Problem Sets Linear Methods 2020

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2020

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1 Exercise Set 1

1.1 Problem 1

Dodo: check solutions.

Let X, Y and Z be sets

- Show that $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

Answer. Recall the definitions

Definition 1.1. *For two sets X and Y is*

$$X \cap Y = \{x \in X \quad \textit{and} \quad x \in Y\}$$

$$X \cup Y = \{x \in X \quad \textit{or} \quad x \in Y\}$$

Let $x \in X$ and $x \in (Y \cup Z)$. Then is $x \in X \cap Y$ and $x \in X \cap Z$ which ratifies $x \in (X \cap Y) \cup (X \cap Z)$.

- Show that $X \setminus (Y \cup Z) = (X \setminus Y) \cap (X \setminus Z)$

Answer. Recall the definition

Definition 1.2. *For two sets X and Y, then is*

$$X \setminus Y = \{x \in X \quad \textit{and} \quad x \notin Y\}$$

We can then use the same argumentation as in the previous proof. Let $x \in X$ and not in $x \in (Y \cup Z)$. Then is $x \in X \setminus Y$ and $x \in X \setminus Z$

1.2 Problem 2

Let $f : X \to Y$ be a function, let B be a subset of Y, and let $\{B_i\}_{i \in I}$ be a family of subsets of Y.

- Prove that

$$f^{-1}\left(\bigcap_{i \in I} B_i\right) = \bigcap_{i \in I} f^{-1}(B_i)$$

Answer.
Since f^{-1} is mapped $Y \to X$ and B_i is a subset of Y. Obviously is

$$\bigcap_{i \in I} B_i \to B$$

And since

- Prove that $f\left(f^{-1}(B)\right) \subseteq B$ and if f is surjective then equality holds. Show by example that equality need not to hold if f is surjective.

Answer. f is surjective if $f : X \to Y$ and there exist one $x \in X$ does it exists at least one $y \in Y$. Since by the definition of an inverse function is $f^{-1}(f(x)) = x$ for every $x \in X$,

2 References