An Innovative Approach to the Determination of Uncertainty in Measurements Based on Fuzzy Variables

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***Abstract*—The determination of the uncertainty of the result of a measurement is presently attained by means of a statistical inference, as suggested by the ISO Guide to the Expression of Uncertainty in Measurement. This paper proposes an approach compatible with that of the ISO Guide, though based on a different mathematical tool. This approach is based on the application of the fuzzy variables and mathematics to the determination of the uncertainty, since they appear to be able to describe the possible distribution of the results of a measurement in a more effective way than the statistical approach. The fundamentals of the employed mathematics are recalled, and applied, as a first example, to the determination of the uncertainty in power measurements. The results obtained by the proposed method are compared with those obtained experimentally.**

***Index Terms*—Fuzzy variables, uncertainty in measurement.**

I. INTRODUCTION

THE most qualifying point of the measurement practice is the estimation of the uncertainty that must be associated with the result of a measurement so that it can be usefully employed in any technical, commercial, or legal activity.

This point is not only suggested by the good practice of measurement, but has become a strict requirement since the early 1990s, when the guidelines for the expression and evaluation of uncertainty in measurements have been published in the ISO Guide [1], which has been more recently encompassed in several International and National Standards (IEC, UNI-CEI, DIN, AFNOR).

The approach followed by the ISO Guide is basically a statistical approach where the uncertainty is defined as "*a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand*" [1]. This parameter is defined as the standard deviation of the probability density function that characterizes this dispersion, and is called standard uncertainty of the result of the measurement.

Furthermore, the ISO Guide [1] states that "*the ideal method for evaluating and expressing measurement uncertainty should be capable of readily providing a confidence interval*" which is defined as "*an interval about the result of a measurement within which the values that could reasonably be attributed to the measurand may be expected to lie with a high level of confidence*."

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Such an interval is called by the Guide "*expanded uncertainty*" and can be derived from the standard uncertainty, by applying a suitable coverage factor, only if the probability density function that characterizes the dispersion of the measured values is known, or suitable assumptions can be made about it.

When indirect measurements are considered, the result y is obtained from a number of other measurement results through a relationship $y = f(x_1, x_2, \ldots, x_\mathrm{n})$ and, under the assumption that the standard uncertainty $u(x_\mathrm{i})$ of the single measurement has been estimated, the ISO Guide [1] defines the combined standard uncertainty $u_\mathrm{c}(y)$, as seen in (1) at bottom of the next page, where $r(x_\mathrm{i}, x_\mathrm{j})$ are the correlation factors, and

$$-1 \leq r(x_\mathrm{i}, x_\mathrm{j}) \leq 1; \quad r(x_\mathrm{i}, x_\mathrm{j}) = r(x_\mathrm{j}, x_\mathrm{i})$$

and $r(x_\mathrm{i}, x_\mathrm{j}) = 0$ when quantities $x_\mathrm{i}, x_\mathrm{j}$ are uncorrelated.

The measurement practice proves that (1) gives good results in most practical situations, but may become quite troublesome when the relationship $f(\cdot)$ between the final measurement result and the results of the single measurements is no longer a continuous function.

A second point, not often considered, is that (1) provides only a standard deviation obtained as a combination of standard deviations. It cannot provide a confidence interval unless the probability distribution function that characterizes the dispersion of y is known. This knowledge may come either from the knowledge of the probability distribution functions that characterize the dispersions of each measurement result x_i, or from reference to the central limit theorem. Unfortunately, both these assumptions might fail under practical situations. It may also happen that two different probability distributions have the same standard deviation, and this lead to consider as compatible the results of two measurements that show a quite different distribution.

The above limitations are clearly evident when the result of a measurement comes from a digital elaboration of the input signals, as in many modern instruments based on digital signal processing (DSP) techniques. In fact, these techniques require, even in the simplest measurement procedures, to process a large number of samples (from several tenths to some thousands), where each of them must be considered as a single measurement result. The application of (1) becomes quite troublesome in this case.

In order to find more significant ways to estimate the uncertainty in complex measurement systems and devices, different approaches have been proposed recently [2]–[4], still based on statistical considerations. These approaches provide results that appear more significant than those obtained by the strict application of the guidelines recommended by the ISO Guide, but they

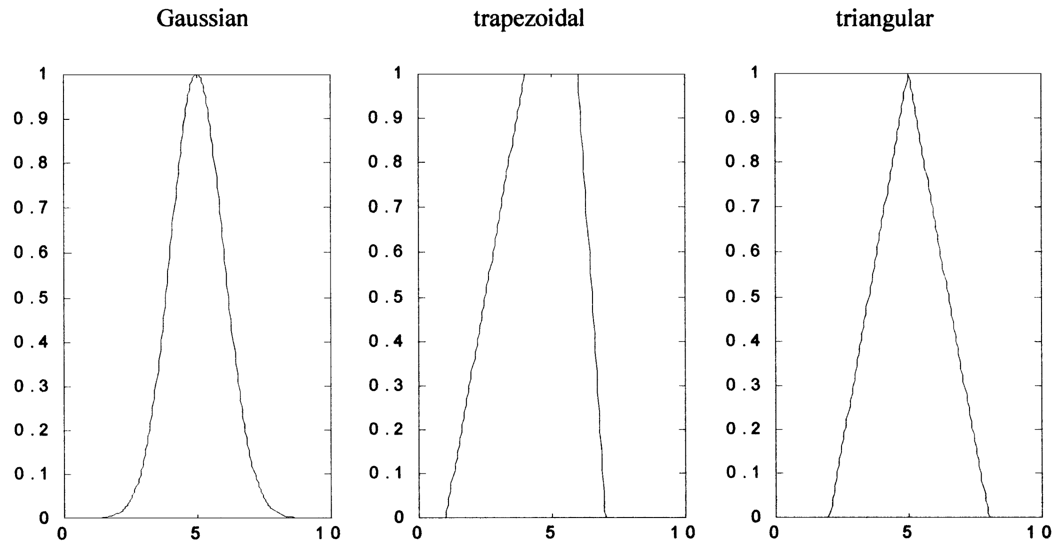

Fig. 1. Example of typical membership functions associated with a fuzzy variable.

require to process a large amount of data and might also show some convergence problem, especially when a Montecarlo-like approach is followed [4].

This paper moves away from the statistical processing of the uncertainty associated with the input data and considers the mathematics of the fuzzy variables, since the very definition of a fuzzy variable appears to describe the uncertainty concept in a more effective mathematical way. Although this approach refers to a different mathematical framework, it is still totally compatible with the definitions given by the ISO Guide [1].

The idea of employing the fuzzy variables for the representation of the measurement ucertainty has been already proposed in [5], [6]: that approach is focused, in particular, on the use of the fuzzy variables for type B evaluation of uncertainty.

This paper introduces the Random-Fuzzy Variables and extends that approach, taking into account all possible contributions to uncertainty. The paper also shows how the mathematics of the fuzzy and random-fuzzy variables can be effectively employed in estimating how the uncertainty associated with input raw data propagates through measurement algorithms and contributes to the uncertainty associated to the final measurement result.

II. Fuzzy Variables

Fuzzy variables and fuzzy sets have been widely used, in the last decades, especially in the field of automatic controls, after Zadeh introduced the basic principles of fuzzy logic and approximate reasoning [7]–[9].

When the traditional approach is followed, a variable may only belong or not belong to the set it is defined into. The function describing the membership of such a crisp variable to its appertaining set can therefore take only the value 1, if the variable belongs to the set, or 0 if the variable does not belong to the set.

When fuzzy variables and fuzzy sets are considered, the function describing the membership of a variable to its appertaining set is allowed to take all values within the 0–1 interval. This means that, given the referential set $\Re$ of the real numbers, a fuzzy variable X is defined by its membership function $\mu_{\mathrm{X}}(x)$, where $0 \leq \mu_{\mathrm{X}}(x) \leq 1, \mu_{\mathrm{X}}(x)$ is convex and normal, and $x \in \Re$. It is generally written: $X = \{x, \mu_{\mathrm{X}}(x)\}$. Fig. 1 shows some typical examples of membership functions.

Taking into account the definition of uncertainty given by the ISO Guide [1] ("*a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand*"), the result of a measurement, together with the associated uncertainty, could be represented by a fuzzy number and its membership function.

The main advantage of this representation is its intrinsic capability of composing the uncertainties in indirect measurements. In fact, the relationship $y = f(x_1, x_2, \ldots, x_{\mathrm{n}})$ can be considered as a relationship $Y = f(X_1, X_2, \ldots, X_{\mathrm{n}})$ between the fuzzy variables $X_1, X_2, \ldots, X_{\mathrm{n}}$ that represent the single measurement results, together with their uncertainties.

This approach requires that a suitable mathematics is defined for the fuzzy variables [10], [11]. This mathematics can be directly derived by the mathematics of the intervals. If a number α, $0 \leq \alpha \leq 1$, is considered, the α-cut of the fuzzy variable X can be defined as

$$X_\alpha = \{x | \mu_{\mathrm{X}}(x) \geq \alpha\} \tag{2}$$

which defines an interval $\lfloor x_1^\alpha, x_2^\alpha \rfloor$, where $x_1^\alpha \leq x_2^\alpha$.

$$u_c(y) = \sqrt{\sum_{\mathrm{i}=1}^{\mathrm{n}} \left(\frac{\partial f}{\partial x_{\mathrm{i}}}\right)^2 \cdot u^2(x_{\mathrm{i}}) + 2 \sum_{\mathrm{i}=1}^{\mathrm{n}-1} \sum_{\mathrm{j}=\mathrm{i}+1}^{\mathrm{n}} \frac{\partial f}{\partial x_{\mathrm{i}}} \frac{\partial f}{\partial x_{\mathrm{j}}} u(x_{\mathrm{i}}) u(x_{\mathrm{j}}) r(x_{\mathrm{i}}, x_{\mathrm{j}})} \tag{1}$$

The sum of two fuzzy variables A and B can be defined as the fuzzy variable $C = A + B$ whose generic α-cut C_α defines an interval

$$\lfloor c_1^\alpha, c_2^\alpha \rfloor = \lfloor a_1^\alpha + b_1^\alpha, a_2^\alpha + b_2^\alpha \rfloor ; \quad \forall\alpha. \tag{3}$$

Similarly, the difference between two fuzzy variables can be defined as the fuzzy variable $C = A - B$ whose generic α-cut C_α defines an interval

$$\lfloor c_1^\alpha, c_2^\alpha \rfloor = \lfloor a_1^\alpha - b_2^\alpha, a_2^\alpha - b_1^\alpha \rfloor ; \quad \forall\alpha. \tag{4}$$

The product between two fuzzy variables can be yet defined by means of the mathematics of the intervals as the fuzzy variable $C = A \cdot B$ whose generic α-cut C_α defines an interval

$$\begin{aligned}\lfloor c_1^\alpha, c_2^\alpha \rfloor = \lfloor &\min\left(a_1^\alpha b_1^\alpha, a_1^\alpha b_2^\alpha, a_2^\alpha b_1^\alpha, a_2^\alpha b_2^\alpha\right), \\ &\max\left(a_1^\alpha b_1^\alpha, a_1^\alpha b_2^\alpha, a_2^\alpha b_1^\alpha, a_2^\alpha b_2^\alpha\right)\rfloor ; \quad \forall\alpha. \end{aligned} \tag{5}$$

The quotient between two fuzzy variables can be defined as the product of the first variable by the inverse of the second one

$$C = \frac{A}{B} = A \cdot B^{-1} \tag{6}$$

where B^{-1} can be defined only if the limits of its generic α-cut have the same sign for any α, which means that none of the possible α-cuts include the zero value. The generic α-cut of B^{-1} is, therefore, defined as

$$B_\alpha^{-1} = \left[\left(b_2^\alpha\right)^{-1}, \left(b_1^\alpha\right)^{-1}\right]; \quad \forall\alpha. \tag{7}$$

More complex operations can be defined as well, if needed, following the same approach. For instance, the square root Y of a fuzzy variable X, $Y = X^{0.5}$, is defined as the fuzzy variable whose generic α-cut Y_α defines an interval

$$Y_\alpha = [y_1^\alpha, y_2^\alpha] = \left[\left(x_1^\alpha\right)^{0.5}, \left(x_2^\alpha\right)^{0.5}\right]; \quad \forall\alpha \tag{8}$$

provided that $x_1^\alpha \geq 0$ and $x_2^\alpha \geq 0\ \forall\alpha$.

A traditional crisp variable can be defined as a particular case of a fuzzy variable X, whose generic α-cut $X_\alpha = \lfloor x_1^\alpha, x_2^\alpha \rfloor$ is defined so that $x_1^\alpha = x_2^\alpha\ \forall\alpha$.

The above definitions allow also to give a deeper glance in the way a fuzzy variable can represent the result of a measurement together with the associated uncertainty. Any α-cut, as defined by (2), can be seen as a confidence interval, whose confidence level is 1-α [10]. As already mentioned, the ISO Guide [1] states that the method for evaluating and expressing the measurement uncertainty should be capable of providing a confidence interval, and defines such an interval as "*an interval about the result of a measurement within which the values that could reasonably be attributed to the measurand may be expected to lie with a high level of confidence*."

Since an α-cut provides such an interval, together with the associated level of confidence 1-α, by its very definition, this not only enforces the idea of using the fuzzy variables for the representation of the uncertainty in measurement, but also suggests that the membership function of a measurement result can be obtained starting from its experimental or assumed distribution.

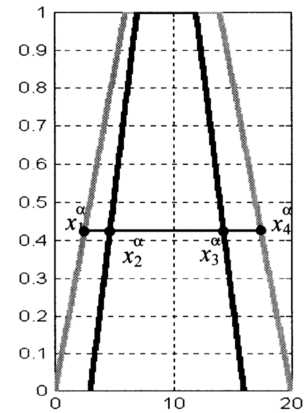

Fig. 2. Example of membership function for a RFV.

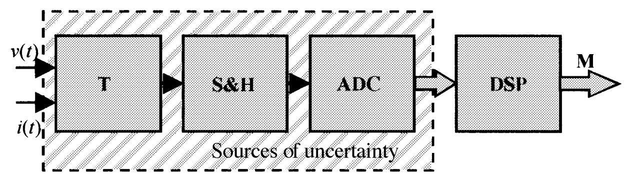

Fig. 3. Block diagram of a DSP-based instrument for the measurement of the electric active power. T: transducers; S&H: sample and hold; ADC: analog-to-digital converter; DSP: digital signal processor.

III. Random-Fuzzy Variables

The definitions from (3) to (8) reported in the previous section sketch a sort of deterministic mathematic framework. It can be readily recognized that the sum of two fuzzy variables shows α-cuts whose width is given by the sum of the widths of the corresponding α-cuts of the two factors.

If the two factors are supposed to represent the results of two single measurements, this situation reflects quite well a measurement condition characterized by systematic, though unknown, effects. Since those effects are supposed to give always the same contribution to the result of the measurement, the result of a measurement coming from the sum of two such measurement results is "reasonably" supposed to lie in an interval which is wider than the interval within which are supposed to lie the single factors of the sum.

From the ISO Guide point of view, this situation is also representative of an uncertainty combination with correlation factors equal to 1.

The situation, however, changes totally when the measurement conditions are characterized by random effects. In this case, due to the possible compensation effects, the result of a measurement coming from the sum of two such measurement results is "reasonably" supposed to lie in an interval which is narrower than the sum of the intervals within which are supposed to lie the single factors of the sum.

From the ISO Guide point of view, this situation is representative of an uncertainty combination with correlation factors lower than 1.

In order to keep into account also this situation, and all possible combination of systematic and random effects, the random-fuzzy variables (RFVs) are introduced [10], [11].

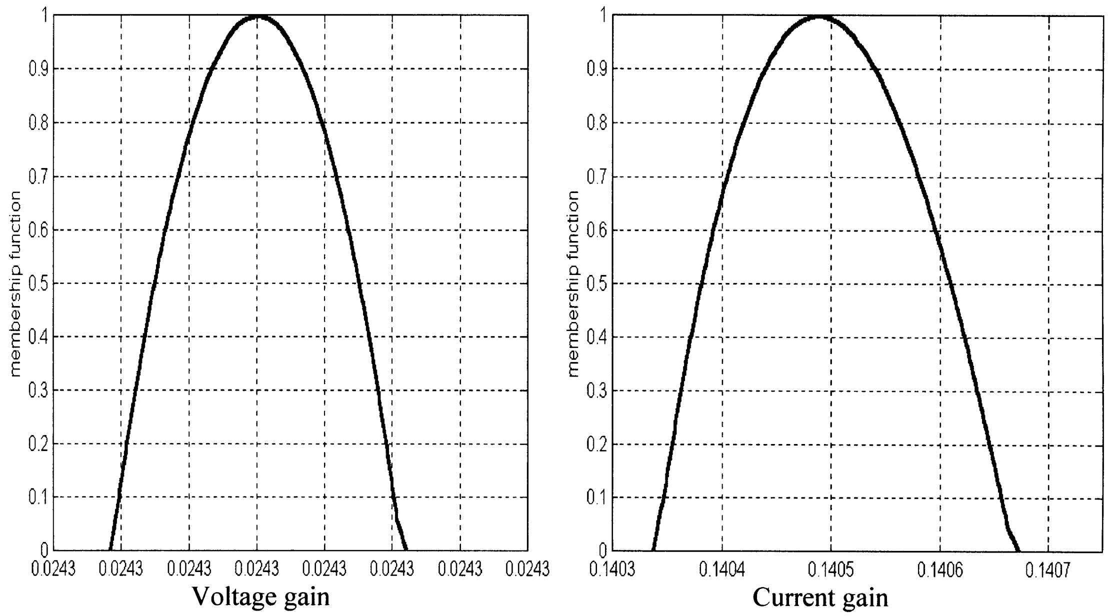

Fig. 4. Representation of the gain distribution of the voltage and current transducers in terms of RFV.

The membership function of a RFV X can be still defined in terms of α-cuts, where an α-cut is now represented by four numbers

$$X_\alpha = \lfloor x_1^\alpha, x_2^\alpha, x_3^\alpha, x_4^\alpha \rfloor \tag{9}$$

where $x_1^\alpha \le x_2^\alpha \le x_3^\alpha \le x_4^\alpha \; \forall \alpha$, which define three intervals as shown in the example of Fig. 2.

The interval $\lfloor x_2^\alpha, x_3^\alpha \rfloor$ represents a confidence interval with confidence level α, in the same way as the simple fuzzy variables defined in the previous section.

Intervals $\lfloor x_1^\alpha, x_2^\alpha \rfloor$ and $\lfloor x_3^\alpha, x_4^\alpha \rfloor$ represent confidence intervals too, again with confidence level α. In this case, the possible values x, $x_1^\alpha \le x < x_2^\alpha$ and $x_3^\alpha < x \le x_4^\alpha$, are supposed to be randomly distributed, according to given probability density functions. These probability density functions can be chosen according to the characteristic of the quantity to be represented by the RFV [10]. In order to represent the random effects that may characterize a measurement process, these distributions can be supposed to have their maximum in x_2^α and x_3^α, decrease monotonically to zero from x_2^α to x_1^α and from x_3^α to x_4^α, and be null before x_1^α and after x_4^α.

It can be readily recognized that if $x_1^\alpha = x_2^\alpha$ and $x_3^\alpha = x_4^\alpha$, the RFV becomes a simple fuzzy variable; if $x_2^\alpha = x_3^\alpha$, no systematic effects are represented by the RFV, and if $x_1^\alpha = x_2^\alpha = x_3^\alpha = x_4^\alpha$, the RFV degenerates into a traditional crisp variable.

As far as the operations between RFVs are considered, the sum of two RFVs A and B, with normal distribution in the random intervals, can be defined as the RFV $C = A + B$ whose generic α-cut is given by $C_\alpha = \lfloor c_1^\alpha, c_2^\alpha, c_3^\alpha, c_4^\alpha \rfloor$, where

$$\begin{aligned} c_1^\alpha &= (1-\beta)\,(a_1^\alpha + b_1^\alpha) + \beta\,(a_2^\alpha + b_2^\alpha) \\ c_2^\alpha &= a_2^\alpha + b_2^\alpha \\ c_3^\alpha &= a_3^\alpha + b_3^\alpha \\ c_4^\alpha &= (1-\beta)\,(a_4^\alpha + b_4^\alpha) - \beta\,(a_3^\alpha + b_3^\alpha) \end{aligned} \tag{10}$$

and where β is a factor that depends on the chosen distributions for the random parts of the RFV. A value $\beta = 1 - 1/\sqrt{2}$ has been considered, since the confidence intervals provided by the α-cuts C_α when $a_2^\alpha = a_3^\alpha$ and $b_2^\alpha = b_3^\alpha$ (that is only random effects are represented by A and B), are the same as those given by the distribution of the sum of two statistical variables associated with normal probability density functions.

A quite similar procedure can be applied to (4) and (5) in order to define the difference, the product and the quotient between two RFVs, as well as other more complex operations [10], [11].

According to the above considerations, the proper choice of the limits $x_1^\alpha, x_2^\alpha, x_3^\alpha, x_4^\alpha$ of each α-cut, as well as the probability density function associated to the outer intervals $\lfloor x_1^\alpha, x_2^\alpha \rfloor$ and $\lfloor x_3^\alpha, x_4^\alpha \rfloor$, allows to take into account all contributions to uncertainty and also the possible correlations. As it has been stated, the effect of total correlation is reproduced by the combination of simple fuzzy variables, while the effect of total uncorrelation is reproduced by the combination of RFVs with null inner intervals for each α-cut. Therefore, the proper positioning of x_1^α and x_2^α limits within each α-cut is expected to take into account the correlation effects.

IV. Practical Application

In order to provide a first experimental validation of the approach described in the previous sections, a DSP-based system for the measurement of the electric active power has been considered. The block diagram of such a system is shown in Fig. 3.

A. Characterization of the Sources of Uncertainty

The sources of uncertainty of the system shown in Fig. 3 can be located in the voltage and current transducers, and the devices used to sample the analog input signals and convert them into digital values.

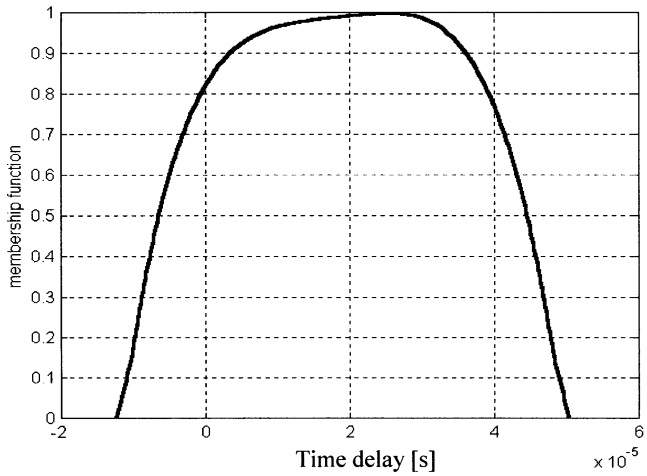

Fig. 5. Representation of the distribution of the time delay between the voltage and current channels in terms of RFV.

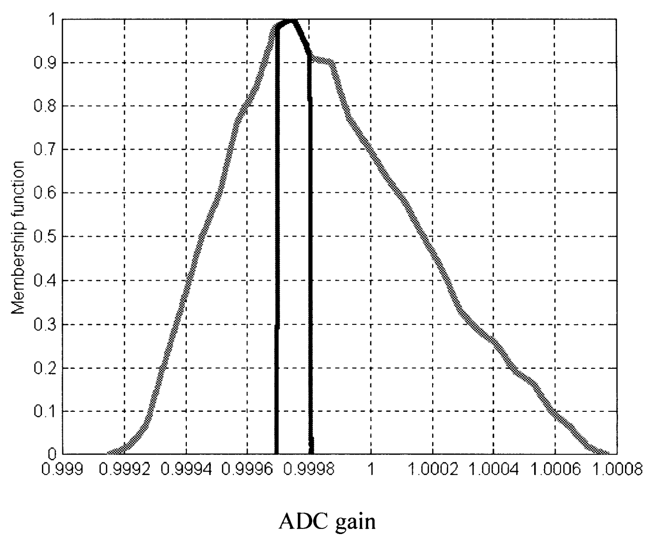

Fig. 6. Representation of the distribution of the ADC gain in terms of RFV.

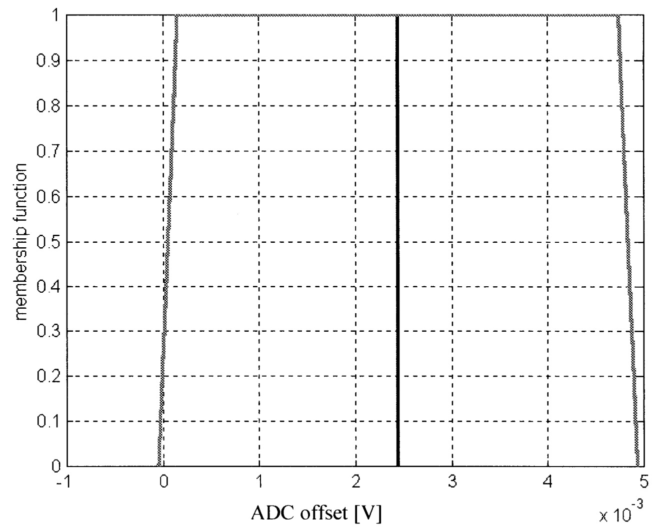

Fig. 7. Representation of the distribution of the ADC offset in terms of RFV.

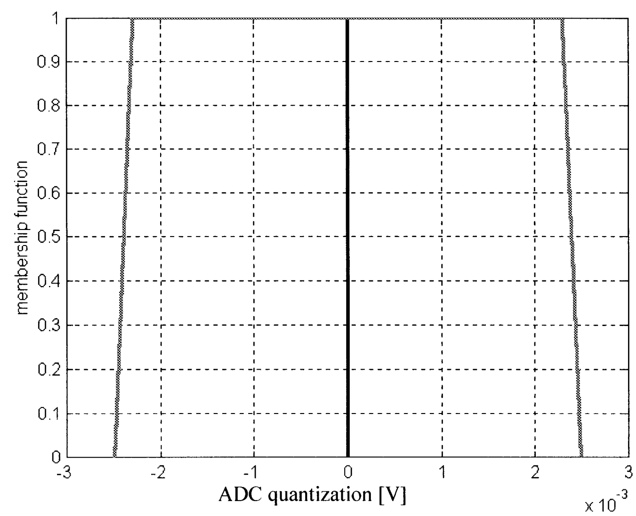

Fig. 8. Representation of the distribution of the ADC quantization error in terms of RFV.

Closed-loop Hall-effect current transducers have been employed that ensure a nominal ratio relative error of 0.4% and a nominal time delay between the output and input signals shorter than 1 μs over a frequency band from dc to 100 kHz and with primary rated currents up to 200 A.

As for the voltage transducers, they are based on a noninductive resistive divider followed by an isolation amplifier that ensures a 1.5-kV insulation level on a frequency band from dc to 120 kHz and with a primary rated voltage up to 400 V.

The analog-to-digital conversion system is based on a commercial board, featuring eight input channels with simultaneous sampling up to 500 kS/s on a single channel, 12-bit resolution and $\pm$10-V input range.

The signal samples coming from the ADC are processed by a PC, equipped with a Pentium III, 200-kHz CPU, and the measurement algorithms have been developed in a LabVIEW environment.

The transducers and the analog-to-digital conversion board have been characterized following the procedure reported in [4] and the distributions of the transducer gain, the time delay between the current and voltage channels, the gain, offset and quantization of the analog-to-digital conversion systems were obtained. The effects of the ADC nonlinearity were found to be negligible with respect to the quantization effects.

These distributions are given as relative frequency histograms of the considered quantities and have been obtained by keeping into account both the results of repeated measurements and the uncertainty of the employed instruments, without making any distinction between the systematic and random effects.

On the other hand, the proper definition of the membership function of a RFV allows to take into account such effects separately. As for the inner interval of the membership function, it has been obtained considering only the contributions to the uncertainty due to the employed instruments and the possible correlations. In fact, the possible deviations of the values given by the instruments from the actual measured value is supposed to be always the same, although it is unknown and only a confidence interval within which it is supposed to lie is known from the instrument uncertainty provided by the manufacturer or obtained from calibration. Therefore, the inner interval of the member-

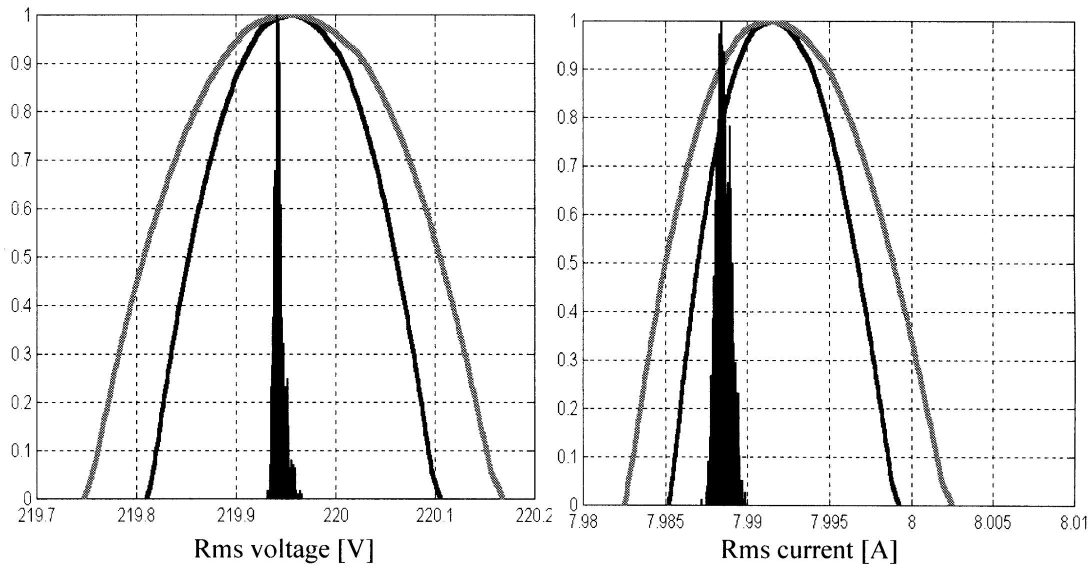

Fig. 9. Comparison between the distribution of the measured voltage and current rms values in terms of simulated RFV and experimental relative frequency histograms.

ship function (9) is obtained by suitably combining the instruments' uncertainty.

As for the side intervals of the membership functions, that take into account the random effects, they can be obtained from the original distributions. In fact, the global width of each interval of confidence $x_4^\alpha - x_1^\alpha$ can be obtained by suitably processing the original cumulative frequency histograms [4]. By properly placing the inner interval $\lfloor x_2^\alpha, x_3^\alpha \rfloor$ within the $\lfloor x_1^\alpha, x_4^\alpha \rfloor$ interval, the side intervals are obtained.

Fig. 4 shows the RFV associated with the gain of the voltage and current transducers. Similarly, Fig. 5 shows the RFV associated with the time delay between the current and voltage channels.

It can be noted that these RFVs are simple fuzzy variables, since the systematic effects introduced by the instruments used in the transducers' calibration process are prevailing over any other effect [4].

Fig. 6 shows the RFV associated with the ADC gain, and Fig. 7 and 8 show the RFVs associated with the ADC offset and quantization respectively.

Fig. 6 gives clear evidence that a RFV is able to represent both the systematic and random effects on the distribution of the result of a measurement.

On the other hand, Fig. 7 and 8 show how a RFV can represent quite well also a distribution of measurement results that is due only to random effects, as is the case of the ADC offset and quantization, whose measurement procedure was found to be much more sensitive to random effects than to the systematic effects introduced by the employed instruments [4].

B. Uncertainty Estimation

Once the RFVs G_{tv}, G_{ti} and T_{t} representing the voltage and current transducer gains and the time delay between the voltage and current channels, respectively, and the RFVs G_{AD}, O and Q representing the ADC gain, offset, and quantization, respectively, have been determined as shown in the previous section, it is possible to estimate the RFV associated with the result of the measurement of the active power.

The input signals $v(t)$ and $i(t)$ are simulated, and each input sample $v(t_{\mathrm{k}})$ and $i(t_{\mathrm{k}})$ taken at time t_{k} is associated with a RFV obtained as

$$\begin{aligned} V_{\mathrm{k}} &= (v(t_{\mathrm{k}}) \cdot G_{\mathrm{tv}} + O) \cdot G_{\mathrm{AD}} + Q \\ I_{\mathrm{k}} &= (i(t_{\mathrm{k}} + T_{\mathrm{t}}) \cdot G_{\mathrm{ti}} + O) \cdot G_{\mathrm{AD}} + Q. \end{aligned} \tag{11}$$

By processing the RFVs defined in (11), according to the measurement algorithm

$$P = \frac{1}{n} \sum_{k=0}^{n-1} V_{\mathrm{k}} I_{\mathrm{k}} \tag{12}$$

n being the number of samples over the observation interval, the measurement result P is obtained as a RFV that represents its distribution over a confidence interval.

The above procedure has been applied by simulating two sinusoidal waveforms of voltage and current, with 50-Hz frequency and 220-V and 8-A rms amplitude, respectively. 256 samples per period were taken, and an observation interval of four periods of the input signals was considered. The current signal was considered both in phase with the voltage signal and lagging $\pi/3$, $\pi/6$, and $\pi/2$ rad. In order to compare the results of the proposed procedure with the experimental results provided by the instrument, a Fluke 5500 calibrator was employed to generate real signals with the same characteristics as the simulated ones, and 1000 repeated measurements were performed.

Fig. 9 shows the RFVs representing the simulated measurement results for the rms values of the voltage and current waveforms. The same figure shows also the histogram of the relative frequencies for the distribution of the experimental results, normalized to 1 in order to have the same scale for the two plots.

Fig. 10 shows the RFVs representing the simulated measurement results for the active power in the four different situations considered for the phase shift between the current and voltage waveforms. The same figure shows also the histogram of the rel-

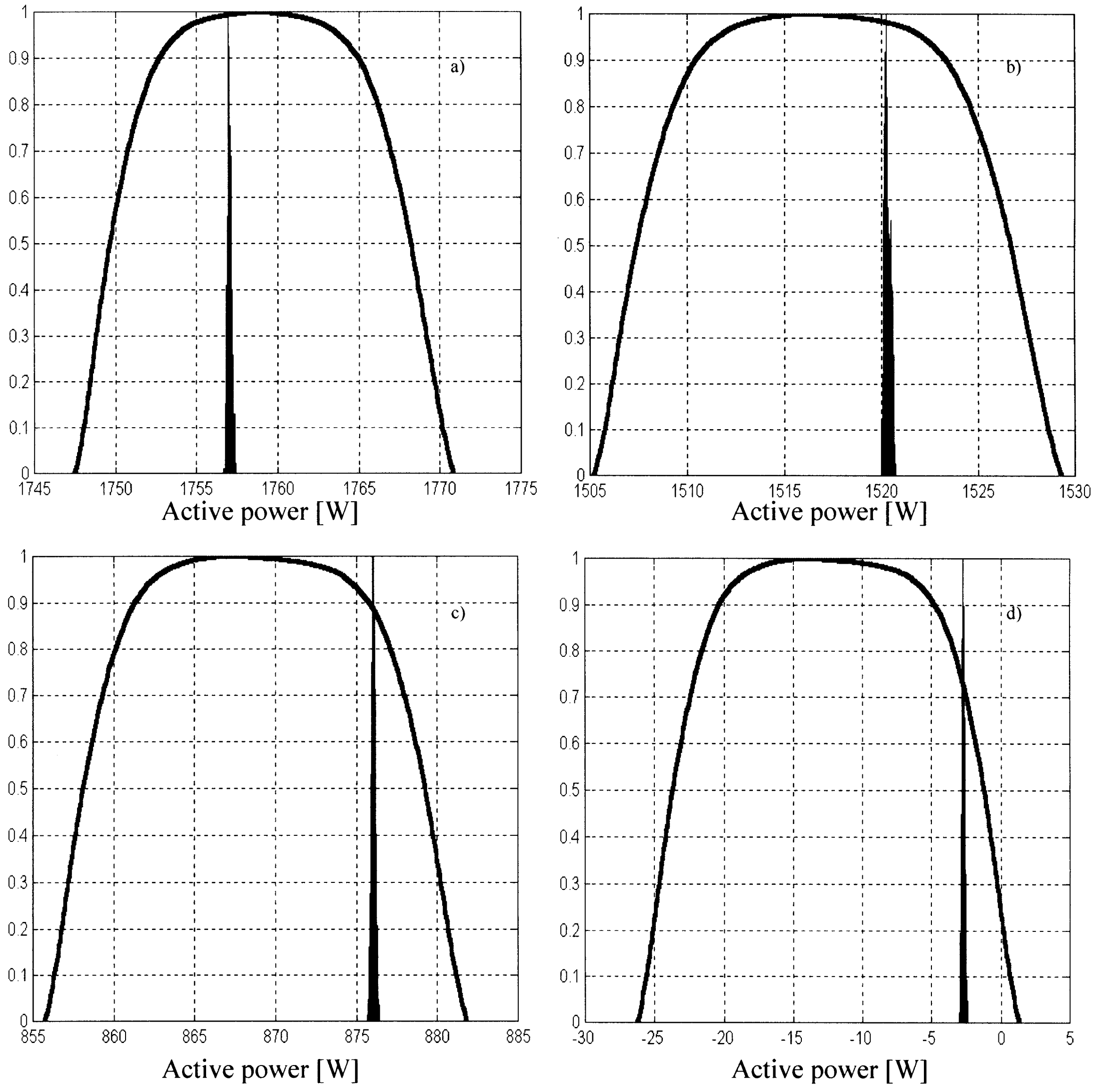

Fig. 10. Comparison between the distribution of the measured active power in terms of simulated RFV and experimental relative frequency histograms. (a) Current in phase with the voltage; (b) current lagging $\pi/3$ rad; (c) current lagging $\pi/6$ rad; (d) current lagging $\pi/2$ rad.

ative frequencies for the distribution of the experimental results, again normalized to 1.

The distribution of the experimental results is always included within the confidence interval given by the RFVs representing the measurement results, thus showing the correctness of the proposed method.

It can be noted that the distribution of the measured rms values is still affected by random effects, mainly due to the ADC gain, offset and quantization. On the other hand, the systematic effects due to the time delay introduced by the transducers prevail over all other effects in the measurement of the active power, and this is well reflected by the membership function of the RFVs representing the distribution of the measured active power values.

V. Conclusion

A method has been presented for representing the result of a measurement, together with the associated uncertainty, in terms of a random-fuzzy variable. The method is consistent with the recommendations of the ISO Guide [1], since a RFV represents the distribution of the possible measurement results over intervals of confidence with given confidence levels. Moreover, RFVs can also naturally separate the contributions to the measurement uncertainty of the systematic and random effects, thus improving the uncertainty representation suggested by the ISO Guide [1].

The proposed method can be more efficiently implemented than other available methods based on a statistical approach [4], since it does not require processing a large number of data extracted from given distributions. Due to its good computation efficiency, this method could even be implemented onboard the instruments, so that a real-time estimation of the measurement uncertainty could be obtained.

The first experimental results show that the obtained RFVs are a good estimation of the distributions of the possible measurement results and show that better estimations can be obtained by means of a better calibration of the input stages of the employed instrument.

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