Problem Sets 19

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2020

Contents

1 Exercise 1

1.1 Problem 2

Define functions $\mathbb{R}$ with values in $\mathbb{R}$.

1. A function that is not left invertible.
2. A function that is not right invertible.

Show that the given functions have their respective properties.

function is left inverrtible if there exists a function f_l^{-1} such that

$$x = f\left(f_l^{-1}(x)\right)$$

or formally

$$id_x = f \circ f_l^{-1}$$

Same for right invertible function which can be written as

$$id_x = f_r^{-1} \circ f$$

A function $h = x^2$ is a function that does not support both right and left invertible.

1.2 Problem 3

Given the linear mapping $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $Tx = Ax$ with

$$A = \begin{pmatrix} 3 & -4 \\ 1 & 6 \\ 1 & 1 \end{pmatrix}$$

1. Show that the matrix

$$A_l^{-1} = \frac{1}{9}\begin{pmatrix} -11 & -10 & 16 \\ 7 & 8 & -11 \end{pmatrix}$$

Is inducing a left invere T_l^{-1} of T . This left inverse is not unique. show that

$$\frac{1}{2}\begin{pmatrix} 0 & -1 & 6 \\ 0 & 1 & -4 \end{pmatrix}$$

gives another left inverse.

(a) We can show it by computing $T \circ T_l^{-1}$ such that

$$A \cdot A_l^{-1} = I$$

(b) THe right inverse can be computed be analysing the transpose of A.

$$AA_l^{-1} = I \quad \leftrightarrow \quad I = I^T = \left(AA_l^{-1}\right)^T = A_{lT}^{-1} A^T$$

At least this is the solution given. Not sure since finding an right inverse to A^T answer the question.

1.3 Problem 4

Show that cartesion product of two (infinite) countable sets is countable.

Solution. A set is countable if it exist a integer which can be allocated for every $\mathbb{N}^+$. Let $A = \left\{a_1, a^2, a^3, \ldots\right\}$ and $B = \{b_1, b_2, \ldots\}$ be two infinite countable sets. Let us define the product $C = B \times A$ such that

$$C = \{a_1 b_1, a_2 b_2, \ldots\}$$

If we compare it with N^+ can we observe that

$$\begin{aligned} C &= \{a_1 b_1, a_2 b_2, \ldots\} \\ N^+ &= \{1, 2, \ldots\} \end{aligned}$$

Which means that there exists one element in $\mathbb{N}$ for every element in C, which shows that C has to be countable.

1.4 Problem 5

Show that the sets $\mathbb{Z}$ of integeres and $\mathbb{Q}$ of rational numbers are countable.
Solutions.

- To show that $\mathbb{Z}$ is countable can we describe the set such that

  $$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2 \ldots\}$$

  By comparing every element in N^+ such that

  $$\begin{aligned} N^+ &= \{1, 2, \ldots\} \\ \mathbb{Z} &= \{\ldots, -2, -1, 0, 1, 2 \ldots\} \end{aligned}$$

  Lets every odd element in N^+ be N_{ODD} and every even element be N_{EVEN}, then can we make

  $$N^{ODD} = \{1, 3, \ldots\} \, \mathbb{Z}^- = \{\ldots, -2, -1, 0\}$$

  and

  $$N^{EVEN} = \{2, 4, 6, \ldots\} \, \mathbb{Z}^+ = \{1, 2, 3, \ldots \ldots\}$$

  We have then showed it exists a element in $\mathbb{N}^+$ for every element in $\mathbb{Z}$, which makes it countable.

- For the rational numbers $\mathbb{Q}$ such that $\frac{a_1}{a_2} \in \mathbb{Q}$ where $a_1, a_2 \in \mathbb{Z}$. We can then use the fact that $\mathbb{Z}$ is countable such that both the nominator and demonitor is countable. In practice can we write the rational numbers as a set such that is

$$.$$

2 References