Norwegian University of Science and Technology
Deptartment of Mathematical Sciences

# TMA4190 Introduction to Topology
# Spring 2018

**Exercise set 4**

1 Let $f\colon X \to Y$ be a submersion and $U$ an open subset of $X$. Show that $f(U)$ is open in $Y$. (In other words, submersions are open maps.)

2 **a)** If $X$ is compact and $Y$ connected, show that every (nontrivial) submersion $f\colon X \to Y$ is surjective. (Recall that a space $Y$ is called connected if $Y$ cannot be written as the union of two nonempty disjoint open subsets; or equivalently, if $Y$ and $\emptyset$ are the only subsets which are both open and closed in $Y$).

**b)** Show that there exist no submersions of compact manifolds into $\mathbb{R}^n$ for any $n$.

3 Show that the orthogonal group $O(n)$ is compact. (Hint: Show that if $A = (a_{ij})$ lies in $O(n)$, then for each $i$, $\sum_j a_{ij}^2 = 1$.)

4 Show that the tangent space to $O(n)$ at the identity matrix $I$ is the vector space of skew symmetric $n \times n$-matrices, i.e. matrices $B$ satisfying $B^t = -B$.

5 Prove that the set $R_1$ of all $2\times 2$-matrices of rank 1 is a three-dimensional submanifold of $\mathbb{R}^4 = M(2)$. (Hint: Show that the determinant function is a submersion on the manifold of nonzero $2 \times 2$-matrices $M(2) \setminus \{0\}$.)