Notes Numerical Maths

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1 Singular Value Decomposition

Definition 1.1. *Let $A \in \mathbb{C}^{m\times n}$, there exist two unitary matrices $U \in C^{m\times m}$ and $V \in \mathbb{C}^{n\times n}$ such that*

$$U^H AV = \Sigma = diag\,(\sigma_1, \sigma_2, \ldots, \sigma_p) \in \mathbb{R}m \times n \quad with \quad p = min\,(m, n)$$

and $\sigma_1 \geq \ldots \geq \sigma_p \geq 0$. The formula called SVD of A and the numbers σ_i or $\sigma_i\,(A)$ are called the singular values of A.

Definition 1.2. *Suppose that $A \in \mathbb{C}^{m\times n}$ has rank equal to r and that it admits a SVD of the type $U^H AV = \Sigma$. The matrix $A^\dagger = V\Sigma^\dagger H^H$ is the* ***Moore-penrose pseudo inverse matrix*** *being*

$$\Sigma^\dagger = diag\left(\frac{1}{\sigma_1}, \ldots, \frac{1}{\sigma_r}, 0, \ldots, 0\right)$$

The matrix $A^\dagger$ is also called the ***generalized inverse*** *of A. Indeed, if $rank\,(A) = n < m$, then $A^\dagger = \left(A^T A\right)^{-1} A^T$, while if $n = m = rank\,(A)$, $A^\dagger = A^{-1}$.*

Definition 1.3. *A matrix $A \in \mathbb{C}^{n\times n}$ is called* ***hermition*** *of* ***self-adjoint*** *if $A^T = \overline{A}$, that is if $A^H = A$, while it is called* ***unitary*** *if $A^H A = AA^H = I$. Finally, if $AA^H = A^H A$, A is called normal.*

A unitary matrix is one such that $A^{-1} = A^H$ and is normal.

2 References