Repetition

Theorem

Let $\{X_n\}$ be a discrete-time Markov chain with state space $S = \{0, 1, \dots, N\}$ and transitition probability matrix $\mathbf{P}$. Let $A \subset S$ be the set of absorbing states. Then

2. If v_i is the expected time to absorption conditional on $X_0 = i$, then

$$v_i = 0, \quad i \in A,$$
$$v_i = 1 + \sum_{k \in A^{\mathrm{C}}} P_{ik} v_k, \quad i \in A^{\mathrm{C}}.$$

Note: This arises from 1) conditioning on the first transition, and 2) using that the Markov property implies no memory of the past conditional on the present.

Definition

Consider a Markov chain $\{X_n : n = 0, 1, \dots\}$ with finite state space $\{0, 1, \dots, N\}$ and transition probability matrix $\mathbf{P}$. If there exists an integer $k > 0$ so that all elements of $\mathbf{P}^k$ are strictly positive, we call $\mathbf{P}$ and $\{X_n\}$ **regular**.

Note: This means that the discrete-time Markov chain is regular if there exist a power $k > 0$ for which all k-step transition probabilities are positive. I.e., $P_{ij}^{(k)} > 0$ for all pairs of states i and j.