0.1. **Construction of the spectral sequence.** In the sections that follow, let (E, μ, e) be a monoid object (**??**) and X and Y be objects in $\mathcal{SH}$. From now on we will freely use the coherence theorem for symmetric monoidal categories without comment, in particular, we will assume unitality and associativity hold up to strict equality.

Definition 0.1. Let $\overline{E}$ be the fiber of the unit map $e : S \to E$ (**??**). Let $Y_0 := Y$ and $W_0 := E \otimes Y$. Then for $s > 0$, define

$$Y_s := \overline{E}^s \otimes Y, \qquad W_s := E \otimes Y_s = E \otimes \overline{E}^s \otimes Y,$$

where $\overline{E}^s$ denotes the s-fold tensor product $\overline{E} \otimes \cdots \otimes \overline{E}$. Then we get fiber sequences

$$Y_{s+1} \xrightarrow{i_s} Y_s \xrightarrow{j_s} W_s \xrightarrow{k_s} \Sigma Y_{s+1}$$

obtained by applying $- \otimes Y_s$ to the fiber sequence

$$\overline{E} \to S \xrightarrow{e} E \to \Sigma \overline{E}.$$

We can splice these sequences together to get the *(canonical) Adams filtration of* Y:

$$\begin{tikzcd}
\cdots \arrow[r] & Y_3 \arrow[r, "i_2"] \arrow[d, "j_3"] & Y_2 \arrow[r, "i_1"] \arrow[d, "j_2"] & Y_1 \arrow[r, "i_0"] \arrow[d, "j_1"] & Y_0 = Y \arrow[d, "j_0"] \\
& W_3 & W_2 \arrow[ul, dashed, "k_2"] & W_1 \arrow[ul, dashed, "k_1"] & W_0 \arrow[ul, dashed, "k_0"]
\end{tikzcd}$$

where here each k_s is of degree $-\mathbf{1}$ (in particular, the above diagram does not commute in any sense), and each i_s and j_s have degree 0. We can extend this diagram to the right by setting $Y_s = Y$, $W_s = 0$, and $i_s = \mathrm{id}_Y$ for $s < 0$. Then we may apply the functor $[X, -]_*$, and by **??**, we obtain the following A-graded unrolled exact couple (**??**):

$$\begin{tikzcd}
\cdots \arrow[r] & {[X, Y_{s+2}]_*} \arrow[r, "i_{s+1}"] \arrow[d, "j_{s+2}"] & {[X, Y_{s+1}]_*} \arrow[r, "i_s"] \arrow[d, "j_{s+1}"] & {[X, Y_s]_*} \arrow[r, "i_{s-1}"] \arrow[d, "j_s"] & {[X, Y_{s-1}]_*} \arrow[r] \arrow[d, "j_{s-1}"] & \cdots \\
& {[X, W_{s+2}]_*} & {[X, W_{s+1}]_*} \arrow[ul, "\partial_{s+1}"] & {[X, W_s]_*} \arrow[ul, "\partial_s"] & {[X, W_{s-1}]_*} \arrow[ul, "\partial_{s-1}"] &
\end{tikzcd}$$

where here we are being abusive and writing $i_s : [X, Y_{s+1}]_* \to [X, Y_s]_*$ and $j_s : [X, Y_s]_* \to [X, W_s]_*$ to denote the pushforward maps induced by $i_s : Y_{s+1} \to Y_s$ and $j_s : Y_s \to W_s$, respectively. Each i_s, j_s, and ∂_s are A-graded homomorphisms of degrees 0, 0, and $-\mathbf{1}$, respectively.

By **??**, we may associate a $\mathbb{Z} \times A$-graded spectral sequence $r \mapsto (E_r^{*,*}(X, Y), d_r)$ to the above A-graded unrolled exact couple, where d_r has $\mathbb{Z} \times A$-degree $(r, -\mathbf{1})$. We call this spectral sequence the *E-Adams spectral sequence for the computation of* $[X, Y]_*$.

For those who would rather not lose themselves in the appendix, we give a brief unravelling of how **??** applies to the present situation. Given some $s \in \mathbb{Z}$ and some $r \geq 1$, we may define the following A-graded subgroups of $[X, W_s]$:

$$Z_r^s := \partial_s^{-1}(\mathrm{im}[i^{(r-1)} : [X, Y_{s+r}]_* \to [X, Y_{s+1}]_*])$$

and

$$B_r^s := j_s(\ker[i^{(r-1)} : [X, Y_s]_* \to [X, Y_{s-r+1}]_*]),$$

where we adopt the convention that $i^{(0)}$ is simply the identity. This yields an infinite sequence of inclusions

$$0 = B_1^s \subseteq B_2^s \subseteq B_3^s \subseteq \cdots \subseteq \mathrm{im}\, j_s = \ker \partial_s \subseteq \cdots \subseteq Z_3^s \subseteq Z_2^s \subseteq Z_1^s = [X, W_s]_*.$$

Then for $r \geq 1$, we define E_r^s to be the A-graded quotient group

$$E_r^s := Z_r^s / B_r^s.$$

Thus taking the direct sum of all the E_r^s's yields the r^{th} page of the spectral sequence

$$E_r := \bigoplus_{s\in\mathbb{Z}} E_r^s,$$

which is a $\mathbb{Z} \times A$-graded abelian group.

The differential $d_r : E_r \to E_r$ is a map of $\mathbb{Z} \times A$-degree $(r, \mathbf{1})$, and is constructed as follows: an element of $E_r^s = Z_r^s/B_r^s$ is a coset represented by some $x \in Z_r^s$, so that $\partial_s(x) = i^{(r-1)}(y)$ for some $y \in [X, Y_{s+r}]_*$. Then we define $d_r([x])$ to be the coset $[j_{s+r}(y)]$ in Z_r^{s+r}/B_r^{s+r}.

In the case $r = 1$, since $B_1^s = 0$ and $Z_1^s = [X, W_s]_*$, we have that $E_1^s = [X, W_s]_*$, and given some $x \in E_1^s = [X, W_s]_*$, the differential d_1 is given by $d_1(x) = j_{s+1}(\partial_s(x))$, so that $d_1 = j \circ \partial$.

In **??**, it is shown in explicit detail that all of these definitions make sense and are well-defined. In particular, it is shown that the differentials are well-defined A-graded homomorphisms, that $d_r \circ d_r = 0$, and that

$$\ker d_r^s / \operatorname{im} d_r^s = \frac{Z_{r+1}^s/B_r^s}{B_{r+1}^s/B_r^s} \cong Z_{r+1}^s/B_{r+1}^s = E_{r+1}^s.$$

0.2. **The E_1 page.**

0.3. **The E_2 page.**