PCMI

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Contents

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1. Suppose f is a continuous function of bounded variation on $[0,1]$ which is absolutely continuous on $[c,1]$ for any $c \in (0,1)$. Is f necessarily absolutely continuous on $[0,1]$?

Proof. □

2. Suppose $f \in L^2(0,\infty)$.

(i) Prove that, for every $x > 0$, $\left|\int_0^x f(t)\,\mathrm{d}t\right| \leq \sqrt{x}\|f\|_2$.

Proof. □

(ii) Prove that $\lim_{x\to\infty} x^{-1/2}\int_0^x f(t)\,\mathrm{d}t = 0$.

Proof. □

Hint: For (ii), approximate f by a function with compact support.

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