NTNU
Norwegian University of
Science and Technology

Week 36: Lecture 1

Gambler's ruin and introduction to regular Markov chains

Geir-Arne Fuglstad

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Information

— Reference group meeting on Friday September 11.

— Send your comments/suggestions to the reference group or to me.

— Contact information for the reference group members is found on Blackboard under course information.

Section 3.4

We start by finishing the topic of first step analysis.

Example

Let $\{X_n\}$ be a Markov chain with transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.4 & 0.3 & 0.2 & 0.1 \\ 0.1 & 0.3 & 0.3 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The starting state is $x_0 = 1$. Calculate the expected time to absorption.

Summary

Theorem (Part 2)

Let $\{X_n\}$ be a discrete-time Markov chain with state space $S = \{0, 1, \ldots, N\}$ and transitition probability matrix $\mathbf{P}$. Let $A \subset S$ be the set of absorbing states. Then

2. *If v_i is the expected time to absorption conditional on $X_0 = i$, then*

$$v_i = 0, \quad i \in A,$$
$$v_i = 1 + \sum_{k \in A^{\mathrm{C}}} P_{ik} v_k, i \in A^{\mathrm{C}}.$$

Example: Gambler's ruin

A gambler has \$10 and bets \$1 each round. If he wins the round, his fortune increases by \$1. If he loses the round, his fortune decreases by \$1. The probability of winning each round is $0 < p < 1$, and the probability of losing each round is $q = 1 - p$. The gambler will continue gambling until his fortune is \$$N$ or \$0, where $N > 10$. What is the probability that the gambler will be ruined?

Read yourselves

The rest of the examples in Sections 3.5 and 3.6.

Section 4.1

Regular Markov chains

Definition (Regular Markov chain)

Consider a Markov chain $\{X_n : n = 0, 1, \ldots\}$ with finite state space $\{0, 1, \ldots, N\}$ and transition probability matrix $\mathbf{P}$. If there exists an integer $k > 0$ so that all elements of $\mathbf{P}^k$ are strictly positive, we call $\mathbf{P}$ and $\{X_n\}$ **regular**.