s(\cdot)$ is constantly equal to $1-\alpha(T_{(\alpha/2)},\varepsilon_M)$ with $\alpha(T_{(\alpha/2)},\varepsilon) \leq \alpha_M$. This value is usually quite close to 1 and occurs when $\delta = -\varepsilon$, i.e., when the process deviation counterbalances the effect of the systematic error. On the other hand, if $|\delta| \geq \varepsilon_M$, the maxima are reached when $\varepsilon = \varepsilon_M$. In this context, the TUR of the measurement instrument can be referred to as $R \triangleq \sigma_x/\sigma_\varepsilon = 3\sigma_x/\varepsilon_M$, where σ_ε is the standard deviation of the systematic contribution in the domain of its influence factors under the hypothesis of normal distribution, as explained in Section II. In Fig. 1(b) some worst-case operating characteristic curves are shown for various TUR values, after setting $\alpha_M = 0.05$. Such curves provide an estimate of the maximum false acceptance risk achievable with (9) whenever the systematic error ε makes most difficult the detection of an out-of-control condition.

C. Comparison Between the Effects of Random Contributions and Systematic Errors on a Mean Control Chart

A complete comparison between the effects of random contributions and systematic errors on the false acceptance risk is a difficult task because of the many degrees of freedom involved in the problem. When random contributions are considered, estimating the measurement uncertainty σ_ε is sufficient to determine a single operating characteristic curve for any given set of parameters α, N and R. On the other hand, the lack of information about the exact values of the systematic errors enables only a worst-case assessment of the false acceptance risk associated with the same values of $\alpha = \alpha_M, N$, and R. This worst-case risk is always greater than the risk associated with random contributions. Of course, if the TUR grows, the difference between every couple of corresponding $\beta.(\cdot)$ curves decreases. In fact, when $R \rightarrow \infty$ the operating characteristic curves coincide, as expected. Nevertheless, since the achievement of a large TUR may be very expensive, an appropriate tradeoff needs to be found between the costs associated with the management of a large TUR and the costs caused by wrong decisions. The point-by-point differences between the $\beta.(\cdot)$ curves obtained for $R = 5$ and $R = 10$ and the ideal limit related to a negligible uncertainty condition ($R \rightarrow \infty$) are shown in Fig. 2. The continuous lines refer to the random contribution analysis, whereas the dashed lines are associated with the systematic error worst-case. Note that, if measurements are affected only by random errors, the employment of a measurement instrument characterized by a TUR greater than 5 is probably unnecessarily expensive, because the maximum difference between actual and ideal risks is lower than 0.018, that is, very small. However, if measurements are affected prevailingly by systematic errors the same difference can be kept below 0.15 only if the TUR is greater than 10. Therefore, unless the systematic errors can be correctly measured and corrected, it is advisable to randomize as much as possible the conditions in which measurements take place. In conclusion, not only can the decision risks associated with random errors be univocally determined, but it is also possible to achieve a lower false rejection risk at a lower TUR.

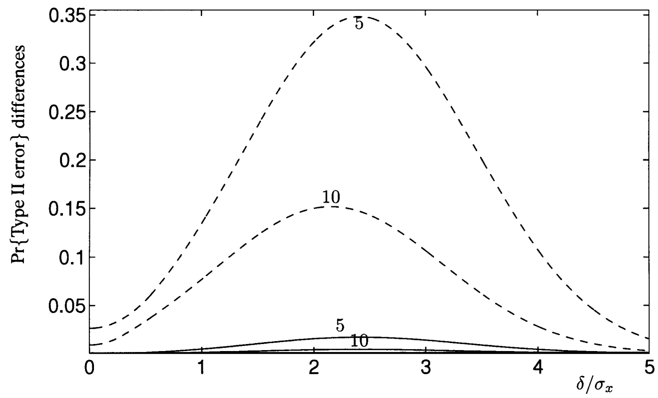

Fig. 2. Differences between the operating characteristic curves obtained for $R = 5$ and $R = 10$ and the ideal curve associated with the condition $\mathrm{R} \rightarrow \infty$. The continuous lines refer to the random error case, while the dashed ones correspond to the systematic error worst-case.

D. Example of Control Chart Design Based on a Preliminary Uncertainty Analysis

In order to validate the proposed approach, a practical example of control chart design under the assumption that either random contributions or systematic errors affect the measured data is presented in this subsection. The considered case of study is based on the monitoring of the printed circuit line width obtained using the *inner-layer* process described in [17]. Such a process is one of the first manufacturing stages in the production of multilayer printed circuit boards and it consists of a sequence of photolithographic operations that eventually return a single layer of circuitry. Given that the width of the conductive lines of each layer must usually adhere stringent customer requirements, the width deviations from a preset specification value need to be carefully checked. In particular, the 44 normally distributed deviation values reported in [17] have been collected over a three week period. Suppose, at first, that the random contributions affecting all measurements are negligible and that the systematic errors can be determined and compensated. As the expected deviation should be ideally equal to zero, the mean chart central line should be $\mathrm{CL} = 0$. If the process is in statistical control, the sampling standard deviation $\hat{\sigma}_x \simeq 7.4$ μm calculated using only the available data, provides an estimate of the intrinsic uncertainty σ_x of the process itself. Therefore, by setting the probability of type I errors α equal to 1% and by assuming $N = 1$, the ideal chart limits LCL_i and UCL_i are given by

$$\begin{aligned} \mathrm{LCL}_i &= Z_{\frac{0.01}{2}}\hat{\sigma}_x = -19.0\ \mu\mathrm{m} \\ \mathrm{UCL}_i &= -Z_{\frac{0.01}{2}}\hat{\sigma}_x = 19.0\ \mu\mathrm{m}. \end{aligned} \tag{10}$$

The mean control chart resulting from this analysis is shown in Fig. 3. The ideal control limits LCL_i and UCL_i are plotted as dotted horizontal lines. Suppose now that the measured line width deviations are affected by a normally-distributed random contribution characterized by a zero mean value and a standard deviation $\sigma_\varepsilon = 3.8$ μm. In this case, it may happen that some critical experimental outcomes, such as those associated with the points 34, 41 and 44, can assume values that are out of the ideal control limits, thus evidencing a false out-of-control condition. In order to avoid the occurrence of this kind of decisional risk, the chart limits have to be widened in accordance with what

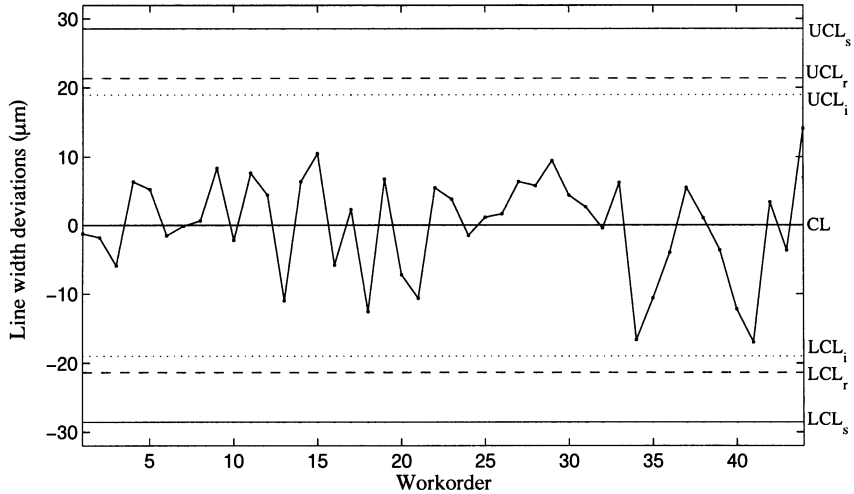

Fig. 3. Mean control chart reporting 44-line width deviations (expressed in μm) obtained at the end of an inner-layer process. The dotted chart limits refer to the ideal case. The dashed and continuous limits are instead widened to keep into account the effects of random contributions and systematic errors, respectively. In all cases, the maximum probability of type I errors is set equal to $\alpha = 1\%$.

stated in Section III.A (horizontal dashed lines in Fig. 3). It follows that, for the same value of α, the new chart limits LCL_r and UCL_r are equal to

$$\begin{aligned}\mathrm{LCL}_r &= Z_{\frac{0.01}{2}}\hat{\sigma}_x\sqrt{1+1/R^2} = -21.4\ \mu\mathrm{m}\\ \mathrm{UCL}_r &= -Z_{\frac{0.01}{2}}\hat{\sigma}_x\sqrt{1+1/R^2} = 21.4\ \mu\mathrm{m}\end{aligned} \tag{11}$$

where $R \simeq \hat{\sigma}_x/\sigma_\varepsilon \simeq 1.9$. Notice that, although the probability of type I errors is the same in both cases, the false acceptance risk estimated with (5) is considerably higher than the risk achievable in the ideal case, because of the small value of the TUR employed. This problem can be solved either by choosing a more accurate instrument or by improving the measurement procedure.

Similar considerations can be repeated even when an unknown systematic error affects the original data. In particular, if both the maximum probablity of type I errors α_M and the TUR value are the same as before (i.e., $\alpha_M = 1\%$ and $R = 1.9$), the chart limits LCL_s and UCL_s are given by

$$\begin{aligned}\mathrm{LCL}_s &= T_{\frac{0.01}{2}}\hat{\sigma}_x = -28.6\ \mu\mathrm{m}\\ \mathrm{UCL}_s &= -T_{\frac{0.01}{2}}\hat{\sigma}_x = 28.6\ \mu\mathrm{m}\end{aligned} \tag{12}$$

where $T_{(0.01/2)} = -3.88$ is the solution of (8), as described in Subsection III.B. Observe that LCL_s and UCL_s (continuous horizontal lines in Fig. 3) are much larger than the control limits associated with both the ideal and the random case.

IV. Conformance Testing and Systematic Errors

Conformance testing is the procedure by which a quality characteristic is measured against pre-set limits. Thus, a product fails the test if the measured quantity is outside a given interval. The measurement uncertainty may alter the final decision about an item in two possible ways: either an out-of-limit product can be wrongly accepted or a valid one can be wrongly rejected. The former probability is referred to as *consumer risk* (CR), while the latter is referred to as *producer risk* (PR). Under the assumption that $\mathbf{x}$ is a Gaussian random variable, CR and PR depend on the uncertainty features. While in [7] the normally and uniformly distributed random contributions are analyzed, in Appendix A it is shown that, when a systematic error $-\varepsilon_M \le \varepsilon \le \varepsilon_M$ affects the measurement results, CR and PR are given by

$$\mathrm{CR} = \begin{cases} \Phi\left(\overline{\mathrm{SL}} + \frac{|\varepsilon|}{\sigma_x}\right) - \Phi\left(\overline{\mathrm{SL}}\right) & \frac{|\varepsilon|}{\sigma_x} \le 2\overline{\mathrm{SL}} \\ \Phi\left(\overline{\mathrm{SL}} - \frac{\varepsilon}{\sigma_x}\right) - \Phi\left(-\overline{\mathrm{SL}} - \frac{\varepsilon}{\sigma_x}\right) & \frac{|\varepsilon|}{\sigma_x} > 2\overline{\mathrm{SL}} \end{cases} \tag{13}$$

$$\mathrm{PR} = \begin{cases} \Phi(\overline{\mathrm{SL}}) - \Phi\left(\overline{\mathrm{SL}} - \frac{|\varepsilon|}{\sigma_x}\right) & \frac{|\varepsilon|}{\sigma_x} \le 2\overline{\mathrm{SL}} \\ 2\Phi(\overline{\mathrm{SL}}) - 1 & \frac{|\varepsilon|}{\sigma_x} > 2\overline{\mathrm{SL}} \end{cases} \tag{14}$$

where $\overline{\mathrm{SL}} \triangleq SL/\sigma_x$ is the positive normalized specification limit. It can be shown that if $\varepsilon_M/\sigma_x \ge 2\overline{\mathrm{SL}}$, the worst-case values both of CR and PR are reached when $\varepsilon = 2\overline{\mathrm{SL}}$ regardless of the TUR. However, when $\varepsilon_M/\sigma_x < 2\overline{\mathrm{SL}}$, the maximum values of (13) and (14) are achieved when $|\varepsilon| = \varepsilon_M = 3\sigma_x/R$, so that CR and PR depend on both the TUR and $\overline{\mathrm{SL}}$. Note that, as these parameters are defined through a ratio of quantities, CR and PR do not change when ε and σ_x vary by the same factor. In Fig. 4 the worst-case CR and PR curves are shown as a function of the TUR for various values of $\overline{\mathrm{SL}}$. It can easily be proved that the resulting curves are always larger than the corresponding risks associated with random contributions.

V. Conclusion

A comparison between the influence of random contributions and systematic errors on some of the best-known statistical control techniques, such as the $\hat{\mu}$—$\hat{\sigma}$ control charts and conformance testing, is presented in this paper. Since the measurement uncertainty associated with most electronic measurement instruments is prevailingly due to systematic errors, their influence on decisional risks should be carefully estimated. The results reported in this paper show that the risks due to systematic errors can be considerably larger than those caused by random contributions. However, in both cases such risks can be kept under control using measurement instruments

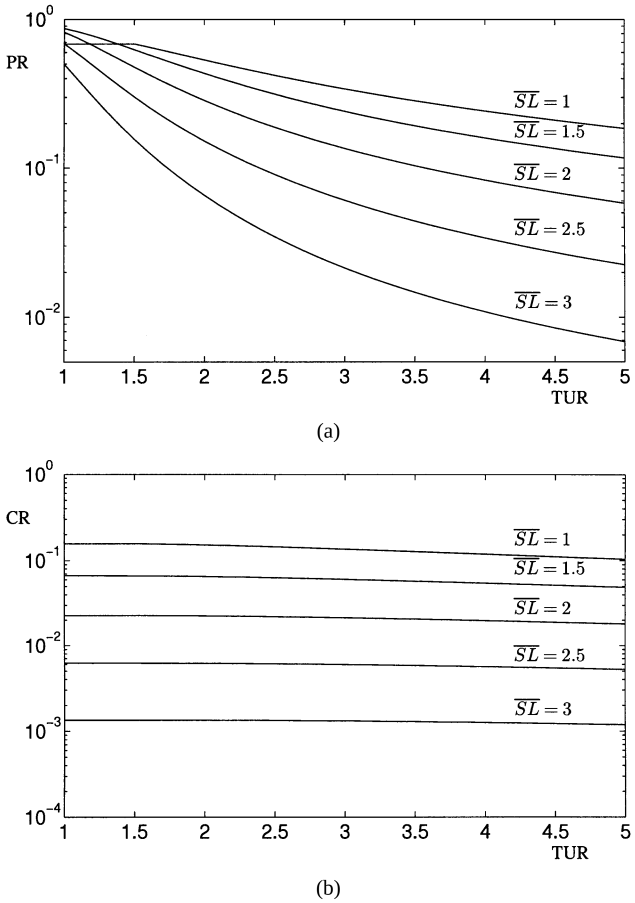

Fig. 4. Worst-case (a) producer and (b) consumer risks when an unknown systematic error $\varepsilon \in [-\varepsilon_M, \varepsilon_M]$ affects a conformance test procedure.

characterized by a suitable value of the TUR. This goal can be achieved more effectively when the systematic errors are minimized, namely when the most important factors affecting the experimental outcomes are randomized.

APPENDIX A
DERIVATION OF EXPRESSIONS (13) AND (14)

If a systematic error affects a measurement, the corresponding consumer risk (CR) is given by

$$\begin{aligned}\mathrm{CR} \triangleq\ & \Pr\{\mathbf{x} > \mu_x + \mathrm{SL}; \mu_x - \mathrm{SL} < \mathbf{x} + \varepsilon \le \mu_x + \mathrm{SL}\} \\ & + \Pr\{\mathbf{x} \le \mu_x - \mathrm{SL}; \mu_x - \mathrm{SL} < \mathbf{x} + \varepsilon \le \mu_x + \mathrm{SL}\}\end{aligned} \tag{A1}$$

where ε is the systematic error and $\mathbf{x}$ is the normal random variable modeling the product feature under analysis. As ε can either be negative or positive, the following three cases can occur, as shown in (A2) at the bottom of the page. By subtracting μ_x from all terms in (A2), as seen at the bottom of the page, normalizing by σ_x and keeping into account the symmetry of the problem, (13) can easily be obtained. A similar approach can be used to deduce the expression of the producer risk. In fact, from the producer risk definition, it results that

$$\begin{aligned}\mathrm{PR} \triangleq\ & \Pr\{\mu_x - \mathrm{SL} < \boldsymbol{x} \le \mu_x + \mathrm{SL}; \boldsymbol{x} + \varepsilon \le \mu_x - \mathrm{SL}\} \\ & + \Pr\{\mu_x - \mathrm{SL} < \boldsymbol{x} \le \mu_x + \mathrm{SL}; \boldsymbol{x} + \varepsilon > \mu_x + \mathrm{SL}\}\end{aligned} \tag{A3}$$

Again, depending on both the sign and the value of ε, three different cases can occur

$$\mathrm{PR} = \begin{cases} \Pr\{\mu_x + \mathrm{SL} - \varepsilon < \boldsymbol{x} \le \mu_x + \mathrm{SL}\} & 0 < \varepsilon \le 2\mathrm{SL} \\ \Pr\{\mu_x - \mathrm{SL} < \boldsymbol{x} \le \mu_x - \mathrm{SL} - \varepsilon\} & -2\mathrm{SL} \le \varepsilon \le 0 \\ \Pr\{\mu_x - \mathrm{SL} < \boldsymbol{x} \le \mu_x + \mathrm{SL}\} & |\varepsilon| > 2\mathrm{SL} \end{cases}. \tag{A4}$$

In conclusion, (14) can be obtained simply repeating the same steps followed in the consumer risk case.

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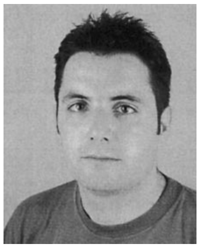
$$\mathrm{CR} = \begin{cases} \Pr\{\mu_x - \mathrm{SL} - \varepsilon < \mathbf{x} \le \mu_x - \mathrm{SL}\} & 0 < \varepsilon \le 2\mathrm{SL} \\ \Pr\{\mu_x + \mathrm{SL} < \mathbf{x} \le \mu_x + \mathrm{SL} - \varepsilon\} & -2\mathrm{SL} \le \varepsilon \le 0 \\ \Pr\{\mu_x - \mathrm{SL} - \varepsilon < \mathbf{x} \le \mu_x + \mathrm{SL} - \varepsilon\} & |\varepsilon| > 2\mathrm{SL} \end{cases} \tag{A2}$$



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