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Magic Formula Tyre Model Application for a Tyre-Ice Interaction

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Abstract

Magic Formula (MF) tyre model is widely used for the analysis of tyre behavior in different driving situations. The model is mostly used to fit the simulation and experimental data slip curves. As there are many modifications of the MF, this paper presents a literature analysis of MF application and fitting techniques. A short review of other empirical tyre models was also made. At the end, a simple least squares minimisation technique was used to fit the experimental data of longitudinal tyre performance on ice. Measurements were performed with the inner drum test rig in the laboratory. In general, the fitting showed a very good accuracy.

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Keywords: Magic Formula, tyre, slip ratio, ice, least squares minimisation

1. Introduction

Tyre models are a prerequisite for any vehicle dynamics simulation and range from the simplest mathematical models that consider only the cornering stiffness to a complex set of formulae. Among all the steady-state tyre models that are in use today, the Magic Formula (MF) tyre model is unique and most popular. Though the MF tyre model is widely used, obtaining the model coefficients from either the experimental or the simulation data is not straightforward due to its nonlinear nature and the presence of a large number of coefficients. A common procedure used for this extraction is the least-squares minimisation that requires considerable experience for initial guesses [1].

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The parameterization of MF model sometimes aren' simply adaptable to other track surfaces, like winter tracks. This paper presents a review of other empirical models, MF tyre model application and techniques of coefficient determination. Then a least squares minimisation technique is used for fitting the experimental tyre-ice interaction data obtained from the indoor measurements.

2. Review of empirical models

Burkhardt [2] developed the model, where friction coefficient μ is expressed as a function of the wheel slip ratio s, and the vehicle velocity v.

$$\mu(s,v) = \left[C_1\left(1-e^{-C_2 s}\right) - C_3 s\right] e^{-C_4 sv}, \tag{1}$$

where C_1 is the maximum value of friction curve; C_2 is the friction curve shape; C_3 is the friction curve difference between the maximum value and the value at $s = 1$; C_4 is wetness characteristic value. By changing values of parameters $C_1 - C_4$, many different tire-road friction conditions can be modelled. The parameters for different road surfaces are listed in Table 1.

Table 1. Burckhardt tyre model parameters [2].

Surface conditions	C_1	C_2	C_3	C_4
Dry asphalt	1.029	17.16	0.523	0.03
Dry concrete	1.197	25.168	0.5373	0.03
Snow	0.1946	94.129	0.0646	0.03
Ice	0.05	306.39	0	0.03

Kiencke and Daiss [3] expanded and approximated previous Burkhardt model and suggested a simpler model.

$$F(s) = k_s \frac{s}{c_1 s^2 + c_2 s + 1}, \tag{2}$$

where k_s is the slope of the $F(s)$ versus s curve at $s = 0$, and c_1 and c_2 properly chosen parameters.

All the previous friction models are highly nonlinear in the unknown parameters, and thus they are not well-adapted to be used for on-line identification. For this reason, simplified models like

$$F(s) = c_1\sqrt{s} - c_2 s, \tag{3}$$

have been proposed [4].

It is also well known that the 'constant' $c'_i s$ in the above models, are not really invariant, but they may strongly depend on the tire characteristics (e.g., compound, tread type, tread depth, inflation pressure, temperature), on the road conditions (e.g., type of surface, texture, drainage, capacity, temperature, lubricant, etc.), and on the vehicle operational conditions (velocity, load) [4].

Jazar [5] suggested empirical equation to show the effects of pressure p and load F_z on the rolling friction coefficient μ_r.

$$\mu_r = \frac{K}{1000}\left(5.1 + \frac{5.5\cdot 10^5 + 90F_z \cdot}{p} + \frac{1100 + 0.0388F_z}{p} v_x^2\right). \tag{4}$$

The parameter K is equal to 0.8 for radial tires, and is equal to 1.0 for non-radial tires.

Navin et al. [6] expressed the effect of winter aggregate and ambient temperature on the friction coefficient from vehicle traction experiments on ice and snow. Field testing was conducted with a variety of vehicles to represent different vehicle types. Vehicles included passenger cars and light trucks. Ice temperature ranged from –6°C to –35°C. The average vehicle coefficient of friction was expressed as:

$$f_x(ice, \mathrm{car}) = 0.11 - 0.0052T + 0.0002A\,,$$

$$f_x(ice, truck) = 0.10 - 0.0052T + 0.00016A\,. \tag{5}$$

Where A is the aggregate application (g/m^2) and T is the temperature in °C. From the above relations it is clear that the addition of aggregates improves the friction coefficient.

All empirical models are based on the coefficients determination from the experimental or simulation data. So it is and MF model which is presented in the next chapter.

3. Review of Magic Formula application

An initial version of MF tyre model was presented by Bakker et al. [7]. It was a formula with coefficients which described some of the typifying quantities of a tyre, such as slip stiffness at zero slip, force and torque peak values. The formula was capable to describe the characteristics of side force, brake force and self aligning torque with great accuracy. This mathematical representation was limited to steady-state conditions during either pure cornering or pure braking and formed the basis for a model describing tyre behaviour during combined braking and cornering.

Pacejka and Bakker [8] presented the newer version of the model mentioned above. It was called the Magic Formula tyre model. The model was able to describe an accurate steady-state tyre behaviour. By selecting the proper values, the characteristics for either side force, aligning torque or longitudinal force can be obtained. The newer version of model contained physically based formulations to avoid the introduction of correcting factors. Double-sided, possibly non-symmetric pure slip curves were employed as the basis for combined slip calculations. The general MF for pure slip is

$$y(x) = D \sin\left[C \arctan\left\{Bx - E(Bx - \arctan Bx)\right\}\right], \tag{6}$$

where

$$Y(x) = y(x) + S_v \;\text{ and }\; x = X + S_h\,, \tag{7}$$

where Y is the output variable: longitudinal force F_x, side force F_y or aligning moment M_z. Longitudinal (μ_x) and lateral (μ_y) force coefficients can be as output parameters as well. X is the input parameter: it can be tangent of slip angle α or slip ratio S_x.

B, C, D, E refers to stiffness, shape, peak and curvature factors respectively. S_h and S_v refers to horizontal and vertical shift respectively. The meaning of curve parameters is showed in the Fig. 1.

Pacejka and Besselink [10] presented a newer version of the MF tyre mode called 'Delft Tyre 97'. The non-steady state tyre behavior was included.

Rao et al. [11] used mixed Lagrangian-Eulerian finite element technique to simulate the steady-state cornering behaviour and predicted force and moment characteristics are represented as MF tire model parameters through non-linear least-squares fitting technique. Tread pattern, tread material properties, belt angle, inflation pressure, frictional behaviour at the tyre-road contact interface and their interactions were found significantly influence vehicle handling characteristics.

Maclaurin [12] described how the MF can be adapted to describe the tractive force-slip relationships of tyres in soft cohesive soil. The MF coefficients derived from experimental results were made as functions of Mobility

Number instead of normal load. Mobility Number is an empirical system for estimating the tractive performance of tyres in soft soils at a single value of slip.

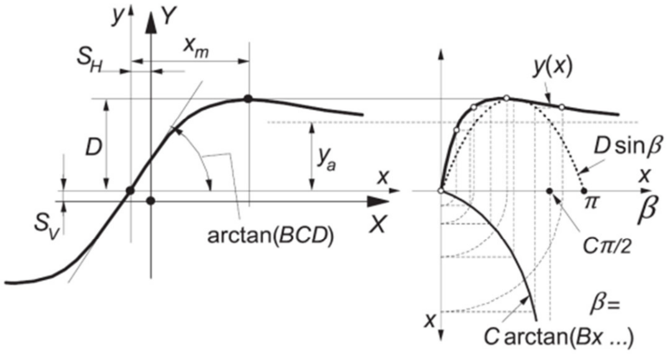

Fig. 1. The meaning of Magic Formula tyre model parameters [9].

Wassertheurer and Gauterin [13] used MF tire model to fit the experimental data of tire longitudinal and lateral measurements on ice, snow and water. After fitting procedure, some parameters were extracted from the MF and effects on the tire characteristics were empirically identified using extensive statistical methods. The main effects of track and ambient conditions were identified as well.

Li and He [14] established a closed-loop DVRES model on the basis of multi-body system dynamics using Adams/Car and Matlab/Simulink. The MF was applied to calculate the magnitude of dynamic responses of lateral tyre forces in the model. The dynamic responses of lateral tyre forces were used to investigate vehicle side slip estimations, in which the root mean square and the maximum value are calculated to quantify the safety margin of tyre side friction.

As MF tyre model is widely used to fit the experimental and simulation data, fitting techniques will be discussed further.

Van Oosten and Bakker [15] presented a simple MF tyre model fitting technique based on least square method. The authors found that the calculation of MF parameters is not difficult for pure slip conditions. For combined cornering and braking, however, the number of points to be fitted and the empirical character of the formulae may led problems when calculating the parameters.

Ortiz et al. [16] presented a new method, called IMMa Optimization Algorithm (IOA) based on genetic techniques to determine MF tyre model parameters. The main advantages of the method are its simplicity of implementation and its fast convergence to optimal solution, with no need of deep knowledge of the searching space. To start the search, it is not necessary to know a set of starting values of the MF parameters (null sensitivity to starting values).

Braghin et al. [17] presented a methodology aimed at identifying MF tyre model coefficients of a single tire for pure cornering conditions. The identification procedure was divided into three parts. The first phase was based on an extended Kalman filter and produced an estimate of the average MF coefficients. In the second step, the vertical loads and the slip angles for each tire were evaluated. The results of the first two phases were used as inputs for last part of the procedure, during which, through a constrained minimization approach, the MF coefficients are identified for each individual tire.

Alagappan et al. [1] highlighted the issues that are commonly encountered in obtaining MF coefficients with different algorithms, namely, least-squares minimisation using trust region algorithms, Nelder-Mead simplex, pattern search, different evolution, particle swarm optimisation, cuckoo search and etc. The main observation was found that not all algorithms give the same MF coefficients for a given data. Only the algorithms that can fit all types of data and all types of models such as longitudinal force, lateral force, self-aligning torque, etc. without tweaking any of the parameters for each case separately are suitable for fitting MF tyre model. There was three algorithms that gave a good fits for all the tested cases – trust region reflective, differential evolution and bounded cuckoo search. Another important condition was the initial or starting values. In case the start values are well known or can be guessed, it is always better to use the trust region or other gradient-based methods.

4. Experimental setup

Tyre-ice measurements were performed at Insitute of Vehicle System Technology of Karslruhe Institute of technology and inner drum test rig was used. It consists of the drum with a diameter of 3.8 m, wherein the tyre, mounted on a rigid wheel suspension, rolls on the installed track. Tyre forces and moments are measured with a six-component measurement hub. The test bench is surrounded by a climate chamber and is equipped with an air conditioning system that enables the whole testing room to be cooled down to an ambient temperature of –20°C.

The ice track is produced by pouring tap water on slowly rotating drum at low temperature (~–10°C) and about 10 mm thickness ice layer is built. The produced ice track is polished with a special fine siped tyre and can be seen in the Fig. 2a.

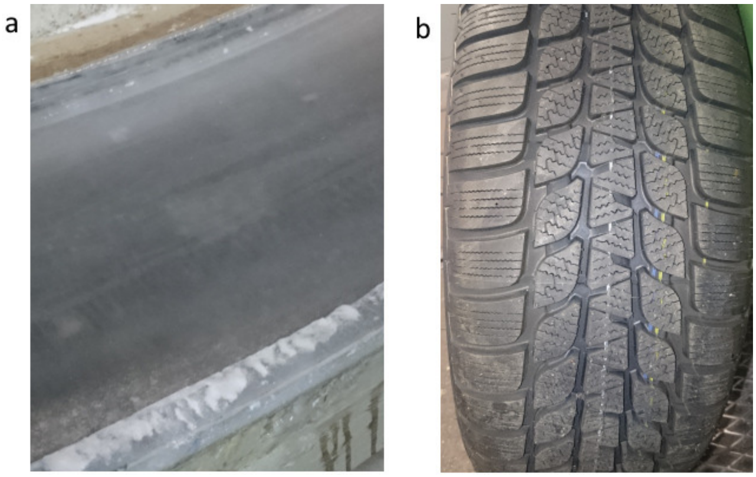

Fig. 2. Produced ice track (a); Measurements tyre (b).

A 205/55 R16 winter tyre (Fig. 2b) was chosen with a typical winter tread pattern of 7.8 mm tread height and shore A hardness with the value of 65. Measurements were performed at these conditions: driving (drum) speed = 30 km/h; tire inflation pressure = 2.2 bar; wheel load = 4260 N; inclination angle = 0°; ambient (drum) temperature = –4°C.

5. Results and discussion

Measurements of longitudinal tyre force transmission were performed and an average longitudinal force coefficient (μ_x) versus slip ratio curve of 5 measurements was calculated and presented in Fig. 3 for tyre acceleration, and in Fig. 4 for braking.

Then least-squares minimization technique was used to fit the experimental data with the MF and coefficients of eq. (6) were determined.

$$\min \sum_{i=1}^{n} \{\mu_{measured}(x_i) - \mu_{MF}(x_i)\}^2 , \quad (8)$$

where $\mu_{measured}$ is the averaged force coefficient value from the measurements and μ_{MF} is the value calculated using the predicted MF coefficients for each data point.

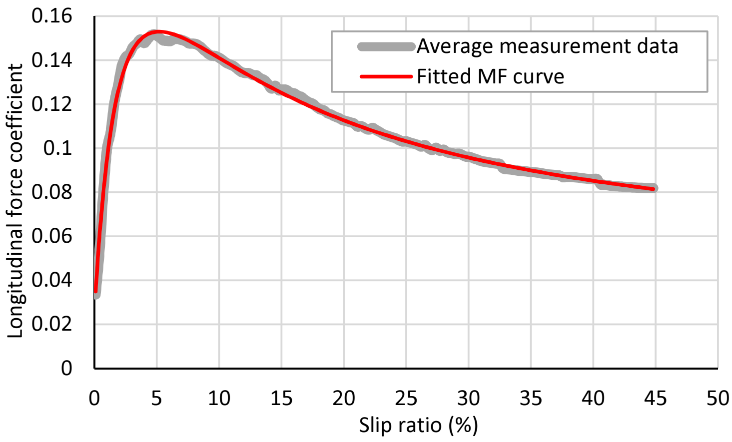

Fig. 3. Fitted Magic Formula on average measurement data. Acceleration. $R^2 = 0.997$.

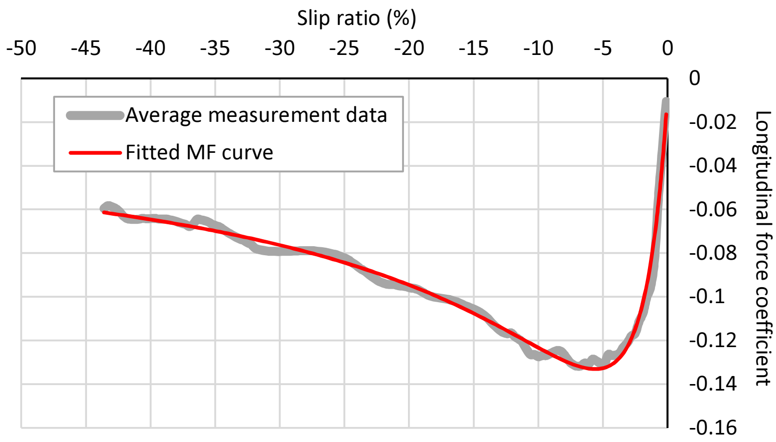

Fig. 4. Fitted Magic Formula on average measurement data. Braking. $R^2 = 0.989$.

The high values of R-square (R^2) showed a very good fit for both acceleration and braking curves. A lower R^2 was observed for a braking curve as it is seen some fluctuations in the average curve which can be caused by the hydraulics of the test rig.

The curves were split and fitted separately because of asymmetric tyre behavior. It is clearly seen that the peak value of maximum force coefficient is higher when tyre is accelerating rather than braking. One of the reason could be that tyre transmits higher shear forces when it accelerates. The investigated tyre reaches maximum force coefficient within a short range of slip. For both braking and accelerating it is only 5%, and clear decreasing trend can be observed when tyre starts sliding. The maximum force coefficient values are 0.15 for accelerating and 0.13 for braking.

6. Conclusions

All empirical models are based on the determination of coefficients from the experimental or simulation tyre-road interaction data and Magic Formula tyre model is mostly used. The literature review showed that there is a wide application of this model and many modifications. As model is mainly used for fitting the experimental tyre-road behavior, there are many fitting techniques and most popular is the least squares minimisation, which was used to fit the longitudinal tyre-ice slip curves obtained from the indoor measurements with the inner drum test rig. The high R^2 values showed a very good fit for tyre acceleration and braking situations. The maximum force coefficient values were reached within a very short slip (5%). The low force coefficient values indicate that icy road condition is one the most dangerous and tyre plays an important role being the most important component of transmitting forces to the track.

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