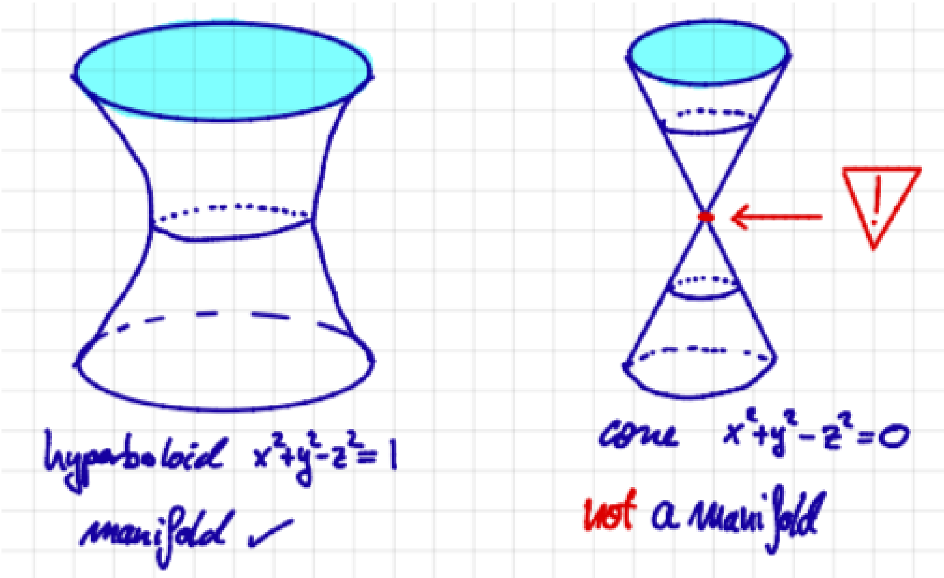
Norwegian University of Science and Technology Deptartment of Mathematical Sciences	TMA4190 Introduction to Topology Spring 2018 **Exercise set 1**

$\boxed{1}$ Show that every k-dimensional vector subspace V of $\mathbb{R}^N$ is a manifold diffeomorphic to $\mathbb{R}^k$ and that any linear map $V \to \mathbb{R}^m$ is smooth.
(Remember that choosing a basis for V corresponds to choosing a linear isomorphism $\phi\colon \mathbb{R}^k \to V$. Expressing a vector in V in terms of this basis, means to attach coordinates to this vector. Since ϕ is linear, we refer to the corresponding coordinates as linear coordinates.)

$\boxed{2}$ **a)** Prove that the subspace of $\mathbb{R}^3$, defined by $x^2 + y^2 - z^2 = a$, is a manifold if $a > 0$.

b) Explain why $x^2 + y^2 - z^2 = 0$ does not define a manifold.



$\boxed{3}$ The torus $T(a, b)$ is the set of points in $\mathbb{R}^3$ at distance b from the circle of radius a in the xy-plane, where $0 < b < a$. Prove that each $T(a, b)$ is diffeomorphic to $S^1 \times S^1 \subset \mathbb{R}^4$. What happens when $b = a$?

$\boxed{4}$ Let $N = (0, \ldots, 0, 1) \in S^k$ be the “north pole" on the k-dimensional sphere. The stereographic projection ϕ_N^{-1} from $S^k \setminus \{N\}$ onto $\mathbb{R}^k$ is the map which sends a point

p to the point at which the line through N and p intersects the subspace in $\mathbb{R}^{k+1}$ defined by $x_{k+1} = 0$. (See the picture for $k = 2$.)

a) Show that ϕ_N^{-1} is given by the formula

$$(x_1, \ldots, x_{k+1}) \mapsto \frac{1}{1 - x_{k+1}}(x_1, \ldots, x_k).$$

b) Find a formula for the inverse ϕ_N of ϕ_N^{-1}, and check that both maps are smooth.

c) Let $S = (0, \ldots, 0, -1) \in S^k$ be the "south pole". Describe the parametrization using the stereographic prjoction starting in S instead of N, and conclude that S^k is a k-dimensional manifold.