2. Methods

2.1. Shade Projected by Direct Sunlight

The following chapters describe how the shaded area on the ground as well as on PV panels was calculated.

2.1.1. Ground-Shading

To calculate the shaded area projected on the ground, the position of the four corners of the polygon projected by the PV panel was calculated. All shade polygons were subsequently merged and the total area was calculated. This will be described in detail in the following sections.

To calculate the position of the polygons corners incomming direct sunbeams were split into components corresponding to two orthogonal planes. For these calculations the used planes are those defined by vectors pointing north and up as well as east and up respectively. An illustration of how incomming sunbeams were split can be seen in figure 2.1.

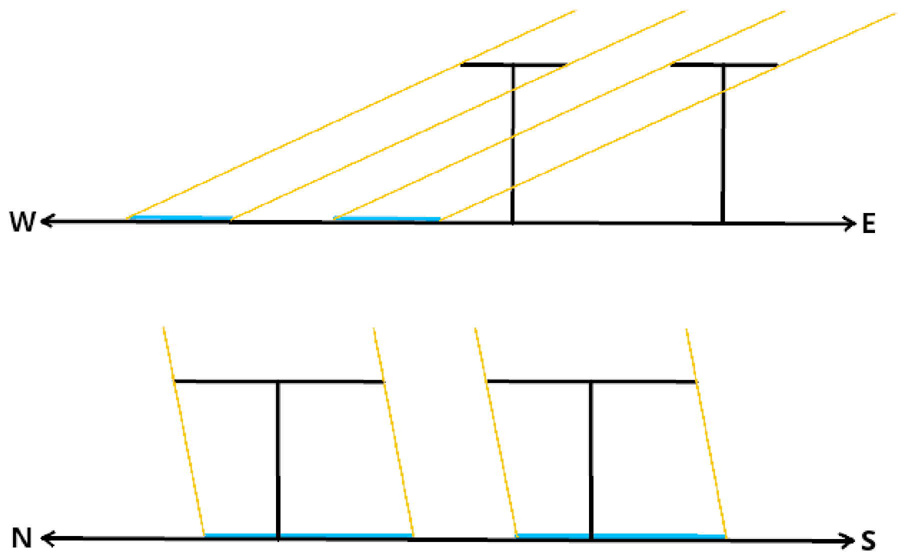

Figure 2.1.: Sunbeams split into components

Calculating height differences between edges

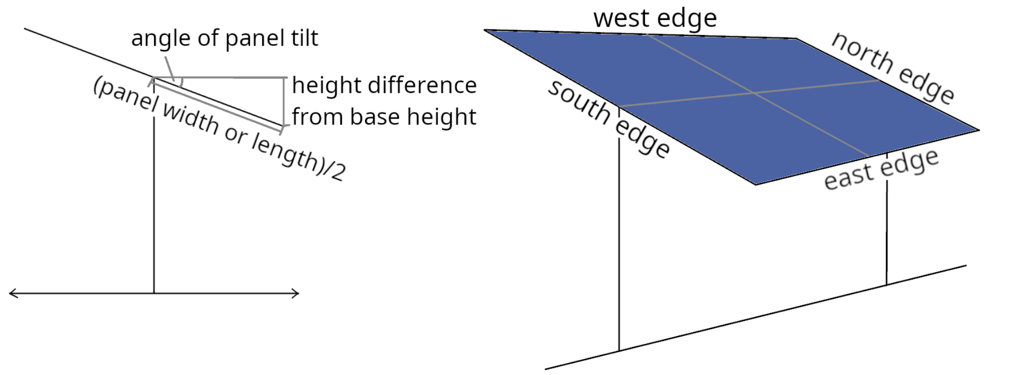

Figure 2.2.: Height difference of edges from base height

The basis for the calculation of the height differences h_{diff} between the panel edges and the base height of the PV panel can be seen in figure 2.2. The difference is calculated via the following formulas:

$$h_{\text{diff, W}} = sin(\alpha) * \frac{w}{2} \tag{2.1}$$

$$h_{\text{diff, E}} = -sin(\alpha) * \frac{w}{2} \tag{2.2}$$

α is the tilt of the PV panel in east/west plane for the calculations of height differences of east and west edges. w is the variable describing the measurement of the panel in east/west direction, also called width here.

$$h_{\text{diff, N}} = sin(\beta) * \frac{l}{2} \tag{2.3}$$

$$h_{\text{diff, S}} = -sin(\beta) * \frac{l}{2} \tag{2.4}$$

β is the tilt of the PV panel in north/south plane for the calculations of height differences of south and north edges. l is the variable describing the measurement of the panel in north/south direction, also called length here.

Calculating corner heights

The corner heights are calculated by simply adding the height differences of the adjoining edges to the base height. Note that the height differences calculated in equations 2.1, 2.2, 2.3 and 2.4 may be negative.

$$h_{\text{N/E}} = h_{\text{base}} + h_{\text{diff, N}} + h_{\text{diff, E}} \tag{2.5}$$

$$h_{\text{N/W}} = h_{\text{base}} + h_{\text{diff, N}} + h_{\text{diff, W}} \tag{2.6}$$

$$h_{\text{S/E}} = h_{\text{base}} + h_{\text{diff, S}} + h_{\text{diff, E}} \tag{2.7}$$

$$h_{\text{S/W}} = h_{\text{base}} + h_{\text{diff, S}} + h_{\text{diff, W}} \tag{2.8}$$

The variables $h_{\text{N/E}}$, $h_{\text{N/W}}$, $h_{\text{S/E}}$ and $h_{\text{S/W}}$ stand for the heights of the north-east, north-west, south-east and south-west corners respectively. h_{base} is the base height of the PV system, meaning the height of the center point of the panel.

Calculating shade corner positions

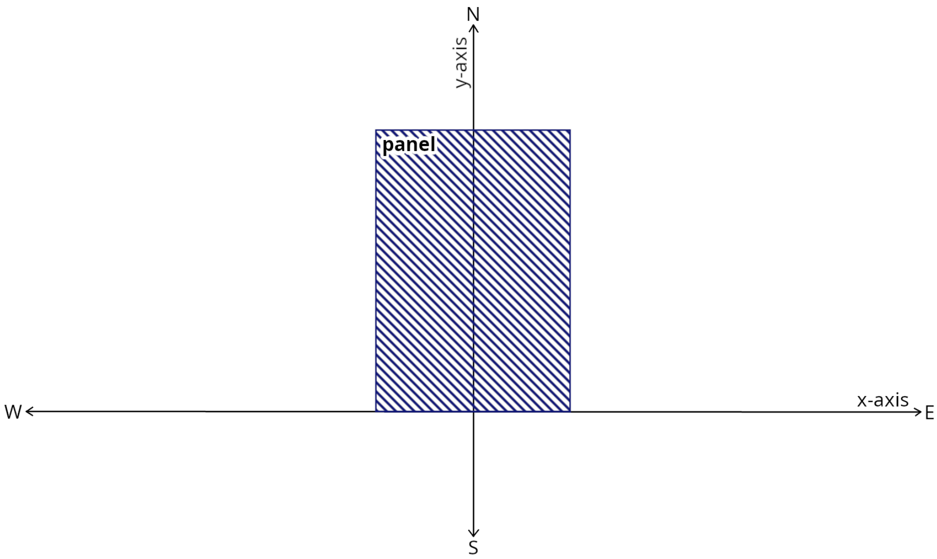

Figure 2.3.: Coordinate system and position of PV panel

In the following section the calculation of the position of the corners of the shade projected on the ground will be discussed. Figure 2.3 shows the coordinate system used for these calculations as well as the position of the PV panel in it.

The x-axis position of the western corners, $x_{\text{N/W}}$ and $x_{\text{S/W}}$, are determined via the construction shown in figure 2.4. They are calculated via formulas 2.9 and 2.10.

$$x_{\text{N/W}} = -|\cos \alpha_{\text{panel}}| \cdot \frac{w}{2} - \frac{h_{\text{N/W}}}{\tan \alpha_{\text{sun}}} \tag{2.9}$$

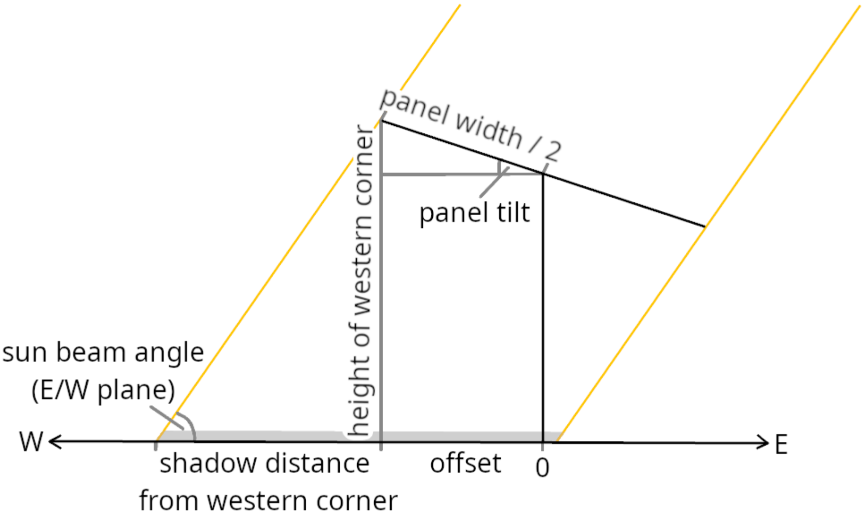

Figure 2.4.: Position of western shade corners

$$x_{\mathrm{S/W}} = -|\cos \alpha_{\mathrm{panel}}| \cdot \frac{w}{2} - \frac{h_{\mathrm{S/W}}}{\tan \alpha_{\mathrm{sun}}} \tag{2.10}$$

The x-axis position of the eastern corners, $x_{\mathrm{N/E}}$ and $x_{\mathrm{S/E}}$, are determined via the construction shown in figure 2.5. They are calculated via formulas 2.11 and 2.12.

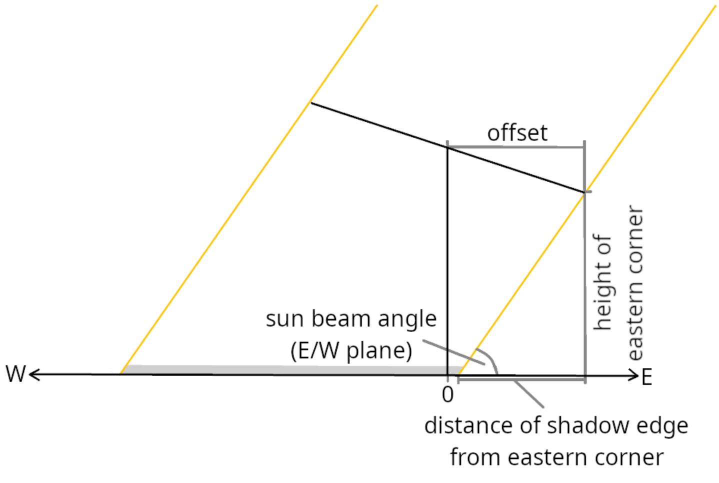

Figure 2.5.: Position of eastern shade corners

$$x_{\mathrm{N/E}} = |\cos \alpha_{\mathrm{panel}}| \cdot \frac{w}{2} - \frac{h_{\mathrm{N/E}}}{\tan \alpha_{\mathrm{sun}}} \tag{2.11}$$

$$x_{\mathrm{S/E}} = |\cos\alpha_{\mathrm{panel}}| \cdot \frac{w}{2} - \frac{h_{\mathrm{S/E}}}{\tan\alpha_{\mathrm{sun}}} \tag{2.12}$$

In the equations 2.9, 2.10, 2.11 and 2.12 α_{panel} describes the panel tilt in E/W direction, α_{sun} the angle of incoming sun beams in the E/W plane. As in the previous section w, $h_{\mathrm{N/W}}$, $h_{\mathrm{S/W}}$, $h_{\mathrm{N/E}}$ and $h_{\mathrm{S/E}}$ stand for the width of the PV panel and the heights of the PV panel's corners respectively.

The y-axis position of the northern corners, $y_{\mathrm{N/W}}$ and $y_{\mathrm{N/E}}$, are determined via the construction shown in figure 2.6. They are calculated via formulas 2.13 and 2.14.

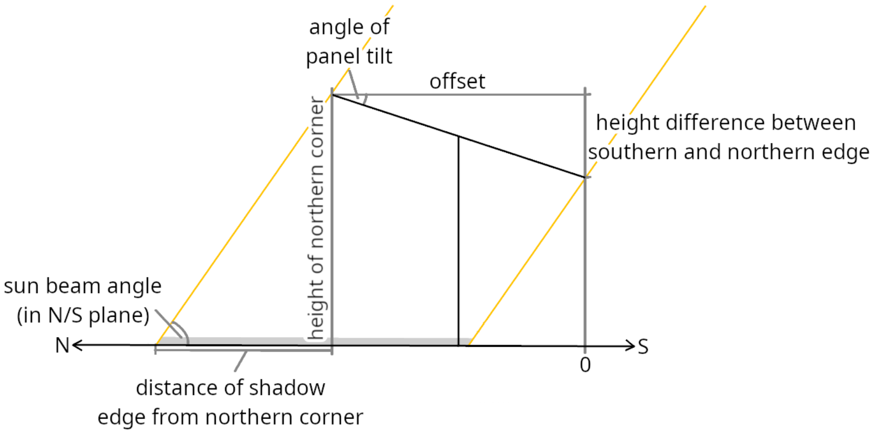

Figure 2.6.: Position of northern shade corners

$$y_{\mathrm{N/W}} = \frac{h_{\mathrm{N/W}}}{\tan\beta_{\mathrm{sun}}} + |\cos\beta_{\mathrm{panel}}| \cdot l \tag{2.13}$$

$$y_{\mathrm{N/E}} = \frac{h_{\mathrm{N/E}}}{\tan\beta_{\mathrm{sun}}} + |\cos\beta_{\mathrm{panel}}| \cdot l \tag{2.14}$$

The y-axis position of the southern corners, $y_{\mathrm{S/W}}$ and $y_{\mathrm{S/E}}$, are determined via the construction shown in figure 2.7. They are calculated via formulas 2.15 and 2.16.

$$y_{\mathrm{S/W}} = \frac{h_{\mathrm{S/W}}}{\tan\beta_{\mathrm{sun}}} \tag{2.15}$$

$$y_{\mathrm{S/E}} = \frac{h_{\mathrm{S/E}}}{\tan\beta_{\mathrm{sun}}} \tag{2.16}$$

In the equations 2.13, 2.14, 2.15 and 2.16 β_{panel} describes the panel tilt in N/S direction, β_{sun} the angle of incoming sun beams in the N/S plane. As in the previous section l, $h_{\mathrm{N/W}}$, $h_{\mathrm{S/W}}$, $h_{\mathrm{N/E}}$ and $h_{\mathrm{S/E}}$ stand for the length of the PV panel and the heights of the PV panel's corners respectively.

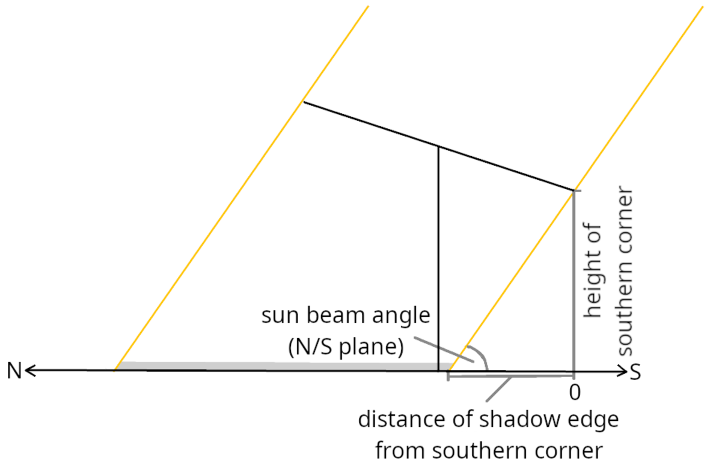

Figure 2.7.: Position of southern shade corners

Calculating total shade on field

In the final step of the calculation the shade cast by a single is turned into a polygon using the previously calculated shade corners.

First, a copy of the polygon is then added for all simulated PV panels. The distances between shadow polygons being the distances between PV panels. In this context the distances refer to the distances between center points of both shadows and panels.

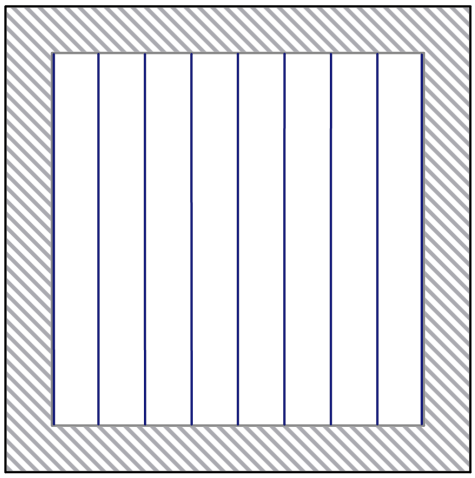

Figure 2.8.: Field layout with vertical PV panels

Second, the field limits are defined. In this thesis a standard field is defined as a 100 by 100 meter square, an area of one hectare. This includes a buffer area with a width of 10 meters. An illustration of this can be seen in figure 2.8, the hatched area represents the buffer, the dark blue lines columns of vertical PV panels.

In a last step, intersection area of the shade polygons with the field is calculated. This intersection is expressed in percent of the total field area. All calculations in relation to the created polygons were done using the python module shapely.