Exercise 1

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Problem A

Question 1

We first want to write an R function that generates samples from a exponential distribution with rate parameter λ. We know that the cumulative exponential distrobution takes the form

$$F(x;\lambda) = 1 - \lambda e^{-\lambda x}, \quad x \geq 0$$

Computing the inverse cumulative function exploiting the uniform distribution $U \sim Unif(0,1)$ we can get

$$X = F^{-1}(u) = -ln(u)/\lambda, \quad 0 \leq u \leq 1$$

We now want to show that this is true using simulation from the inverse cumulative distribution

```
library(ggplot2)
# n: number of samples to generate
# rate: the rate
# expdist returns a vector with generated random numbers from exponential distribution

expdist <- function(rate, n) {
   u <- runif(n)
   x <-  -log( (1/rate)*( 1 - u))/rate
   list = list(x, u)
   return(list)
}

n <- 60000
lambda <- 1
outval <- expdist(lambda, n)
x = outval[[1]]
y = outval[[2]]

ggplot(data.frame(x=x))+geom_histogram(aes(x=x, y = ..density..),
bins = 50)  +geom_line(data = data.frame(x=x,y=dexp(x, rate = 1)),
aes(x=x, y=y) ,
color = "blue")+xlim(0,5)
```

From the plot one can see that the exponential distribtion that we generated and the exponential distribution in R closely follow each other, which means that our implementation is correct.

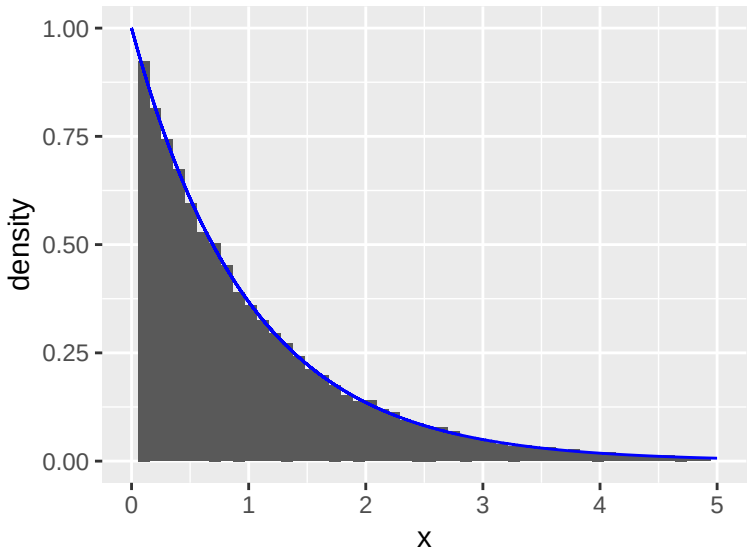

Figure 1: Analytical and inverse sampled Exponential Distribution where lambda = 1, n=60000