Nomenclature

a1, a2, a3 Distances for six-mass approximation

a, b Distance from CM to front and rear axles, respectively

a₁₁...**a**₂₂ Elements of a matrix (generic)

{a_I}₁ Unit vector at marker I resolved parallel to frame 1 (GRF)
 {a_I}₁ Unit vector at marker J resolved parallel to frame 1 (GRF)

a_x Longitudinal acceleration (Wenzel model)
 a_y Lateral acceleration (Wenzel model)

b Longitudinal distance of body mass centre from front axle

c Damping coefficient

c Longitudinal distance of body mass centre from rear axle

c Specific heat capacity of brake rotor

d Wire diameter

 $\left(\frac{dB}{dE_{z}}\right)$ Variation in scaling factor with load (Harty Model)

 $\{d_{IJ}\}_1$ Position vector of marker I relative to J resolved parallel to frame 1

(GRF)

 e_1 Path error

f Natural frequency (Hz)
g Gravitational acceleration

h Brake rotor convection coefficient

h Height of body mass centre above roll axis

 $\begin{array}{lll} \textbf{i} & & \text{Square root of } -1 \\ \textbf{k} & & \text{Path curvature} \\ \textbf{k} & & \text{Radius of gyration} \end{array}$

k Stiffness

k Spring constant in hysteretic model

k Tyre spring constant

k₁, **k**₂ Front and rear ride rates, respectively

k_s Spring stiffness

k_w Stiffness of equivalent spring at the wheel centre

Length of pendulumMass of a body

m{g}₁ Weight force vector for a part resolved parallel to frame 1 (GRF)

m_t Mass of tyre

n Number of active coils

n Number of friction surfaces (pads)

p Brake pressure

q_i Set of part generalised coordinates

r Yaw rate

 $\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}$ Coupler constraint rotations

 $\{\mathbf{r}_{\mathbf{I}}\}_{\mathbf{I}}$ Position vector of marker I relative to frame i resolved parallel to

frame 1 (GRF)

 $\{\mathbf{r}_{\mathbf{I}}\}_{\mathbf{I}}$ Position vector of marker J relative to frame j resolved parallel to

frame 1 (GRF)

xviii Nomenclature

r_u	Unladen radius
$\mathbf{r_l}$	Laden radius
$\mathbf{r}_{\mathbf{w}}$	Wheel radius

s₁, s₂, s₃ Coupler constraint scale factors

 $\begin{array}{cc} t_f & & \text{Front track} \\ t_r & & \text{Rear track} \end{array}$

 $\begin{array}{ll} v_{cog} & \text{Centre of gravity (Wenzel model)} \\ v_x & \text{Longitudinal velocity (Wenzel model)} \\ v_y & \text{Lateral velocity (Wenzel model)} \end{array}$

x Generic variable for describing tanh function

 $\mathbf{x_i}, \mathbf{y_i}, \mathbf{z_i}$ Coordinates of each of the six masses in the six-mass

approximation

x_i, y_i, z_i Components of the ith eigenvector
 x(t) Function of time (generic)

x_{CM}, **y**_{CM}, **z**_{CM} Coordinates of body centre of mass

 $\{x_I\}_1$ Unit vector along x-axis of marker I resolved parallel to frame 1

(GRF)

{y_I}₁ Unit vector along y-axis of marker I resolved parallel to frame 1

(GRF)

 $\{x_I\}_I$ Unit vector along x-axis of marker J resolved parallel to frame 1

(GRF)

 $\{y_J\}_1$ Unit vector along y-axis of marker J resolved parallel to frame 1

(GRF)

y_s Asymptotic value at large slip (Magic Formula)

z Auxiliary state variablez Heave displacement variable

 $\{z_I\}_1$ Unit vector along z-axis of marker I resolved parallel to frame 1

(GRF)

 $\{z_I\}_1$ Unit vector along z-axis of marker J resolved parallel to frame 1

(GRF)

A Area

A Linear acceleration

A, B, C Intermediate terms in a cubic equation

A Scaling for solution form of a differential equation (generic)

A Step height

 A_c Convective area of brake disc $[A_{1n}]$ Euler matrix for part n

 $\{A_n\}_1$ Acceleration vector for part n resolved parallel to frame 1 (GRF)

A^p Centripetal acceleration

{A^pPO}₁ Centripetal acceleration vector P relative to Q referred to frame 1

(GRF)

 $\{A_{PO}^t\}_1$ Transverse acceleration vector P relative to Q referred to frame 1

(GRF)

{A^c_{PO}}₁ Coriolis acceleration vector P relative to Q referred to frame 1

(GRF)

{A^s_{PO}}₁ Sliding acceleration vector P relative to Q referred to frame 1 (GRF)

A_{vehicle} Acceleration of vehicle

A_X Longitudinal curvature factor

Ay Lateral acceleration
AyG Lateral acceleration gain

B Load scaling factor (Harty Model)
B Stiffness factor (Magic Formula)

[B] Transformation matrix from frame O_e to O_n

BKid Bottom kingpin marker
BM Bump movement
BT Brake torque

C Shape factor (Magic Formula)

[C] Compliance matrix

 C_{D0} Drag coefficient at zero aerodynamic yaw angle C_{D0} Drag coefficient sensitivity to aerodynamic yaw angle

C_F Front axle cornering stiffness

 \mathbf{C}_{γ} Camber coefficient

C_{L0} Coefficient of lift at zero angle of attack

 $C_{L\alpha}$ Variation in coefficient of lift with angle of attack

C_{MX} Overturning moment coefficient C_r Rolling resistance moment coefficient

C_R Rear axle cornering stiffness
C_S Tyre longitudinal stiffness
C_n Process capability

Cp Process capability
CP Centre of pressure

 $\begin{array}{lll} C_{\alpha} & & \text{Tyre lateral stiffness due to slip angle} \\ C_{\alpha f} & & \text{Front tyre lateral stiffness due to slip angle} \\ C_{\alpha r} & & \text{Rear tyre lateral stiffness due to slip angle} \\ C_{\gamma} & & \text{Tyre lateral stiffness due to camber angle} \\ \end{array}$

D Clipped camber scale constant

D Mean coil diameter

D Peak value (Magic Formula)
DZ Displacement variable (generic)

DM(I,J)Magnitude of displacement of I marker relative to J marker **DX(I,J)**Displacement in X-direction of I marker relative to J marker parallel

to GRF

DY(I,J) Displacement in Y-direction of I marker relative to J marker parallel

to GRF

DZ(I,J) Displacement in Z-direction of I marker relative to J marker parallel

to GRF

E Camber clip curvature constant
E Young's modulus of elasticity
E Curvature factor (Magic Formula)

F Aerodynamics force
F Applied force

F Force generated by hysteretic model

F Spring force

Fhyst Amplitude of hysteretic force

Fhyst Final outcome from sequence of hysteretic calculations

 $\{F_{nA}\}_1$ Applied force vector on part n resolved parallel to frame 1 (GRF)

 $\{F_{nC}\}_1$ Constraint force vector on part n resolved parallel to frame 1 (GRF)

 $\mathbf{F}_{\mathbf{FRC}}$ Lateral force reacted by front roll centre $\mathbf{F}_{\mathbf{RRC}}$ Lateral force reacted by rear roll centre

Frictional force

 $\mathbf{F}_{\mathbf{x}}$ Longitudinal tractive or braking tyre force

Fx₁ Friction moderated longitudinal load in moderate slip Fx₂ Friction moderated longitudinal load in deep slip

 $\mathbf{F}_{\mathbf{y}}$ Lateral tyre force

FY1
 Friction moderated lateral load at moderate slip angles
 FY2
 Friction moderated lateral load at deep slip angles

 F'_y Lagged (relaxed) side force $F_{y\alpha}$ Lateral load due to slip angle

 $\begin{array}{ll} F_{y\alpha'} & \text{Friction moderated side force due to slip angle} \\ F_{v\gamma} & \text{Lateral load due to camber/inclination angle} \end{array}$

 $\mathbf{F}_{\mathbf{y}\gamma'}$ Friction moderated side force due to camber/inclination angle

 $\frac{\widehat{\mathbf{F}}\mathbf{y}}{\mu \widehat{\mathbf{F}}\mathbf{z}}$ Lateral capacity fraction

 $\begin{array}{ll} F_z & \text{Normal force} \\ F_z & \text{Vertical tyre force} \\ F_z & \text{Time varying tyre load} \\ F_{z0} & \text{Static corner load} \end{array}$

 $\mathbf{F}_{\mathbf{zc}}$ Vertical tyre force due to damping $\mathbf{F}_{\mathbf{zk}}$ Vertical tyre force due to stiffness

 $\{F_A\}_1 \{F_B\}_1...$ Applied force vectors at points A, B,... resolved parallel to frame

1 (GRF)

[F_E] Elastic compliance matrix (concept suspension)

F_D Drag force

FG Fixed ground marker
G Shear modulus
GC Gravitational constant
GO Ground level offset
GRF Ground reference frame

 $\{H\}_1$ Angular momentum vector for a body

 $\begin{array}{ccc} H(\omega) & & \text{Transfer function} \\ HTC & & \text{Half track change} \\ I & & \text{Mass moment of inertia} \\ I & & \text{Second moment of area} \\ I_2 & & \text{Pitch inertia of vehicle} \end{array}$

I₁, I₂, I₃ Principal mass moments of inertia of a body

I_{wheel} Mass moment of inertia of road wheel in the rolling direction

Jz Vehicle body yaw inertia (Wenzel model)

K Drive torque controller constant

K Spring stiffness

 $\begin{array}{ll} \textbf{K} & \text{Stability factor} \\ \textbf{K} & \text{Understeer gradient} \\ \textbf{K}_{\textbf{Z}} & \text{Tyre radial stiffness} \\ \textbf{K}_{\textbf{T}} & \text{Torsional stiffness} \end{array}$

 K_{Ts} Roll stiffness due to springs K_{Tr} Roll stiffness due to anti-roll bar

L Contact patch length

L Length L Wheelbase

{L}1Linear momentum vector for a particle or bodyLPFZ2Pneumatic lead scaling factor with load squaredLPFZPneumatic lead scaling factor with load

 $\begin{array}{lll} L_{PC} & & \text{Pneumatic lead at reference load} \\ LPRF & & Local part reference frame} \\ L_{R} & & \text{Tyre relaxation length} \end{array}$

M_{FRC} Moment reacted by front roll centre

 $\{M_{nA}\}_e$ Applied moment vector on part n resolved parallel to frame e $\{M_{nC}\}_e$ Constraint moment vector on part n resolved parallel to frame e

M_s Equivalent roll moment due to springs

M_x Tyre overturning moment

 $\mathbf{M}_{\mathbf{X}\gamma\kappa}$ Overturning moment due to longitudinal forces

M_v Moment about y-axis

 $\begin{array}{ll} M_y & \quad \text{Tyre rolling resistance moment} \\ M_z & \quad \text{Tyre self aligning moment} \end{array}$

 $\mathbf{M}_{\mathbf{z}\alpha}$ Friction moderated side force due to slip angle

M₂ Friction moderated side force due to camber/inclination angle

M_{ZYK} Aligning moment due to longitudinal forces

MRF Marker reference frame

M_{RRC} Moment reacted by rear roll centre

N_r Vehicle yaw moment with respect to yaw rate

[N_t] Norsieck vector

N_{vv} Vehicle yaw moment with respect to lateral velocity

 $egin{array}{ll} O_1 & & \text{Frame 1 (GRF)} \\ O_e & & \text{Euler axis frame} \\ \end{array}$

O_i Reference frame for part i O_i Reference frame for part j

O_n Frame for part n

 $egin{array}{ll} \mathbf{O_P} & \text{Lateral offset of contact patch} \\ \mathbf{P_0} & \text{Initial tyre pressure at zero load} \\ \overline{P} & \text{Average footprint pressure} \\ \end{array}$

 $\{P_{nr}\}_1$ Rotational momenta vector for part n resolved parallel to frame 1

(GRF)

{P_{nt}}₁ Translational momenta vector for part n resolved parallel to frame

1 (GRF)

Pt Constant power acceleration P_{4z} Change in nominal pressure

 $P_{\Delta z}$ Pressure due to tyre vertical deflection

QG Position vector of a marker relative to the GRF

QP Position vector of a marker relative to the LPRF

R Radius (generic)
R Radius of turn

R Fraction of roll moment distributed between front and rear axles

R₁ Unloaded tyre radiusR₂ Tyre carcass radius

 $egin{array}{ll} R_d & ext{Radius to centre of brake pad} \ R_e & ext{Effective rolling radius} \end{array}$

 $\{\mathbf{R}_{\mathbf{i}}\}_{\mathbf{1}}$ Position vector of frame i on part i resolved parallel to frame 1

(GRF)

 $\{R_i\}_1$ Position vector of frame j on part j resolved parallel to frame 1

(GRF)

R_I Loaded tyre radius

 $\{R_n\}_1$ Position vector for part n resolved parallel to frame 1 (GRF) $\{R_p\}_1$ Position vector of tyre contact point P relative to frame 1, referenced

to frame 1

R_u Unloaded tyre radius

 $\{R_w\}_1$ Position vector of wheel centre relative to frame 1, referenced to

frame 1

 $\{R_{AG}\}_n$ Position vector of point A relative to mass centre G resolved parallel

to frame n

 $\{R_{BG}\}_n$ Position vector of point B relative to mass centre G resolved parallel

to frame n

 ${RC}_{front}$ Front roll centre ${RC}_{rear}$ Rear roll centre

 $\begin{array}{ll} RCY & Y\text{-coordinate of roll centre} \\ RCZ & Z\text{-coordinate of roll centre} \\ R_Z & Reference load (Harty Model) \end{array}$

S Distance travelled

SA Spindle axis reference point

 $egin{array}{ll} S_{CX} & & \text{Critical slip ratio} \\ S_e & & \text{Error variation} \end{array}$

Sh Horizontal shift (Magic Formula) Sv Vertical shift (Magic Formula)

S_L Longitudinal slip ratio

S_L* Critical value of longitudinal slip

 $\begin{array}{lll} SN & & \text{Signal-to-noise ratio} \\ S_T & & \text{Total variation} \\ S_{\alpha} & & \text{Lateral slip ratio} \\ S_{L\alpha} & & \text{Comprehensive slip ratio} \\ S_{\alpha}^* & & \text{Critical slip angle} \end{array}$

 S_{κ} Variation due to linear effect T Camber clipping threshold fraction

T Kinetic energy for a part

T_{env} Environmental temperature

T_{PFZ2} Pneumatic trail scaling factor with load squared

T_{PFZ} Pneumatic trail scaling factor with load

T_{PC} Pneumatic trail scaling constant

T_S Spin up torque

T₀ Initial brake rotor temperature

 $\{T_A\}_1 \{T_B\}_1...$ Applied torque vectors at points A, B,... resolved parallel to frame

1 (GRF)

TK Top kingpin marker
TR Suspension trail

{U_r} Unit vector normal to road surface at tyre contact point

 $\{U_s\}$ Unit vector acting along spin axis of tyre

UCF Units consistency factor

US Understeer V Forward velocity

 $egin{array}{ll} egin{array}{ll} egi$

 $\{V_n\}_1$ Velocity vector for part n resolved parallel to frame 1 (GRF) $\{V_p\}_1$ Velocity vector of tyre contact point P referenced to frame 1

 V_s Desired simulation velocity

 V_x Sliding velocity

V_{xc} Longitudinal slip velocity of tyre contact point
 Vy Lateral slip velocity of tyre contact point
 Vz Vertical velocity of tyre contact point
 Vref Reference velocity in hysteretic model

VR(I,J) Radial line of sight velocity of I marker relative to J marker

 $egin{array}{ll} VZ & ext{Velocity variable (generic)} \ V_{dz} & ext{Reduced tyre cavity volume} \ \end{array}$

W Tyre width

WB Wheelbase marker
WC Wheel centre marker
WF Wheel front marker
WR Wheel recession

XP Position vector of a point in a marker xz-plane

 $\{X_{sae}\}_1$ Unit vector acting at tyre contact point in X_{sae} direction referenced

to frame 1

Y_r Vehicle side force with respect to yaw rate
Y_{vv} Vehicle side force with respect to lateral velocity

YRG Yaw rate gain

 $\{Y_{sae}\}_1$ Unit vector acting at tyre contact point in Y_{sae} direction referenced

to frame 1

 $\{Z_{sae}\}_1$ Unit vector acting at tyre contact point in Z_{sae} direction referenced

to frame 1

ZP Position vector of a point on a marker z-axis

 $\begin{array}{ccc} \alpha & & \text{Angle of attack} \\ \alpha & & \text{Tyre slip angle} \end{array}$

xxiv Nomenclature

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$\alpha_{\rm CY}$	Critical slip ar	igle (Harty Model)

 $\{\alpha_n\}_1$ Angular acceleration vector for part n resolved parallel to frame 1

(GRF)

 $egin{array}{ll} α_{f} & Front axle slip angle \\ α_{r} & Rear axle slip angle \\ \end{array}$

β Aerodynamic yaw angle (or body slip angle surrogate)

 β Side slip angle

 $\dot{\beta}$ Rate of change of side slip angle (Beta Dot)

 δ Steer or toe angle

 $oldsymbol{\delta_0}$ Steer angle of outer wheel $oldsymbol{\delta_i}$ Steer angle of inner wheel

 δ_{mean} Average steer angle of inner and outer wheels

γ Camber angle

 $\{\gamma_n\}_e$ Set of Euler angles for part n

 ζ Damping ratio

κ Longitudinal slip (Pacjeka) κ Sensitivity of process θ 2nd Euler angle rotation

 θ Pendulum displacement variable θ Pitch displacement variable

 θ_1 Orientation of the first principal axis within a plane of symmetry

λ Eigenvalue (generic)

 $\{\lambda\}_1$ Reaction force vector resolved parallel to frame 1 (GRF)

 $\begin{array}{ll} \lambda_d & \text{Magnitude of reaction force for constraint d} \\ \lambda_p & \text{Magnitude of reaction force for constraint p} \\ \lambda_\alpha & \text{Magnitude of reaction force for constraint } \alpha \end{array}$

 μ Friction coefficient

 μ_0 Tyre to road coefficient of static friction μ_1 Tyre to road coefficient of sliding friction

 η Signal-to-noise ratio

η Hysteresis constant/loss factor

ρ Density

 σ Standard deviation

 $\sigma_{
m d}$ Standard deviation of attribute d Φ 3rd Euler angle rotation ψ 1st Euler angle rotation

 ψ Ist Euler angle rotation ψ Compass heading angle ψ Yaw rate (Wenzel model) ω Angular frequency (rads s⁻¹)

 ω Yaw rate

 $egin{array}{lll} \omega_{d} & & {
m Damped \ natural \ frequency} \ \omega_{d} & & {
m Demanded \ yaw \ rate} \ \omega_{err} & & {
m Yaw \ rate \ error} \ \end{array}$

 ω_{fns} Front axle no-slip yaw rate $\omega_{friction}$ Yaw rate from limiting friction ω_{geom} Yaw rate from geometry ω_{n} Undamped natural frequency

 $\{\omega_e\}_1$ Angular velocity vector for part n resolved parallel to frame e

$\{\omega_{\mathbf{n}}\}_{1}$	Angular velocity vector for part n resolved parallel to frame 1
	(GRF)
ω_0	Angular velocity of free rolling wheel
$\omega_{ m D}$	Angular velocity of driven wheel
$\Delta_{\mathbf{d}}$	Allowable range for attribute d
∆x	Change in longitudinal position of wheel (concept suspension)
∆y	Change in lateral position (half/track) of wheel (concept
	suspension)
Δz	Deformation of tyre
△V	Change in tyre cavity volume
$\Delta \varepsilon$	Change in steer angle (toe in/out) of wheel (concept suspension)
$\Delta\gamma$	Change in camber angle of wheel (concept suspension)
$\{oldsymbol{\Phi}_{\mathrm{a}}\}_{\mathrm{1}}$	Vector constraint equation resolved parallel to frame 1 (GRF)
$oldsymbol{\Phi}_{\mathbf{d}}$	Scalar constraint expression for constraint d
$oldsymbol{\Phi}_{ m p}$	Scalar constraint expression for constraint p
Φ_{α}	Scalar constraint expression for constraint α
$\dot{\Omega}_{ ext{wheel}}$	Angular acceleration of road wheel