Repetition

Simulation of discrete-time Markov chains

Input:

- i_0: starting state
- $\mathbf{P}$: transition probability matrix
- T: number of time steps

Algorithm:

1. Set $x_0 = i_0$
2. for $n = 1 \dots T$
3. Simulate x_n from $X_n | X_{n-1} = x_{n-1}$
4. end

Output: One realization $x_0, x_1, \dots, x_T$.

Definition

For a Markov chain, a state i such that $\mathrm{P}_{ij} = 0 \ \forall j \neq i$ is called **absorbing**.

Theorem

Let $\{X_n\}$ be a discrete-time Markov chain with state space $S = \{0, 1, \dots, N\}$ and transitition probability matrix $\mathbf{P}$. Let $A \subset S$ be the set of absorbing states. Then

1. If u_i is the probability of absorption in state $j \in A$ conditional on $X_0 = i$, then

$$\begin{aligned} u_i &= 1, \quad i = j, \\ u_i &= 0, \quad i \in A, i \neq j, \\ u_i &= P_{ij} + \sum_{k \in A^{\mathrm{C}}} P_{ik} u_k, \quad i \in A^{\mathrm{C}}. \end{aligned}$$