0.1. **Homological (co)algebra.** The primary reference for this section will be the nLab page on derived functors in homological algebra ([1]).

Recall that given abelian categories $\mathcal{A}$ and $\mathcal{B}$, given an additive functor $F : \mathcal{A} \to \mathcal{B}$, if F is left exact and $\mathcal{A}$ has enough injectives, we may form the *right derived functors* $R^n F : \mathcal{A} \to \mathcal{B}$ of F, for $n \in \mathbb{N}$. Given an object A in $\mathcal{A}$, we may compute $R^n F(A)$ to be the object (defined only up to isomorphism) which is obtained as follows: First, fix an injective resolution $i : A \to I^*$ of A, i.e., the data of a long exact sequence

$$0 \longrightarrow A \xrightarrow{i} I^0 \xrightarrow{d^0} I^1 \xrightarrow{d^1} I^2 \xrightarrow{d^2} I^3 \longrightarrow \cdots$$

where each I^n is an injective object in $\mathcal{A}$. Such a sequence is guaranteed to exist since $\mathcal{A}$ has enough injectives. Then we define $R^n F(A)$ to be the n^{th} cohomology group $H^n(F(I^*))$ of the sequence

$$0 \longrightarrow F(I^0) \xrightarrow{F(d^0)} F(I^1) \xrightarrow{F(d^1)} F(I^2) \xrightarrow{F(d^2)} F(I^3) \longrightarrow \cdots .$$

It is a standard result that this definition of $R^n F(A)$ does not depend on the choice of injective resolution $i : A \to I^*$.

Definition 0.1. Given an abelian category $\mathcal{A}$ with enough injectives and an object A in $\mathcal{A}$, we denote the right derived functors of the left exact functor $\mathrm{Hom}_{\mathcal{A}}(A, -) : \mathcal{A} \to \mathbf{Ab}$ by

$$R^n \mathrm{Hom}_{\mathcal{A}}(A, -) := \mathrm{Ext}^n_{\mathcal{A}}(A, -).$$

Remark 0.2. It is not uncommon to instead define $\mathrm{Ext}^n_{\mathcal{A}}(-, A)$ to be the right derived functor of the functor $\mathrm{Hom}_{\mathcal{A}}(-, A) : \mathcal{A}^{\mathrm{op}} \to \mathbf{Ab}$, in which case we may compute $\mathrm{Ext}^n_{\mathcal{A}}(B, A)$ by means of *projective* resolutions of A in $\mathcal{A}$. It is a standard result that these definitions of $\mathrm{Ext}^n_{\mathcal{A}}(A, B)$ coincide.

Now, the first result we will state is that in order to compute the values of the right derived functors $R^n F(A)$, we do not need to consider strictly injective resolutions of A, rather, we may consider more generally "F-acyclic resolutions". First, we define F-acyclic objects:

Definition 0.3 ([1, Definition 3.8])**.** Let $F : \mathcal{A} \to \mathcal{B}$ be a left or right exact additive functor between abelian categories, and suppose $\mathcal{A}$ has enough injectives. An object A in $\mathcal{A}$ is called an *F-acyclic object* if $R^n F(A) = 0$ for all $n > 0$.

Definition 0.4. Let $F : \mathcal{A} \to \mathcal{B}$ be a left exact additive functor between abelian categories, and suppose $\mathcal{A}$ has enough injectives. Then given an object A in $\mathcal{A}$, an *F-acyclic resolution* $i : A \to I_F^*$ is the data of a long exact sequence in $\mathcal{A}$

$$0 \longrightarrow A \xrightarrow{i} I_F^0 \xrightarrow{d^0} I_F^1 \xrightarrow{d^1} I_F^2 \xrightarrow{d^2} I_F^3 \longrightarrow \cdots$$

such that each I_F^n is an F-acyclic object in $\mathcal{A}$.

The reasons that F-acyclic objects are useful is that they allow you to compute the right derived functors of F without having to use strictly injective resolutions:

Proposition 0.5 ([1, Theorem 3.15])**.** *Let $F : \mathcal{A} \to \mathcal{B}$ be a left exact additive functor between abelian categories. Then for each object A in $\mathcal{A}$, given an F-acyclic resolution $i : A \to I_F^*$ of A, for each $n \in \mathbb{N}$ there is a canonical isomorphism*

$$R^n F(A) \cong H^n(F(I_F^*))$$

between the n^{th} right derived functor of F evaluated on A and the cohomology of the sequence obtained by applying F to I_F^*.