Project 1

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2020

Contents

1 Problem 1: Modelling the outbreak of measles

1.1 Answer a

Definition 1.1. *Consider a stochastic process* $\{X_n\}_{n\in\mathbb{N}_0}$*, where* n *is a random variable. Then is the process said to be a* ***discrete-time markov chain*** *if, and only if*

$$Pr\{X_{n+1} = i_{n+1} \mid X_n = i_n, \ldots, X_0 = i_0\} = Pr\{X_{n+1} \mid X_0\}$$

for all states $i_0, \ldots, i_n, i_{n+1}$.

Using the fact that $\{X_n : n = 0, 1, \ldots\}$ has well defined probabilities which does not depend on time, and makes it possible to determine the state X_n at any time n makes this problem equivalent to the definition of a discrete markov chain 1.1. Using this fact, can we conclude that it exists a common transition matrix such that

$$\mathbf{P} = \begin{bmatrix} p_{00} & p_{01} & p_{12} \\ p_{10} & p_{11} & p_{10} \\ p_{20} & p_{21} & p_{22} \end{bmatrix}$$

where p_{ij} is the stationary transition probability from state i to state j in one trial for all n states. Note that that the sum of rows has to be $\sum_j p_{ij} = 1$. However, given the transition information in the problem description, 1), 2) and 3), can this be simplified to

$$\mathbf{P} = \begin{bmatrix} 1-\beta & \beta & 0 \\ 0 & 1-\gamma & \gamma \\ \alpha & 0 & 1-\alpha \end{bmatrix}.$$

Where β, α and γ are real constants which satisfies the conditions mentioned above.

1.2 Answer b

- Is this a reducible or irreducable Markov chain?

  Answer. If all state

- Determine the euivalent classes and determine wherher they are recurrent of transient.

- What is the period of each state?

2 References