Superfluidity as Bose-Einstein Condensation

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ABSTRACT

Bose-Einstein Condensate is a quantum phenomena that happens when a macroscopic number of bosons occupy the lowest quantum state at a temperature as close to absolute zero as possible. This leads to quantum behaviours like super fluidity, the ability for a fluid to flow without any viscosity. The relationship between Bose Einstein condensate and super fluidity is still a major question being asked in physics. The properties of super fluidity in Helium-4 are explored, with a focus on the experimental observations known as lambda transitions and quantized vortices and the thermal conductivity being infinite. Additionally, the derivation for Bose-Einstein condensates critical temperature is discussed. As well as a brief history as to how Bose-Einstein condensate was discovered and current research in the field and limitations, such as the difficulty in creating such extreme environments like absolute zero temperature.

1. INTRODUCTION

Bose-Einstein condensate (BEC) first proposed by Einstein in 1925, is formed by the condensation of a large (macroscopic) number of bosons at temperatures close to absolute zero. The bosons begin to occupy the same quantum state (the lowest possible quantum state). The result leads to a microscopic quantum mechanical phenomena, where wave function interference becomes apparent at the macroscopic level, along with the origin of superfluidity and superconductivity.

Superfluidity often occurs in unison with Bose-Einsten condensation. However, it is still unknown whether both phenomena are directly related to each other (1). Superfluidity is the property of a fluid with zero viscosity, which leads to more phenomenena such as the fountain effect, the lambda point, discontinuities in thermal conductivity, and the formation of vortices within superfluids (2).

2. HISTORY

Bose-Einstein condensate is a form of matter that was predicted by Einstein in 1924 based on quantum-mechanical formulations by Satyendra Nath Bose. Einstein extended Boses ideas, which resulted in the discovery of Bose gas, which are a part of Bose-Einstein statistics, which describe the probability distribution of Bosons. Although Einstein and Bose formulated this in 1924, it was not until 1995, that the first atomic Bose-Einstein condensate was produced, at the University of Colorado Boulder, where a gas of rubidium atoms was cooled down to 170 nanokelvins above absolute zero.

3. DERIVATION OF THE CRITICAL TEMPERATURE

Bose-Einstein condensation was first theorized for an ideal gas composed of bosons. The chemical potential μ, comes from the conditions

$$n_k = \frac{1}{e^{\beta(\epsilon_k - \mu)} - 1}, \quad \sum_k n_k = N$$

with $\epsilon_k = \hbar^2 k^2/2m$, and $\beta = \frac{1}{kT}$. Here, μ goes to 0 at some temperature, T_c, the critical temperature. Below T_c, some number of particles N_0 begin to occupy the lowest state since μ is stuck at zero.

However, this only works when the ideal Bose gas is in three spatial dimensions, $d = 3$. But what if $d < 3$?

T_c is determined by solving the following equation where the particle density of states $\nu(\epsilon) \approx \epsilon^{\frac{d}{2}-1}$ and setting it equal to N,

$$\int d\epsilon \;\; \nu(\epsilon)\frac{1}{e^{\beta_c \epsilon}-1} = N,$$

if $d < 3$, the integral diverges and Bose-Einstein condensate doesn't occur, such that $T_c = 0$ (3).

The critical temperature for an ideal gas is know to be

$$T_c = \left(\frac{n}{\zeta(3/2)}\right)^{2/3} \frac{2\pi\hbar^2}{mk_B} \approx 3.31\frac{\hbar^2}{mk_B}n^{2/3}$$

where n is the particle density, m is the particle mass, and $\zeta(3/2)$ is the Riemann zeta function, where $\zeta(3/2) \approx 2.612$ (4).

4. SUPERFLUIDITY IN HELIUM -4

The liquefaction of Helium was first accomplished in 1908 by Heike Kamerlingh Onnes. The liquefaction of helium would prove to be a major advancement in the study of elements near absolute zero, as many strange behaviours have been observed with Liquid Helium. Helium has a very low melting point as a result of its weak interatomic forces. At low temperatures as an ordinary fluid Helium I, it transitions to the superfluid Helium II (also know as the superfluid state of Helium -4) phase. The temperature at which this transition occurs is known as the *Lambda point*, and has an approximate value of $T_\lambda = 2.18\,K$.

In an experiment done by W.H. Keesom and A.P. Keesom in 1932, the thermal conductivity of Helium II was observed to be nearly infinite. As heat is added to a chamber of Helium I, the temperature initially spikes before stabilizing over a period of time as the heat diffuses through Helium I. However, for Helium II as heat is added to the chamber, the temperature stabilizes instantly (2).

Even though many strange properties are observed with liquid Helium II, its appearance of

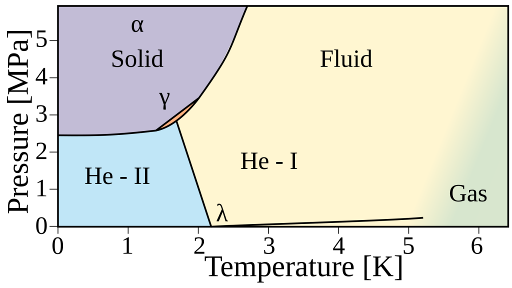

Fig. 1.— Phase Diagram of helium near absolute 0

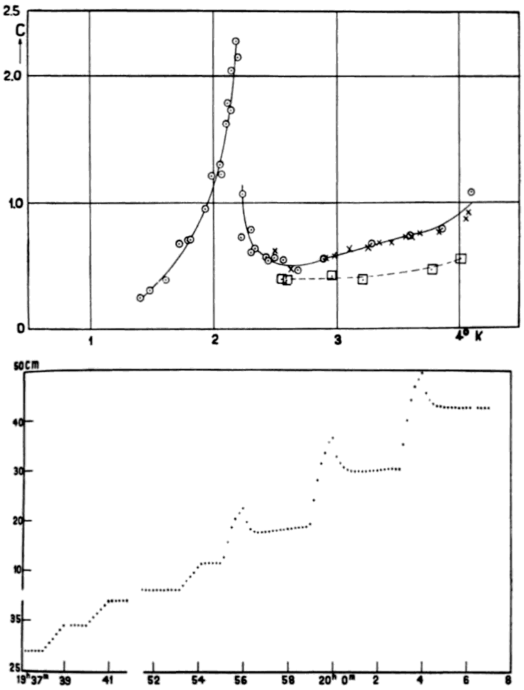

Fig. 2.— Top: Specific heat of Liquid Helium II, showing a discontinuity at T_λ. Bottom: Temperature of Liquid Helium II vs time. Below T_λ, immediate stabilization is observed. Above T_λ, temperature spikes initially before stabilization (2).

having zero viscosity is the main lead behind dubbing it as a superfluid. The lack of viscosity in Helium II allows it to flow without dissipating any energy, and allows for quantized vortices to form. Helium II is able to flow with almost no internal friction or dissipation of energy as a result of having zero viscosity. The various observations of the superfluid were first explained as the system consisting of a superfluid (0 viscosity) and a normal viscous fluid (5). The transition observed at the lambda transition boundary was confirmed to be caused by Bose-Einstein condensate by neutron scattering experiments (6).

5. CURRENT RESEARCH

There is plenty of research still on going at the time on Bose-Einstein condensates and their relations to superfluids. In an article from March 2024 (8), magnon BECs are investigated for use as probes and components for room-temperature quantum devices. The research is based on the BEC systems of Helium II. The study mentions recent advances in the applications of magnon BEC systems to study areas ranging from particle physics and cosmology to new phases of condensed matter (8). A magnon is simply a quasi-particle, as the excitation of the spin structure of an electron in a crystal lattice.

Much of the current research related to BEC is still aimed at improving understanding of the structure and behaviour of Bose-Einstein condensate systems. Severe challenges are faced when trying to perform experimentation around superfluids and BECs, simply due to the nature of having to achieve and maintain temperatures near absolute zero, and being able to handle the liquid at such low temperatures can be of extreme challenge. During Heike Oneness's 1908 experiment, where the liquefaction of helium was first achieved, it took more than 14 hours in the laboratory to achieve the liquefaction of approximately 10 cm^3 of liquid Helium, which lasted less than an hour while still completely isolated within the apparatus (9).

6. CONCLUSION

While it is still in question as to whether Super fluidity is a result of Bose-Einstein condensate, they are both quantum phenomena that arise under extreme and similar conditions. The actual relationship between the two still remains an open ended question in physics, although Bose-Einstein condensate provides a theoretical framework for super fluidity. Experiments with helium-4 and recent advancements when studying magnons Bose-Einstein condensates holds promise for the advancements in condensed matter physics.

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