Repetition

Definition

The stochastic variable X has a **Poisson distribution** with (mean) parameter $\mu > 0$ if

$$p(x) = \frac{\mu^x}{x!}\mathrm{e}^{-\mu}, \quad x = 0, 1, \ldots.$$

We write $X \sim \text{Poisson}(\mu)$.

Note: $\mathrm{E}[X] = \mu$ and $\mathrm{Var}[X] = \mu$.

Theorem 5.1

If $X \sim \text{Poisson}(\mu)$, $Y \sim \text{Poisson}(\nu)$, and X and Y are independent, then

$$X + Y \sim \text{Poisson}(\mu + \nu).$$

Theorem 5.2

If $N \sim \text{Poisson}(\mu)$ and $M|N \sim \text{Binomial}(N, p)$, then

$$M \sim \text{Poisson}(\mu p).$$

Definition

A **Poisson process** with **rate (intensity)** $\lambda > 0$ is an integer-valued stochastic process $\{X(t) : t \geq 0\}$ for which

1. for any integer $n > 1$ and any time points $0 = t_0 < t_1 < \cdots < t_n$, the increments

$$X(t_1) - X(t_0), X(t_2) - X(t_1), \ldots, X(t_n) - X(t_{n-1})$$

   are independent.

2. for $s \geq 0$ and $t > 0$,

$$X(s+t) - X(s) \sim \text{Poisson}(\lambda t).$$

3. $X(0) = 0$.

Note: We have $X(t) \sim \text{Poisson}(\lambda t)$.
Note 2: We have $\mathrm{E}[X(t)] = \lambda t$ and $\mathrm{Var}[X(t)] = \lambda t$.

Definition

An **inhomogeneous Poisson process** with **rate (intensity)** $\lambda(t) \geq 0$, $t \geq 0$, is an integer-valued stochastic process $\{X(t) : t \geq 0\}$ for which

1. for any integer $n > 1$ and any time points $0 = t_0 < t_1 < \cdots < t_n$, the increments

$$X(t_1) - X(t_0), X(t_2) - X(t_1), \ldots, X(t_n) - X(t_{n-1})$$

   are independent.

2. for $s \geq 0$ and $t > 0$,

$$X(s+t) - X(s) \sim \text{Poisson}\left(\int_s^{s+t} \lambda(y)\, \mathrm{d}y\right).$$

3. $X(0) = 0$.