Modeling for IMU-based Online Estimation of a Ship's Mass and Center of Mass

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Abstract: A ship's roll dynamics is very sensitive to changes in the loading conditions and a worst-case scenario is the loss of stability. This paper proposes an approach for online estimation of a ship's mass and center of mass. Instead of focusing on a sensor-rich environment where all possible signals on a ship can be measured and a complete model of the ship can be estimated, a minimal approach is adopted. A model of the roll dynamics is derived from a well-established model in literature and it is assumed that only motion measurements from an inertial measurement unit together with measurements of the rudder angle are available. Furthermore, identifiability properties and disturbance characteristics of the model are presented. Due to the properties of the model, the parameters are estimated with an iterative instrumental variable approach to mitigate the influence of the disturbances and it uses multiple datasets simultaneously to overcome identifiability issues. Finally, a simulation study is presented to investigate the sensitivity to the initial conditions and it is shown that the sensitivity is low for the desired parameters.

Keywords: Modelling, Identification, Operational safety, Inertial measurement unit

1. INTRODUCTION

There are several factors that influence the dynamic behavior of a ship and the mass and the center of mass (CM) are properties that have a particularly large impact on the ship's dynamical behavior and especially on the roll dynamics (Tannuri et al., 2003; Fossen, 2011). Mathematical models are typically used to enhance performance or safety, for instance, to simulate the ship's response in an advisory system in order to aid the crew in the operation of the ship. However, loading conditions may change over time and large variations can be critical for the stability of the ship. Consider, for instance, the change in mass of a fishing vessel or a bulk tanker that changes due to the loads in the cargo holds. Online estimation is one way to improve the model accuracy if the variations have a large impact on the dynamic behavior.

The main challenge in both online and offline ship modeling is the complex interaction with water. This complexity makes it difficult to compute which forces that are acting on the ship during normal operation unless special sensors are introduced. Without knowledge about these forces, estimation of the inertial properties becomes challenging, for example, since large forces acting on a large mass gives the same behavior as small forces acting on a small mass.

Online mass and CM estimation for vehicles is a hot topic, especially in automotive applications where roll over accidents is a common type of accident, see for instance, Fathy et al. (2008) or Sadeghi Reineh et al. (2013). However, due to the complex interaction with the water, these methods cannot be used directly for estimation of a ship's mass and CM. The two major differences are that the environmental disturbances have a larger impact on the motion in a ship application and that the couplings between the degrees of freedom (DOF) typically are stronger for a ship.

An approach to address the disturbance issue was proposed in Linder et al. (2014b). A model of the *decoupled* roll dynamics was used and it was assumed that only motion data from an inertial measurement unit (IMU) together with the rudder angle were available. An instrumental variable (IV) method was used to mitigate the influences of the environmental disturbances where the rudder angle measurements were used to create the instruments. To overcome identifiability issues, multiple datasets were used sequentially. In Linder et al. (2014a), it was shown that the estimation problem is similar to closed-loop estimation and that the variance properties of the IV estimator could be improved by considering this.

This paper presents an approach for online estimation of a ship's mass and CM. The approach can be seen as an extension of the method presented in Linder et al. (2014a). The contribution of the extension is threefold. 1) A model of a ship's roll dynamics is derived from a well-established maneuvering model. This model can be seen as a generalization of the model in Linder et al. (2014b) that considers the strong coupling to the other DOF. 2) The identifiability properties and disturbance characteristics are presented together with the implications for the parameter estimation. 3) Due to the disturbance characteristics, an iterative IV approach is formulated to estimate the parameters. Multiple datasets are used simultaneously to overcome the identifiability issues. In addition to the formulation of the approach, a Monte Carlo simulation is performed to investigate the estimator's sensitivity to initial conditions. This paper presents theoretical aspects, but the proposed method has also been validated on data from a scale model of a fishing vessel, see Linder et al. (2015).

The remainder of this paper is organized as follows: In Section 2 the model is derived. Section 3 presents an analysis of the model's key properties. Section 4 describes an iterative IV approach for estimating the parameters. The estimator's sensitivity to the initial conditions is investigated in Section 5. Finally, in Section 6, the paper is summarized with conclusions and suggestions for future work.

2. SHIP MODELING

The basis for the approach in this paper is the maneuvering model developed and discussed in Blanke and Christensen (1993) and Perez (2005). The model describes the planar motion, i.e. position and heading, and the roll dynamics. This model is particularly useful for describing the rudder induced roll motion and is as such a good foundation for the simplified model developed in this paper. Here, the model will be expressed in a vectorial setting described by Fossen (1991). The model can be written as

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{\nu} \\ \boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} - \boldsymbol{N}(\boldsymbol{\nu},\boldsymbol{\eta}) + \boldsymbol{g}(\boldsymbol{\eta}) &= \boldsymbol{R}(\delta) + \boldsymbol{\tau} \end{aligned} \tag{1}$$

where $\boldsymbol{\eta}^T = [X, Y, \phi, \psi]$ is the generalized position described in an Earth-fixed coordinate system (assumed to be inertial), $\boldsymbol{\nu}^T = [u, v, p, r]$ is the generalized velocity described in a body-fixed coordinate system, δ is the rudder angle, $\boldsymbol{\tau}$ is the environmental forces acting on the ship and the variables of $\boldsymbol{\eta}$ and $\boldsymbol{\nu}$ are defined in Table 1 (Fossen, 2011). A sketch relating the variables and coordinate systems can be seen in Fig. 1. The generalized position $\boldsymbol{\eta}$ and velocity $\boldsymbol{\nu}$ are related through the Euler angle velocity transformation matrix which in this case is given by

$$\boldsymbol{J}(\boldsymbol{\eta}) = \begin{bmatrix} c_\psi & -c_\phi s_\psi & 0 & 0 \\ s_\psi & c_\phi c_\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & c_\phi \end{bmatrix} \tag{2}$$

where $c_i = \cos(i)$ and $s_i = \sin(i)$. The system inertia matrix (including added mass and moment of inertia) is given by

$$\boldsymbol{M} = \begin{bmatrix} \tilde{M} - X_{\dot{u}} & 0 & 0 & 0 \\ 0 & \tilde{M} - Y_{\dot{v}} & -\tilde{M}\tilde{z}_g - Y_{\dot{p}} & \tilde{M}\tilde{x}_g - Y_{\dot{r}} \\ 0 & -\tilde{M}\tilde{z}_g - K_{\dot{v}} & \tilde{I}_x - K_{\dot{p}} + \tilde{M}\tilde{z}_g^2 & -K_{\dot{r}} \\ 0 & \tilde{M}\tilde{x}_g - N_{\dot{v}} & -N_{\dot{p}} & \tilde{I}_z - N_{\dot{r}} + \tilde{M}\tilde{x}_g^2 \end{bmatrix} \tag{3}$$

where it is assumed that the ship is port–starboard symmetric ($\tilde{y}_g = 0$) and $\tilde{M}$ is the ship's total mass that has its center of mass (CM) located at $\tilde{\boldsymbol{r}}_g^T = [\tilde{x}_g, 0, \tilde{z}_g]$. Furthermore, $\tilde{I}_x$ and $\tilde{I}_z$ are the moments of inertia about the CM and the other parameters are added mass and moment of inertia. The Coriolis-centripetal matrix is given by

$$\boldsymbol{C}_{RB}(\boldsymbol{\nu}) = \begin{bmatrix} 0 & -\tilde{M}r & \tilde{M}\tilde{z}_g r & -\tilde{M}\tilde{x}_g r \\ \tilde{M}r & 0 & 0 & 0 \\ -\tilde{M}\tilde{z}_g r & 0 & 0 & 0 \\ \tilde{M}\tilde{x}_g r & 0 & 0 & 0 \end{bmatrix}, \tag{4}$$

where the $(3, 4)$ and $(4, 3)$ elements of $\boldsymbol{C}_{RB}(\boldsymbol{\nu})$ are 0 due to the to the chosen representation (Fossen, 2011). The nonlinear hydrodynamic damping and Coriolis effects due to added mass are assumed to be described by

$$\boldsymbol{N}(\boldsymbol{\nu},\boldsymbol{\eta}) = \begin{bmatrix} X_{|u|u}|u|u \\ \hline Y_{|u|v}|u|v + Y_{ur}ur + Y_{v|v|}v|v| + Y_{v|r|}v|r| + Y_{r|v|}r|v| \\ +Y_{\phi|uv|}\phi|uv| + Y_{\phi|ur|}\phi|ur| + Y_{\phi uu}\phi u^2 \\ \hline K_{|u|v}|u|v + K_{ur}ur + K_{v|v|}v|v| + K_{v|r|}v|r| + K_{r|v|}r|v| \\ +K_{\phi|uv|}\phi|uv| + K_{\phi|ur|}\phi|ur| + K_{\phi uu}\phi u^2 + K_{|u|p}|u|p \\ +K_{p|p|}p|p| + K_p p + K_{\phi\phi\phi}\phi^3 \\ \hline N_{|u|v}|u|v + N_{|u|r}|u|r + N_{r|r|}r|r| + N_{r|v|}r|v| \\ +N_{\phi|uv|}\phi|uv| + N_{\phi u|r|}\phi u|r| + N_{\phi|uv|}\phi|uv| + N_p p \\ +N_{|p|p}|p|p + N_{|u|p}|u|p + N_{\phi u|u|}\phi u|u| \end{bmatrix} \tag{5}$$

where the coefficients are assumed to be constant (Perez, 2005). The hydrostatic forces and moments are, assuming that the roll angle ϕ is small, given by

$$\boldsymbol{g}^T(\boldsymbol{\eta}) = \begin{bmatrix} 0 & 0 & \rho g \nabla \overline{GM}_T s_\phi & 0 \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & \rho g \nabla \overline{GM}_T \phi & 0 \end{bmatrix} \tag{6}$$

where ρ is the density of water, g is acceleration due to gravity, ∇ is the displaced water volume and $\overline{GM}_T$ is the transversal metacentric height (Journée and Massie, 2001). The only actuators acting on the ship are assumed to be a rudder that has a force proportional to the rudder angle and a constant force acting as the forward propulsion. The actuator contribution is

$$\boldsymbol{R}(\delta) = [C_3 - C_1\delta,\ C_2\delta,\ -z_r C_2\delta,\ x_r C_2\delta]^T \tag{7}$$

Table 1. The positions and velocities used to describe the ship's position and orientation.

		Position	Velocity
Generalized		$\boldsymbol{\eta}$	$\boldsymbol{\nu}$
Linear	Surge	X	u
	Sway	Y	v
Angular	Roll	ϕ	p
	Yaw	ψ	r

where the coefficients $C_i \geq 0, i = 1, 2, 3$ and $\boldsymbol{r}_r^T = [x_r, 0, z_r]$ is the position of the rudder in the body-fixed frame.

2.1 Sensors

The motion of the ship is assumed to be measured by an IMU, and in the next section it is shown that the key to the model developed in this paper is to eliminate the unknown signal $\dot{v}$ using the tangential acceleration measurement. Assuming that the roll angle ϕ is small, the (tangential) acceleration sensed by the IMU is

$$a_s = z_s\ddot{\phi} + g\phi - a_y, \tag{8}$$

where $-z_s = -\bar{z}_s - z_f$ is the distance from the center of rotation (CR) to the origin of the IMU coordinate system, see Fig. 1. The tangential acceleration has three contributions, the first term from the angular acceleration, the second term due to gravity and the third term due to acceleration of the CR in the xy–plane in the Earth-fixed frame. Note that the identity $p = \dot{\phi}$ only holds due to the model assumptions and that it is not valid in a general model. The IMU measurements are assumed to be

$$\begin{aligned} y_{1,t} &= p_t \quad + b_{1,t} + e_{1,t} = \dot{\phi}_t + b_{1,t} + e_{1,t} && \text{(9a)} \\ y_{2,t} &= a_{s,t} \, + b_{2,t} + e_{2,t} && \text{(9b)} \\ y_{3,t} &= -r_t + b_{3,t} + e_{3,t} && \text{(9c)} \end{aligned}$$

where $p_t = \dot{\phi}_t$ is the sampled system's angular velocity about the roll axis, $a_{s,t}$ is the tangential acceleration after sampling, r_t is the sampled system's angular velocity about the yaw axis, and $b_{i,t}$ and $e_{i,t}, i = 1, 2, 3$, are sensor biases and measurement noises, respectively.

2.2 A Limited Sensor Approach – Indirect Model

Assuming that the surge velocity u is constant and equal to U, and by introducing the states $x^T = [\phi, \psi, v, p, r]$, the surge–sway–roll–yaw model (1) can be written in the nonlinear state-space form

$$\tilde{\boldsymbol{M}}\dot{x} = F(x, \delta) + \boldsymbol{\tau} \tag{10}$$

where $\tilde{\boldsymbol{M}}$ is the inertia matrix and $F(x, \delta)$ is the nonlinear state transition function. Furthermore, it is assumed that the ship is fore-aft symmetric ($\tilde{x}_g = 0$), that the total mass $\tilde{M}$ can be split into a nominal mass M and a

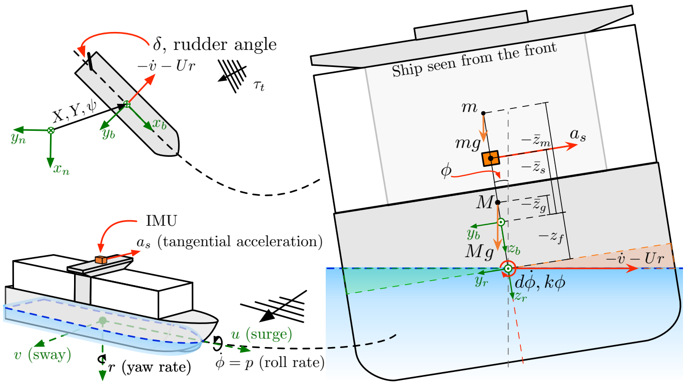

Fig. 1. A sketch of a ship performing a turning maneuver which results in an unknown acceleration a_y is acting on the CR. The shift in displaced water (green/red area) forces the ship back towards its equilibrium.

load mass m with centers of gravity given by $[0, 0, z_g]^T$ and $[0, 0, z_m]^T$, respectively, and the inertia of the load mass is neglected. Linearization of the nonlinear model (10) about $\bar{x}=0$ and $\bar{\delta}=0$ gives

$$\tilde{\boldsymbol{M}}\dot{x} = \left.\frac{\partial F}{\partial x}\right|_{\bar{x},\bar{\delta}=0} x + \left.\frac{\partial F}{\partial \delta}\right|_{\bar{x},\bar{\delta}=0} \delta + \boldsymbol{\tau} \tag{11}$$

where

$$\tilde{\boldsymbol{M}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & M+m-Y_{\dot{v}} & -Mz_g-mz_m-Y_{\dot{p}} & -Y_{\dot{r}} \\ 0 & 0 & -Mz_g-mz_m-K_{\dot{v}} & I_x-K_{\dot{p}}+Mz_g^2+mz_m^2 & -K_{\dot{r}} \\ 0 & 0 & -N_{\dot{v}} & -N_{\dot{p}} & I_z-N_{\dot{r}} \end{bmatrix}, \tag{12}$$

$$\left.\frac{\partial F}{\partial x}\right|_{\bar{x},\bar{\delta}=0} = \left[\begin{array}{cc|ccc} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \hline Y_{\phi uu}U^2 & 0 & Y_{|u|v}|U| & 0 & (Y_{ur}-M-m)U \\ K_{\phi uu}U^2-\rho g\nabla\overline{GM}_T & 0 & K_{|u|v}|U| & K_p+K_{|u|p}|U| & (K_{ur}+Mz_g+mz_m)U \\ N_{\phi u|u|}U|U| & 0 & N_{|u|v}|U| & N_p+N_{|u|p}|U| & N_{|u|r}|U| \end{array}\right] \tag{13}$$

and

$$\left.\frac{\partial F}{\partial \delta}\right|_{\bar{x},\bar{\delta}=0} = [0 \;\, 0 \;\, C_2 \;\, -z_rC_2 \;\, x_rC_2]^T \tag{14}$$

The fourth row of (11) is

$$\begin{aligned} &A_1\dot{p} - (K_{\dot{v}} + Mz_g + mz_m)\dot{v} - K_{\dot{r}}\dot{r} \\ &\quad = (K_{\phi uu}U^2 - \rho g\nabla\overline{GM}_T)\phi + (K_p + K_{|u|p}|U|)p \\ &\quad\quad + K_{|u|v}|U|v + (K_{ur} + Mz_g + mz_m)Ur - z_rC_2\delta + \tau \end{aligned} \tag{15}$$

which is a model of the roll dynamics in component form and

$$A_1 = A_x + Mz_g^2 + mz_m^2 = I_x - K_{\dot{p}} + Mz_g^2 + mz_m^2 \tag{16}$$

To get an identifiable model structure, the parameters

$$k = -K_{\phi uu}U^2 + \rho g\nabla\overline{GM}_T - Mgz_g - mgz_m, \tag{17a}$$
$$d = -K_p - K_{|u|p}|U| \tag{17b}$$

and

$$K_\delta = -z_rC_2, \tag{17c}$$

are introduced together with the parameter A_x. Furthermore, $K_{\dot{r}}\dot{r}$ and $K_{|u|v}|U|v$ are neglected since the influences from these terms are assumed to be small. This gives

$$\begin{aligned} A_1\ddot{\phi} = &- (k + Mgz_g + mgz_m)\phi - d\dot{\phi} \\ &+ (K_{\dot{v}} + Mz_g + mz_m)\dot{v} \\ &+ (K_{ur} + Mz_g + mz_m)Ur + K_\delta\delta + \tau \end{aligned} \tag{18}$$

Here, the parameter k can be interpreted as representing the physical restoring properties of the ship, for instance, being dependent on the hull shape. The other two terms Mgz_g and mgz_m are in this model representing the influence by the mass and its location on the restoring properties and a big change in loading condition is assumed to be captured by mgz_m. Note that the parameters k and d are dependent on the speed and given a speed U, assumed to be fixed and independent of the loading condition.

The model (18) can be seen as a mass-spring-damper model with three inputs. With this point of view, the largest issue is the unknown signal $\dot{v}$. Since neither $\dot{v}$ nor a model of it is known, an alternative model can be formed by eliminating $\dot{v}$. The key to this elimination is the measured tangential acceleration a_s defined in (8) and its relation to the signal $\dot{v}$ (Linder, 2014). The sway acceleration $\dot{v}$ is related to the tangential acceleration a_s through the third term in (8). This third term, i.e. the acceleration a_y of the ship in the Earth-fixed xy–plane, has two parts. The contributions emanate from the sway motion and the angular velocity about the yaw axis. The total acceleration is given by

$$a_y = \dot{v} + Ur \tag{19}$$

and combing (8) with (19) gives

$$a_s = z_s\ddot{\phi} + g\phi - a_y = z_s\ddot{\phi} + g\phi - \dot{v} - Ur \tag{20}$$

Solving (20) for $\dot{v}$ and substituting it into (18) give

$$\begin{aligned} A_2\ddot{\phi} = &- (k - K_{\dot{v}}g)\phi - d\dot{\phi} \\ &- (K_{\dot{v}} + Mz_g + mz_m)a_s + (K_{ur} - K_{\dot{v}})Ur \\ &+ K_\delta\delta + \tau \end{aligned} \tag{21}$$

where

$$A_2 = A_x + Mz_g(z_g - z_s) + mz_m(z_m - z_s) - K_{\dot{v}}z_s \tag{22}$$

Further simplifications can be obtained since the surge velocity U is assumed to be constant and by introducing the lumped parameter $K_r = (K_{ur} - K_{\dot{v}})U$, giving the model

$$\begin{aligned} A_2\ddot{\phi} = &- (k - K_{\dot{v}}g)\phi - d\dot{\phi} \\ &- (K_{\dot{v}} + Mz_g + mz_m)a_s + K_rr + K_\delta\delta + \tau \end{aligned} \tag{23}$$

In most cases, the true center of rotation is not known due to the complex interaction with the water (Balcer, 2004). Instead, a known body-fixed coordinate system can be introduced and the CR z_f can be estimated relative to this body-fixed coordinate system by introducing

$$z_g = \bar{z}_g + z_f, \quad z_m = \bar{z}_m + z_f \quad \text{and} \quad z_s = \bar{z}_s + z_f \tag{24}$$

where it is assumed that the x_b–axis of the body-fixed frame is parallel to the rotation axis but shifted in the z_b–direction. Finally, the model (23) with the output $y = \dot{\phi}$ can be written on the transfer function form

$$y = G(\mathsf{p})(a_s + F_rr + F_\delta\delta + \tau) \tag{25}$$

where

$$\begin{aligned} &G(\mathsf{p}) = \frac{\beta_1\mathsf{p}}{\mathsf{p}^2 + \alpha_1\mathsf{p} + \alpha_2}, \quad F_\delta = \frac{\gamma_1}{\beta_1}, \quad F_r = \frac{\kappa_1}{\beta_1}, \\ &\alpha_1 = \frac{d}{A_2}, \;\; \alpha_2 = \frac{k - K_{\dot{v}}g}{A_2}, \;\; \beta_1 = -\frac{K_{\dot{v}} + Mz_g + mz_m}{A_2}, \\ &\gamma_1 = \frac{K_\delta}{A_2} \quad \text{and} \quad \kappa_1 = \frac{K_r}{A_2} \end{aligned} \tag{26}$$

where p is the differentiation operator, A_2 is defined in (22) and z_g, z_m and z_s are defined in (24).

3. ANALYSIS OF MODEL PROPERTIES

From a system identification perspective it is important to understand the model's properties. Firstly, an identifiability analysis is performed to investigate if the parameters in the chosen model structure can be determined uniquely. Secondly, to make an appropriate choice of the estimation method, the signals' dependency on the process disturbance τ is analyzed.

3.1 Identifiability Issues – Using Multiple Datasets

The question whether the parameters in the model can be uniquely estimated has two aspects, the informativity of the data and the parameterization of the model (Ljung and Glad, 1994; Bazanella et al., 2010).

Firstly, let us assume that the data is informative enough. The goal is to estimate the change in mass m and change in CM $\bar{z}_m$ but there are additional nuisance parameters that have to be estimated along with the desired parameters. It would thus be preferable if the parameters

$$\vartheta_p = \big[M, \; \bar{z}_g, \; k, \; \bar{A}_x, \; d, \; K_r, \; K_{\dot{v}}, \; K_\delta, \; z_f, \; m, \; \bar{z}_m\big]^T \tag{27}$$

of the model (26) could be estimated using a single dataset, i.e. all parameters except for g and $\bar{z}_s$ that are assumed to be known. However, the model (26) is not identifiable with respect to the parameters in (27) (Linder, 2014). To gain identifiability, either the model structure must be changed or more information must be introduced. Here, more information is introduced through *a priori* knowledge of parameters and by using multiple datasets.

In a first calibration phase, the two datasets

$$Z_n = (y_t, u_t, \delta_t)_{t=1+t_n}^{N_n+t_n} \text{ and } Z_c = (y_t, u_t, \delta_t)_{t=1+t_c}^{N_c+t_c} \tag{28}$$

called the nominal and calibration datasets, respectively, are collected. The nominal dataset has a known mass M and CM $\bar{z}_g$. The calibration dataset has a different known mass and CM expressed in terms of the load mass $m = m_c$ and its CM $z_m = z_c$ relative to the nominal case. These masses and center of masses can, for instance, be estimated using pressure sensors in the harbor, or any other place where the dynamic pressure is not influencing the readings, together with a ballast system, assuming that the ballast system's tanks have a known weight and position.

The parameters for the two cases are

$$\bar{\vartheta}_p^n = \bar{\vartheta}_p|_{m=\bar{z}_m=0} \quad \text{and} \quad \bar{\vartheta}_p^c = \bar{\vartheta}_p|_{m=m_c, \bar{z}_m=\bar{z}_c} \tag{29}$$

The vector of parameters (27) is extended to

$$\tilde{\vartheta}_p = [\overbrace{M,\ \bar{z}_g,\ m_c,\ \bar{z}_c}^{\tilde{\vartheta}_{p,1}\ \text{(known)}},\ \overbrace{k,\ A_x,\ d,\ K_r,\ K_{\dot{v}},\ K_\delta,\ z_f,\ m,\ \bar{z}_m}^{\tilde{\vartheta}_{p,2}\ \text{(unknown)}}]^T, \tag{30}$$

and the datasets (28) together with the known parameters $\tilde{\vartheta}_{p,1}$ are then used simultaneously with the loaded dataset

$$Z_l = (y_t, u_t, \delta_t)_{t=1+t_l}^{N_l+t_l}, \tag{31}$$

collected during *normal operational conditions*, to estimate the unknown parameters $\tilde{\vartheta}_{p,2}$.

Secondly, now when the chosen model structure using the three datasets is identifiable, the data have to be informative enough. The key to getting informative datasets is excitation by the rudder. Note that the rudder is the only true input to the system except for the disturbances. This implies that the motion induced by the rudder and observed in a_s and r is uniquely determined through the dynamics. Here, we assume that the complexity of the system is sufficient and thus, that a_s and r will supply more information than δ does by itself. This means that the inputs to (23) are informative if the roll dynamics is sufficiently excited by the rudder (Bazanella et al., 2010; Linder, 2014).

3.2 Identification Issues – Correlation with τ

The ship's motion is assumed to be affected by two inputs, the rudder and the disturbances acting on the ship. The model (18) can be expressed in terms of (23) and the subsystems of the linearized system (11) which results in the structure seen in Fig. 2. Analyzing this model reveals the dependencies between the measured signals and shows that even though the proposed method avoids building a model of the entire ship, it introduces some new challenges.

As mentioned in the previous section, the rudder is the only true (actuator) input acting on the system, which implies that both a_s and r are dependent on δ, i.e. all measurements (9) are correlated with δ. Due to coupling in the system, the measurements (9) are also correlated with the process disturbance τ. This means that there are similarities with identification in closed loop and it is important to understand these dependencies to make the correct choices in the identification procedure. On top of the correlation, there is also a direct term in the loop gain from τ to a_s which might introduce a bias for certain closed-loop identification methods if this is not considered (Linder, 2014).

Finally, in addition to the process disturbance τ, also the measurement noises and sensor biases in (9) have to be considered. Hence, the inputs will be noisy and the identification problem will be of errors-in-variables type.

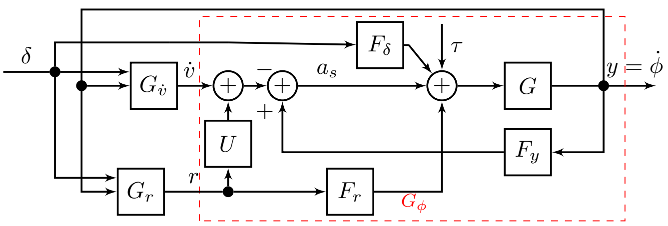

Fig. 2. The model (11) expressed in terms of its subsystems. Note that due to coupling in the system, all signals depend on each other which means that all signals are dependent on τ.

4. ESTIMATION OF A SHIP'S ROLL DYNAMICS

There are a lot of details to consider when estimating the unknown parameters in (23). Firstly, a discrete-time model is introduced. Secondly, the properties of the signals are considered to avoid pitfalls when choosing and tuning the estimation approach.

4.1 Discretization Using Physical Parameters

The transfer function (26) is discretized using the bilinear transform $\mathsf{p} = (2\mathsf{q}-2)/(T\mathsf{q}+T)$ where T is the sample period and q is the shift operator. Applying the transform gives

$$G_d(\mathsf{q}) = \frac{\bar{\beta}_0(1-\mathsf{q}^{-2})}{1+\bar{\alpha}_1\mathsf{q}^{-1}+\bar{\alpha}_2\mathsf{q}^{-2}},\ F_{\delta,d} = \frac{\bar{\gamma}_0}{\bar{\beta}_0},\ F_{r,d} = \frac{\bar{\kappa}_0}{\bar{\beta}_0} \tag{32}$$

with

$$\begin{aligned} &\bar{\alpha}_1 = \frac{2(k-K_{\dot{v}}g)T^2 - 8A_2}{\mathcal{A}_2(\bar{\vartheta}_p)},\ \bar{\alpha}_2 = \frac{-2dT + (k-K_{\dot{v}}g)T^2 + 4A_2}{\mathcal{A}_2(\bar{\vartheta}_p)} \\ &\bar{\beta}_0 = -\frac{2T(K_{\dot{v}} + Mz_g + mz_m)}{\mathcal{A}_2(\bar{\vartheta}_p)},\ \bar{\kappa}_0 = \frac{2TK_r}{\mathcal{A}_2(\bar{\vartheta}_p)},\ \bar{\gamma}_0 = \frac{2TK_\delta}{\mathcal{A}_2(\bar{\vartheta}_p)}, \end{aligned} \tag{33}$$

and $\mathcal{A}_2(\bar{\vartheta}_p) = 2dT + (k-K_{\dot{v}}g)T^2 + 4A_2$. By introducing

$$\mu_t^T = [a_{s,t} - a_{s,t-2},\ r_t - r_{t-2},\ \delta_t - \delta_{t-2}], \tag{34}$$

the discrete-time model can be written as

$$y_t = \varphi_t^T g_\vartheta(\bar{\vartheta}_p) + \tilde{\tau}_t \tag{35}$$

where

$$g_\vartheta(\bar{\vartheta}_p) = \left[\bar{\alpha}_1(\bar{\vartheta}_p),\ \bar{\alpha}_2(\bar{\vartheta}_p),\ \bar{\beta}_0(\bar{\vartheta}_p),\ \bar{\kappa}_0(\bar{\vartheta}_p),\ \bar{\gamma}_0(\bar{\vartheta}_p)\right]^T \tag{36}$$

and

$$\varphi_t^T = [-y_{t-1}, -y_{t-2}, \mu_{1,t}, \mu_{2,t}, \mu_{3,t}], \tag{37}$$

In Linder et al. (2014a) it was possible to solve several linear problems sequentially and obey the original physical parameterization by linear constraints to overcome the identifiability issues. In this paper, this is unfortunately not possible due to extra complexity. Instead, the model is extended and all datasets defined in Section 3.1 are used simultaneously, i.e. the models

$$y_t^i = (\varphi_t^i)^T g_\vartheta(\bar{\vartheta}_p^i) + \tilde{\tau}_t^i, \quad i = n, c, l \tag{38}$$

and the parameters $\vartheta_{p,2}$ are estimated simultaneously. Here, the subscripts $i = n, c, l$ correspond to the nominal, calibration and loaded datasets, respectively. Note that additional datasets can be added if the parametric variation is known, for instance, multiple shorter datasets of the same loading condition can be used.

4.2 The Iterative Instrumental Variable Method

An instrumental variable method uses instruments to extract the interesting information from the data. In principle, the information is estimated by requiring that

$$\frac{1}{N_i}\sum_{t=1}^{N_i} \zeta_t^i (y_t^i - (\varphi_t^i)^T g_\vartheta(\bar{\vartheta}_p^i)) = 0, \quad i = n, c, l, \tag{39}$$

i.e. that the sample covariance between ζ_t^i and the residual for dataset i, $i = n, c, l$, should be zero. There are two terms contributing to the output of (38), one containing information about the interesting input-output relation and the second containing a contribution from disturbances. A good instrument should in this case be correlated with the motion induced by the rudder but be uncorrelated with the process disturbance, the sensor biases and the measurement noises. This idea is generalized in the extended IV method, where the parameters are found by computing

$$\hat{\vartheta}_{p,2} = \underset{\vartheta_{p,2}}{\operatorname{argmin}} \sum_{i=n,c,l} \left\| Y^i - \Phi^i g_\vartheta(\bar{\vartheta}_p^i) \right\|_Q^2 \tag{40}$$

where $\|x\|_Q^2 = x^T Q x$, $Q \succeq 0$ is a weighting matrix,

$$Y^i = \frac{1}{N_i}\left[\zeta_1^i \dots \zeta_{N_i}^i\right]\begin{bmatrix} \bar{y}_1^i \\ \vdots \\ \bar{y}_{N_i}^i \end{bmatrix},\ \Phi^i = \frac{1}{N_i}\left[\zeta_1^i \dots \zeta_{N_i}^i\right]\begin{bmatrix} (\bar{\varphi}_1^i)^T \\ \vdots \\ (\bar{\varphi}_{N_i}^i)^T \end{bmatrix}, \tag{41}$$

$\bar{y}_t = L^i(\mathsf{q})y_t$, $\bar{\varphi}_t^T = L^i(\mathsf{q})\varphi_t^T$ and $L^i(\mathsf{q})$ is a stable prefilter.

See, for instance, Söderström and Stoica (1989) or Ljung (1999) for more details on the extended IV method.

An iterative method based on Gilson et al. (2006) is used in this paper. As above, z^i indicates that z belongs to the dataset i. For brevity, $i = \{n, c, l\}$ is not explicitly written at all places. In the j^{th} iteration, the parameters are estimated using the instruments and prefilters obtained from the $j-1^{\text{th}}$ iteration. ARMA noise models $\bar{H}^i_d(\mathsf{q}, \hat{\eta}^{i,j})$ are estimated from the residuals

$$\varepsilon^{i,j}_t = y^i_t - (\varphi^i_t)^T g_\vartheta(\hat{\bar{\vartheta}}^{i,j}_p) \tag{42}$$

and the prefilters are calculated as

$$L^{i,j}(\mathsf{q}, \bar{\vartheta}^{i,j}_p) = \bar{H}^i_d(\mathsf{q}, \hat{\eta}^{i,j})^{-1} \tag{43}$$

The transfer functions (46) and (47) are then simulated with δ^i_t as input, which gives the signals

$$\hat{y}^{i,j}_t = \hat{G}^{i,j}_{\delta y,d}(\mathsf{q})\delta^i_t,\ \hat{a}^{i,j}_{s,t} = \hat{G}^{i,j}_{\delta a_s,d}(\mathsf{q})\delta^i_t \text{ and } \hat{r}^{i,0}_t = \hat{G}^{i,0}_{\delta r,d}(\mathsf{q})\delta^i_t \tag{44}$$

Finally, the instrument vectors are created according to

$$\begin{aligned}\zeta^{i,j}_t = L^{i,j}(\mathsf{q}, \bar{\vartheta}^{i,j}_p)\big[&\hat{y}^{i,j}_t \cdots \hat{y}^{i,j}_{t-n_y+1},\ \hat{\mu}^{i,j}_{1,t} \cdots \hat{\mu}^{i,j}_{1,t-n_{a_s}+1},\\ &\hat{\mu}^{i,j}_{2,t} \cdots \hat{\mu}^{i,j}_{2,t-n_r+1},\ \mu^{i,0}_{3,t} \cdots \mu^{i,0}_{3,t-n_\delta+1}\big]^T\end{aligned} \tag{45}$$

where the constants $n_i, i{=}y, a_s, r, \delta$, are the number of time lags (including the non-delay signal) included in $\zeta^{i,j}_t$. For instance, $n_\delta{=}0$ means that $\mu_{3,t}$ is not included in $\zeta^{i,j}_t$.

In the initializing (0^{th}) iteration, the transfer functions in (44) are estimated blackbox models and in the refining iterations, the first two transfer functions of (44) are given by

$$\hat{G}^{i,j}_{\delta y,d} = \frac{G^{i,j}_d}{1 - G^{i,j}_d F^{i,j}_{y,d}} \left[(F^{i,j}_{r,d} - U)\hat{G}^{i,0}_{\delta r,d} + F^{i,j}_{\delta,d}\right], \tag{46}$$

$$\hat{G}^{i,j}_{\delta a_s,d} = \frac{G^{i,j}_d}{1 - G^{i,j}_d F^{i,j}_{y,d}} \left[F^{i,j}_{\delta,d} + F^{i,j}_{r,d}\hat{G}^{i,0}_{\delta r,d}\right] + \frac{U\hat{G}^{i,0}_{\delta r,d}}{1 - F^{i,j}_{y,d}G^{i,j}_d}, \tag{47}$$

while $\hat{G}^{i,0}_{\delta r,d}$ are given by the blackbox models from the 0^{th} iteration. Here, the dependencies on q and $\hat{\bar{\vartheta}}^{i,j}_p$ have been dropped for brevity. The method is summarized in Algorithm 1 and more details can be found in Linder (2014).

4.3 Summary of Approach

The approach presented in this paper is based on two stages, firstly, a calibration phase and secondly, an operational phase. In the first calibration phase, the nominal and calibration datasets (28) are collected. The mass and CM relative the user-chosen body-fixed coordinate system have to be *known* while collecting both datasets, i.e. $\tilde{\vartheta}_{p,1}$. Note that the masses also have to be different, see Section 3.1. In the second phase, the loaded dataset (31) is collected *during normal operation* and Algorithm 1 is used together with all three datasets to estimate the unknown parameters $\tilde{\vartheta}_{p,2}$ *online*.

In addition to the datasets (28), only the position $\bar{z}_s$ of the IMU in relation to the user-chosen body-fixed coordinate system and the acceleration of gravity g have to be known.

Finally, we emphasize that the rudder angle δ not only is vital to reduce the effects of the disturbances but also to excite the ship sufficiently to get informative data.

Table 2. The parameters used in the simulation.

Description	Parameters and Values
General	$U = 1$, $M = 22.04$, $\bar{z}_g = 0$, $m_c = 0.2$, $\bar{z}_c = -0.172$, $m = 0.4$, $\bar{z}_m = -0.182$, $z_f = -0.028$, $\bar{z}_s = -0.2$, $g = 9.82$
Sway Dynamics	$Y_{\dot{v}} = -100$, $Y_{\dot{p}} = K_{\dot{v}}$, $Y_{\dot{r}} = -1$, $Y_{\phi uu} = 0$, $Y_{\lvert u \rvert v} = 35$, $Y_{ur} = 0$
Roll Dynamics	$K_{\dot{v}} = 0.1$, $A_x = 0.1385$, $K_{\dot{r}} = 0$, $k = 10.38$, $d = 0.2067$, $K_{ur} = -0.4$ ($\Leftrightarrow K_r = -0.5$)
Yaw Dynamics	$N_{\dot{v}} = Y_{\dot{r}}$, $N_{\dot{p}} = K_{\dot{r}}$, $A_z = 3A_x$, $N_{\lvert u \rvert r} = 1.4$, $N_{\phi u \lvert u \rvert} = N_p = N_{\lvert u \rvert p} = 0$

Algorithm 1 The iterative joint IV method

(A) Initialize:
 (a) Set initial value of $\bar{\vartheta}^{i,0}_p$ and set prefilters $L^{i,0}(\mathsf{q}, \bar{\vartheta}^{i,0}_p) = 1$
 (b) Create initial instruments
  (i) Estimate blackbox models of the transfer functions $\hat{G}^{i,0}_{\delta y,d}(\mathsf{q})$, $\hat{G}^{i,0}_{\delta a_s,d}(\mathsf{q})$ and $\hat{G}^{i,0}_{\delta r,d}(\mathsf{q})$ in (44)
  (ii) Simulate $\hat{y}^{i,0}_t$, $\hat{a}^{i,0}_{s,t}$ and $\hat{r}^{i,0}_t$ according to (44)
  (iii) Create the instruments $\zeta^{i,0}_t$ according to (45)
 (c) Set $j = 1$
(B) Estimate parameters:
 (a) Compute $\hat{\bar{\vartheta}}^j_{p,2}$ using (40) with $L^{i,j-1}(\mathsf{q}, \hat{\bar{\vartheta}}^{i,j-1}_p)$ and $\zeta^{i,j-1}_t$
 (b) Estimate $\eta^{i,j}$ of the models $\bar{H}^i_d(\mathsf{q}, \eta^{i,j})$ from (42)
 (c) Create the prefilters $L^{i,j}(\mathsf{q}, \hat{\bar{\vartheta}}^{i,j}_p)$ according to (43)
 (d) Simulate the signals $\hat{y}^{i,j}_t$ and $\hat{a}^{i,j}_{s,t}$ according to (44)
 (e) Create the instruments $\zeta^{i,j}_t$ according to (45)
(C) Terminate: Increase j and go to Step B while $\|\hat{\bar{\vartheta}}^j_{p,2} - \hat{\bar{\vartheta}}^{j-1}_{p,2}\|_2 + \|\hat{\eta}^{n,j} - \hat{\eta}^{n,j-1}\|_2 + \|\hat{\eta}^{c,j} - \hat{\eta}^{c,j-1}\|_2 + \|\hat{\eta}^{l,j} - \hat{\eta}^{l,j-1}\|_2$ is above a threshold or as long as a certain number of iterations is not reached. Otherwise, return $\hat{\bar{\vartheta}}_{p,2} = \hat{\bar{\vartheta}}^j_{p,2}$ and terminate.

Note that $i = \{n, c, l\}$ is not explicitly written at all places and if the superscript i is used, it should be understood as for $i = \{n, c, l\}$.

5. SENSITIVITY TO INITIAL CONDITIONS

A challenging aspect is that the proposed estimator is nonconvex in the parameters and in this section, a brief simulation study will be presented to evaluate the estimated parameters' sensitivity to the initial conditions. To simplify the analysis, all disturbances were set equal to zero. For a discussion on the estimator's disturbance rejection, see Linder et al. (2014b), Linder et al. (2014a) and Linder et al. (2015). The system was assumed to be given by (11) and the parameters, given in Table 2, were chosen to resemble the parameters of the scale model in Linder et al. (2015). Note that (11) has 29 parameters while (38) has 15 parameters where only the six parameters listed in Section 4.3, i.e. M, $\bar{z}_g$, m_c, $\bar{z}_c$, g and $\bar{z}_s$, are assumed to be known.

The datasets described in Section 3.1 were synthesized by simulating (11) with the masses and CMs given by Table 2. The length of the datasets were chosen to be 60 seconds and they were sampled at 50 Hz. Fig. 3 shows the nominal dataset as an example.

A Monte Carlo (MC) simulation with 10 000 runs was performed to test the solution's sensitivity to the initial condition. In each run, the initial condition was sampled uniformly between the upper and lower bound given in Table 3. In all iterations, the instruments (45) were created using the constants $n_y{=}n_{a_s}{=}n_r{=}16$ and $n_\delta{=}2$.

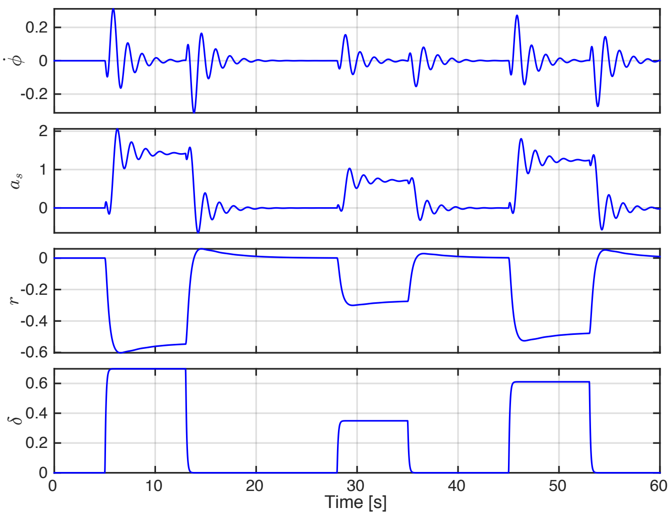

Fig. 3. Nominal data used in the simulation study.

Table 3. Results for Monte Carlo simulation. T: true, U/L: upper/lower bounds, M: mean and S: standard deviation.

	A_x	d	k	K_δ	K_r	$K_{\dot{v}}$	z_f	m	$\bar{z}_m$
T	0.1385	0.2067	10.380	−0.405	−0.5	0.1	−0.028	0.4	−0.182
U	100	100	100	0	0	10	1	1	0
L	0	0	0	1	10	0	−1	0	0.5
M	0.1261	0.2194	13.297	−0.4302	−0.5318	0.3396	−0.0403	0.4007	−0.1821
S	0.0296	0.0233	5.3579	0.0454	0.0584	0.4367	0.0225	0.0018	0.0004

The means and standard deviations of the solutions were calculated and can be seen on the two last rows of Table 3. Five of the runs resulted in unrealistic solutions (that could be easily identified) and were not used in the calculations. Note that the approach was surprisingly robust for finding the mass m and the CM $\bar{z}_m$ despite the large variations in the other parameters. This was fortunate since these were the desired parameters. The other parameters were in some sense only estimated out of necessity for the estimator and should be treated with care due to the variations.

Fig. 4 shows the cost function of the estimation problem as a function of the mass m and the CM $\bar{z}_m$ for one solution of the MC simulation (the other parameters were fixed to their estimated values). Note that the gradient is largest orthogonal to the dotted line and that there is a unique minimum (red cross). However, the cost function is quite flat close to the minimum and due to the gradient of the cost function, it is easier to detect that there has been a change in the loading condition than to separate the effect between the mass and the CM.

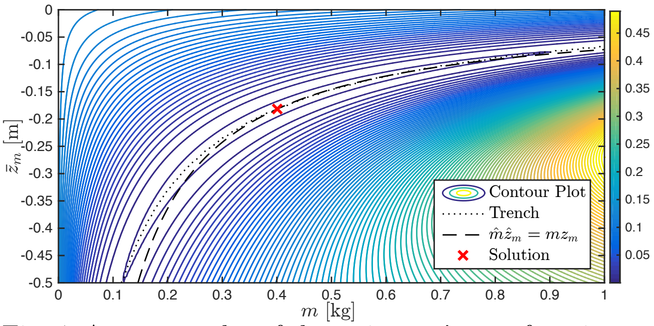

Fig. 4. A contour plot of the estimator's cost function as a function of m and $\bar{z}_m$ with the other parameters fixed.

6. CONCLUSIONS AND FUTURE WORK

In this paper, an extension of a previously proposed online estimation approach for mass and CM estimation has been presented. The method relies on measurement of rudder angle and motion measurement from an IMU. The model of the roll dynamics, parameterized with physical parameters, was derived from a well-established maneuvering model. Due to identifiability issues, *a priori* information was introduced in the form of known parameters and calibration datasets. To mitigate environmental disturbances, sensor biases and measurement errors, an iterative closed-loop instrumental variable approach, using all datasets simultaneously, was proposed to estimate the parameters.

A limited MC simulation was performed to investigate the estimator's sensitivity to initial conditions. It was shown that the estimator is surprisingly robust in the desired parameters corresponding to mass and center of mass but that the other parameters had large variations. Although it has not been discussed in this paper, the proposed method has also been validated on data from scale model with good results, see Linder (2014) or Linder et al. (2015).

Future work includes an investigation of the underlying cause of the sensitivity to the initial condition and to perform a more exhaustive simulation study including a full nonlinear model and realistic process disturbances.

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