TMA4165 Differential Equations and Dynamical Systems: Problem Sheet I

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1. By first diagonalizing the following matrix $\mathbf{A}$, find $\exp(\mathbf{A}t)$:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 2 & -4 & 2 \end{pmatrix}.$$

2. Sketch the phase diagram for the following linear systems and classify the fixed/equilibrium point:

(i) $\dot{x} = x - 5y$, $\dot{y} = x - y$;
(ii) $\dot{x} = x + y$, $\dot{y} = x - 2y$;
(iii) $\dot{x} = -4x + 2y$, $\dot{y} = 3x - 2y$;
(iv) $\dot{x} = x + 2y$, $\dot{y} = 2x + 2y$;
(v) $\dot{x} = 4x - 2y$, $\dot{y} = 3x - y$.

3. The following systems are degenerate in some way. Sketch the phase diagram for the following systems.

(i) $\dot{x} = 3x - y$, $\dot{y} = x + y$;
(ii) $\dot{x} = x - y$, $\dot{y} = 2x - 2y$;
(iii) $\dot{x} = x$, $\dot{y} = 2x - 3y$;
(iv) $\dot{x} = x$, $\dot{y} = x + 3y$;
(v) $\dot{x} = -y$, $\dot{y} = 2x - 4y$.

4. Find the nonsingular matrix $\mathbf{P}$ that reduces the following matrix to a Jordan normal form and state its Jordan normal form:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix}.$$