Repetition

Definition

Let $\{N(t) : t \geq 0\}$ be a Poisson process. The **waiting time** W_n is the time of occurrence of the n-th event. We define $W_0 = 0$.

Definition

The differences $S_n = W_{n+1} - W_n$ are called the **sojourn times** (interarrival times).

Note: It is **very** common to call these interarrivial times, but I will follow the book and call them sojourn times.

Definition

The stochastic variable Y has an **exponential distribution** with **rate parameter** $\lambda > 0$ if

$$f(y) = \lambda e^{-\lambda y}, \quad y > 0.$$

We write $Y \sim \text{Exp}(\lambda)$.

Note: There also exists a parametrization using a **scale parameter** $\beta = 1/\lambda$. We will **not** use that in this course.

Theorem 5.5

Let $\{N(t) : t \geq 0\}$ be a Poisson process with rate λ. Then $S_0, S_1, \ldots, S_{n-1} \overset{\text{iid}}{\sim} \text{Exp}(\lambda)$.

Note: A valid **third** definition of the Poisson process would be that you start in state 0 at time 0, and spend a time in each state that follows an exponential distribution with rate λ before jumping from state i to state $i + 1$.

Definition

The stochastic variable Y has a **gamma distribution** with **shape parameter** $\alpha > 0$ and **rate parameter** $\lambda > 0$ if

$$f(y) = \frac{\lambda^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \mathrm{e}^{-\lambda y}, \quad y > 0.$$

We write $Y \sim \text{Gamma}(\alpha, \lambda)$.

Note: There also exists a parametrization using a **scale parameter** $\beta = 1/\lambda$. We will **not** use that in this course.

Theorem 5.4

For a Poisson process with rate $\lambda > 0$, $W_n \sim \text{Gamma}(n, \lambda)$ for all integers $n > 0$.