TMA4165: Problem Sheet V

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1. Use the Poincaré-Lindsted method to find a perturbation expansion to first order in the following systems:

(i) $\ddot{x} - \varepsilon x\dot{x} + x = 0$,
(ii) $(1 + \varepsilon\dot{x})\ddot{x} + x = 0$.

2. Find the index about the critical points in the following diagrams:

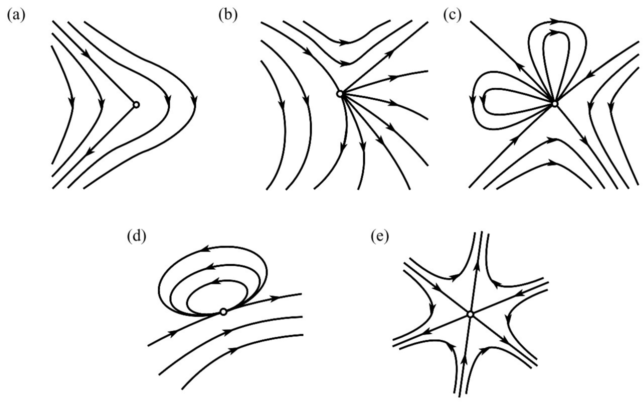

Figure 1. Taken from *Jordan and Smith*

3. Find the index at the critical points of the following systems:

(i) $\dot{x} = 2xy, \;\; \dot{y} = 3x^2 - y^2$;
(ii) $\dot{x} = y^2 - x^4, \;\; \dot{y} = x^3 y$;
(iii) $\dot{x} = x - y, \;\; \dot{y} = x - y^2$.

4. Show that for linear planar systems, saddles always have index -1, stable foci always have index 1, centres always have index 1, and stable nodes always have index 1.

5. Show that the following systems have no periodic solutions:

(i) $\dot{x} = y, \;\; \dot{y} = 1 + x^2 - (1 - x)y$;
(i) $\dot{x} = -(1 - x)^3 + xy^2, \;\; \dot{y} = y + y^3$;
(i) $\dot{x} = 2xy + x^3, \;\; \dot{y} = -x^2 + y - y^2 + y^3$;
(i) $\dot{x} = x, \;\; \dot{y} = 1 + x + y^2$;
(i) $\dot{x} = y, \;\; \dot{y} = -1 - x^2$;
(i) $\dot{x} = 1 - x^3 + y^2, \;\; \dot{y} = 2xy$;
(i) $\dot{x} = y, \;\; \dot{y} = (1 + x^2)y + x^3$.