Cheat Sheet

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1 Introduction

2 Continious maps

3 Topological spaces

Definition 3.1 (Topological spaces.). *Recall that a topological space is a set X together with a collection Y of subsets of X that are open in X s.t.*

- ***T1**. $\emptyset, X \in \tau$*
- ***T2.** τ is closed under union if $U_\lambda \in \tau$ for all $\lambda \in \Lambda$, then*

$$\bigcup_{\lambda \in \Lambda} U_\lambda \in \tau$$

- ***T3.** τ is under finite intersections if $U_1, U_2, \ldots, U_n \in \tau$, then*

$$U_1 \cap U_2 \cap \ldots \cap U_n \in \tau$$

Definition 3.2 (Open and closed sets). . *Let (X, τ), $U \subseteq X$*

- ***Open set**. If $U \in \tau$, then is U open.*
- ***Closed set.** If $U^c = X - U \in \tau$, then is U closed*

Remark. Let $X = \{a, b, c\}$ and let $U = \{a, b\}$. Then if $\tau = \{X, \emptyset\}$, U is not open nor closed.

Definition 3.3 (Neighbourhoods). *Let X be a topological space, U a subset* *of X and $x \in X$. We say U is a neighborhood of x if $x \in U$ and U is open in X.*

Theorem 3.1. ***Continuity between topological spaces.*** *Let X, Y be topological spaces. A map $f : X \to Y$ is said to be continious if preimages of open sats are open, i.e., if V is an open set in Y then the preimage $f^{-1}(V)$ of V is open in X.*

4 Generating topologies

Definition 4.1 (Basis)**.** *A* ***basis*** *for a topology on X is a collection of subsets of X s.t.*

- ***B1.*** *For each $x \in X$ there is a $B \in \mathfrak{B}$ s.t. $x \in B$.*
- ***B2.*** *B_1, B_2 and $x \in B_1 \cap B_2$ there is a $B_3 \in \mathfrak{B}$ s.t. $x \in B_3 \subseteq B_1 \cap B_2$*

5 Constructing topological spaces

6 Topological properties

Definition 6.1 (Connected space)**.** *Let X be a topological space. A* ***seperation*** *of X is a pair of non-empty subsets U and V that are open in X, disjoint and whose union equal X. We say that X is* ***connected*** *if there are no seperations of X. Otherwise it is* ***disconnected.***

Theorem 6.1 (Connectivity)**.** *Let X be a topological space. Then X is connected if and only if the are no non-empty proper subsets of X that are both open and closed.*

7 The fundamental group

8 The fundamental group of the circle

9 References