Repetition

Little's law (Part 2)

For a (stable) queueing system

$$L_0 = \lambda W_0,$$

where

L_0: Average number of customers queueing in the system.

λ: Rate of arrival to the system.

W_0: Average waiting time queuing in the system.

Notation

We write **"A/B/c queue"** for a queueing system with

1) Interarrival distribution "A"

2) Individual service time distribution "B"

3) Number of servers "c"

Note: We only consider A=B=M, where M denotes "memoryless". This means interarrival distribution and service distribution are exponential distributions.

Definition

A **M/M/1 queue** with arrival rate $\lambda > 0$ and expected service time $1/\mu > 0$ has

1) Interarrival times are independent and identically distributed as $\text{Exp}(\lambda)$.
2) Service times are independent and identically distributed as $\text{Exp}(\mu)$.
3) One server, and service times are independent of the arrival process.

Note: The number of customers in the queueing system is a birth and death process with birth rates $\lambda_i = \lambda$, $i = 0, 1, \ldots$, and death rates $\mu_0 = 0$ and $\mu_i = \mu$, $i = 1, 2, \ldots$.

Definition

A **M/M/∞ queue** with arrival rate $\lambda > 0$ and expected service time $1/\mu > 0$ has

1) Interarrival times are independent and identically distributed as $\text{Exp}(\lambda)$.
2) Service times are independent and identically distributed as $\text{Exp}(\mu)$.
3) Infinitely many servers, and service times are independent of the arrival process.

Note: The number of customers in the queueing system is a birth and death process with birth rates $\lambda_i = \lambda$, $i = 0, 1, \ldots$, and death rates $\mu_i = i\mu$, $i = 0, 1, \ldots$.

Definition

A **M/M/s queue** with arrival rate $\lambda > 0$ and expected service time $1/\mu > 0$ has

1) Interarrival times are independent and identically distributed as $\text{Exp}(\lambda)$.
2) Service times are independent and identically distributed as $\text{Exp}(\mu)$.
3) s servers, and service times are independent of the arrival process.

Note: The number of customers in the queueing system is a birth and death process with birth rates $\lambda_i = \lambda$, $i = 0, 1, \ldots$, and death rates

$$\mu_i = \begin{cases} i\mu, & i = 0, 1, \ldots, s, \\ s\mu, & i = s+1, s+2, \ldots \end{cases}$$