

Глава 6.

Задача 1.

$$H = \begin{pmatrix} 1 & \alpha^1 & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 & \alpha^7 & \alpha^8 & \alpha^9 & \alpha^{10} & \alpha^{11} & \alpha^{12} & \alpha^{13} & \alpha^{14} \\ 1 & \alpha^2 & \alpha^4 & \alpha^6 & \alpha^8 & \alpha^{10} & \alpha^{12} & \alpha^{14} & \alpha^1 & \alpha^3 & \alpha^5 & \alpha^7 & \alpha^9 & \alpha^{11} & \alpha^{13} \end{pmatrix} =$$

$$p(x) = 1 + x + x^4$$

$$= \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad d = 3$$

Задача 3.

Таблицы и циклотомические классы посчитаны программно (Task3).

Для $n = 31$:

$$C_0 = \{0\}$$

$$C_1 = \{1, 2, 4, 8, 16\}$$

$$C_3 = \{3, 6, 12, 24, 17\}$$

$$C_5 = \{5, 10, 20, 9, 18\}$$

$$C_7 = \{7, 14, 28, 25, 19\}$$

$$C_{11} = \{11, 22, 13, 26, 21\}$$

$$C_{15} = \{15, 30, 29, 27, 23\}$$

n	Generator polynomial	k_{\min}	d	d_{best}
31	M_0	30	2	2
	M_15	26	3	3
	M_0M_1	25	4	4
	M_7M_15	21	5	5
	M_0M_1M_3	20	6	6
	M_7M_11M_15	16	7	8
	M_0M_1M_3M_5	15	8	8
	M_3M_7M_11M_15	11	11	11
	M_0M_1M_3M_5M_7	10	12	12
	M_3M_5M_7M_11M_15	6	15	15
	M_0M_1M_3M_5M_7M_11	5	16	16
	M_1M_3M_5M_7M_11M_15	1	31	31

Для $n = 63$:

$$C_0 = \{0\}$$

$$C_1 = \{1, 2, 4, 8, 16, 32\}$$

$$C_3 = \{3, 6, 12, 24, 48, 33\}$$

$$C_5 = \{5, 10, 20, 40, 17, 34\}$$

$$C_7 = \{7, 14, 28, 56, 49, 35\}$$

$$C_9 = \{9, 18, 36\}$$

$$C_{11} = \{11, 22, 44, 25, 50, 37\}$$

$$C_{13} = \{13, 26, 52, 41, 19, 38\}$$

$$C_{15} = \{15, 30, 60, 57, 51, 39\}$$

$$C_{21} = \{21, 42\}$$

$$C_{23} = \{23, 46, 29, 58, 53, 43\}$$

$$C_{27} = \{27, 54, 45\}$$

$$C_{31} = \{31, 62, 61, 59, 55, 47\}$$

n	Generator polynomial	k_{min}	d	d_{best}
63	M_0	62	2	2
	M_31	57	3	3
	M_0M_1	56	4	4
	M_15M_31	51	5	5
	M_0M_1M_3	50	6	6
	M_15M_23M_31	45	7	6-7
	M_0M_1M_3M_5	44	8	8
	M_7M_15M_23M_31	39	9	9-11
	M_0M_1M_3M_5M_7	38	10	10-12
	M_7M_15M_23M_27M_31	36	11	11-12
	M_0M_1M_3M_5M_7M_9	35	12	12-13
	M_7M_13M_15M_23M_27M_31	30	13	13-16
	M_0M_1M_3M_5M_7M_9M_11	29	14	14-16
	M_7M_11M_13M_15M_23M_27M_31	24	15	16-19
	M_0M_1M_3M_5M_7M_9M_11M_13	23	16	16-20
	M_3M_7M_11M_13M_15M_23M_27M_31	18	21	21-22
	M_0M_1M_3M_5M_7M_9M_11M_13M_15	17	22	22-23
	M_3M_7M_11M_13M_15M_21M_23M_27M_31	16	23	23-24
	M_0M_1M_3M_5M_7M_9M_11M_13M_15M_21	15	24	24
	M_3M_5M_7M_11M_13M_15M_21M_23M_27M_31	10	27	27-28
	M_0M_1M_3M_5M_7M_9M_11M_13M_15M_21M_23	9	28	28
	M_3M_5M_7M_9M_11M_13M_15M_21M_23M_27M_31	7	31	31
	M_0M_1M_3M_5M_7M_9M_11M_13M_15M_21M_23M_27	6	32	32
	M_1M_3M_5M_7M_9M_11M_13M_15M_21M_23M_27M_31	1	63	63

Задача 4.

Для $n = 17$:

$$C_0 = \{0\}$$

$$C_1 = \{1, 2, 4, 8, 16, 15, 13, 9\}$$

$$C_3 = \{3, 6, 12, 7, 14, 11, 5, 10\}$$

n	Generator polynomial	k_{min}	d	d_{best}
17	M_0	16	2	2
	M_1	9	3	5
	M_3	9	4	6
	M_1M_3	1	17	17

Для $n = 21$:

$$C_0 = \{0\}$$

$$C_1 = \{1, 2, 4, 8, 16, 11\}$$

$$C_3 = \{3, 6, 12\}$$

$$C_5 = \{5, 10, 20, 19, 17, 13\}$$

$$C_7 = \{7, 14\}$$

$$C_9 = \{9, 18, 15\}$$

n	<i>Generator polynomial</i>	k_{min}	d	d_{best}
21	M_0	20	2	2
	M_7M_9	16	3	3
	M_0M_1	14	4	4
	M_5M_9	12	5	5
	M_0M_1M_3	11	6	6
	M_1M_5M_9	6	7	8
	M_0M_1M_3M_5	5	8	10
	M_1M_5M_7M_9	4	9	10
	M_0M_1M_3M_5M_7	3	10	12
	M_1M_3M_5M_7M_9	1	21	21