In the sections that follow, let (E, μ, e) be a monoid object (**??**) and X and Y be objects in $\mathcal{SH}$. From now on we will freely use the coherence theorem for symmetric monoidal categories without comment, in particular, we will assume unitality and associativity hold up to strict equality.

Definition 0.1. Let $\overline{E}$ be the fiber of the unit map $e : S \to E$ (**??**). Let $Y_0 := Y$ and $W_0 := E \otimes Y$. Then for $s > 0$, define

$$Y_s := \overline{E}^s \otimes Y, \qquad W_s := E \otimes Y_s = E \otimes \overline{E}^s \otimes Y,$$

where $\overline{E}^s$ denotes the s-fold tensor product $\overline{E} \otimes \cdots \otimes \overline{E}$. Then we get fiber sequences

$$Y_{s+1} \xrightarrow{i_s} Y_s \xrightarrow{j_s} W_s \xrightarrow{k_s} \Sigma Y_{s+1}$$

obtained by applying $- \otimes Y_s$ to the fiber sequence

$$\overline{E} \to S \xrightarrow{e} E \to \Sigma \overline{E}.$$

We can splice these sequences together to get the *(canonical) Adams filtration of* Y:

$$\begin{tikzcd}
\cdots \arrow[r] & Y_3 \arrow[r, "i_2"] \arrow[d, "j_3"] & Y_2 \arrow[r, "i_1"] \arrow[d, "j_2"] & Y_1 \arrow[r, "i_0"] \arrow[d, "j_1"] & Y_0 = Y \arrow[d, "j_0"] \\
& W_3 & W_2 \arrow[ul, dashed, "k_2"] & W_1 \arrow[ul, dashed, "k_1"] & W_0 \arrow[ul, dashed, "k_0"]
\end{tikzcd}$$

where here each k_s is of degree $-\mathbf{1}$ (in particular, the above diagram does not commute in any sense), and each i_s and j_s have degree 0. We can extend this diagram to the right by setting $Y_s = Y$, $W_s = 0$, and $i_s = \mathrm{id}_Y$ for $s < 0$. Then we may apply the functor $[X, -]_*$, and by **??**, we obtain the following A-graded unrolled exact couple (**??**):

$$\begin{tikzcd}
\cdots \arrow[r] & {[X, Y_{s+2}]_*} \arrow[r, "i_{s+1}"] \arrow[d, "j_{s+2}"] & {[X, Y_{s+1}]_*} \arrow[r, "i_s"] \arrow[d, "j_{s+1}"] & {[X, Y_s]_*} \arrow[r, "i_{s-1}"] \arrow[d, "j_s"] & {[X, Y_{s-1}]_*} \arrow[r] \arrow[d, "j_{s-1}"] & \cdots \\
& {[X, W_{s+2}]_*} & {[X, W_{s+1}]_*} \arrow[ul, "\partial_{s+1}"] & {[X, W_s]_*} \arrow[ul, "\partial_s"] & {[X, W_{s-1}]_*} \arrow[ul, "\partial_{s-1}"] &
\end{tikzcd}$$

where here we are being abusive and writing $i_s : [X, Y_{s+1}]_* \to [X, Y_s]_*$ and $j_s : [X, Y_s]_* \to [X, W_s]_*$ to denote the pushforward maps induced by $i_s : Y_{s+1} \to Y_s$ and $j_s : Y_s \to W_s$, respectively. Each i_s, j_s, and ∂_s are A-graded homomorphisms of degrees 0, 0, and $-\mathbf{1}$, respectively.

By **??**, we may associate a $\mathbb{Z} \times A$-graded spectral sequence $r \mapsto (E_r^{*,*}(X, Y), d_r)$ to the above A-graded unrolled exact couple, where d_r has $\mathbb{Z} \times A$-degree $(r, -\mathbf{1})$. We call this spectral sequence the *E-Adams spectral sequence for the computation of* $[X, Y]_*$.