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A Concept for Estimation and Prediction of the Tire-Road Friction Potential for an Autonomous Racecar*

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Abstract— **This paper presents a concept to estimate and predict the friction potential between a vehicle's tires and the road surface. An important aspect of this research project is a local high-resolution evaluation and storage of the obtained data to gain precise knowledge of the road conditions. Next to a current-state friction potential estimation algorithm, a concept is introduced which aims to predict the tire-road friction potential for a predefined horizon ahead of the vehicle. The concept focuses on an autonomous racecar, which will drive on different racetracks during the Roborace Season Alpha events. After a brief overview of state-of-the-art methods in tire-road friction potential estimation, the overall concept is presented and its application on the real-world racecar is outlined. The effects of the high-resolution friction potential information on the raceline trajectory planning algorithms are shown. Lastly, aspects of future work regarding research and implementation of the presented concept and a transfer to public road cars are discussed.**

I. INTRODUCTION

Knowledge of the friction potential μ_{pot} between a vehicle's tires and the road is essential for both safety (road vehicles) and vehicle performance (racecars). In both cases, situations arise where maximum utilization of the possible transferable tire forces is essential. On public roads, these states occur when a vehicle encounters dangerous situations, which require an emergency stop or sudden steering-action. In motor-racing, the human driver constantly strives to utilize the full tire force potential in order to achieve minimal lap time.

Coming from the perspective of autonomous driving, we need to plan safe trajectories that permit all types of vehicle maneuvers on different road surface conditions. In order to plan trajectories at the limits of handling (e.g. icy roads), a precise knowledge of the possible maximum tire forces is necessary [1]. These maximum tire forces rely on the friction potential between the vehicle's tires and the road surface. Besides the estimation of the tire-road friction potential at the current vehicle position, knowledge of the road sections ahead is also vital. Otherwise, a certain friction potential drop could lead to an unfeasible trajectory, which has been planed beforehand for an expected, higher friction potential. The driver and the vehicle's assistance systems, or the vehicle completely on its own when driving autonomously, could react to this drop, but is restricted by the low friction surface and the resulting maximum tire forces. This could lead to dangerous situations where a collision or driving off the road is unavoidable.

Our team at the Technical University of Munich develops software for an autonomous racecar [2]. Since a human race driver always tries to maximize the tire forces, an autonomous driving software has to fulfill the same task. One important aspect is the precise knowledge of local track-related data of the friction potential. This requires a high-resolution "friction potential map" of the racetrack, which should store and provide different types of data for use in subsequent laps. Furthermore, a prediction of the friction potential is necessary to obtain information about the track area ahead, or about changing track conditions over the complete race duration. A focus of the prediction is gathering information about areas where the vehicle does not often drive. Usually, the vehicle will drive on or close to the raceline. Therefore, data of areas beside the raceline is not available. Nevertheless, situations can arise which require the vehicle to drive on these specific areas, e.g. when an obstacle or a slower opponent vehicle is blocking the raceline. This urges the need for prediction, not only for the area ahead but also for the area beside the raceline.

Within the scope of the TUM-Roborace project, we develop software to participate in the Roborace Championship [3], a supporting series of the Formula E Championship. The goal of Roborace is to initialize the first racing series for autonomous vehicles. The teams taking part in this competition develop software for provided autonomous racecars. [2] and [4] provide an overview of the overall software concept, the idea behind Roborace and the autonomous racing vehicle itself. The software stack of the vehicle's control module was published in January 2019 and is available as open-source code on github [5].

II. STATE OF THE ART

The following section gives an overview of different methods to estimate the friction potential at the contact point of a vehicle's tire and the road surface, which depends on a variety of factors. In general, methods to estimate the tire-road friction potential can be divided into two groups based on what information they rely on [6] (Fig. 1). Effect-based methods utilize the effects that a varying friction potential has on the vehicle. Cause-based methods utilize information about the vehicle's environment to examine the causes for a varying friction potential between tire and road. [7] proposes another categorization and divides into experiment- and model-based approaches.

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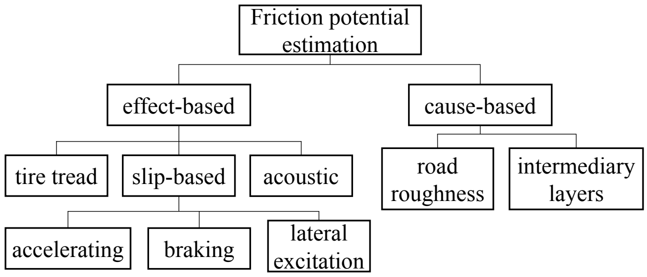

Fig. 1. A sample of different approaches to estimate the tire-road friction potential grouped into effect- and cause-based methods (cf. [6]).

A. Effect-based methods

Effect-based methods observe effects resulting from varying friction potential and give an estimate based on measurements. These observed effects can be measured at the vehicle level or at the individual tire level with standard vehicle sensors (e.g. acceleration, wheel speed, torque) and special sensors which are not standard in road cars (e.g. tire-tread sensor, acoustic sensor) [6]. The most common approaches are slip based, use standard vehicle sensors, and retrieve the friction potential by modeling vehicle dynamics and tire characteristics (e.g. Pacejka tire-model) [8], [9]. To obtain an estimation for the vehicle states and friction potential, different methods are used to process the measured signals. Often, non-linear Kalman Filters are used, i.e. the Extended Kalman Filter (EKF) [10], [11] and the Unscented Kalman Filter (UKF) [12]–[14]. Next to experimental models which represent tire characteristics, machine learning methods are utilized to model the tire behavior. Some of these approaches use Support Vector Machines [15], Neural Networks [11], [13] or ANFIS (adaptive-network-based fuzzy inference system) models [16], [17].

Effect-based methods require the vehicle to drive over the road surface with a specific friction potential to allow an estimate. This leads to a certain vehicle state which can then be measured. An estimation in advance, and thus a contact-less estimation which is necessary for prediction, is not possible with this type of methods.

B. Cause-based methods

Cause-based methods do not measure the effects of the friction potential. The focus of cause-based methods lies in detecting and assessing the road surface roughness and texture, or the presence of an intermediary layer and its properties (e.g. wet roads) [6]. Cause-based methods assess the road surface condition in front of the car with different measurement horizons, i.e. the size of the observed road area. Some methods only consult the area immediately in front of the vehicle, e.g. when using laser scanners mounted at the vehicle's front bumper [18]. The considered measurement horizon is small compared to other approaches. For assessing road surface texture as well as intermediary layers, [19] uses camera images with a measurement horizon of approximately $50\,\mathrm{m}$. Nolte [20] trains Deep Convolutional Neural Networks to classify the road surface into six categories and derive information about the friction potential.

Cause-based friction potential estimation relies on machine learning techniques to a lager extent than effect-based approaches because the data of the latter approaches is less specific in comparison to vehicle sensor data. In many cases, image processing and feature detection tasks have to be performed to obtain sufficient data for estimation.

C. Prediction of the tire-road friction potential

Cause-based methods require sensors which provide a measurement horizon that allows visibility of the road ahead of the vehicle, since this is the only way to obtain relevant information about the road condition. The area covered by the sensors can be of different size. The measurement horizon has to be large enough to provide enough time for computation of an adjusted trajectory and thus to react to changing conditions. If it is too small, the time between a measurement and the point in time where the vehicle passes the examined area is too short to react appropriately.

This forward-looking friction estimation is a prediction of the to-expect friction potential of the area in front, which can be only achieved by cause-based methods. Therefore, the focus of the concept for predicting the tire-road friction potential relies on these specific estimation methods.

D. Major differences in driving on public road and racetrack

Driving on a racetrack requires special attention to some aspects but also allows simplifications compared to public road traffic. Two important differences are the level of longitudinal and lateral acceleration and vehicle velocity. In both situations, the friction potential is essential for safe operation of the vehicle. On public roads, the friction potential exploitation is low in most cases but becomes high in special situations (e.g. emergency braking). On racetracks, the friction potential is almost fully utilized for most of the time. A major drawback of this high utilization is the small range for dealing with uncertainties compared to road vehicles. If the maximum tire forces are exceeded, the vehicle becomes unstable and hard to control. As a consequence, overdriving the limit must be prevented at any time, but safety margins must also be kept as small as possible to avoid performance loss due to conservative driving. In comparison to public roads, the racetrack's dimensions are known in advance. Furthermore, the environment, aside from the opponent vehicles, is static and simple because the number and variety of unexpected scenarios is small.

In conclusion, the encountered scenarios are largely known in advance which simplifies the complexity. However, due to much smaller safety margins than in public road traffic, both accuracy and robustness have to be much higher than in public road applications. The high average velocity implies strict real-time constraints on computational performance.

III. CONCEPT FOR A FRICTION ESTIMATION AND PREDICTION ALGORITHM

In this section, the main concept is presented for the friction estimation and prediction algorithms as well as the storage, processing and distribution of the gathered data. Fig. 2 shows the overall layout of the so-called Tire Performance Assessment (TPA) module with its submodules, friction potential estimation and friction potential prediction. This is the architecture of a software module that can be implemented in any autonomous driving software. The TPA map is a central part of the process that manages and distributes data as required. An interface is provided to distribute the stored data to several other algorithms running on the vehicle which require high-resolution local data.

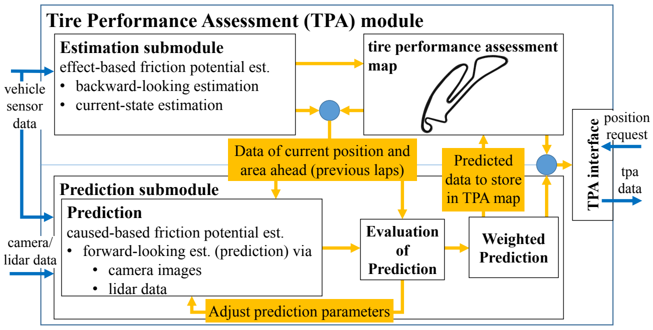

Fig. 2. Concept and layout of the Tire Performance Assessment module.

For the scope of this research project, three different friction potential estimation algorithms are defined:

- Backward-looking (Fig. 3, top): utilize vehicle sensor data acquired over a certain period of time to calculate an average estimation for a position behind the current vehicle position.
- Current-state (Fig. 3, middle): utilize current state vehicle sensor data to calculate an estimation of the current position's friction potential.
- Forward-looking (Fig. 3, bottom): utilize vehicle sensor, camera and LIDAR data to calculate a friction potential estimation for the road area ahead of the vehicle.

The "backward-looking" and "current-state" estimation algorithms are located in the estimation submodule, the "forward-looking" estimation, from now on referred to as "prediction", is located in the prediction submodule (Fig. 2).

A. Estimation submodule

The estimation submodule consists of two different estimation methods. One uses data which is collected over a specific period of time, the other one uses real-time vehicle sensor data to calculate an estimation for the current position.

Backward-looking friction potential estimation: This method gathers a continuous data stream over a specific period of time. The main intention of the "backward-looking" estimation is that an estimation does not need to be immediately available, because the racecar will pass every track section multiple times during a race. Therefore, the purpose of storing the data is given. The vehicle will pass the respective section only after a specific, prior known period of time. The data processing could take this amount of time. Both facts enable this estimation method. The major advantage arising from this method of processing the acquired data is that the data can be processed, smoothed and filtered more extensively. After preprocessing the raw vehicle data, virtual sensor signals like longitudinal and lateral tire slip and tire forces can be calculated using a vehicle dynamics model and a tire-model, respectively. Knowing the longitudinal, lateral and vertical tire forces, the friction potential can be calculated by consulting the tire-model to get the maximum of possible tire forces at the current tire state.

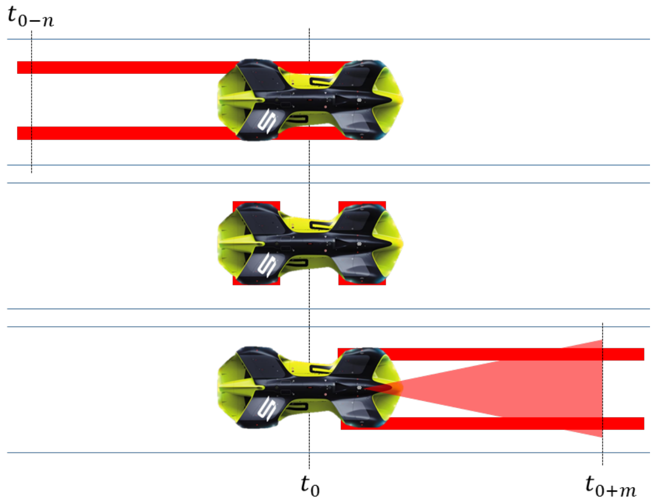

Fig. 3. Three different friction potential estimation algorithms: "backward-looking" (top), "current-state" (middle), "forward-looking" (bottom).

Current-state friction potential estimation: This estimation method is capable of estimating the friction potential in real-time for the current vehicle position. The current-state estimator will focus on calculating the friction potential per axle and not per tire, unlike the backward-looking estimator. This is mainly due to increased uncertainty in real-time estimation. Furthermore, the estimated values of this method are not inserted into the friction potential map. The resulting estimation can be used for several tasks, e.g. to precondition the prediction. Additionally, the accuracy of the vehicle state estimation can be increased with an adjusted friction potential estimation, since the tire behavior will be more accurately modeled compared to a fixed friction potential value determined in advance [21]. A non-linear Kalman Filter (EKF) will be implemented for this type of estimation.

B. High-resolution friction potential map (TPA map)

The friction potential map is the central part of the Tire Performance Assessment module, which stores and distributes local data of the racetrack. This data is not limited to friction potential information, but is comprised of any type of data which provides benefits when local knowledge is available. This includes road surface characteristics obtained from camera and LIDAR data, e.g. luminance or reflectivity. The generated map is essential for data management within the module. The stored data is aggregated every time an updated friction potential value is available (every lap), and can be accessed in subsequent laps. This information is used to compare the current estimations to previous ones and to precondition the prediction algorithm with data from previous laps. A simple plausibility check is possible because values can be expected to change minimally during a lap.

The resolution of the map is variable. Next to a rectangular grid which is aligned to the x- and y-axis of the track coordinate grid, other grid orientations are possible, e.g. aligned to the racetrack's centerline. The TPA map is designed to have two parts, a static part and a dynamic data storage part. The static part acts as a look-up table which links specific spatial coordinates (x,y) to a grid cell. The look-up table is static for a given racetrack, that allows offline processing of the TPA map in advance. The grid cells have a specific size and shape and store data for the area they cover. This data-storage part of the TPA map changes every time the vehicle gathers new data. Once the vehicle passes over an area, the data of corresponding cells gets updated.

C. Prediction submodule

Cause-based friction potential estimation methods are necessary to achieve a prediction functionality. Furthermore, camera and LIDAR are necessary to enable an examination of the road surface, and thus a prediction. Both are available on the Roborace vehicle and on most of the future autonomous vehicles. The focus of the prediction module is on developing algorithms which process data of both sensor-types to obtain information about the friction potential.

The results of the prediction will be compared to the data from previous laps to provide an online validation of the prediction. Also, the predicted values can be validated once the vehicle passes the predicted area and estimates the friction potential using the effect-based methods. These estimated values are compared to the predicted values to adjust the prediction algorithm in order to achieve more accurate results. The predicted values for the whole area ahead are not inserted into the TPA map, since data of the estimation module will be available after the prediction and is expected to be more accurate. An important task of the prediction is to provide data for areas where the vehicle does not usually drive, especially areas beside the raceline. The prediction submodule is able to provide data for these areas, as far as they are within the sight of view of the sensors. Data of these areas are highly relevant for the vehicle, and will be integrated into the friction potential map once the prediction has been evaluated with the mentioned validation methods. The used techniques will be of both types, conventional methods (e.g. for image processing) and machine-learning methods (e.g. for classification and training of a model to link camera/LIDAR data to road surface condition).

IV. IMPLEMENTATION ON AN AUTONOMOUS RACECAR

The following section presents the working progress of the presented concept. Already-implemented algorithms are outlined and first results are shown. The estimation submodule runs on a Speedgoat mobile real-time target machine and is implemented in MATLAB and Simulink. The prediction submodule (C++ and Python) and the TPA map (Python) run on a NVIDIA Drive PX2. [2] provides detailed hardware specifications and layout. Whereas parts of the estimation submodule will already be operated during the 2019 season, the prediction submodule is still a work in progress. Following subsections focus on the estimation submodule, the TPA map, and the effects of local varying friction potential provided by the map on planed trajectories.

A. Friction potential estimation

Out of both friction potential estimation methods contained in the estimation submodule, the backward-looking friction potential estimation is already implemented on the racecar. For the 2019 season, the TPA will only provide data collected during previous laps. Data of the current lap (via current state-estimation) will not be collected, nor will a prediction be made.

The "backward-looking" estimation gathers a continuous data stream which is then smoothed by applying Savitzky-Golay filtering [22] to minimize the influence of high-frequency noise. The main advantage of this is the possibility to filter the data forwards and backwards, which is not possible when using a real-time estimation method since future timesteps are unknown. After preprocessing the raw vehicle data, a dual-track model is used to calculate virtual signals, e.g. the tires' slip in longitudinal and lateral direction [12]. In addition, the longitudinal and lateral forces acting on the front and rear axle, as well as the dynamic vertical tire load distribution are calculated [12], [13]. Lastly, the tire forces are calculated utilizing the Pacejka tire-model [9]. Knowing the longitudinal, lateral and vertical tire forces, the friction exploitation of the current state can be determined. By consulting the tire-model, the maximum of possible tire forces at the current state of the tire, and thus the current friction potential, are calculated.

B. High-resolution friction potential map (TPA map)

The mapping algorithm is divided into two parts: the static look-up table and the dynamic data structure containing various data for each map cell. The TPA map is implemented using the Python package for spatial algorithms and data structures (*scipy.spatial*) and the included *cKDtree*, which is a nearest neighbor binary search tree algorithm written in C code [23]. This algorithm provides a quick way to search for the nearest neighbor of an input position (vehicle position), and to output the index value of the nearest grid cell center point. This index value is used to access a data structure with the particular grid cell's data. The data structure is implemented as a Python dictionary, a hash map which provides quick access even for huge data structures.

To increase computational performance, three different data request types are available: tire-individual data request, axle-individual data request, and a request for data within a circle of predefined radius around the vehicle's Center of Gravity (CoG). This aims to decrease the number of requested positions (e.g. four positions for each tire-individual request, one for CoG request), and thus decrease computation time. At the same time, accuracy is lost. The Berlin racetrack

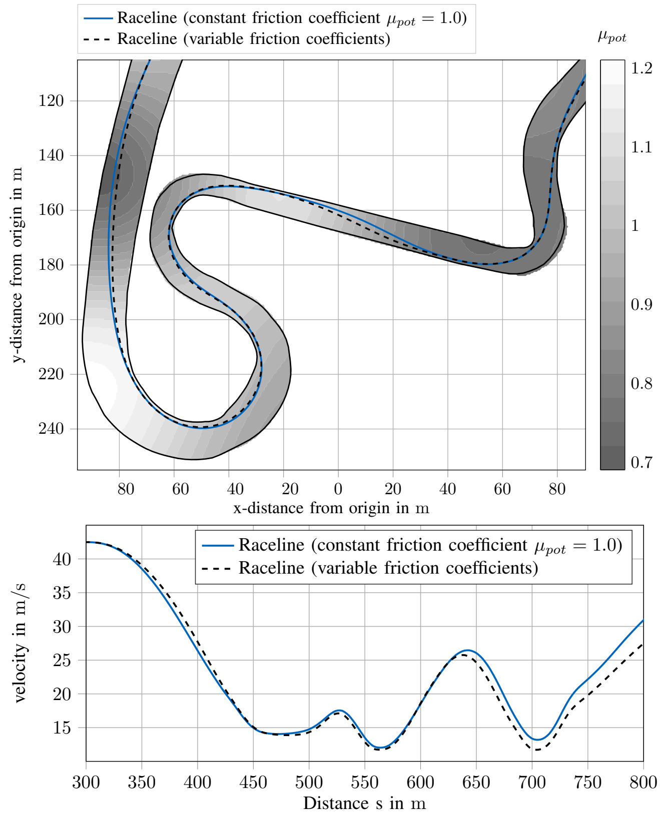

Fig. 4. Calculated racelines and corresponding velocity profiles from consulting TPA map data with varying friction potential.

is 2.3 km long and is covered by a TPA map of 120 000 grid cells when applying a map resolution of 0.5 m.

C. Trajectory planning based on TPA map data

The most important task of the TPA module is to provide the friction potential for every position the vehicle encounters during a race. With this knowledge, both planning algorithms for global raceline [24] and local trajectory [25] adjust their output. The raceline is calculated before driving on the track, what allows a more extensive computation than calculating online. The algorithm uses a dual-track model and takes the friction potential into account to adjust both path coordinates and velocity profile. In contrast, the local trajectory planner runs online and has strict time constraints to guarantee an update frequency of approximately 15 Hz, and a constant planning horizon of approximately 200 m. As a consequence, computational complexity must be reduced, which leads to a less complex consideration of the friction potential. Within the local trajectory planner, the path is calculated independently of the friction potential. This is due to the huge number of paths sampled for one timestep. After an assessment of these paths for being collision-free, a set of five paths is considered for further planning. The velocity profiles for the chosen paths are calculated taking the friction potential provided by the map into account. If the local trajectory planner encounters a low-μ area, it will adjust the velocity to decrease longitudinal and lateral acceleration. One trajectory is chosen by the behavior planner and then propagated to the vehicle's control unit. This adjustment happens every iteration step and results in feasible trajectories.

Fig. 4 demonstrates the effects of a varying friction potential in specific areas on the global raceline's path and velocity. The raceline path is not substantially affected by the varying friction potential. The velocity profile plot shows the part of the above track section from 300 m, starting in the top left to 800 m, in the top right corner. The adjusted velocity profile shows the effects of areas with lower traction, especially in the last turn, where the spread between both velocity profiles increases. The vehicle has to brake earlier and accelerate softer.

Fig. 5 demonstrates an extreme example of a varying friction potential only for specific areas. The global raceline with $\mu_{pot} = 0.25$ differs clearly from the one calculated with a friction potential of $\mu_{pot} = 1.0$. The low-μ areas have an effect on the raceline before and after the specific area. The global trajectory planer minimizes the lateral acceleration by passing the low-μ area with low steering-angle and curvature, and uses the high-μ areas afterwards to correct the trajectory. After the correction, the path is nearly the same as in the constant-μ case. In the second section, the trajectory pulls to the right of the track and drives off the low-μ area to allow higher tire forces. The change of sides occurs later when the friction potential is high. The low-μ area effects the trajectory even afterwards, where the vehicle enters and passes the subsequent turn in a different way.

V. CONCLUSION AND FUTURE WORK

We presented and outlined a concept to estimate and predict the tire-road friction potential for application on an autonomous racecar. A central aspect of the proposed design is a high-resolution map to store, manage and distribute local data to algorithms relying on this information. In a subsequent paper, the friction potential estimation submodule and

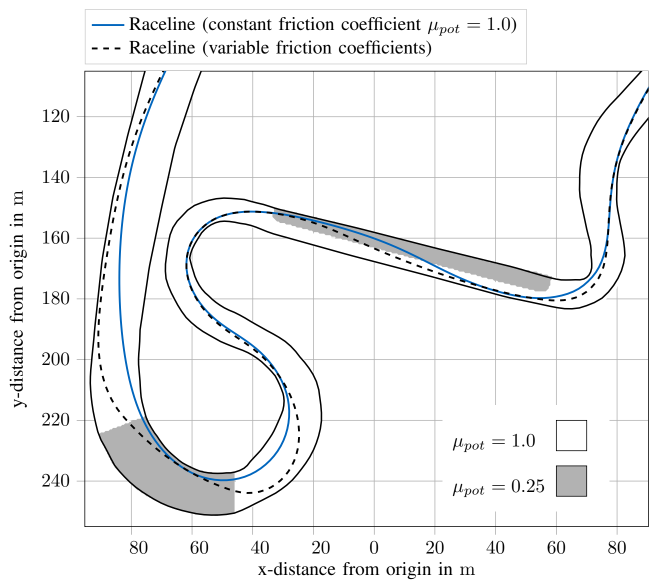

Fig. 5. Calculated racelines from consulting TPA map data with an extreme case of local varying friction potential.

the high-resolution TPA map will be outlined in more detail. This includes both estimation methods, their application, and first measurements of the racecar with a comparison between a human race driver's lap and a fully autonomous lap.

Future work will focus on the presented prediction submodule with the goal of developing a method to predict the tire-road friction potential on a racecar, as well as on road cars. First of all, the entire vehicle software stack will be extended to enable the vehicle to reach the handling limits completely on its own. In a first step, this will rely only on effect-based friction potential estimation methods. A prediction will not be applied in this development stage, but will be integrated later. During the 2019 season, camera data and corresponding vehicle data will be collected. This data is the basis for developing a prediction functionality and is highly important because the visual data and the vehicle dynamics data is connected. Image or LIDAR streams without synchronized vehicle sensor data are not sufficient due to the lack of information about the vehicle's behavior, and thus, the road condition. Data gathering is planed with the racecar and a public road testing vehicle at the Institute to ensure transferability of the developed algorithms.

For the upcoming 2020 season, the estimation submodule as well as the TPA map are finalized. The prediction submodule will be subject to research for the next two years, however, a first working version should run on the vehicle in the 2020 season. The development process will be accompanied by further publications.

Acknowledgments and Contributions

Research was supported by the Bavarian Research Foundation (BFS). As the first author, Leonhard Hermansdorfer initiated the idea of this paper and is responsible for the presented concept and its implementation on the racecar. His contribution was essential to the overall system design of the Tire Performance Assessment module. Johannes Betz contributed to the overall concept and system design. Markus Lienkamp contributed to the conception of the research project and revised the paper critically for important intellectual content. He gave final approval of the version to be published and agrees to all aspects of the work. As guarantor, he accepts responsibility for the overall integrity of the paper.

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References

[1] N. A. Spielberg, M. Brown, N. R. Kapania, J. C. Kegelman, and J. C. Gerdes, "Neural network vehicle models for high-performance automated driving," *Science Robotics*, vol. 4, no. 28, p. eaaw1975, 2019.

[2] J. Betz, A. Wischnewski, A. Heilmeier, F. Nobis, T. Stahl, L. Hermansdorfer, B. Lohmann, and M. Lienkamp, "What can we learn from autonomous level-5 motorsport?," in *9th International Munich Chassis Symposium 2018* (P. Pfeffer, ed.), Proceedings, pp. 123–146, Wiesbaden: Springer Fachmedien Wiesbaden, 2019.

[3] R. Ltd., "Roborace." `https://roborace.com/`, 2018.

[4] J. Betz, A. Wischnewski, A. Heilmeier, F. Nobis, T. Stahl, L. Hermansdorfer, and M. Lienkamp, "A software architecture for an autonomous racecar," in *2019 IEEE 89th Vehicular Technology Conference (VTC2019-Spring)*, pp. 1–6, IEEE, Apr. 2019.

[5] L. fuer Fahrzeugtechnik, "Autonomous driving control software of tum roborace team." `https://github.com/TUMFTM/veh_passenger`, 2019.

[6] M. R. Uchanski, *Road Friction Estimation for Automobiles Using Digital Signal Processing Methods*. Dissertation, University of California, Berkeley, 2001.

[7] S. Khaleghian, A. Emami, and S. Taheri, "A technical survey on tire-road friction estimation," *Friction*, vol. 5, no. 2, pp. 123–146, 2017.

[8] M. Acosta, S. Kanarachos, and M. Blundell, "Road friction virtual sensing: A review of estimation techniques with emphasis on low excitation approaches," *Applied Sciences*, vol. 7, no. 12, pp. 1230–1277, 2017.

[9] H. B. Pacejka and I. Besselink, *Tire and vehicle dynamics*. Oxford: Elsevier and Butterworth-Heinemann, 3 ed., 2012.

[10] M. Acosta, S. Kanarachos, and M. E. Fitzpatrick, "Optimized tire force estimation using extended kalman filter and fruit fly optimization," in *Proceedings, IECON 2017 - 43rd Annual Conference of the IEEE Industrial Electronics Society*, (Piscataway, NJ), pp. 4074–4079, IEEE, 2017.

[11] M. Acosta and S. Kanarachos, "Tire lateral force estimation and grip potential identification using neural networks, extended kalman filter, and recursive least squares," *Neural Computing and Applications*, vol. 30, no. 11, pp. 3445–3465, 2018.

[12] M. Wielitzka, A. Busch, M. Dagen, and T. Ortmaier, "Unscented kalman filter for state and parameter estimation in vehicle dynamics," in *Kalman Filters - Theory for Advanced Applications* (G. Serra, ed.), InTech, 2018.

[13] M. Acosta, S. Kanarachos, and M. E. Fitzpatrick, "A virtual sensor for integral tire force estimation using tire model-less approaches and adaptive unscented kalman filter," in *Proceedings of the 14th International Conference on Informatics in Control, Automation and Robotics*, pp. 386–397, SCITEPRESS - Science and Technology Publications, Jul. 2017.

[14] Long Chen, Mingyuan Bian, Yugong Luo, Keqiang Li, "Real-time identification of the tyre–road friction coefficient using an unscented kalman filter and mean-square-error-weighted fusion," *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 230, no. 6, pp. 788–802, 2016.

[15] E. Regolin and A. Ferrara, "Svm classification and kalman filter based estimation of the tire-road friction curve," *IFAC (International Federation of Automatic Control)*, vol. 50, no. 1, pp. 3382–3387, 2017.

[16] J.-S. Jang, "Anfis: adaptive-network-based fuzzy inference system," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 23, no. 3, pp. 665–685, 1993.

[17] M. Acosta, S. Kanarachos, and M. E. Fitzpatrick, "Robust virtual sensing for vehicle agile manoeuvring: A tyre-model-less approach," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 3, pp. 1894–1908, 2018.

[18] L. Raslavičius, A. Pakalnis, A. Keršys, R. Skvireckas, and D. Juodvalkis, "Investigation of asphalt texture roughness on friction evolution for wheeled vehicles," *TRANSPORT*, vol. 31, no. 2, pp. 133–141, 2016.

[19] Frederic Holzmann, Mario Bellino, Roland Siegwart, Heiner Bubb, "Predictive estimation of the road-tire friction coefficient," in *IEEE International Symposium on Computer-Aided Control Systems Design, 2006*, (Piscataway, NJ), pp. 858–890, IEEE, Oct. 2006.

[20] Marcus Nolte, Nikita Kister, Markus Maurer, "Assessment of deep convolutional neural networks for road surface classification," *CoRR*, vol. abs/1804.08872, 2018.

[21] M. Wielitzka, M. Dagen, and T. Ortmaier, "State and maximum friction coefficient estimation in vehicle dynamics using ukf," in *2017 American Control Conference (ACC)*, (Piscataway, NJ), pp. 4322–4327, IEEE, 2017.

[22] A. Savitzky and M. Golay, "Smoothing and differentiation of data by simplified least squares procedures," *Analytical chemistry*, vol. 36, pp. 1627–1639, 07 1964.

[23] SciPy.org, "scipy.spatial.cKDTree." `https://docs.scipy.org/doc/scipy/reference/generated/scipy.spatial.cKDTree.html`, 2019.

[24] F. Christ, A. Wischnewski, A. Heilmeier, and B. Lohmann, "Time-optimal trajectory planning for an autonomous race carwith variable friction coefficients," 2019. Unpublished manuscript.

[25] T. Stahl, J. Betz, and M. Lienkamp, "Multilayer graph-based trajectory planning for race vehicles in dynamic scenarios," 2019. Unpublished manuscript.