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ORIGINAL PAPER

Identification of tire forces using Dual Unscented Kalman Filter algorithm

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Abstract Nowadays, application of active control systems in vehicles has been developed in order to increase safety and steerability. In these systems, using an appropriate dynamic model can be very effective in increasing the accuracy of simulations and analysis. Tire-road forces are crucial in vehicle dynamics and control since they are the only forces that a vehicle experiences from the ground and have maximum uncertainty on vehicle dynamic model. In order to simulate the non-linear regimes of vehicle motion, the 'Pacejka' tire model is being utilized. In this paper, a dynamic model with Dual Unscented Kalman Filter algorithm has been utilized to identify the lateral forces, side slip angle, and normal forces of tires. In order to solve the non-linear least squares problem, these parameters were given as input to the hybrid Levenberg–Marquardt and quasi Newton algorithm to find the Pacejka tire model coefficients in the offline mode. Four degrees of freedom vehicle model combined with Pacejka tire model are used for simulation in various maneuvers. Results show appropriate compatibility with CarSim software.

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Keywords Dual Unscented Kalman Filter · Uncertainty · Pacejka tire model · Levenberg–Marquardt algorithm · Carsim

List of symbols

a_x	Longitudinal acceleration
a_y	Lateral acceleration
m	Vehicle mass
m_s	Sprung mass
I_z	Moment of inertia about z axis
I_x	Moment of inertia about x axis
I_{xz}	Product of moment of inertia
a	Distance of vehicle C.G from front axle
b	Distance of vehicle C.G from rear axle
l	Distance between two axle
h	Height of C.G
h_s	Height of roll center
T	Track
ρ	Density of air
C_d	Drag coefficient
A_p	Frontal area projection
μ_R	Rolling resistance coefficient
K_ϕ	Roll stiffness
D_ϕ	Roll damping
δ	Wheel steer angle
α	Tire side slip angle
F_z	Tire normal load
F_y	Lateral tire force
σ	Relaxation length

BCD	Tire lateral stiffness
C	Shape factor
D	Peak factor
B	Stiffness factor
E	Curvature factor
S_h	Horizontal shift
S_v	Vertical shift
$a_0, \ldots, a_{13}$	Parameters of Pacejka -tire model
γ	Camber angle
τ	Step size
u	Longitudinal velocity
v	Lateral velocity
r	Yaw rate
ϕ	Roll angle
p	Roll rate

1 Introduction

Recent studies show that most of the road accidents are due to driving mistakes like losing control, exceeding the permitted speed range, getting out of the road at high speeds, vehicle sliding due to severe braking, etc. Today, in order to minimize such accidents, using control systems in vehicles has become very common. These systems have been able to prevent accidents and their damages during crucial conditions.

These control systems require that certain information about vehicle kinematic parameters is specified which can be used as feedbacks in these systems. Some of these parameters like lateral and longitudinal acceleration and rotational velocities around vehicle longitudinal and vertical axes can be measured by simple sensors such as accelerometers and gyros; whereas, some of these parameters such as lateral and longitudinal velocities and in result side slip angle, roll angle, and road slope angles cannot be measured due to technical and economical reasons.

However, some of these quantities can be obtained by devices such as global positioning systems or optical sensors. As mentioned above, the technical and economic problems prevent their use extensively in control systems. Due to high cost of these systems, their applications are limited to luxury cars only. In addition, the technical reasons can be cited for less accuracy of these systems. For example, the Global Positioning System technology cannot be used when crossing the road nearby mountains, bridges, towers, and structures or for unfavorable atmospheric conditions because the signals may be disrupted. The optical sensors are less accurate in addition to their technical problems. For example, these sensors require road image effect to measure vehicle longitudinal and lateral velocities. Also, in unfavorable physical condition and when the image resolution is not high, the sensors error rate is significantly high.

Considering the mentioned reasons, in control systems, it is preferred to estimate kinematic parameters instead of measuring them directly. It is clear that there will be difference between the actual and the estimated quantities. Therefore, this error must be minimized in a way. There are various mathematical algorithms and procedures for estimating such parameters, but the best choice of these methods is the main issue.

Furthermore, it is necessary to apply a proper vehicle dynamic model in these algorithms. So, using an appropriate vehicle model can be very effective in the precision of the estimation process. Also, in the case of using more complex models, there will be differences between simulation and real results. This is because that, some physical phenomena which are being described in these models cannot be modeled well. Of the most important phenomena that are difficult to describe are the tire forces.

Modeling of these forces by simply using the fundamental laws of physics is not possible. However, in some cases, physical description of this phenomenon has been calculated, but due to the extreme complexity, it cannot be used in simple vehicle dynamic models. On the other hand, this phenomenon has non-linear and complex relationship with the parameters of tire such as roughness, tire geometric size, and physical characteristics, its pressure, temperature, wear, normal forces, longitudinal and lateral slip, and camber angle. Nevertheless, there are semi-experimental or experimental models which are very common. For instance, Pacejka tire model is one of the models, which has been taken into consideration during recent years [1]. There are various Pacejka tire models with different complexity degrees. In one model which is used widely, tire forces are considered relative to normal forces and slip, non-linearly. In this model, the coefficients are obtained through experiments. Therefore, it is called experimental model. Numerous and expensive tests and the absence of dynamic behavior have led to necessity of identification and estimation of its parameters.

Doumiati et al. [2] estimate the amount of lateral weight transfer of the vehicle and the tire vertical forces to use in vehicle safety algorithms. Initially, the vehicle mass is estimated by measuring the displacement of suspension springs. Thus, by measuring the displacement of the spring Δ_{ij} and regarding that there is a linear relationship between the spring force and stiffness with the displacement, the additional load on each of the four corners of the vehicle is obtained as $\Delta m_{sij} = \frac{K_s \Delta_{ij}}{g}$, where K_s and g represent spring stiffness and gravity, respectively. Also, the equivalent mass in all four corners and total mass of the vehicle are calculated as $m_{ij} = m_{eij} + \Delta m_{sij}$, $m_v = \sum_{i,j} m_{ij}$, respectively, where, m_{eij} is equivalent to static mass of the vehicle without load at all four corners, which is an available value. m_{ij} is the same quantity but during driving which is subject to change. Finally, m_v is the vehicle total mass. Having these parameters and considering vehicle roll dynamics, measuring the lateral acceleration, and using Kalman Filter, the amount of lateral weight transfer is estimated. At this point, the change of tire normal forces is obtained. This value along with the estimated mass and the measured longitudinal and lateral accelerations by sensors—considering that the sum of normal forces acting on tires must be equal to the vehicle's whole estimated mass—are used as input to identification system to find tire normal forces.

A simple method to estimate the longitudinal and lateral tire forces using the Extended Kalman Filter was presented by Wilkin et al. [3]. In Lakshmanan et al. [4], the sampled-data state estimation problem is investigated for neural networks with time-varying delays. Instead of the continuous measurement, the sampled measurement is used to estimate the neuron states, and a sampled-data estimator is constructed.

The estimation process consists of two blocks, and its duty is to estimate normal and lateral forces at each tire/road level and then to evaluate the lateral friction coefficient. Moreover, vehicle side slip, the longitudinal and lateral tire forces, and tire lateral stiffness were estimated using two separate blocks in [5] by Baffet et al. The first block contains a sliding mode observer, which is based on vehicle bicycle model to describe vehicle dynamics. Principle role of this block is to calculate tire forces which are sent to the second block. In the second block, the amount of tire force and the rate of rotation angle around the vertical axis which are obtained from the first block with the longitudinal vehicle velocity and the steering angle as input and using an Extended Kalman Filter, the side slip angle, and the cornering stiffness are estimated. In other words, in the second block, instead of sensor data as a viewer, the amount of the lateral force which is obtained from the first block is used. Juang [6] discusses the identification of continuous-time bilinear systems using a family of unit pulses, of fixed amplitude but varying widths, as inputs in a series of experiments the number of which is proportional to the order of the system.

Best et al. [7] consider a method for estimating vehicle handling dynamic states in real time using a reduced sensor set. At the first step, the lateral stiffness of the tire in a linear dynamic model with two degrees of freedom is being estimated by using a least squares method. In the second step, an Extended Kalman Filter is designed to estimate the rapidly varying handling state vector. This employs a 4-DOF handling model which is augmented to include adaptive states (cornering stiffnesses) to compensate for tire force nonlinearities. Finally, to evaluate the effect of non-linear tire, this method used an adaptive model for lateral stiffness.

In Mashadi et al. [8], according to vehicle handling parameters, an optimal controller has been developed for tracking of the driver intended path of an integrated driver/vehicle system. Furthermore, Genetic Algorithm procedure is utilized in order to adapt a set of optimized controller parameters suitable for vehicle path following.

Cho et al. [9] present a scheme for longitudinal/lateral tire force estimation. At first, distribution of normal forces acting on tires obtained by the roll dynamics calculated lateral weight transfer, lateral and longitudinal accelerations, together with initial roll rates. In the second step, using the relationship between angular velocity and output torque of the engine combined by the transmission dynamics and a Kalman Filter, torque at the wheels has been estimated. Then, longitudinal tire force estimation based on a simplified wheel dynamics model, which uses tire dynamics, was conducted. Furthermore, lateral forces on the tire, using a dynamic model with two degrees of freedom, are estimated. Finally, in order to model lateral/longitudinal forces in the combined slip condition, states estimated in the previous stages have been used as input to 3-DOF vehicle dynamic model, which uses a random-walk Kalman Filter.

Further papers like [10–17] demonstrate other studies done in the field of tire forces and their relative parameters estimation. In [18], Observer/Kalman Filter Identification (OKID), a linear time invariant (LTI) system identification algorithm, is used to identify the aforementioned equivalent linear model from which the original bilinear model is recovered.

In this paper, using a vehicle dynamic model and Dual Unscented Kalman Filter (DUKF) method, the tire forces are identified. Measured signals by sensors—from CarSim Software—along with steering angle are considered as input to the algorithm. The outputs of the algorithm are side slip, lateral, and normal forces. Then, the coefficients of Pacejka tire model are obtained using non-linear least square problem.

2 Vehicle dynamic model

The dynamic model used in the identification process is a 4-DOF vehicle model which consists of longitudinal, lateral, roll, and yaw motions (Fig. 1).

The state space equations of the model are [19]

$$\begin{aligned}
&m(\dot{u}-rv)+m_s h_s r p=-(F_{yfl}+F_{yfr})\sin\delta \\
&-\frac{1}{2}\rho C_d A_p u^2-\mu_R m g \\
&m(\dot{v}+ru)-m_s h_s \dot{p}=(F_{yfl}+F_{yfr})\cos\delta \\
&+F_{yrl}+F_{yrr} \\
&I_z\dot{r}-I_{xz}\dot{p}=a(F_{yfl}+F_{yfr})\cos\delta \\
&-b(F_{yrl}+F_{yrr}) \\
&+\frac{T}{2}(F_{yfl}-F_{yfr})\sin\delta \\
&\dot{\phi}=p \\
&I_x\dot{p}-I_{zx}\dot{r}=m_s h_s(\dot{v}+ru)+m_s h_s g\sin\phi \\
&-K_\phi\phi-D_\phi p.
\end{aligned} \tag{1}$$

The longitudinal and lateral accelerations of the vehicle are calculated as in (2):

$$\begin{aligned}
a_x&=\frac{1}{m}\sum F_x=\frac{1}{m}\Big(-(F_{yfl}+F_{yfr})\sin\delta \\
&\quad-\frac{1}{2}\rho C_d A_p u^2-\mu_R m g\Big), \\
a_y&=\frac{1}{m}\sum F_y=\frac{1}{m}((F_{yfl}+F_{yfr})\cos\delta \\
&\quad+F_{yrl}+F_{yrr}).
\end{aligned} \tag{2}$$

3 Tire model

The Magic Formula of Pacejka is being used under pure slip condition to identify the tire model. The tire lateral forces in steady-state condition are obtained as in (3):

$$\begin{aligned}
F_y&=D\sin\left[C\tan^{-1}\left\{B(\alpha+S_h)-E(B(\alpha+S_h)\right.\right. \\
&\quad\left.\left.-\tan^{-1}(B(\alpha+S_h)))\right\}\right]+S_v.
\end{aligned} \tag{3}$$

In Eq. (3), the unknown coefficients are functions of tires normal forces and their relationship can be demonstrated as [1]

$$\begin{aligned}
D&=a_1F_z^2+a_2F_z, \\
\mathrm{BCD}&=a_3\sin\left[2\tan^{-1}\left(\frac{F_z}{a_4}\right)\right](1-a_5|\gamma|), \\
C&=a_0, \\
B&=\frac{\mathrm{BCD}}{\mathrm{CD}}, \\
E&=a_6F_z+a_7, \\
S_h&=a_9F_z+a_{10}+a_8\gamma, \\
S_v&=a_{11}F_z\gamma+a_{12}F_z+a_{13}.
\end{aligned} \tag{4}$$

In Eqs. (3) and (4), the values of tires side slip angles are as in Eq. (5) and in equation (4), and the values of tires normal forces are as in equation (6):

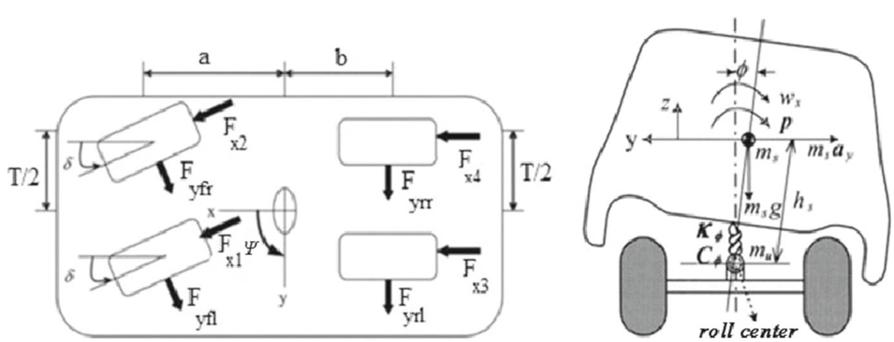


Fig. 1 Vehicle model

$$
\begin{aligned}
\alpha_{fl} &= \delta - \tan^{-1}\left(\frac{v+ar}{u-\frac{T}{2}r}\right)\\
\alpha_{fr} &= \delta - \tan^{-1}\left(\frac{v+ar}{u+\frac{T}{2}r}\right)\\
\alpha_{rl} &= -\tan^{-1}\left(\frac{v-br}{u-\frac{T}{2}r}\right)\\
\alpha_{rr} &= -\tan^{-1}\left(\frac{v-br}{u+\frac{T}{2}r}\right)
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
F_{z_{fl}} &= \frac{mgb}{2l} - \frac{mh}{2l}a_x - \frac{mhb}{lT}a_y - \frac{K_{\phi f}\phi + D_{\phi f}\dot{\phi}}{T}\\
F_{z_{rl}} &= \frac{mgb}{2l} - \frac{mh}{2l}a_x + \frac{mhb}{lT}a_y + \frac{K_{\phi f}\phi + D_{\phi f}\dot{\phi}}{T}\\
F_{z_{rl}} &= \frac{mga}{2l} + \frac{mh}{2l}a_x - \frac{mha}{lT}a_y - \frac{K_{\phi r}\phi + D_{\phi r}\dot{\phi}}{T}\\
F_{z_{rr}} &= \frac{mga}{2l} + \frac{mh}{2l}a_x + \frac{mha}{lT}a_y + \frac{K_{\phi r}\phi + D_{\phi r}\dot{\phi}}{T}.
\end{aligned} \tag{6}
$$

In Eqs. (5) and (6), the indices *fl, fr, rl, rr* show the location of the tires. For example, fl is the front left tire. Having the parameters of the above dynamic model and also by having the angle δ as the steering input in an arbitrary maneuver, the dynamic behavior of the vehicle can be simulated.

4 Identification and estimation

In this part, during identification process, all the measured signals are gathered at first and are being used as input to Dual Unscented Kalman Filter. To identify the lateral forces of the tire, it is better to use the models that include its transient response behavior which are being shown in Eq. (7) [1]:

$$
\dot{F}_{y_{ij}} = \frac{u_{ij}}{\sigma_i}(-F_{y_{ij}} + \bar{F}_{y_{ij}}) \quad ij = fl, fr, rl, rr, \tag{7}
$$

where σ_i describes the lag of the tire force and u_{ij} is the longitudinal velocity of each wheel, which is calculated in Eq. (8):

$$
\begin{aligned}
u_{fl} &= \left(u - \tfrac{T}{2}r\right)\cos\delta + (v + ar)\sin\delta\\
u_{fr} &= \left(u + \tfrac{T}{2}r\right)\cos\delta + (v + ar)\sin\delta\\
u_{rl} &= u - \tfrac{T}{2}r\\
u_{rr} &= u + \tfrac{T}{2}r.
\end{aligned} \tag{8}
$$

In Eq. (3), $\bar{F}_y$ shows the steady-state response of tire forces.

These forces along with Eq. (3) describe the dynamic system of the identification process, so that the state vector of the system is

$$
x = [\, u \quad v \quad r \quad \phi \quad \dot{\phi} \quad F_{y_{fl}} \quad F_{y_{fr}} \quad F_{y_{rl}} \quad F_{y_{rr}} \,]^T. \tag{9}
$$

The vector of the unknown parameters of the system which must be calculated are written as in (10):

$$
w = [\, D_f \quad B_f \quad E_f \quad \sigma_f \quad D_r \quad B_r \quad E_r \quad \sigma_r \,]^T. \tag{10}
$$

The vector z is the value measured by the sensors and is demonstrated in Eq. (11):

$$
z = \begin{bmatrix} u\\ v\\ a_x\\ a_y\\ r\\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} u\\ v\\ \frac{1}{m}(-(F_{yfl}+F_{yfr})\sin\delta - \frac{1}{2}\rho C_d A_p u^2 - \mu_R mg)\\ \frac{1}{m}((F_{yfl}+F_{yfr})\cos\delta + F_{yrl} + F_{yrr})\\ r\\ \dot{\phi} \end{bmatrix} = h(x, w). \tag{11}
$$

Substituting Eqs. (7)–(1) and considering the state vector as (9), the dynamic system equations can be written as $\dot{x} = f(x, u, w) = M^{-1}E$ in which the parameters M and E are obtained from Eqs. (12) and (13).

$$
M = \begin{bmatrix}
m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & m & 0 & 0 & -m_s h_s & 0 & 0 & 0 & 0\\
0 & 0 & I_z & 0 & -I_{xz} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & -m_s h_s & -I_{xz} & 0 & I_x & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \tag{12}
$$

$$
E = \begin{bmatrix}
-(F_{yfl} + F_{yfr})\sin\delta - \frac{1}{2}\rho C_d A_p u^2 - \mu_R mg + mrv\\
(F_{yfl} + F_{yfr})\cos\delta + F_{yrl} + F_{yrr} - mru\\
a(F_{yfl} + F_{yfr})\cos\delta - b(F_{yrl} + F_{yrr}) + \frac{T}{2}(F_{yfl} - F_{yfr})\sin\delta\\
p\\
m_s h_s r u + m_s h_s g \sin\phi - K_\phi \phi - D_\phi p\\
\frac{u_{fl}}{\sigma_f}(-F_{y_{fl}} + \bar{F}_{y_{fl}})\\
\frac{u_{fr}}{\sigma_f}(-F_{y_{fr}} + \bar{F}_{y_{fr}})\\
\frac{u_{rl}}{\sigma_r}(-F_{y_{rl}} + \bar{F}_{y_{rr}})\\
\frac{u_{rr}}{\sigma_r}(-F_{y_{rr}} + \bar{F}_{y_{rr}})
\end{bmatrix}. \tag{13}
$$

Using Euler's approximation method, the derivative of the state vector x can be written as $\dot{x} = \frac{x_{k+1} - x_k}{\tau}$ and the dynamic system can be obtained discretely as $x_{k+1} = x_k + \tau f(x_k, u_k, w) = F(x, u, w)$ in which τ is the time of each step.

Unscented Kalman Filter is an effective method to estimate the states and the parameters of a discrete dynamic system. This filter includes a recursive manner which is based on receiving noisily data and a certain predictive dynamic model. Its solver algorithm consists of a predictive and an update part [20]. The dynamic system can be written as

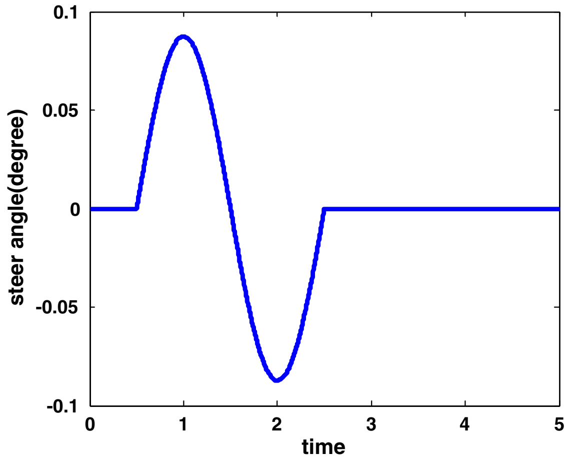


Fig. 2 Steering angle of front tires

$$x_{k+1} = F(x_k, u_k, w) + v_k,$$
$$y_k = H(x_k, w) + n_k, \tag{14}$$

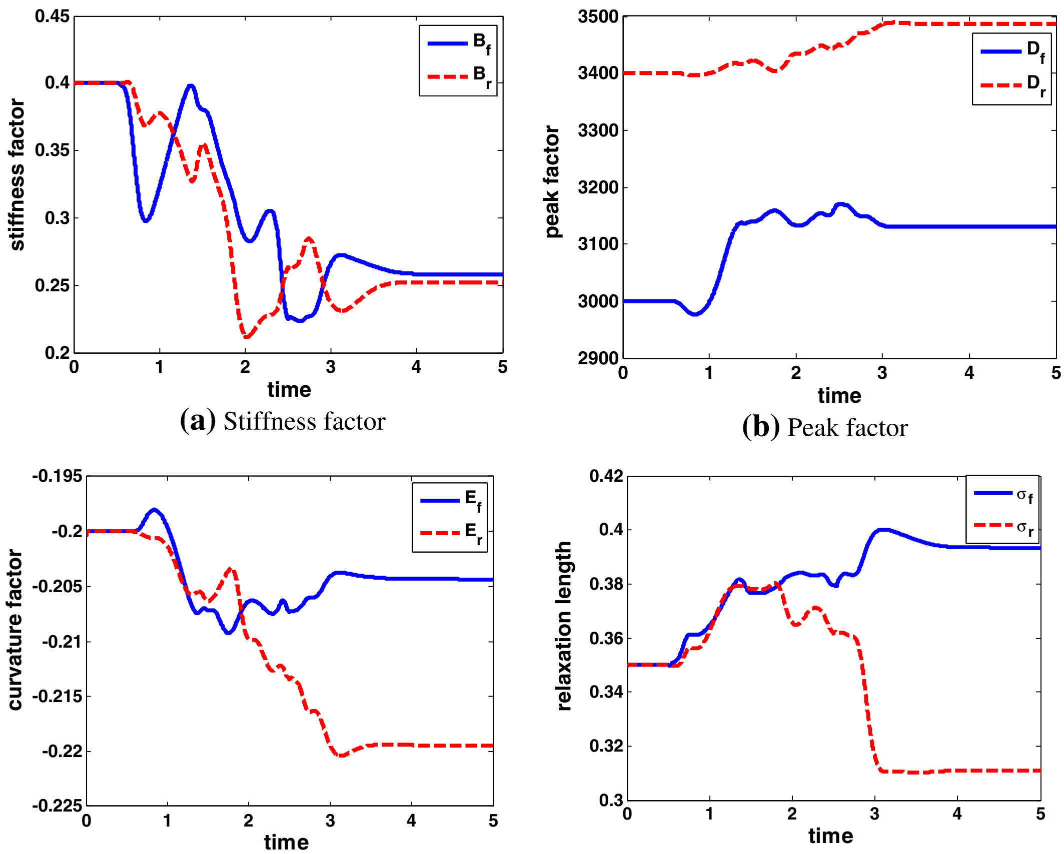
where x_k is the system state, w is the model parameter, y_k is the measured signal, v_k is the system noise, and n_k is the measured noise which has Gaussian distribution with zero mean. Unscented Kalman Filter estimates the states values in a way that the cost function J in Eq. (15) becomes minimum.

$$J(x_t^k) = \sum_{t=1}^{k} \Big\{ [y_t - H(x_t, w)]^T (R^n)^{-1} [y_t - H(x_t, w)] + (x_t - x_t^-)^T (R^v)^{-1} (x_t - x_t^-) \Big\}. \tag{15}$$

$R^v = E[v_k v_k^T]$ and $R^n = E[n_k n_k^T]$ are covariance matrices for system noise and the measured noise. If the values of the estimated states are demonstrated with $\hat{x}_k$ and the estimated parameters are shown with $\hat{w}_k$, then their values are obtained with a recursive method as follows:

Equation (16) is used to calculate the states.

$\hat{x}_0 = E[x_0]$



(a) Stiffness factor

(b) Peak factor

(c) Curvature factor

(d) Relaxation Length

Fig. 3 Mean values of the identified magic formula coefficients

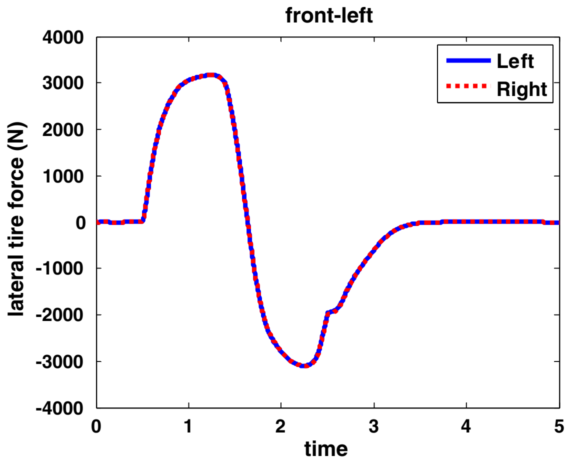


Fig. 4 Lateral forces estimation of front tires

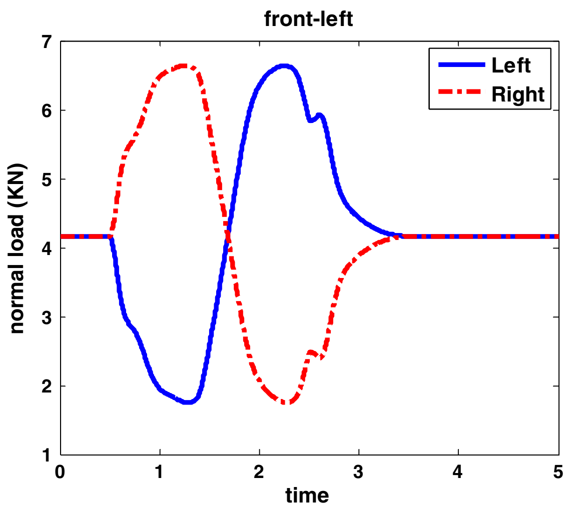


Fig. 5 Normal forces estimation of front tires

$$
\begin{aligned}
P_0 &= E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \\
\chi_{k-1} &= \left[\hat{x}_{k-1} \hat{x}_{k-1} + \gamma\sqrt{P_{k-1}}\hat{x}_{k-1} - \gamma\sqrt{P_{k-1}}\right] \\
\chi^*_{k|k-1} &= F[\chi_{k-1}, u_{k-1}] \\
\hat{x}_k^- &= \sum_{i=0}^{2L} W_i^{(m)} \chi^*_{i,k|k-1} \\
P_k^- &= \sum_{i=0}^{2L} W_i^{(c)} \left[\chi^*_{i,k|k-1} - \hat{x}_k^-\right]\left[\chi^*_{i,k|k-1} - \hat{x}_k^-\right]^T \\
&\quad + R^v.
\end{aligned} \tag{16}
$$

The states estimation and their covariance as in (17):

$$
\chi_{k|k-1} = \left[\hat{x}_k^- \hat{x}_k^- + \gamma\sqrt{P_k^-}\hat{x}_k^- - \gamma\sqrt{P_k^-}\right]
$$

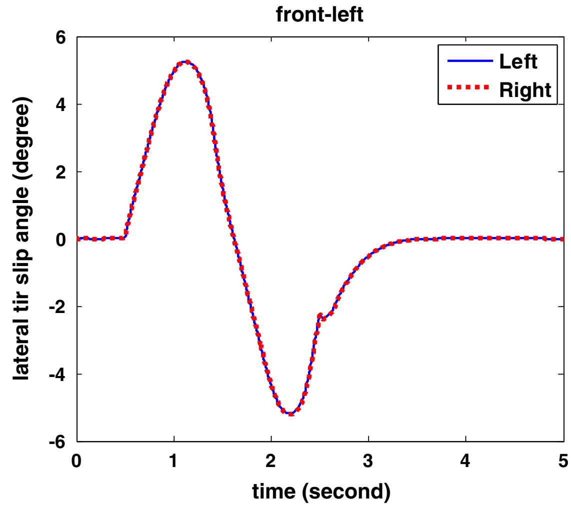


Fig. 6 Slip angle of front tires

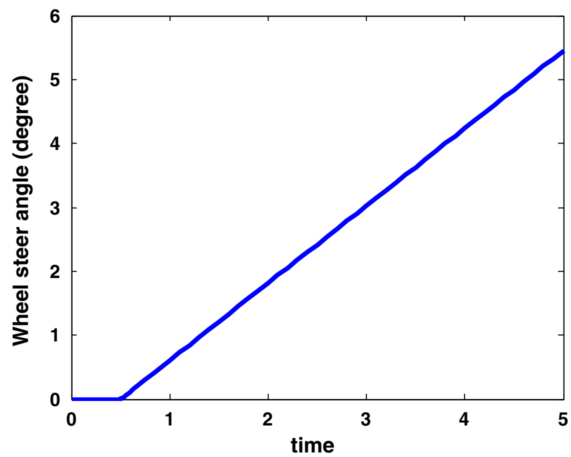


Fig. 7 Ramp input for steering angle

$$
\begin{aligned}
Y_{k|k-1} &= H[\chi_{k|k-1}] \\
\hat{y}_k^- &= \sum_{i=0}^{2L} W_i^{(m)} Y_{i,k|k-1} \\
P_{\tilde{y}_k \tilde{y}_k} &= \sum_{i=0}^{2L} W_i^{(c)} \left[Y_{i,k|k-1} - \hat{y}_k^-\right]\left[Y_{i,k|k-1} - \hat{y}_k^-\right]^T + R^n \\
P_{x_k y_k} &= \sum_{i=0}^{2L} W_i^{(c)} \left[\chi^-_{i,k|k-1} \hat{x}_k^-\right]\left[Y_{i,k|k-1} - \hat{y}_k^-\right]^T \\
\kappa_k &= P_{x_k y_k} P^{-1}_{\tilde{y}_k \tilde{y}_k} \\
\hat{x}_k &= \hat{x}_k^- + \kappa_k \left(y_k - \hat{y}_k^-\right) \\
P_k &= P_k^- - \kappa_k P_{\tilde{y}_k \tilde{y}_k} \kappa_k^T,
\end{aligned} \tag{17}
$$

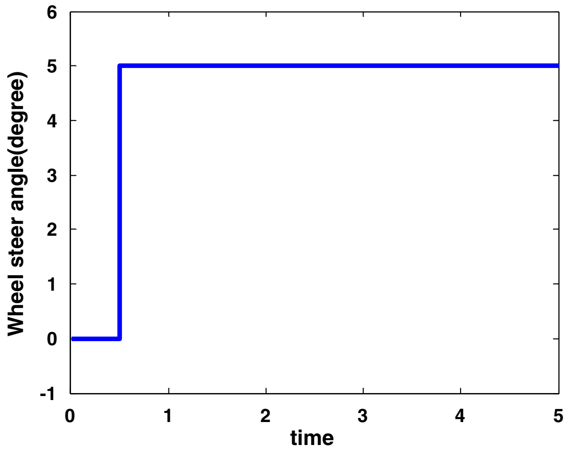


Fig. 8 Step input for steering angle

where $\gamma = \sqrt{L+\lambda}$ is called the combined scale parameter, L is the dimension of x, and W_i is obtained from (18):

$$\begin{aligned} W_0^{(m)} &= \frac{\lambda}{(L+\lambda)} \\ W_0^{(c)} &= \frac{\lambda}{(L+\lambda)} + (1-\alpha^2+\beta) \\ W_i^{(m)} &= W_i^{(c)} = \frac{1}{2\{(L+\lambda)\}} \quad i = 1,\ldots,2L \\ \sum_{i=0}^{2L} W_i^{(m)} &= 1 \\ \sum_{i=0}^{2L} W_i^{(c)} &= 1-\alpha^2+\beta \end{aligned} \tag{18}$$

Equation (19) is used to predict the parameters:

$$\begin{aligned} \hat{w}_0 &= E[w] \\ P_{w_0} &= E\left[(w-\hat{w}_0)(w-\hat{w}_0)^T\right] \\ \hat{w}_k^- &= \hat{w}_{k-1} \\ P_{w_k}^- &= P_{w_{k-1}}^- + R_{k-1}^r, \end{aligned} \tag{19}$$

and for the estimated values of parameters and their covariance, Eq. (20) is used:

$$\begin{aligned} \omega_{k|k-1} &= \left[\hat{w}_k^- \; \hat{w}_k^- + \gamma\sqrt{P_{w_k}^-} \; \hat{w}_k^- - \gamma\sqrt{P_{w_k}^-}\right] \\ D_{k|k-1} &= G[x_k, \omega_{k|k-1}] \\ \hat{d}_k &= \sum_{i=0}^{2L} W_i^{(m)} D_{i,k|k-1} \\ P_{\tilde{d}_k\tilde{d}_k} &= \sum_{i=0}^{2L} W_i^{(c)} \left[D_{i,k|k-1} - \hat{d}_k\right]\left[D_{i,k|k-1} - \hat{d}_k\right]^T + R_k^e \\ P_{w_k d_k} &= \sum_{i=0}^{2L} W_i^{(c)} \left[\omega_{i,k|k-1} - \hat{w}_k^-\right]\left[D_{i,k|k-1} - \hat{d}_k\right]^T \\ \kappa_k &= P_{w_k d_k} P_{\tilde{d}_k\tilde{d}_k}^{-1} \\ \hat{w}_k &= \hat{w}_k^- + \kappa_k\left(d_k - \hat{d}_k\right) \\ P_{w_k} &= P_{w_k}^- - \kappa_k P_{\tilde{d}_k\tilde{d}_k} \kappa_k^T, \end{aligned} \tag{20}$$

where $H(x_k, w) = h(x, w)$. For the measured parameters, the function G can be written as $G = \begin{bmatrix} F_1 & F_2 & H_3 & H_4 & F_5 & F_6 \end{bmatrix}^T$.

By defining the quantities and using Unscented Kalman Filter, the estimated state vector $\hat{x}$, which includes vehicle states and lateral forces of the tires, is obtained. Using the estimated values of the states and using Eqs. (5) and (6), the tires side slip angle and normal forces can be obtained. To apply the so-called algorithm, its parameters must be determined at first.

The optimum value of statistical index β is 2 in the case of Gaussian extension. Substituting values $\alpha = 0.5$ and $\kappa = 3 - L$ in Eq. (18), the sigma points can be determined [21].

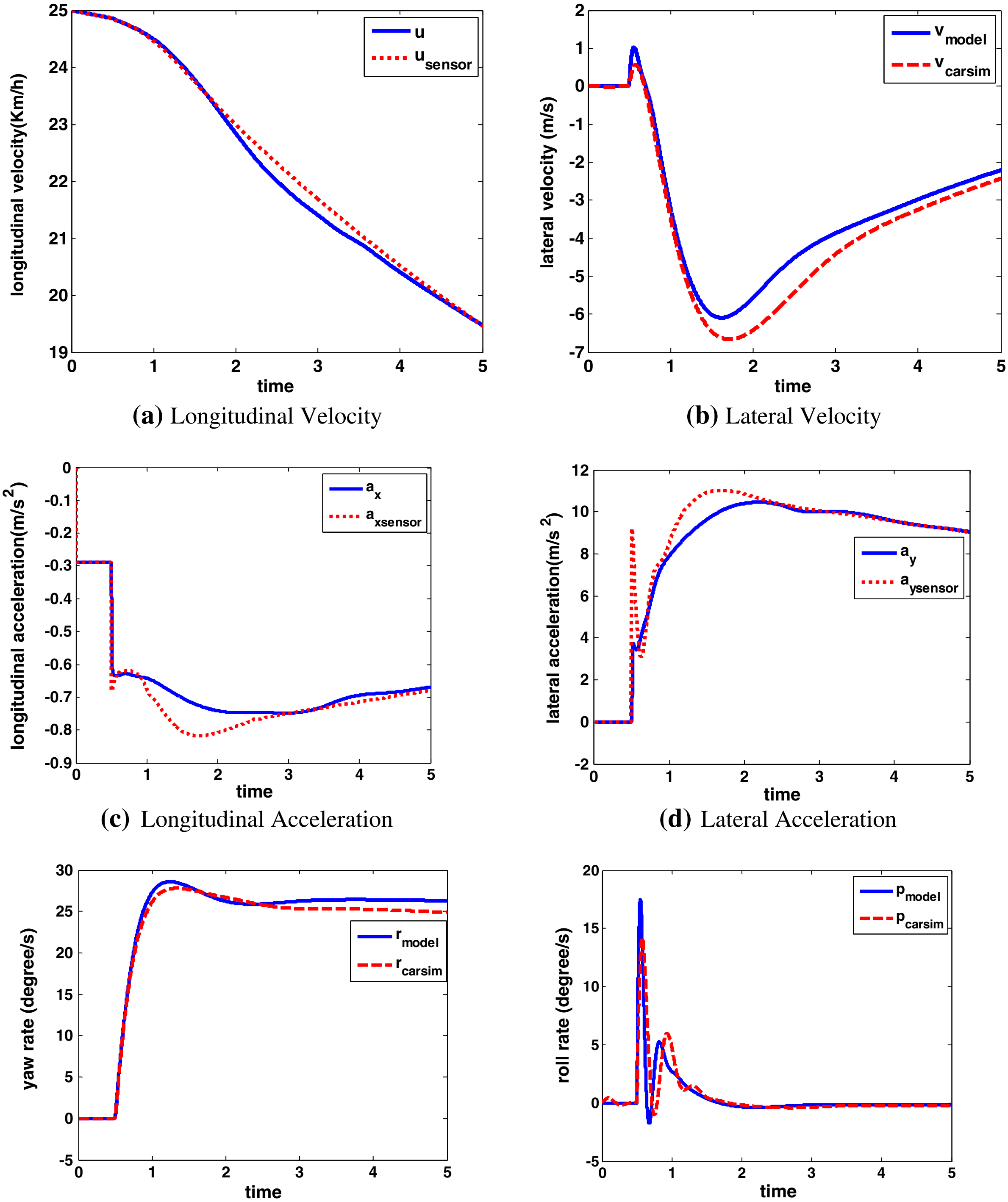
For system noises, $R = 10^{-6} I_{L\times L}$ and the measured noise is considered as

$$R^e = R^n = \mathrm{diag}\left[s_u^2 \quad s_v^2 \quad s_{a_x}^2 \quad s_{a_y}^2 \quad s_r^2 \quad s_{\dot{\phi}}^2\right], \tag{21}$$

where S_i is the variance of the measured error for signal i, which is considered 0.1 for optic sensors, 0.01 for accelerometers and 0.005 for gyro sensors [19]. To

Table 1 Pacejka tire parameters for front and rear tires

	a_0	a_1	a_2	a_3	a_4	a_6	a_7	a_9	a_{10}	a_{12}	a_{13}
Front tires	1.2	−3.9	2500	3730	38.5	0.1	1	0	0	−3.2	7.1
Rear tires	2	−1.6	2000	1700	11	0.1	1.7	−0.2	−0.35	−2	16.3



(a) Longitudinal Velocity **(b)** Lateral Velocity

(c) Longitudinal Acceleration **(d)** Lateral Acceleration

(e) Yaw Rate **(f)** Roll Rate

Fig. 9 Comparison of estimated states values and their corresponding values in CarSim

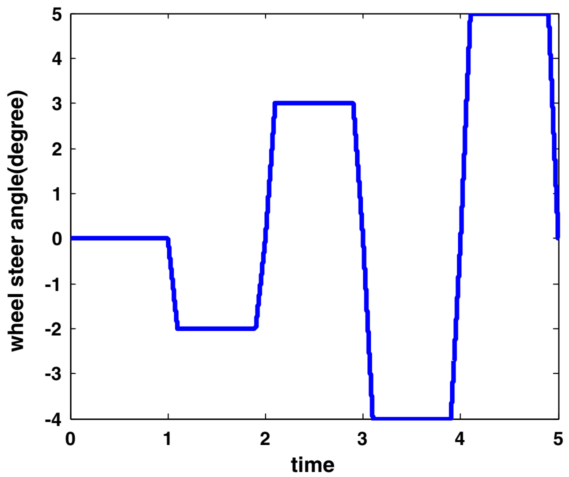


Fig. 10 Arbitrary maneuver

calculate R_k^r, the Robbins–Monro stochastic approximation method in equation (22) will be used whose primary value is as (23)

$$R_k^r = (1-\alpha)R_{k-1}^r + \alpha\kappa_k(d_k - G(x_k, \hat{w}_k)) \times (d_k - G(x_k, \hat{w}_k))^T \kappa_k^T. \tag{22}$$

$$R_0^r = \mathrm{diag}\left[\, S_{D_f}^2 \quad S_{B_f}^2 \quad S_{E_f}^2 \quad S_{\sigma_f}^2 \quad S_{Dr}^2 \quad S_{B_r}^2 \quad S_{E_r}^2 \quad S_{\sigma_r}^2 \,\right]. \tag{23}$$

The measure of S_i in Eq. (23) is equal to 1 % of the numerical value of the parameter i [22].

If a maneuver like Fig. 2 is being considered for an input to the wheels steering angle, the variation of the estimated parameters can be seen in Fig. 3.

As it is seen in Fig. 3, the estimated values of parameters become steady after a while and since the identification method that is being used is a statistical method, the simulations are repeated in order to reach a more precise response. In this way, the final value obtained for any parameter is considered as the primary value for the next simulation. This process is done repeatedly so that the parameter will not have any sensible variation.

Figures 4, 5 and 6 show the results of this estimation for the calculation of front tires lateral and normal forces and also side slip angles due to lane change input of Fig. 2.

This process is done for step and ramp inputs as Figs. 7 and 8, and the estimation results due to these inputs are obtained.

5 Estimation of Pacejka coefficients

In this part, using hybrid Levenberg–Marquardt method and quasi Newton as in [23,24], the non-linear least squares problem (24) is solved to find the unknown coefficients of Pacejka formula which is showed in Eq. (4).

$$\begin{aligned} &x^* = \arg\min\{E_i\}, \\ &E_{ij} = \sum_k E_{ij,k}, \\ &E_{ij,k} = \tfrac{1}{2}\left\| F_{y_{ij,k}} - \bar{F}_{y_{ij,k}} \right\|^2, \\ &ij = fl, fr, rl, rr. \end{aligned} \tag{24}$$

In Eq. (24), $F_{y_{ij,k}}$ is the lateral tire force obtained from the identification problem, and $\bar{F}_{y_{ij,k}}$ is calculated in Eq. (3) by having the estimated values for normal forces of the tire ij and its side slip angle. The parameters of Pacejka tire for front and rear wheels are demonstrated in Table 1. Note that the effect of roll angle on lateral forces is being ignored, and this means that the values of a_5, a_8, and a_{11} for each tire are considered zero.

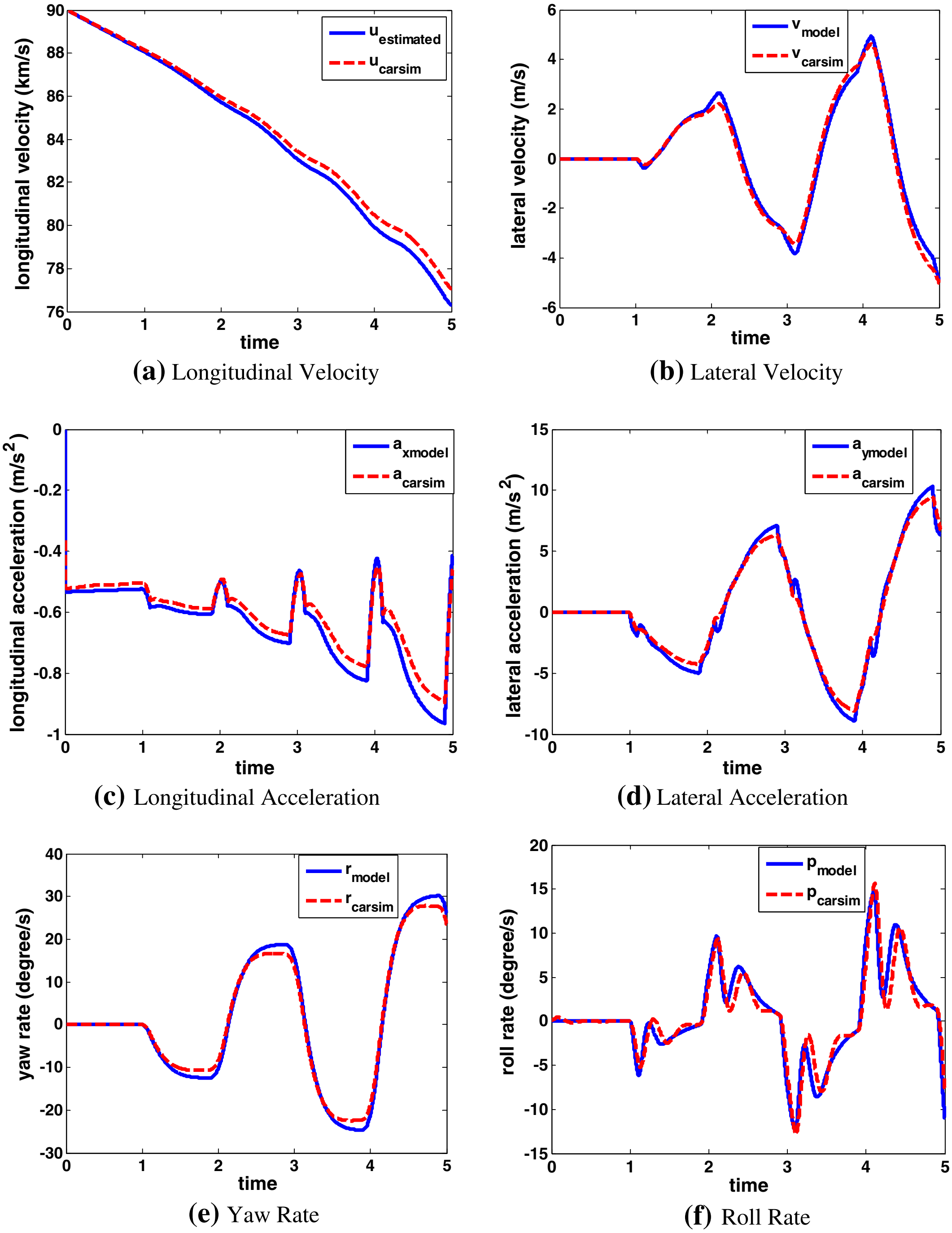
6 Simulations

Having the coefficients of Pacejka formula for front and rear tires and substituting them in Eqs. (3) and (4), the lateral forces applied to tires are obtained according to the value of side slip angle and their normal forces. Substituting these forces in the dynamic system (1), the simulation for different maneuvers can be operated. Figure 9 shows the results of this simulation for Fig. 8 input and their comparison to their corresponding values which come from the sensors (CarSim data are being used as sensor data in this paper).

Also, if an arbitrary maneuver other than the maneuvers which operated the identification process is being considered for front tires steering angle as in Fig. 10, the simulation results can be seen in Fig. 11.

If the value of the estimation error is considered as $e_z = 100 \times \frac{|z - z_{\mathrm{measure}}|}{\max|z_{\mathrm{measure}}|}$ in which z is the estimated value and z_{measure} is the measured value, then the mean and the variance of the error for step and arbitrary maneuvers are demonstrated in Tables 2 and 3, respectively.

The simulation results and studying the errors show the high precision of the simulation and the estimation in this part, and this means that the derived model for



(a) Longitudinal Velocity **(b)** Lateral Velocity

(c) Longitudinal Acceleration **(d)** Lateral Acceleration

(e) Yaw Rate **(f)** Roll Rate

Fig. 11 Comparison of estimated states values and their corresponding values in CarSim

Table 2 Mean and variance of errors for step input

State parameters of error e_z	u	v	a_x	a_y	r	$\dot{\phi}$
Error mean	0.179	3.99	0.966	3.585	3.422	2.318
Variance	0.106	3.61	2.14	2.268	2.032	5.82

Table 3 Mean and variance of errors for arbitrary input

State parameters of error e_z	u	v	a_x	a_y	r	$\dot{\phi}$
Error mean	0.0124	4.276	0.964	3.531	3.596	5.14
Variance	0.009	2.69	1.156	2.352	1.86	4.707

the lateral tire forces is an appropriate model to apply in vehicle dynamic simulation.

7 Conclusions

In this paper, in order to identify the tire forces, a dynamic vehicle model and a set of measured signals by sensors (CarSim Software) are used, and the tire forces are identified using Dual Kalman Filter algorithm. In this algorithm, kinematic parameters of vehicle are estimated simultaneously with identification process by which normal tire forces and tires side slip angles can be calculated. Since Pacejka tire model is one of the models which is very common today in vehicle dynamic modeling and determining its parameters needs many and expensive experiments, so using the data of tire identification forces and fitting them on Pacejka tire curves results in obtaining the coefficients. The reason of using Dual Unscented Kalman Filter for the identification process is that in addition to high precision, it can filter very well the measured data and system noises. So, the fitting process of hybrid Levenberg–Marquardt and quasi Newton algorithm performs properly which makes the coefficient without disturbance.

Having the tire model and also using it in dynamic models, the vehicle motion can be simulated and the vehicle parameters can be calculated. The simulation results for different driving maneuvers and comparing the obtained kinematic parameters to their corresponding parameters from CarSim have little difference, and this shows the high precision of identification and estimation process in this paper. Thus, the provided model is an appropriate model for further investigations in the vehicle dynamic and control field.

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