ICROS-SICE International Joint Conference 2009
August 18-21, 2009, Fukuoka International Congress Center, Japan

Detection of Critical Driving Situations Based on Wheel-Ground Contact Normal Forces

Arjon Turnip, Le Hoa Nguyen, and Keum-Shik Hong

School of Mechanical Engineering, Pusan National University; #30 Jangjeon-dong, Gumjeong-gu,
Busan 609-735, Korea. (Tel: +82-51-510-1481, Fax: +82-51-514-0685,
Emails: {turnip, nglehoa, kshong}@pusan.ac.k)

Abstract: To develop an effective vehicle control system, it is needed to estimate vehicle motions accurately, and the estimation of reliable road frictions is one of the most important steps to achieve this goal. In the absence of commercially available transducers to measure the friction coefficient directly, various types of estimation methods have been investigated in the past. Most models in the literature usually use low degrees of freedom. Also, these models have different values from the real vehicle motions and have a difficulty to adapt with new technologies. In this paper, a new estimation process is proposed to estimate tire forces and vehicle state histories, that is, longitudinal and lateral velocities, angular velocity and rolling radius of wheels, and side slip, pitch and roll angle based on extended Kalman filter, and road friction coefficient based on a recursive least squares method. These methods use the measurements from currently available standard sensors. For such estimation, a fourteen degree-of-freedom nonlinear vehicle model was developed. The estimated results are compared with the results obtained from CarSim using the same parameter values to verify the proposed model.

Keywords: Friction estimation, Tire forces, Road friction coefficient, Extended Kalman filter, Recursive least squares.

1. INTRODUCTION

Since vehicle dynamics concerned with stability and controllability of an automobile, it is important to consider this factor in the design of an accurate vehicle model. A large number of efforts have been devoted previously for the studies and investigations of vehicle modeling [1-14]. In these studies, three typical models have been developed in the researches related to the dynamic behavior of vehicles and its vibration control. The simplest representation of a vehicle is a quarter-car model [1-4]. The quarter-car model is used only when the heave motion need to be considered. A half-car model is a two wheel model (front and rear) that can be used for studying the heave and pitch motions [5-8]. This model is allowed taking into consideration the deflection of tires and suspensions of the vehicles. A more complex model is the full-vehicle model which is four wheel models for studying the heave, pitch, yaw and roll motions. Previous investigation essentially treated the rolling radius of the wheel as a constant [13]. Moreover, the angular velocities of the wheels were measured but the suspension dynamics including pitch motion were neglected. In this paper, all of those variables are considered as vehicle state histories.

To develop an effective vehicle control system, it is necessary to estimate the vehicle motions accurately, and reliable road friction estimation is one of the most important steps to achieve this goal. In the absence of commercially available transducers to measure road friction coefficient directly, various types of estimation methods have been investigated in the past. Most of the reference models usually have low level degree of freedom such as bicycle model. However, these models have some different value from the real vehicle motion and have a difficulty to adapt with new technologies. With accurate estimation of the instantaneous maximum friction coefficient for the current road condition, the performance of the car can be improved by optimization for varying road conditions.

The first objective of this work is to develop an estimator for vehicle modeling by using the extended Kalman filter [15,16] to estimate lateral and longitudinal velocities, pitch, roll, rolling radius, angular velocities, lateral and longitudinal forces of the tire. The estimated results will be used to calculate the normal forces of the wheels. The second is to develop an estimator using recursive least square method [17,18] to estimate friction coefficient of the road. This estimate can be used to give the driver or a closed-loop controller an advance warning when the tire force limit is being approached.

The outline of this paper is the following: in Section 2 a fourteen degree-of-freedom nonlinear full vehicle model is derived. Tire forces and the vehicle state histories are explored in Section 3. The road coefficient of friction estimation is presented in section 4. The comparison between the simulated and the estimated values with the CarSim results are discussed in Section 5. Conclusions are given in section 6.

2. A FULL-VEHICLE MODEL

Fig. 1 and 2 show the full-vehicle model: horizontal, vertical model and tire model. The heave, pitch and roll motions of the sprung mass are included. The horizontal vehicle model is made of three DOF which consist of longitudinal, lateral translation, and rotation of vehicle mass center as depicted in Fig. 1. This model gets the lateral and the longitudinal forces from tire model. Based on these two forces, the horizontal model calculates horizontal vehicle performance. The vertical vehicle model is made of seven DOF which consist of three DOF of mass center (vertical, roll, pitch dynamic)

PR0002/09/0000-2519 ¥400 © 2009 SICE

and four DOF of each wheel (wheel vertical dynamic) as depicted in Fig. 2. Vertical dynamic model calculates the vehicle vertical motion from road information, and pitch, roll motion from tire model using lateral and longitudinal forces. The tire model is made of four DOF as shown in Fig. 2. It is basically consist of longitudinal one DOF of each tire. Tire model received steering input, both vehicle velocity and yaw rate, and vertical forces from driver, horizontal, and vertical models.

The purpose of this work is to develop an estimation method for the friction coefficient of the road based on a fourteen degree-of-freedom vehicle. However, before trying to do the implementation of the estimation, it is critical to specify the model that can be used for the simulation of the vehicle. Such model will be derived in this paper. In the horizontal vehicle model shown in Fig. 1, V is the vehicle speed, v_x and v_y are the longitudinal and lateral velocities, respectively, $\dot{\varphi}$ is the yaw rate, β is the side slip angle, α_{ij} is the slip angle of wheel, respectively. F_x^{ij} is the longitudinal tire force and F_y^{ij} is the lateral tire force. The subscript $i = f, r$ indicates front and rear while the subscript $j = L, R$ indicates left and right sides. In this way, F_x^{fR} and F_y^{fR} indicate the longitudinal and lateral forces of tire-road in front and right corners, respectively. δ_i is the steering angle with $\delta_r = 0$, L_i

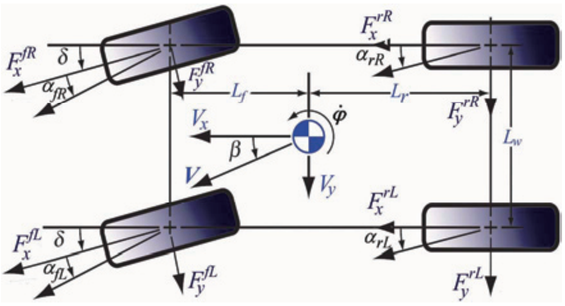

Fig. 1 Three degree-of-freedom full-vehicle model.

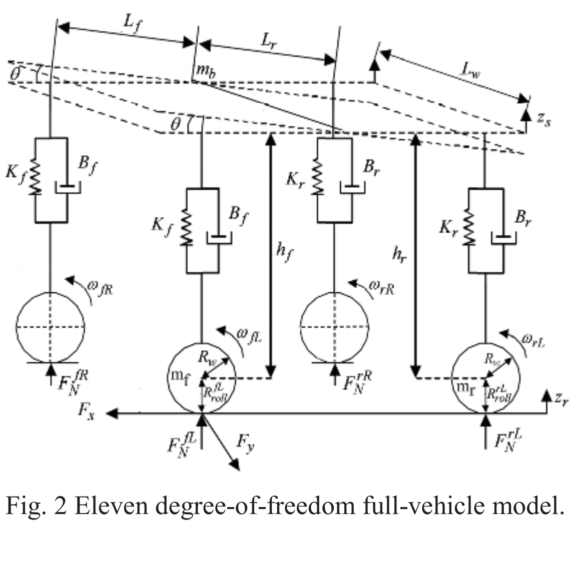

Fig. 2 Eleven degree-of-freedom full-vehicle model.

is the longitudinal distance from the center of front and rear wheels to the body's center of gravity. L_w is the track width. Direct application of Newton's Laws for the system in Fig. 1 yields,

$$\dot{v}_x = \frac{1}{m}\left[\left(F_x^{fL} + F_x^{fR}\right)\cos\delta - \left(F_y^{fL} + F_y^{fR}\right)\sin\delta + \left(F_x^{rL} + F_x^{rR}\right) + \dot{\varphi} v_y\right], \tag{1}$$

$$\dot{v}_y = \frac{1}{m}\left[\left(F_y^{fL} + F_y^{fR}\right)\cos\delta - \left(F_x^{fL} + F_x^{fR}\right)\sin\delta + \left(F_y^{rL} + F_y^{rR}\right) - \dot{\varphi} v_x\right], \tag{2}$$

$$\begin{aligned}\ddot{\varphi} = \frac{1}{I_\varphi}\Big(& L_f\left(F_x^{fL} + F_x^{fR}\right)\sin\delta + L_f\left(F_y^{fL} + F_y^{fR}\right)\cos\delta - L_r\left(F_y^{rL} + F_y^{rR}\right) \\ & + \frac{L_w}{2}\left(F_x^{fR} - F_x^{fL}\right)\cos\delta + \frac{L_w}{2}\left(F_x^{rR} - F_x^{rL}\right) + \frac{L_w}{2}\left(F_y^{fL} - F_y^{fR}\right)\sin\delta\Big),\end{aligned} \tag{3}$$

where m represents the total mass of the vehicle.

In the vertical and tire vehicle model shown in Fig. 2, Let ω_{ij} is the wheel angular velocity, z_s is the sprung mass vertical displacement, z_r is the road profile, θ is the body pitch angle, m_b is the mass of the vehicle without the mass of the front and rear wheels m_i, R_{rol}^{ij} is the rolling radius (tire and wheel), F_N^{ij} is the normal force, and h_i is the vertical distance from the center of gravity to the center of the front and the rear wheel at equilibrium. The spring and damping constants K_i and B_i, respectively, are the lumped parameters associated with the passive suspension system and tires. In the model, ϕ is the roll angle, $I_{\varphi,} I_{\theta,}$ and I_ϕ are the moments of inertia of the vehicle about its yaw, pitch, and roll axis, respectively. I_w is the moment of inertia of the wheel about its axle. For a small value of θ, after applying a force-balance analysis to the model in Fig. 2, the equations of motion can be derived to the static equilibrium positions. The equations for describing the sprung mass are

$$\begin{aligned}\ddot{z}_s = \frac{1}{m_b}[& -K_f z_s^{fL} - B_f \dot{z}_s^{fL} - K_f z_s^{fR} - B_f \dot{z}_s^{fR} \\ & -K_r z_s^{rL} - B_r \dot{z}_s^{rL} - K_r z_s^{rR} - B_r \dot{z}_s^{rR}],\end{aligned} \tag{4}$$

$$\begin{aligned}\ddot{\theta} = \frac{1}{I_\theta}\Big\{& \left(h_{fL} F_x^{fL} + h_{fR} F_x^{fR}\right)\cos\delta + \left(h_{fL} F_y^{fL} + h_{fR} F_y^{fR}\right)\sin\delta \\ & - h_{rL} F_x^{rL} - h_{rR} F_x^{rR} - L_f\left(K_f z_s^{fL} + B_f \dot{z}_s^{fL} + K_f z_s^{fR} + B_f \dot{z}_s^{fR}\right) \\ & + L_r\left(K_r z_s^{rL} + B_f \dot{z}_s^{rL} + K_r z_s^{rR} + B_r \dot{z}_s^{rR}\right)\Big\},\end{aligned} \tag{5}$$

$$\begin{aligned}\ddot{\phi} = \frac{1}{I_\phi}\Big\{& \left(h_{fL} F_y^{fL} - h_{fR} F_y^{fR}\right)\sin\delta + \left(h_{fR} F_x^{fR} - h_{fL} F_x^{fL}\right)\cos\delta \\ & + h_{rR} F_x^{rR} - h_{rL} F_x^{rL} - \frac{L_w}{2}\left(K_f z_s^{fR} + B_f \dot{z}_s^{fR} + K_r z_s^{rR} + B_r \dot{z}_s^{rR}\right) \\ & + \frac{L_w}{2}\left(K_f z_s^{fL} + B_f \dot{z}_s^{fL} + K_r z_s^{rL} + B_r \dot{z}_s^{rL}\right)\Big\},\end{aligned} \tag{6}$$

where $z_s^{fL}=z_s+L_f\theta+L_w/2\phi$, $z_s^{fR}=z_s+L_f\theta-L_w/2\phi$, $z_s^{rL}=z_s-L_f\theta+L_w/2\phi$, $z_s^{rR}=z_s-L_f\theta-L_w/2\phi$, $\dot{z}_s^{fL}=\dot{z}_s+L_f\dot{\theta}+L_w/2\dot{\phi}$, $\dot{z}_s^{fR}=\dot{z}_s+L_f\dot{\theta}-L_w/2\dot{\phi}$, $\dot{z}_s^{rL}=\dot{z}_s-L_f\dot{\theta}+L_w/2\dot{\phi}$, $\dot{z}_s^{rR}=\dot{z}_s-L_f\dot{\theta}-L_w/2\dot{\phi}$, $h_{fL}=h_f+R_{rol}^{fL}$, $h_{fR}=h_f+R_{rol}^{fR}$, $h_{rL}=h_r+R_{rol}^{rL}$, and $h_{rR}=h_r+R_{rol}^{rR}$, and for the wheels is

$$\dot{w}_{ij}=\frac{1}{I_w}\left(T_d-T_b^{ij}-R_{rol}^{ij}F_x^{ij}\right), \tag{7}$$

where T_d is the input torques applied to front wheels and T_b^{ij} is the applied braking torque.

3. TIRE FORCES AND VEHICLE STATES HISTORIES ESTIMATION

An extended Kalman filter (EKF) is used to estimate the tire forces and the vehicle state histories. The estimation model of the vehicle dynamics includes all the equations of motions specified in (1) to (7). The model is represented by a nonlinear state space description incorporating state and measurement difference equations.

$$\begin{aligned} x_{k+1}&=f(x_k,u_k,w_k),\\ z_k&=h(x_k,u_k,v_k). \end{aligned} \tag{8}$$

In (8) the nonlinear function f relates the state vector x and the input vector u at time step k to the state at time step $k+1$. The measurement vector h relates the state to the measurements z_k. Vectors w_k and v_k denote the superimposed process and measurement noise, respectively. The variance of w and v are Q and R, respectively. We defined the state vector as $x(t)=[v_x\ v_y\ R_{rol}^{ij}\ w_{ij}\ \varphi\ \dot{\varphi}\ z_s\ \dot{z}_s\ \theta\ \dot{\theta}\ \phi\ \dot{\phi}\ F_x^{ij}\ F_y^{ij}\ \dot{F}_x^{ij}\ \dot{F}_y^{ij}]^T$, and the measurement vector as $z(t)=\begin{bmatrix}V & \dot{\varphi} & a_x & a_y\end{bmatrix}^T$ that incorporates all the measurement values, and input vector $u(t)=[\delta\ T_b]^T$. a_x and a_y are the longitudinal and lateral accelerations of the vehicle. The control inputs and the outputs are measured using a set of sensors. All of these sensors are assumed to be on the vehicle. The state vector x_k is augmented to include the differential equations of each force to be estimated. To model each force, the following model form [5] is used.

$$\begin{bmatrix}\dot{f}_0\\ \dot{f}_1\\ \dot{f}_2\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}f_0\\ f_1\\ f_2\end{bmatrix}+w, \tag{9}$$

where f_0 is the force, f_1 and f_2 are first and second derivatives of the force. The discrete-time EKF algorithm adopted from [15] is used to develop the complete set of equations in (10) to (16). The EKF measurement update equations are:

$$K_k=P_k^-H_k^T\left(H_kP_k^-H_k^T+R_k\right)^{-1}, \tag{10}$$

$$\hat{x}_k=\hat{x}_k^-+K_k\left(z_k-h(\hat{x}_k^-,u_k)\right), \tag{11}$$

$$P_k=\left(I-K_kH_k\right)P_k^-. \tag{12}$$

The EKF time update equations are as follows:

$$P_k^-=P_{k-1}+\left(A_kP_{k-1}+P_{k-1}A_k^T+Q\right)T, \tag{13}$$

$$\hat{x}_k^-=\hat{x}_{k-1}+f\left(\hat{x}_{k-1},u_k\right)T. \tag{14}$$

The EKF propagates the state and error covariance estimates (14) and (13) by computing the filter gain matrix (10), and by updating the state and covariance estimates based on the measurement residual (11) and (12). The matrices A_k and H_k are computed by linearizing (8) around $\hat{x}_k^-$ at each time step. The matrix A is thus the Jacobian matrix of the partial derivatives of $f()$ with respect to x**:**

$$A_k=\frac{\partial f}{\partial x}(x_k,u_k), \tag{15}$$

and the matrix H is the Jacobian matrix of the partial derivatives of $h()$ with respect to x :

$$H_k=\frac{\partial h}{\partial x}(x_k,u_k). \tag{16}$$

The filter is initialized with a state estimate corresponding to the true state and a large covariance matrix.

4. ROAD FRICTION COEFFICIENT ESTIMATION

The tire model is used to simulate the true tire forces of the vehicle. The front and rear normal forces at the tire-road interface are as follows:

$$F_N^{ij}=-K_iz_s^{ij}-B_i\dot{z}_s^{ij}+m_{ij}g+\frac{m_b}{2}\frac{L_i}{L_f+L_r}g, \tag{17}$$

where g is the gravity acceleration. The front and rear wheel slip angles α_{ij} are as follows:

$$\alpha_{ij}=\delta_i-\tan^{-1}\left(\frac{v_y\pm L_i\dot{\varphi}}{v_x}\right), \tag{18}$$

where $\delta_r=0$. The magnitudes of the front and rear axle velocities are as follows:

$$v_i=\sqrt{\left(v_y\pm L_i\dot{\varphi}\right)^2+v_x^2}. \tag{19}$$

The longitudinal and lateral slips of the front and rear wheel are as follows:

$$\begin{bmatrix}s_x^{ij}\\ s_y^{ij}\end{bmatrix}=\frac{1}{v_i}\begin{bmatrix}r_{ij}w_{ij}\cos\alpha_{ij}-v_i\\ r_{ij}w_{ij}\sin\alpha_{ij}\end{bmatrix}. \tag{20}$$

The resultant of the front and rear wheel slips becomes

$$s_{ij}=\sqrt{\left(s_x^{ij}\right)^2+\left(s_y^{ij}\right)^2}. \tag{21}$$

Now, the road friction coefficients μ_{ij} can be calculated accurately through the following equation,

$$\mu_{ij} = \frac{s_{ij}}{F_N^{ij}} \begin{bmatrix} F_x^{ij} \\ F_y^{ij} \end{bmatrix} \left[\begin{bmatrix} s_x^{ij} \\ s_y^{ij} \end{bmatrix} \right]^{-1}. \tag{22}$$

The tire-road friction model in (22) can be formulated in the parameter identification form as:

$$y(t) = \varphi^T(t)\theta(t) + e(t), \tag{23}$$

where $y(t)$ is the estimated longitudinal or lateral forces discussed in Section 2 as the measured outputs, $\theta(t) = \mu_{ij}$ is the unknown parameter, $\varphi^T(t) = (F_N^{ij})^T = F_N^{ij}$ is the calculated normal force in (17) as the regression vector, $e(t)$ is the identification error between the measured output $y(t)$ and estimated values $\varphi^T(t)\theta(t)$. The RLS (recursive least squares) algorithm updates the unknown parameters so as to minimize the sum of the squares of the modeling errors. The calculations in the RLS algorithm at each time step t are as follows: First, measure the system outputs, $y(t)$, and calculate the regression vector $\varphi(t)$. Second, calculate the identification error, $e(t)$, which is the difference between system actual output at this sample and the predicted model output obtained from the estimated parameters in the previous sample, $\theta(t-1)$, i.e. $e(t) = y(t) - \varphi^T(t)\theta(t-1)$. Third, calculate the updated gain vector, $K(t)$, and the covariance matrix, $P(t)$, as

$$K(t) = \frac{P(t-1)\varphi(t)}{\lambda + \varphi^T(t)P(t-1)\varphi(t)}, \tag{24}$$

$$P(t) = \frac{1}{\lambda}\left[P(t-1) - \frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{\lambda + \varphi^T(t)P(t-1)\varphi(t)}\right]. \tag{25}$$

Fourth, update the parameter estimate vector, $\theta(t)$, as

$$\theta(t) = \theta(t-1) + K(t)e(t), \tag{26}$$

where the parameter λ is called the forgetting factor.

5. SIMULATION RESULTS

The tire forces, the vehicle state histories, and the friction coefficient of the road govern all aspects of the vehicle motions and are important for vehicle simulation, handling evaluation, control system design, and safety measures. In this paper, the simulation model is a fourteen degree-of-freedom of a full-vehicle model that includes the vertical suspension dynamics, the roll, yaw and pitch motion. Using the vehicle speed, yaw rate, longitudinal and lateral accelerations of the vehicle as measurement vectors, the steering angle, and the braking as input control, all those state variables are estimated. The vehicle parameters for CarSim model and simulation are given in Table 1. Then, the estimated results are compared with the obtained responses from CarSim model. Fig. 3 – 8 show the comparison of the CarSim results, the simulated values based on equations (1) to (7), and the estimated values based on an EKF

Tabel 1. Vehicle Parameters.

Parameter	Value	Parameter	Value
m	1,993 kg	I_ϕ	614 kgm^2
m_b	1,653 kg	h_f	0.59 m
m_f	90 kg	h_r	0.59 m
m_r	80 kg	R_w	0.365 m
L_f	1.402 m	I_w	4.07 kgm^2
L_r	1.646 m	K_f	34,000 N/m
L_w	1.6 m	K_r	38,000 N/m
I_φ	2,765 kgm^2	B_f	5,000 Ns/m
I_θ	2,765 kgm^2	B_r	4500 Ns/m

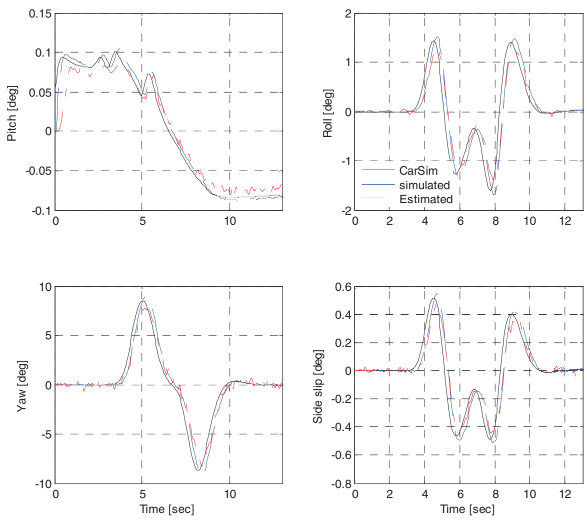

Fig. 3 CarSim results, simulated and estimated values of ϕ, θ, φ, and β.

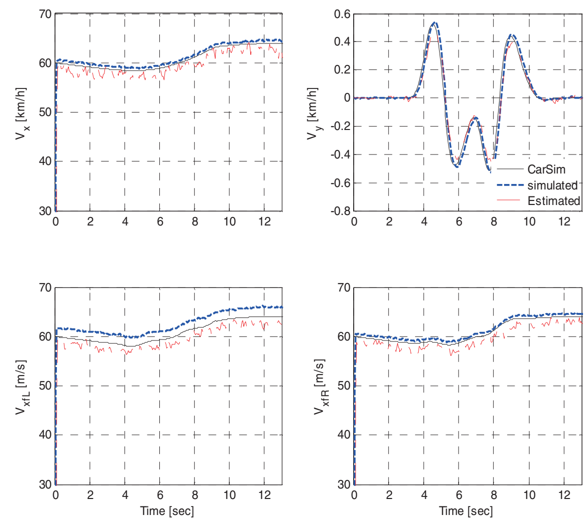

Fig. 4 CarSim results, simulated and estimated values of v_x, v_y, v_x^{fL}, and v_x^{fR}.

algorithm. Through those responses, it can be seen that the derived model could represent the vehicle dynamics well. The estimated tire forces and vehicle state histories using an EKF show excellent tracking and robust properties. The estimated states of EKF follow the actual states properly, even in the face of cornering conditions. Fig. 5 shows that the rolling radiuses and angular velocities of the front right and left, as well as for rear right and left, wheel are different, especially in cornering conditions. It means, these variables can not be assumed as a constant. Since most of the disturbances from the road come through the tires, the information of the tire behavior is very important. Fig. 9 shows the estimated results of the friction coefficient of the road for its wheel based on equations (17) to (22) using the RLS identification method. In the CarSim model, the maximum value of friction coefficient of the

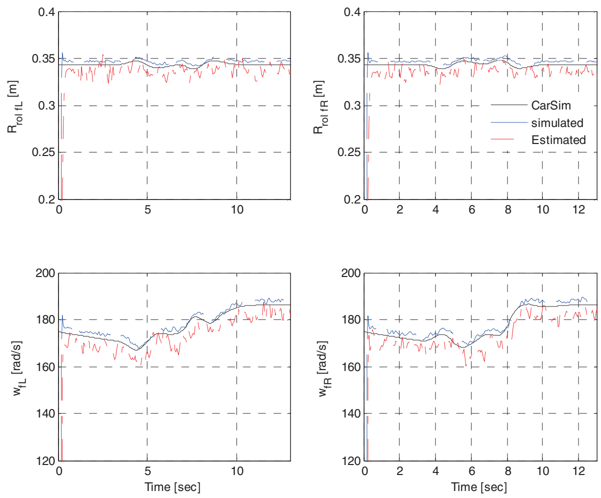

Fig. 5 CarSim results, simulated and estimated values of $R_{roll}^{fL}, R_{roll}^{fR}, w_{fL}$, and w_{fR}.

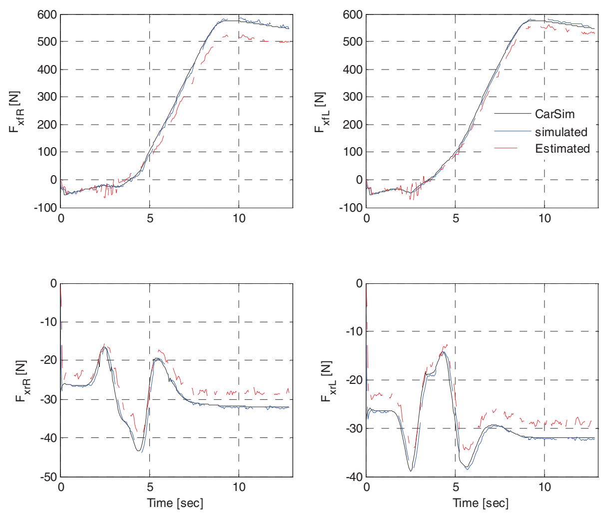

Fig. 6 CarSim results, simulated and estimated values of $F_x^{fL}, F_x^{fR}, F_x^{rL}$, and F_x^{rR}.

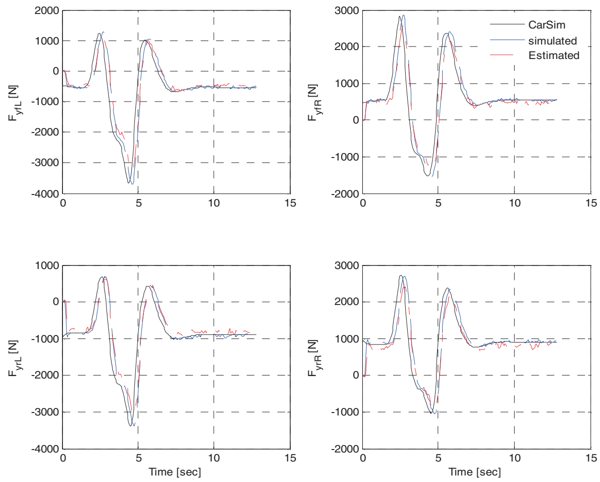

Fig. 7 CarSim results, simulated and estimated values of $F_y^{fL}, F_y^{fR}, F_y^{rL}$, and F_y^{rR}.

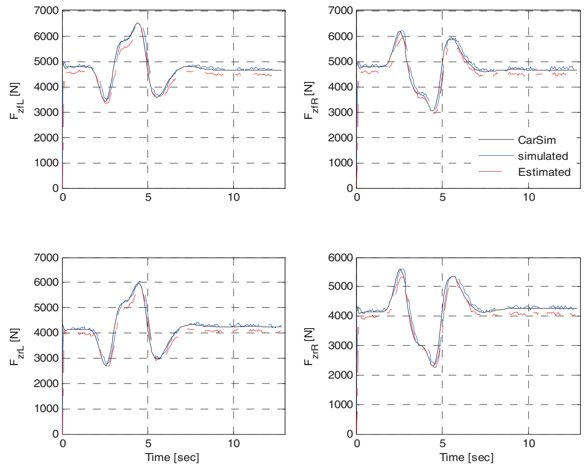

Fig. 8 CarSim results, simulated and estimated values of $F_N^{fL}, F_N^{fR}, F_N^{rL}$, and F_N^{rR}.

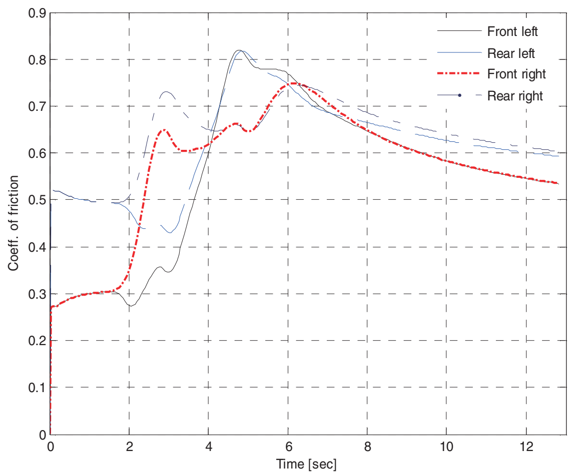

Fig. 9 Simulated and estimated values of $\mu_{fL}, \mu_{fR}, \mu_{rL}$, and μ_{rR}.

road is 0.8. From Fig. 9, it can be seen that tracking of the maximum estimated μ is around 0.82, and these value occurred when the vehicle was in cornering conditions. The differences of these values can be accepted based on some assumption when the dynamic vehicle models were derived and the estimation processes were involved by some noises.

6. CONCLUSION

In this paper, a fourteen degrees-of-freedom nonlinear vehicle model was developed. In the absence of commercially available transducers to measure some state variables, this paper proposed estimation methods for tire forces, vehicle state histories, and road friction coefficient using EKF and RLS algorithms, respectively. The comparison between the estimated results and the simulated CarSim model confirms the validity of the model, in which all the state variables follow the CarSim response well. A robustness study indicates that the μ identification procedure can accommodate reasonable uncertainties in the tire force and vehicle models. The obtained state estimate values can be used for advanced feedback control, whereas the resulted road friction coefficient estimates can be used to determine controller set points and to make intelligent driving decisions.

ACKNOWLEDGEMENT

This work was supported by the Regional Core Technology Development Program granted by the Ministry of Knowledge Economy, Korea.

REFERENCES

[1] E. Onoa, K. Asanoa, M. Sugaia, S. Itob, M. Yamamotob, M. Sawadac, and Y. Yasui, "Estimation of Automotive Tire Force Characteristics Using Wheel Velocity", *Control Engineering Practice*, Vol. 11, pp. 1361–1370, 2003.

[2] A. Pohl, R. Steindl, and L. Reindl, "The Intelligent Tire Utilizing Passive SAW Sensors Measurement of Tire Friction", *IEEE Transactions on Instrumentation and Measurement*, vol. 48, no. 6, pp. 1041-1046, 1999.

[3] C.-S. Liu and H. Peng, "Road Friction Coefficient Estimation for Vehicle Path Prediction", *Vehicle System Dynamics*, Vol.25, pp.413-425, 1996.

[4] F. Gustafsson, "Slip-based Tire-Road Friction Estimation", *Automatica* Vol. 33. No. 6, pp. 1087-1099, 1997.

[5] L. R. Ray, "Nonlinear State and Tire Force Estimation for Advanced Vehicle Control", *IEEE Transactions on Control System Technology*, Vol. 3, No. 1, pp. 117-124, 1999.

[6] T. Hiraoka, H. Kumamoto, and O. Nishihara, "Sideslip Angle Estimation and Active Front Steering System Based on Lateral Acceleration Data at Centers of Percussion with Respect to Front/Rear Wheels", *JSAE Review*, 25 pp. 37-42, 2004.

[7] J. Campos, L. Davis, F. L. Lewis, S. Ikenaga, S. Scully, and M. Evans, “Active Suspension Control of Ground Vehicle Heave and Pitch Motions”, *Proceedings of the 7th Mediterranean Conference on Control and Automation Haifa*, Israel - June 28-30, 1999.

[8] S. Ikenaga, F. L. Lewis, J. Campos and L. Davis, "Active Suspension Control of Ground Vehicle based on a Full-Vehicle Model", *Proceedings of the American Control Conference Chicago*, Illinois June 2000.

[9] B. J. Olson, S. W. Shaw, and G. Stepan, “Nonlinear Dynamics of Vehicle Traction”, *Vehicle Syatem Dynamics*, Vol. 40, No. 6, pp. 377-399, 2003.

[10] R. Rajamani, *Vehicle Dynamics and Control*, Springer, USA, 2006.

[11] R. N. Jazar, *Vehicle Dynamis: Theory and Application,* Springer, 2008.

[12] R. Andrzejewski and J. Awrejcewicz, *Nonlinear Dynamis of a Wheeled Vehicle,* Springer, 2005.

[13] T. Gillespie, *Fundamentals of Vehicle Dynamics,* Society of Automotive Engineers (SAE), Inc, 1992.

[14] L. R. Ray, “Nonlinear Tire Force Estimation and Road Friction Identification: Simulation and Experiment”, *Automatica,* Vol. 33. No. 10, pp. 1819-1833, 1997.

[15] M. S. Grewal and A. P. Andrews, *Kalman Filtering: Theory and Practice Using Matlab*, John Wiley & Sons, Inc., Second Edition, 2001.

[16] R.G. Brown, *Introduction to random signals and applied Kalman filtering*, John Wiley & Sons, Inc., Third Edition, 1997.

[17] K. J. Astrom and B. Wittenmark, *Adaptive Control*, Addison- Wesley Publishing Company, Inc., Second Edition, 1995.

[18] I. D. Landau and G. Zito, *Digital Control Systems: Design, Identification, and Implementation,* Springer, 2006.