NEW PRIORITY CALCULATIONS

Abstract

There are many methods of deriving priority vectors from pairwise comparison matrices, e.g. the standard largest eigenvector, the geometric mean, Harker's method, the least squares method, etc. Through our work on the SimpleAHP web application, we discovered an issue with most of these methods, that confused our users. The issue arises when there are two or more voters on a particular pairwise set, and those users have opposite votes on everything (we call them doppleganger voters). Given doppleganger voters one would expect the resulting priority sets to be inverses, or at least have reversed rankings. This is not always the case (this result has long been known, but the SimpleAHP web application made this idea more apparent).

While the geometric mean method does address this issue, it has its own shortcomings. Therefore in this paper we describe two new priority vector calculations that address doppleganger voters, while retaining the graph theoretic spirit of the eigenvector method. We compare and contrast the results of the priority calculations with some of the standard methods (eigenvector and geoemtric mean) on some differentiating examples. In addition we provide open source implementations of the new calculations in several languages (Python, R, Excel) in a free available github repository.

1 Introduction

The process of deriving a priority vector from a pairwise comparison matrix is a very well studied problem. However, our work creating the SimpleAHP web application (see Adams (2016a) for more information) brought to light some subtleties surrounding the standard largest eigenvector calculation. In the SimpleAHP web application, participants can provide symbolic votes of *better* and *much better*, and the facilitator can adjust the meaning of those symbolic votes. This structure allows for easy construction of doppleganger voters.

Definition 1 (Doppleganger voter pair)**.** *Two voters on a pairwise comparison set are said to be* <u>*dopplergangers*</u> *if the pairwise comparison matrix of one user is the transpose of the other user.*

Doppleganger voters happened naturally with our participants because of the reduced amount of choice. However, a second natural way to construct doppleganger voters in the SimpleAHP web application is to change the meaning of *better* to be a number less than 1, and similarly for *much better*. Effectively we are making an inverse interpretation of what *better* and *much better* mean.

We constructed a doppleganger voter in this fashion on our first participant and attempted to verify our calculations by verifying that the doppleganger voter had reversed the order of the alternatives being compared. In fact we found that it did not reverse the order, and eventually we discovered this fact is indeed well known, (e.g. see Choo and Wedley (2004)). However this behavior of the largest eigenvector was troublesome for our target user, namely youth and those with no experience in AHP theory.

In this paper we put forward two novel methods for calculating priority vectors from a pairwise comparison matrix. Both are adaptations of the standard largest eigenvector calculation, and both are the result of a limiting process very similar to the largest eigenvector's standard computational algorithm.

2 Literature Review

This work is heavily influenced by our work in Adams (2016a) where we create tools and techniques that allow youths, and those inexperienced in AHP, to nonetheless use AHP successfully on their own, without the need of facilitator or expert in the theory. Through that work we discovered some properties of the largest eigenvector priority vector calculation that confused our target audience (and in truth it confused us initially as well).

This behavior of the largest eigenvector, namely that the largest eigenvector of the transpose and the largest eigenvector of the original pairwise matrix need not be related, is well known and can be found in Choo and Wedley (2004) or Saaty (1990). However, because of the simplicity of changing symbolic votes in the SimpleAHP web interface, this property of the largest eigenvector is more readily observable and causes issues for those new to AHP theory. The geometric average is a standard method that remedies the transpose issue, but it has its own shortcomings (see Barzilai and Golany (1994) for details of using geometric averages of columns). Instead we opted to create two new eigenvector inspired, priority vector calculations, both of which respect the doppleganger property.

3 Hypotheses/Objectives

This paper defines two new priority vector calculations, algorithmically. We compare and contrast these new calculations with the standard eigenvector method and provide some examples that elucidate the differences in approach.

4 Research Design/Methodology

For this research, we develop programming libraries for several languages that implement the new algorithm. These libraries are freely available on our github page at Adams (2016b). In addition there is a publicly available gitter room for discussion of these algorithms and results (see Adams (2016c)).

5 Data/Model Analysis

6 Limitations

7 Conclusions

8 Key References

Adams, B. (2016a). Ahp for student decisions in a montessori elementary class.

Adams, B. (2016b). Github ahpnewpriority project. `https://github.com/wjladams/ahpNewPriority`.

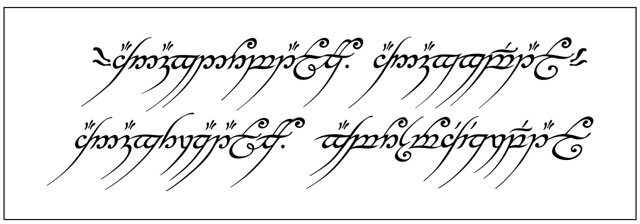
Adams, B. (2016c). Gitter discussion room for the ahpnewpriority project. `https://gitter.im/wjladams/ahpNewPriority`.

Barzilai, J. and Golany, B. (1994). Ahp rank reversal, normalization and aggregation rules. *Infor*, 32(2):57.

Choo, E. U. and Wedley, W. C. (2004). A common framework for deriving preference values from pairwise comparison matrices. *Computers & Operations Research*, 31(6):893–908.

Saaty, T. L. (1990). Eigenvector and logarithmic least squares. *European journal of operational research*, 48(1):156–160.

9 Appendix 1



One ring to rule them all
One ring to find them
One ring to bring them all
And in the darkness bind them