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NONLINEAR OPTIMIZATION OF A RACING LINE FOR AN AUTONOMOUS RACECAR USING PROFESSIONAL DRIVING TECHNIQUES

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ABSTRACT

1 INTRODUCTION

For racecar drivers, careful consideration is used when defining a racing line for a given track. Each driver ultimately seeks to minimize the track time by compromising between the shortest distance around a track and a line that results in the fastest speed along the path. Professional racing strategies could be used to create trajectories for obstacle avoidance in autonomous vehicles if there was a way to analytically represent their techniques.

In order to leverage the computational advantage autonomous systems have over their human counterparts, autonomous racing research has generally focused on various optimization algorithms in an attempt to find the fastest path for the car. Mühlmeier and Müller [1] populate a track with a set of initial paths and then use a genetic algorithm to test and evolve the racing line. Gerdts et al. [2] use optimal control and ultimately a moving horizon approach to iterate and converge to a racing line given a set of road boundaries. The solutions to these methods use a set points to describe the path along the track. However, such methods do not attempt to represent and emulate the techniques described by professional drivers. Velenis and Tsiotras [3] use optimal control to generate velocity profiles that minimize lap time when a geometric path is already defined. This paper shapes the geometry of a path to find a compedative racing line comparable to a professional driver using a curve structure that emulate professional driving techniques.

For a professional racecar driver, the racing line can be broken down to a family of curves of straights, clothoids, and constant radius arcs. The use of clothoids for general vehicle path planning is not uncommon [4], but their use and justification for describing a racing line is an emerging idea [5]. A way to generate racing lines with this family of curves can be accomplished using a nonlinear gradient descent approach. These solutions could offer a way of generating obstacle avoidance trajectories similar to what a professional driver would preform in a given situation.

Figure 1. *Autonomous Audi TTs*

The approach presented in this paper will be used on the Mazda Raceway at Laguna Seca. The first section discusses how straights, clothoids and circular arcs can be used emulate the racing line professional racecar drivers describe. The second section discusses how an initial racing line is derived from the lane edges of a race track. The third section outlines the nonlinear gradient

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descent approach used to converge to paths similar to a professional driver. The last section compares the final result to the path a professional driver, Bruce Canepa, drove with an instrumented racecar during a racing event. The proposed curve structure has been validated on an autonomous Audi TTs (Figure 1).

2 PROFESSIONAL DRIVING TECHNIQUES

A professional driver often separates racing into two objectives. The first is utilizing all the tire friction available to the car at all times [6]. The second is defining and following a successful racing line [7].

2.1 DRIVING AT THE LIMITS

Mitchell et al. [6] discuss the concept of maximizing the performance of a car by "driving the traction circle." The traction or friction circle is a model used to describe the limited friction available in a vehicle's tires. There is a tradeoff between steering and braking when negotiating a corner, as illustrated in Figure 2. Driving at the limits implies remaining on the edge of the friction circle but not exceeding its bounds. While negotiating a turn, the friction circle and a racing line can be described in three phases. These three phases are trail braking, maximum cornering, and throttle on exit.

2.2 PROFESSIONAL RACING LINES

Taruffi [7], who not only won the 1952 Swiss Grand Prix but earned a Doctorate of Industrial Engineering, also describes taking a corner in three phases based on the friction circle. He further described these phases geometrically by connecting a pair of straights to a constant radius arc via two "linking curves." In this paper, clothoids with linearly varying curvature serve as the linking curves Taruffi describes. By plotting the curvature of the path, Figure 3 illustrates how clothoids are used to link straights to constant radius arcs. For the circular arc, the design goal is to minimize the curvature at the apex as much as possible. Figure 4 demonstrates this by approaching a corner from the outside, meeting the inside boundary at the apex and exiting the corner on the outside.

Along the straights, the racecar is either at maximum acceleration or deceleration. The entry clothoid is where the driver balances between braking and steering to remain at the limits. This maneuver is called trail braking. The circular arc is where the racecar is experiencing maximum cornering and cannot accelerate or decelerate. The exit clothoid is where the driver steers out of the turn and feathers the throttle to remain at the limits.

2.3 TESTING ON AN AUTONOMOUS RACECAR

Once the racing line is constructed, a target velocity profile for braking into a corner can be calculated by starting with the required velocity at the constant radius arc. The speed of maximum cornering is dictated by

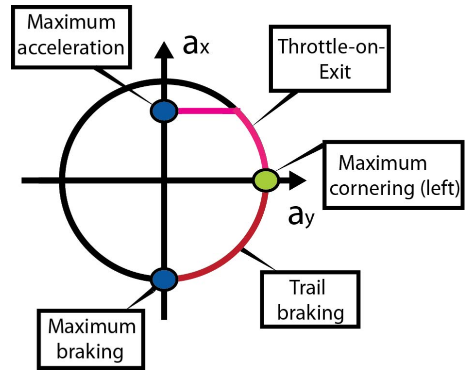

Figure 2. *Maneuvering along the edge of the friction circle*

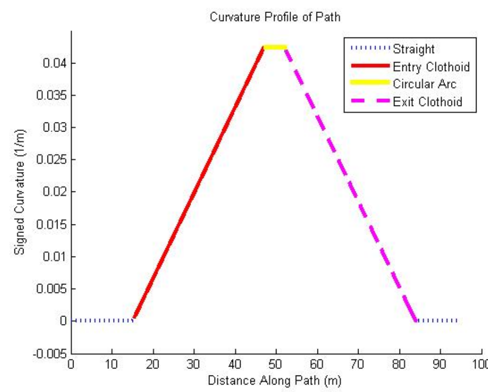

Figure 3. *Curvature of a simple turn*

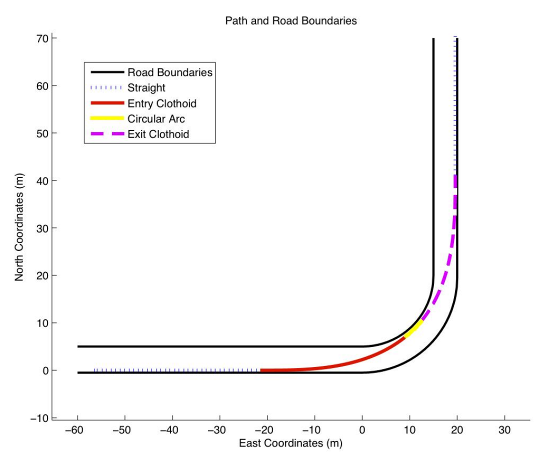

Figure 4. *Racing line of a simple turn*

$$\frac{v^2}{R} = a_y \tag{1}$$

where a_y is the maximum cornering on the friction circle and v is the velocity of the racecar. By integrating the available brak-

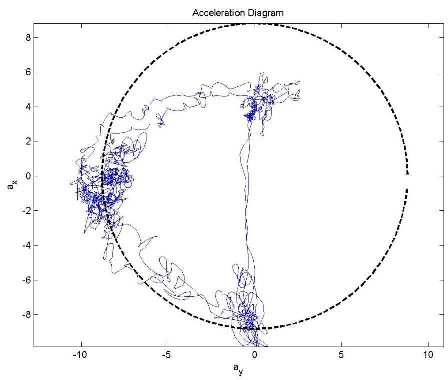

Figure 5. *Acceleration diagram superimposed on a 0.9g friction circle*

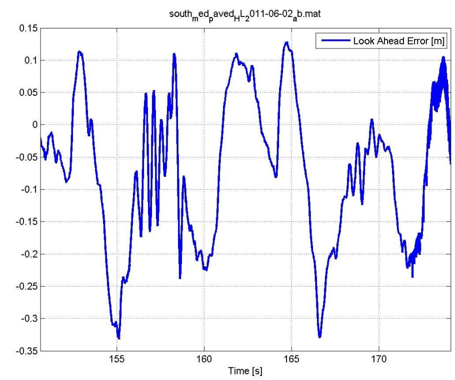

Figure 6. *Lookahead error*

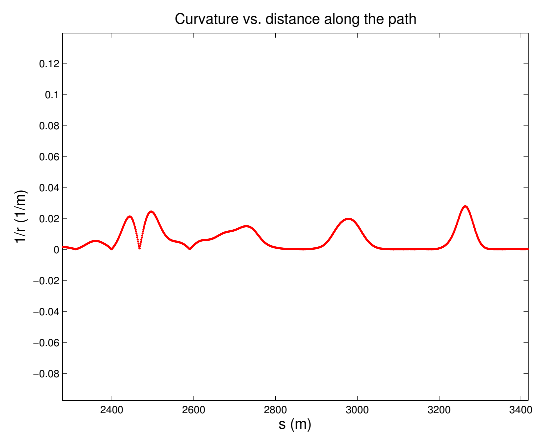

Figure 7. *Filtered and reflected curvature data of the center line*

ing force along the entry clothoid using the friction circle model, a velocity profile can be calculated. From the required velocity at the beginning of the entry clothoid, the braking point on the straight before each turn can be found. As the racecar leaves the apex and exits the turn, the throttle increases as the curvature of the turn decreases. Kritayakirana and Gerdes [8] outline this process using the friction circle model. An important advantage of using the proposed curve structure is that the curvature of each turn can be readily extracted from the racing line making it possible to follow the edge of the friction circle. This again leads to maximizing the total force the tires can use along the track.

The Autonomous Audi TTS (four-wheel drive) in Figure 1, was used to test the curve structure. A controller designed by Kritayakirana and Gerdes [8] sends commands to the electronic power steering motor, active brake booster, and throttle by-wire of the vehicle. The vehicle is equipped with a Differential Global Positioning System (DGPS) and inertial sensors (INS), from which vehicle position and other states can be obtained.

The ability of the racecar to reach maximum performance was analyzed. Measurements from the INS were used to construct acceleration diagrams illustrated on Figure 5. The diagram shows that the racecar is performing at the limits using the purposed curve structure. The Audi was also able to remain at the limits with a reasonable tracking error (Figure 6). The maximum lookahead error from the path was about 30 cm.

Using this family of curves as a set of path primitives to describe the racing line can offer a way to construct aggressive, yet simple, racing trajectories similar to what racecar drivers use on race tracks.

3 INITIALIZING THE RACING LINE

In order to produce a competitive racing line around a track, the optimization algorithm requires an initial path to modify and improve. Creating the initial racing line is essentially the process of taking a point cloud and converting it into a set of path primitives that fit within the boundaries of the track. The center line found from the lane edges of the track is a simple reference line used for this purpose.

3.1 FINDING STRAIGHTS ON A TRACK

The center line is used to define the straights of the initial path. Straights have zero curvature, therefore, any point along the center line where the curvature is small is where a straight is defined. Equation (2)

$$k(s) = \frac{x'y'' - y'x''}{\sqrt{(x'^2 + y'^2)^3}} \tag{2}$$

is used to calculate curvature $k(s)$ where x and y are functions of distance where change in distance is constant along the path. Because $k(s)$ depends on the second derivative of both x and y, $k(s)$ contains a lot of undesirable noise that must be filtered. A low pass filter is used. $k(s)$ is then reflected about the x axis, meaning $k(s) = |k(s)|$. Figure 7 is a sample of the result for the center line at Laguna Seca. From this data, the goal is to extract the places along the track where the curvature is relatively

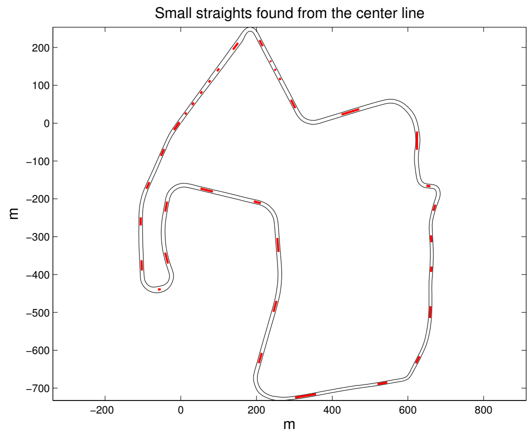

Figure 8. *Small straights found from the center line*

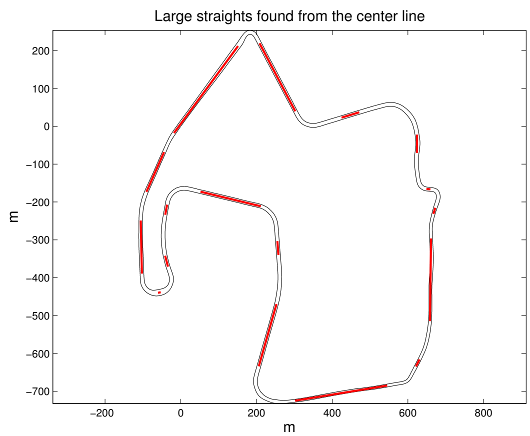

Figure 9. *Large straights combined from small straights*

small which can be initially approximated as straights. This is done by finding all the local minima of the track and creating small straights in these locations along the track. Figure 8 is an example of this result. Once the small straights are found along the track, any straight with approximately the same heading is combined to form larger straights that will be used to initialize the path as demonstrated on Figure 9.

3.2 CLOSING AND INITIALIZING THE PATH

Once the straights are defined, a pair of symmetric clothoids are used to connect the straights and form a closed, continuous path. For each turn, the lengths of the straights are adjusted so that the clothoids between each pair of straights meet the inside boundaries of the track. This step most commonly involves shortening one straight at a time which brings the connected clothoids closer to the inside apex of the track until the path and the boundary meet at a single point. For professional racecar drivers, this is referred to as hitting the apex of a turn. As the straights are being shortened to meet the inside apex of a particular turn, if the length of a particular straight gets very

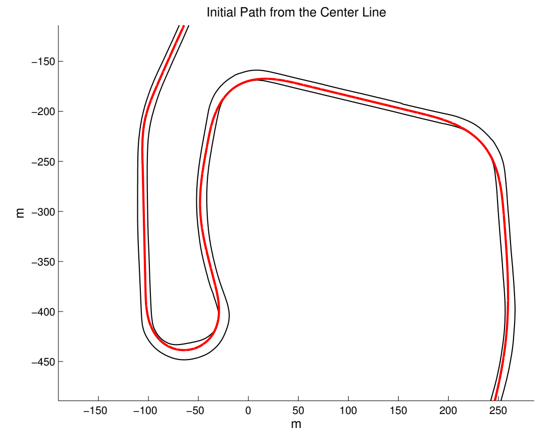

Figure 10. *Initial path derived from the center line*

small, the straight is removed from the track and replaced with a circular arc in the turn sequence. Figure 10 is a sample of how the path fits on the track once this process is complete. Once the initial path is created, the optimization can be applied to the path to converge to a racing line comparable to a professional driver.

4 NONLINEAR PATH OPTIMIZATION

The proposed optimization algorithm is fundamentally a sequential gradient based approach. Lap time is the cost function that is being minimized. There are N parameters that define the geometry of the path which can be organized into two categories: parameters that define the straights and parameters that define the shape of a turn (i.e. the sequence of clothoids and arcs connecting the straights).

4.1 DEFINITION OF PATH PARAMETERS

For straights, there are three parameters that uniquely define a straight: heading, length, and position perpendicular to the heading. An illustration of the parameters that define each straight can be seen on Figure 11. The direction of travel for the racecar is right to left. The turn entry point signifies where the straight ends and the turn begins. Each straight has three degrees of freedom which is depicted on the right side of Figure 11. However, whenever a straight is modified, the length of the straight is adjusted to hit the apex of the turn on the track. Thus two degrees of freedom are used for optimization, perpendicular position and heading, and the other is used to ensure that the turn always meets the inside boundary of the track during a turn.

The parameters that control the shape of each turn are the lengths and curvature values of each clothoid and circular arc within a given turn. Figure 12 illustrates how a curvature profile can change for a particular turn with an entry clothoid, circular arc, and exit clothoid.

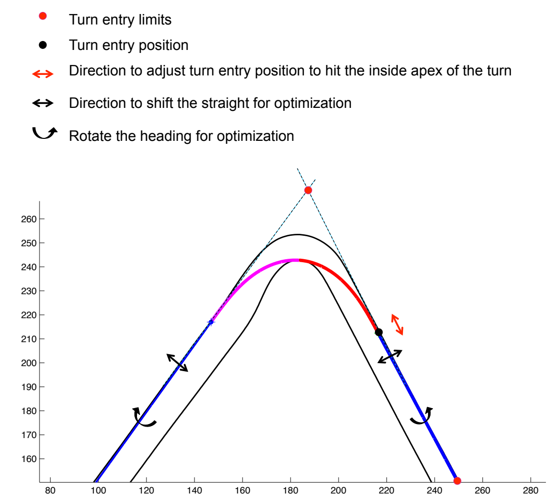

Figure 11. *Parameters that uniquely define a straight*

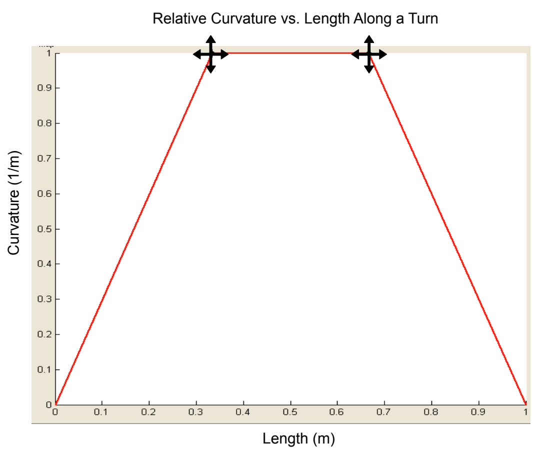

Figure 12. *Parameters that uniquely define a clothoid-arc-clothoid turn*

4.2 OPTIMIZATION OVER THE ENTIRE TRACK

These parameters are the variables being modified by the optimization algorithm to improve the lap time. The performance or lap time of the path can be considered an N dimensional hypersurface where N is the number of parameters that uniquely define the path. Instead of minimizing over the entire hyperspace at one time where all the variables would change simultaneously, a smaller parameter subset is chosen for a given iteration. This is similar to projecting the hypersurface onto a two dimensional plane and finding a local minima for a particular variable subset selected. Thus each parameter set minimized, changes the shape of the path at specific turns along the track. For each subset iteration, the parameter(s) change in one direction with an initial step size until the lap time begins to increase. The parameter(s) then change in the opposite direction with half the step size until the lap time increases again. This process is analogous to a ball being dropped in a large bowl and settling at the bottom. Once a

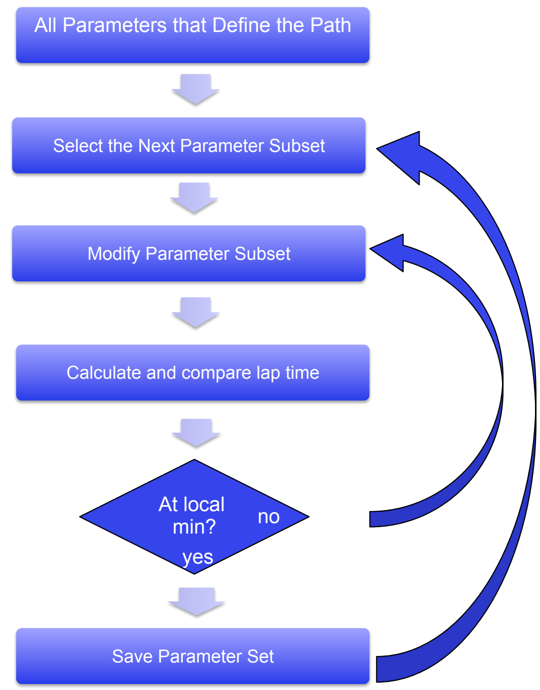

Figure 13. *Flow diagram of the successive gradient descent algorithm*

local minima is found for a particular parameter subset, the next subset is adjusted. Figure 13 illustrates a flow diagram of how the successive gradient descent minimizes lap time for the entire track by iterating over each parameter subset. The order in which the subsets are minimized are as follows:

1. Pairs of neighboring straights
   (a) Shift both positions perpendicular to the path toward the inside or outside boundary
   (b) Counter rotate the headings
2. Single straights
   (a) Shift the position perpendicular to the path
   (b) Rotate the heading
3. Single turn segments (i.e. clothoids or circular arcs)
   (a) Change the relative length
   (b) Change the relative curvature

4.3 CALCULATING THE LAP TIME

The lap time or cost function is found using the friction circle model previously discussed where there are acceleration limits placed on the car. A simple point mass is used to model the forces acting on the car as it brakes, turns, and accelerates. Parameters used to define the vehicle's capabilities is the peak friction limit, and the acceleration limit of the engine as can be seen on Figure 2. This simple model can be used to generate velocity profiles along the track and ultimately a simulated lap time.

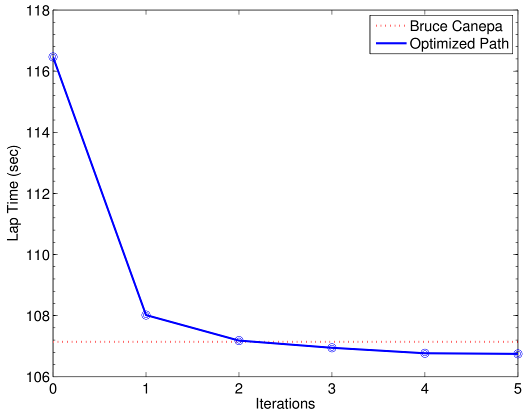

Figure 14. *Lap times after each iteration vs. Bruce's racing line*

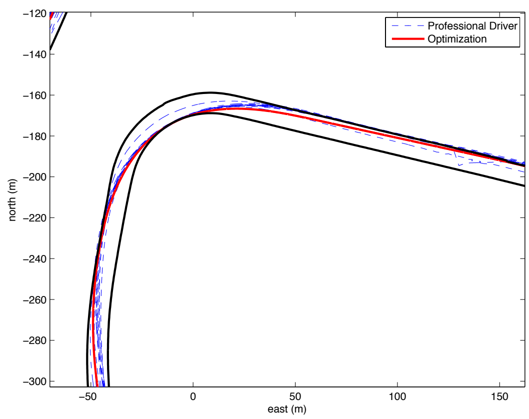

Figure 15. *Turn 3 of the optimized path compared with ten laps of Bruce Canepa*

5 RESULTS OF THE OPTIMIZATION

The work in this paper was applied to the Mazda Raceway at Laguna Seca with eleven turns across 3.602 km. The results were compared to Bruce Canepa's racing line, a professional racecar driver who holds a world record at the Pikes Peak International Hill Climb. Bruce's path was recorded from a GPS data acquisition system instrumented on a 1967 Porsche 910.

5.1 COMPARING LAP TIMES

When the path was first initialized on the track (Figure 10) the lap time was 116.466 seconds. After five iterations over all parameter subsets, the lap time was reduced to 106.749 seconds. Bruce made a total of ten laps around the track. After fitting the path primitives to his fastest lap, Bruce's comparable lap time was 107.142 sec. This lap time was derived from the same vehicle model to ensure that only the performance of the trajectories were being compared. Future work will involve racing the autonomous Audi TTs around Laguna Seca and compaing that lap time with one driven by Bruce Canepa in an Audi TTs.

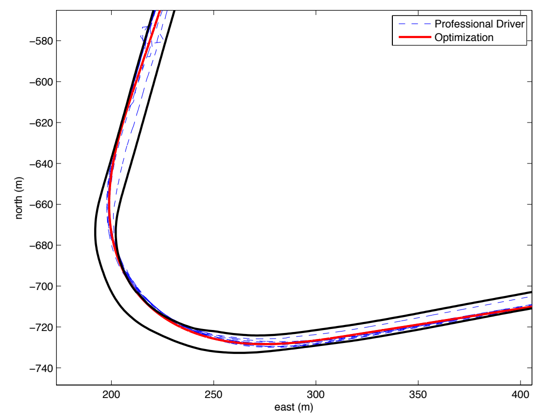

Figure 16. *Turn 5 of the optimized path compared with ten laps of Bruce Canepa*

5.2 COMPARING SIMPLE TURNS

A comparison of the optimized trajectory and all ten of Bruce's laps can be seen on Figures 15 and 16. Turn four on Figure 15 (the corner at the top right, driving left to right) is an example of how the path is modified to take a simple turn by approaching the the corner from the outside, turning to the inside apex, and then returning to the outside edge again to minimize the curvature of the turn.

5.3 COMPARING MORE ADVANCED TURNS

Turn nine on Figure 16 (top right, driving right to left) is a good example of how the optimization converges to a solution that is the same as a professional driver but may not be apparent to the unskilled driver. After turn eight (bottom right, driving down to up) the racecar is at a lower speed and does not need to fade all the way to the outside boundary to minimize the curvature of the turn. This is an example of how the optimization algorithm recognized that minimizing path length is the better strategy in this situation and is an important result for validation.

6 CONCLUSION

This paper demonstrates how a set of simple curves can be used to analyze and emulate professional driving techniques. The proposed work outlines a method for generating competitive racing lines for an autonomous racecar given the lane boundaries of the track. A nonlinear successive gradient descent algorithm was used to modify an initialized path. The resultant path was used to compare with a professional driver and was found to recognize strategies similar to professional drivers.

6.1 FUTURE WORK

Future work will involves testing the optimized path with the autonomous Audi at Laguna Seca and comparing its lap time with a professional driving a Audi TTs. The vehicle model will also incorporate features of the track such as the bank and grade of the road and the weight transfer of the vehicle.

7 ACKNOWLEDGEMENTS

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