

# DD2459 Software Reliability

## Lecture 2

Glass-box Testing and  
Glass-box Coverage  
(see Amman and Offut, Chapter 2,  
also my [online lecture notes](#) )

Part 1 Control Flow Coverage

# Glass-box Testing:

## Basic Idea

- An error may exist at one or more **locations**
  - Line numbers
  - Boolean tests
  - Expressions etc.
- *If tests don't exercise those locations then errors can never be observed*
- So identify and exercise locations
- No loops – **finitely many locations** – good!
- Loops – **infinitely many locations** – bad!
- Loops + branches – **exponential growth** in locations with loop depth – **very bad !!!!**

# Glass-box Testing:

## Definition

*Glass box* or *structural testing* is the process of exercising software with test scenarios written from the source code, not from the requirements.

Usually structural testing has the goal to exercise a minimum collection of (combinations of) locations.

How many locations and combinations are enough?

# Coverage

- *The size of a test suite is an unreliable indicator of the work achieved by testing.*
- Coverage gives a measure of the amount of testing work achieved (c.f. *energy* in physics).
- *“Enough” testing is defined in terms of coverage rather than size.*
- A major advantage of structural testing is that coverage can be easily and accurately defined.
- Structural coverage measures

# Problems of Glass-Box Testing

- What about **sins of omission**?
- Missing code = no path to go down!  
**Unimplemented requirements!**
- What about **dead code** – is a path possible?
- How to avoid **redundant testing**?
- What about testing the **user requirements**?
- How to handle **combinatorial explosion** of locations and their combinations?

# Problems of Requirements-based Testing

- A test set that meets requirements coverage is not necessarily a **thorough** test set
- Requirements may not contain a **complete and accurate** specification of all code behaviour
- Requirements may be **too coarse** to assure that all implemented behaviours are tested
- Requirements-based testing alone cannot confirm that code doesn't include **unintended functionality**. **Need structural testing too!**

# Coverage Model Type 1:

## Control flow

- Model the **flow of control** between statements and sequences of statements
- Examples: *node coverage, edge coverage.*
- Mainly measured in terms of statement invocations (line numbers).
- Exercise main flows of control.
- Oldest and most common method

# Coverage Model Type 2:

## Logic

- Analyse the **influence** of all Boolean variables
- Examples: *predicate coverage, clause coverage, MCDC* (FAA DO178B)
- Exercise Boolean variables at control points.
- Modern method, and increasingly common

# Coverage Model Type 3:

## Data Flow

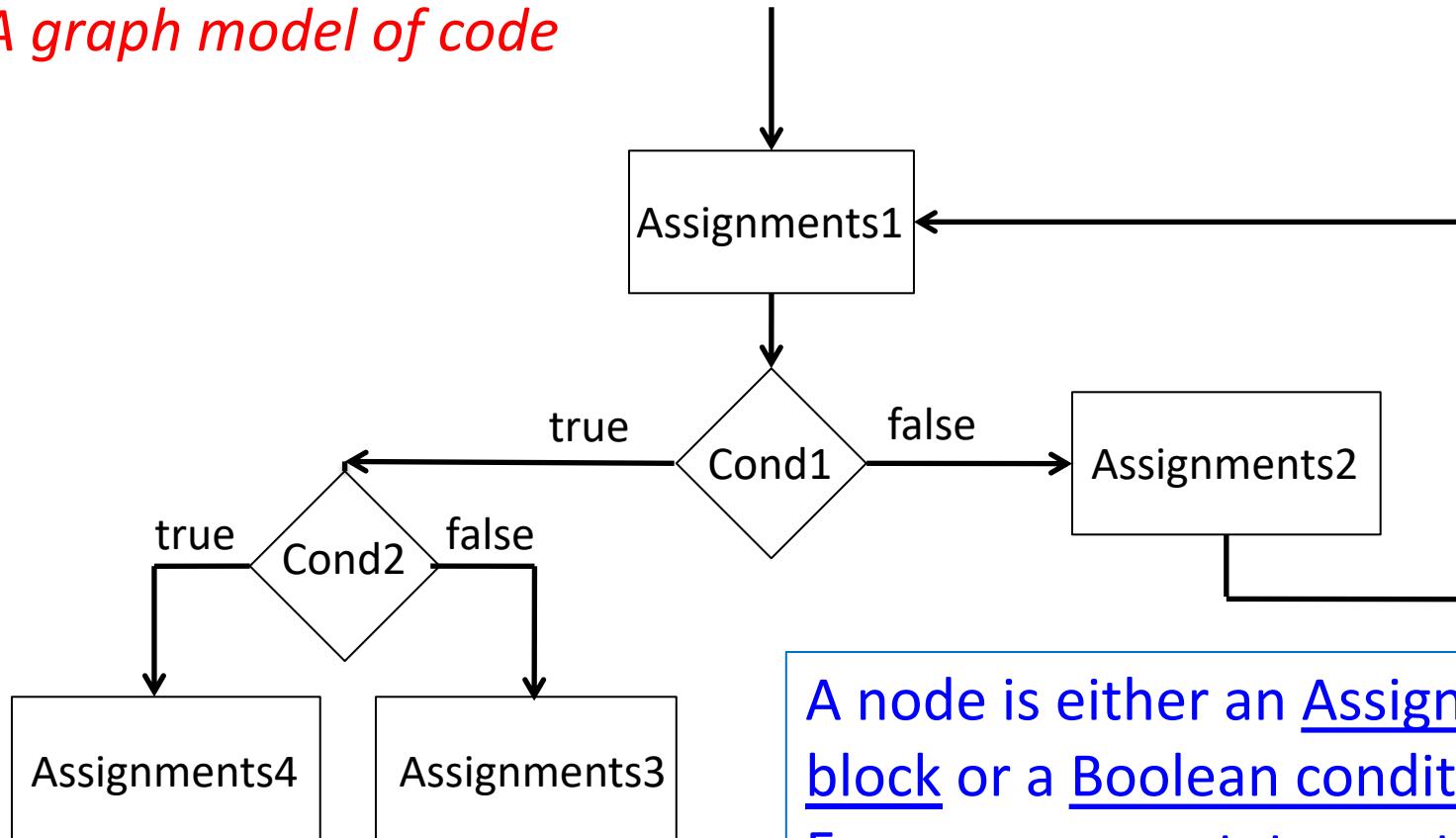
- Data flow criteria measure the **flow of data between variable assignments (*writes*) and variable references (*reads*)**.
- Examples: *all-definitions, all-uses*
- Exercise **paths** between definition of a variable and its subsequent use.
- Still rather rare in industry

# Glass-Box Test Requirements

- Generated by a coverage model
- Requirements on input values to satisfy a property: either
  - Graph-theoretic property (**control & data flow**)
  - data constraint (**logic**)
- Easy to define using graph theory
- Possible to automate generation by **constraint solving**
- Easy to measure **coverage!**
- Oracle is usually just crash (**fail**)/no crash(**pass**)
- Therefore they ignore **functionality!**

# Starting Point: a Condensation Graph

*A graph model of code*



A node is either an Assignment block or a Boolean condition. Every program statement is associated with exactly one node.

# Graph Components

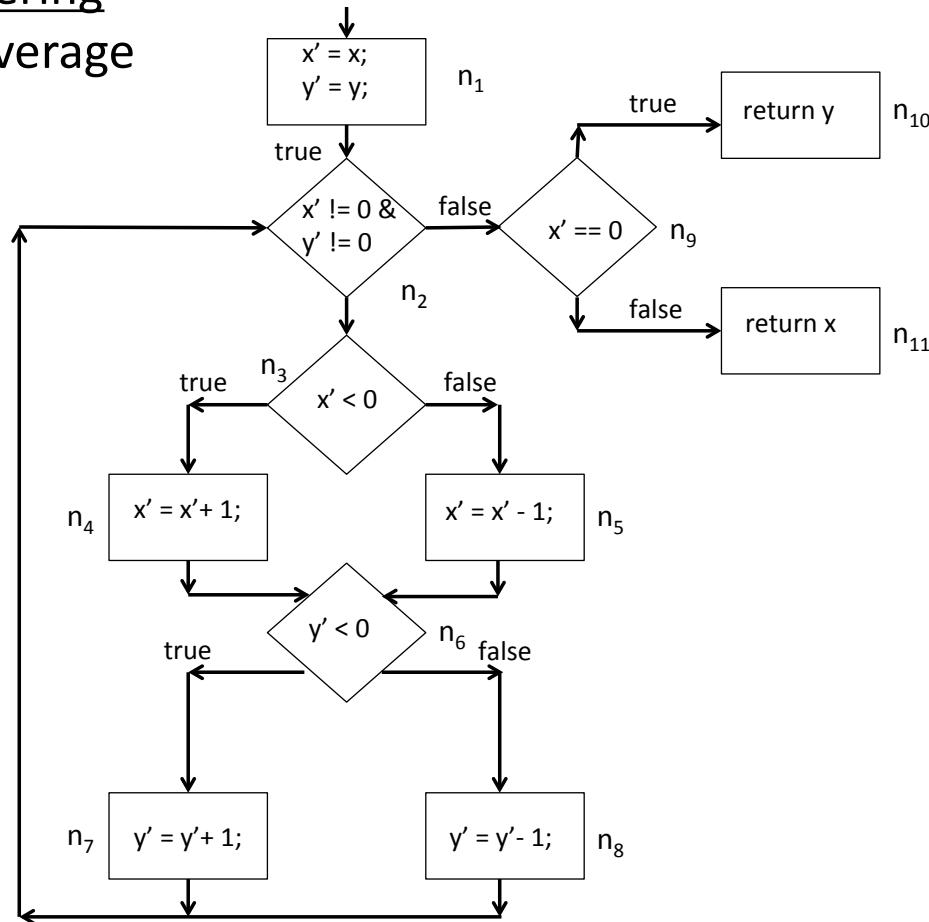
- A Boolean condition models:
  - If-then-else statements `if (bexp) then .. else ..`
  - If statements `if (bexp) ...`
  - Loops (of any kind) `while (bexp) ...`
- An Assignment block contains consecutive
  - assignments `x = exp`
  - return statements `return exp`
  - procedure/method calls `myFunc(...)`
  - expressions e.g. `i++`

# Example Code

```
1. x' = x;  
2. y' = y;  
3. while (x' != 0 & y' != 0) do {  
4.     if (x' < 0) then x' = x' + 1 else x' = x' - 1;  
5.     if (y' < 0) then y' = y' + 1 else y' = y' - 1;  
6. }  
7. if (x' == 0) then return y else return x;
```

# Corresponding Graph

Notice the node numbering  
which is needed for coverage  
definition



# Types 1 and 3 Coverage

- A **path** is a sequence of nodes  $n_0, \dots, n_k$  in a (condensation) graph  $G$ , such that each adjacent node pair,  $(n_i, n_{i+1})$  forms an edge in  $G$ .
- For type 1 and type 3 testing, a **test requirement**  $tr(\cdot)$  is a path

# Satisfying a Coverage Model

**Definition:** Let  $TR$  be a set of test requirements demanded by a coverage model  $C$ . A **test suite  $TC$  satisfies  $C$  on graph  $G$**  if, and only if for every test requirement  $p = n_0, \dots, n_k \in TR$ , there is at least one test case  $t$  in  $TC$  such  $t$  takes path  $p$ .

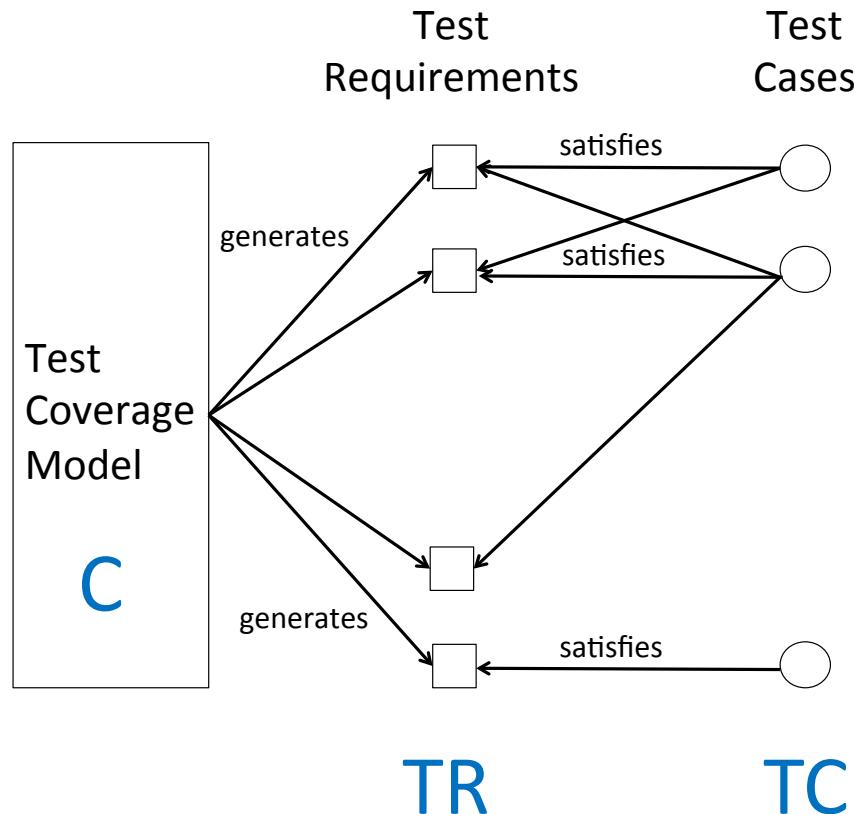
- Implicitly assumes 100% coverage is possible.
- Maybe only < 100% is achievable?

# Why so Formal?

## Answers:

1. Sometimes coverage properties become very technical to define for reasons of accuracy.
2. Precise definitions can be automated to **measure test coverage**.
3. Precise definitions can be automated to make **test case generation tools**.

# Relationship between Coverage Model, Test Requirements and Test Cases



# Type 1 Examples:

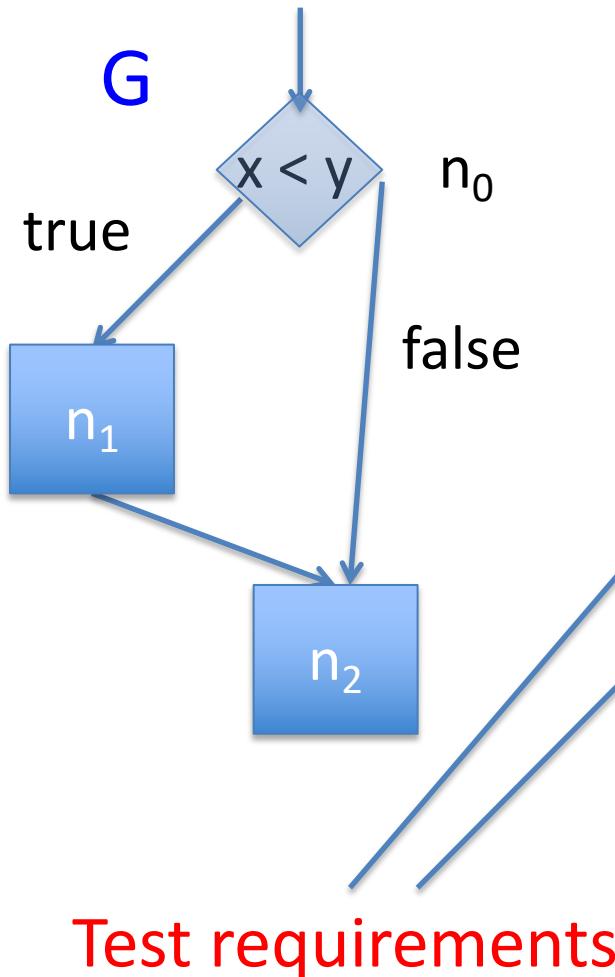
## Control Flow Coverage

2.1. **Node Coverage (NC)** Each **reachable** path  $p$  of length 1 in  $G$  is a test requirement  $p \in \text{TR}_{\text{NC}}(G)$ .

Myers : “NC is so weak that it is generally considered useless”

2.2. **Edge Coverage (EC)** Each **reachable** path  $p$  of length 2 in  $G$  is a test requirement  $p \in \text{TR}_{\text{EC}}(G)$

# Node vs. Edge Coverage



`if ( x < y ) then n1; n2`

$p_1 = n_0, n_1, n_2$

$p_2 = n_0, n_2$

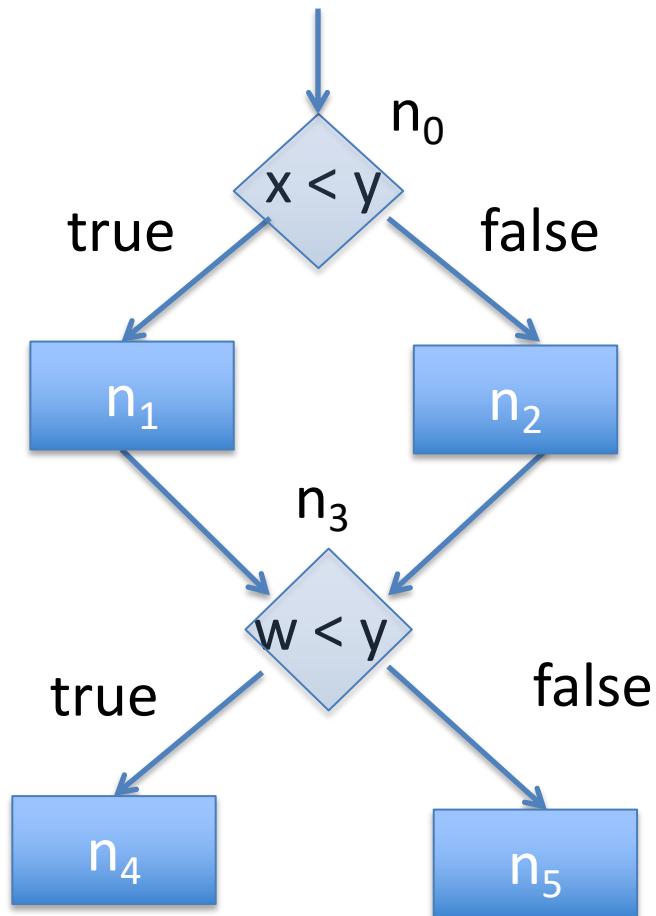
$TR_1 = \{p_1\}$  gives  
100% node coverage  
for  $G$

$TR_2 = \{p_1, p_2\}$  gives 100%  
edge coverage for  $G$

2.3 **Edge-Pair Coverage (EPC = EC<sup>2</sup>)** Each  
reachable path  $p$  of length  $\leq 3$  in  $G$  is a test  
requirement  $p \in \text{TR}_{\text{EC}^2}(G)$ .

- Clearly we can continue this beyond 1,2 to  $\text{EC}^n$
- Combinatorial explosion in TR size!
- $\text{EC}^n$  doesn't deal with *loops*, which have unbounded length.

```
if ( x < y ) then n1 else n2;
if ( w < y ) then n4 else n5;
```



$$p_1 = n_0, n_1, n_3, n_4$$

$$p_2 = n_0, n_2, n_3, n_5$$

$$p_3 = n_0, n_2, n_3, n_4$$

$$p_4 = n_0, n_1, n_3, n_5$$

$TR_1 = \{p_1, p_2\}$  gives 100% edge coverage

$TR_2 = \{p_1, p_2, p_3, p_4\}$  gives 100% edge-pair coverage

# Simple Paths

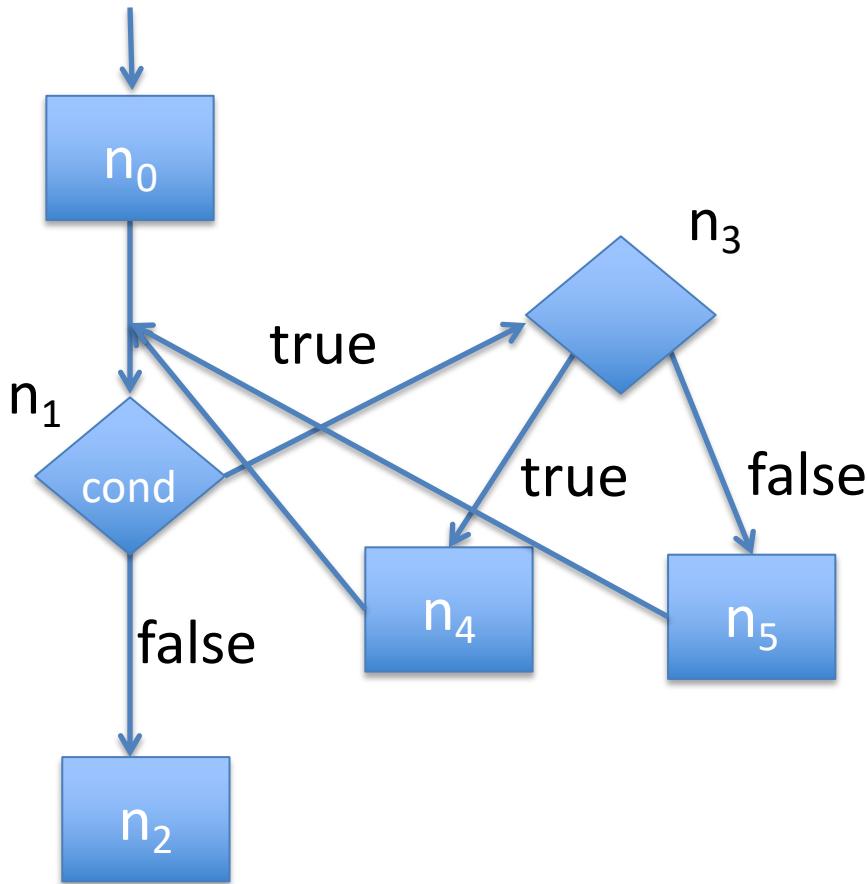
- How to deal with code loops?
- A path  $p$  is **simple** if it has no repetitions of nodes other than (possibly) the first and last node.
- *So a simple path  $p$  has no internal loops, but may itself be a loop*
- **Problem:** there are **too many** simple paths, since many are just sub-paths of longer simple paths.

# Prime Paths

- A path  $p$  is **prime** iff  $p$  is a maximal simple path  
i.e.  $p$  cannot be extended without losing  
simplicity.
- This cuts down the number of cases to consider

2.4. **Prime Path Coverage (PPC)** Each reachable  
prime path  $p$  in  $G$  is a test requirement:

$$p \in TR_{PPC}(G)$$



Prime Paths =  
*Maximal simple paths*  
includes  
 $(n_0, n_1, n_2),$   
 $(n_1, n_3, n_4, n_1), (n_1, n_3, n_5, n_1),$   
 $(n_3, n_4, n_1, n_3), (n_3, n_5, n_1, n_3),$   
 $(n_4, n_1, n_3, n_4), (n_4, n_1, n_3, n_5),$   
 $(n_5, n_1, n_3, n_4), (n_5, n_1, n_3, n_5),$

**Exercise:** complete the set  
 above with any missing  
 maximal simple paths.

# Computing Prime Paths

- One advantage is that the set of all prime paths can be computed by a simple dynamic programming algorithm
- See Amman and Offut Chpt. 2 for details
- Then test cases can be derived manually (heuristic: start from longest paths?) or automatically.

**2.7. Complete Path Coverage (CPC)** Every reachable path in  $G$  is contained in some path  $p \in TR$ .

Infeasible if  $G$  has infinitely many paths

**2.8. Specified Path Coverage (SPC)** Every reachable path in a set  $S$  of test paths is contained in some path  $p \in TR$ . Here  $S$  is supplied as a **parameter**.

**Example heuristic.**  $S$  contains paths that traverse every **loop free path**  $p$  in  $G$  and every **loop** in  $G$  exactly **1** times.

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**Part 2 Logic and Data Flow Coverage**

## Type 2 Examples:

### Logic Coverage

- Graph and data flow coverage force execution of certain paths (branches) through code.
- They don't necessarily exercise **different ways of taking the same branch.**
- We can **partition** the number of ways to be finite and coverable.
- For this, we consider **exercising Boolean conditions in different ways.**

# Clauses and Predicates

- A **clause** is a Boolean valued expression with no Boolean valued sub-expression (i.e. **atomic**)
- Examples:  $p$ ,  $\text{myGuard}$ ,  $x==y$ ,  $x\leq y$ ,  $x>y$
- A **predicate** is a Boolean combination of clauses (i.e. **compound**) e.g.  $\&$ ,  $\text{I}$ ,  $!$ ,
- Let  $P$  be a set of predicates
- For  $p \in P$ , let  $C_p$  be the set of all clauses in  $p$ .

# Type 2 Coverage

- For logic coverage, a test requirement **tr** is a logical constraint on input data values

# Satisfying a Logic Coverage Model

**Definition:** Let  $TR$  be a set of test requirements demanded by a logic coverage model  $C$ . A **test suite  $TC$  satisfies  $C$**  if, and only if for every test requirement  $r \in TR$ , there is at least one test case  $t$  in  $TC$  such  $t$  satisfies  $r$ .

- Again assumes 100% coverage.
- Maybe can only achieve  $< 100\%$ ?

# Logic Coverage Models

- Look at some well known coverage models
- Increasingly sophisticated and subtle,
- Powerful for exactly these reasons!
- Should produce better testing results?
- Since a test requirement is a constraint, it may not be solvable i.e. **dead code**

# Predicate Coverage

- **3.12 Predicate Coverage (PC)** For each predicate  $p \in P$ , the set TR contains: (1) a requirement that implies  $p$  is reached and evaluates to **true**, and (2) a requirement that implies  $p$  is reached and evaluates to **false**.
- Example:  $p = a \mid b$
- TR1  $p = \text{true}$ ,  $TC1 = (a = T, b = F)$
- TR2  $p = \text{false}$ ,  $TC2 = (a = F, b = F)$
- Notice here we never test for  $b = T$

# Clause Coverage

- **3.13 Clause Coverage (CC)** For each predicate  $p \in P$ , and each clause  $c \in C_p$  the set  $TR$  contains: (1) a requirement that implies  $c$  is reached and evaluates to **true**, and (2) a requirement that implies  $c$  is reached and evaluates to **false**.
- Example:  $p = a \mid b$  Satisfy Coverage
- $TR1 \ a = \text{true}, \quad TC1 = (a = T, b = F)$
- $TR2 \ a = \text{false}, \quad TC2 = (a = F, b = T)$
- $TR3 \ b = \text{true}, \quad TC3 = TC2$
- $TR4 \ b = \text{false}, \quad TC4 = TC1$
- Notice  $p$  is always **true**, so we satisfy **CC** but not **PC**
- So **PC** and **CC** are **independent coverage criteria**.

# Distributive vs. Non-Distributive

- Note: these definitions of PC and CC are **non-distributive**, i.e. We don't take all combinations of all predicate or clause values. (**Can be unsolvable combinations even without dead code!**)
- **(Non-distributive) PC** – linear growth.
- **(Non-distributive) PC** implies **EC**, but not  **$EC^n$**  for  $n \geq 2$ .
- **Distributive PC** - exponential growth.
- See my online lecture notes for more detail

# Brute Force Approach to Combining PC & CC

- **3.14 Combinatorial Coverage (CoC)** For each predicate  $p \in P$ , and every possible truth assignment  $\alpha$  to the clauses  $C_p$  of  $p$  the set  $TR$  contains a requirement which implies  $p$  is reached and the clauses evaluate to  $\alpha$ .
- CoC implies both PC and CC.
- CoC is also called **multiple condition coverage (MCC)**.
- Too strong? Too many test cases? Use less?

# Active Clause Coverage

Example:  $p = a \mid b$

$TC1 = (a = T, b = T)$ ,  $TC2 = (a = F, b = F)$

$(TC1, TC2)$  satisfies both PC and CC

- Effect of  $a$  on its own and  $b$  on its own are never considered.
- Notice  $b = T$  masks the effect of  $a$  (and vice versa)
- But  $b = F$  completely exposes the effect of  $a$  (vice versa)
- We say that  $a$  **determines**  $p$  in this latter case
- **Can we find something *more expressive than PC or CC but less expensive than CoC* which handles this?**
- Use notion of **active clause** which **determines** the overall predicate value

# Determination

**Definition:** Given a clause  $c$  in a predicate  $p$  we say that  $c$  **determines**  $p$  under assignment  $\alpha$  iff changing the value of  $c$  under  $\alpha$  (and only this value) changes the truth value of  $p$ .

The idea is that in some contexts (assignments)  $c$  “**has complete control**” of  $p$ , and we should test this context.

Notice determination is a local property depending only on the truth table for  $p$ .

Below, when  $G1(C) = G1(D) = G1(E) = T$  then  $D$  determines  $P$

When  $R1(C) = R1(E) = F$  then  $D$  again determines  $P$ .

C	D	E	P	
T	T	T	T	G1
F	T	T	T	
T	F	T	F	G2
F	F	T	T	
T	T	F	T	
F	T	F	F	R1
T	F	F	T	
F	F	F	T	R2

# Improved Models

**3.43 Active Clause Coverage (ACC)** For each predicate  $p \in P$  and each clause  $c \in C_p$  which determines  $p$  (under some  $\alpha$ ), the set  $TR$  contains two requirements for  $c$ : (1)  $c$  is reached and evaluates to **true**, and (2)  $c$  is reached and evaluates to **false**.

**Example:** For ACC of clause  $D$  above there are 4 possible pairs of test cases

$(G1, G2)$ ,  $(R1, R2)$ ,  $(G1, R2)$ ,  $(G2, R1)$

# Ambiguity

- ACC can be seen as **ambiguous**.
- Do the other clauses get the same assignment when **c** is true and **c** is false, or can they have different assignments?
- We may not be able to *isolate* individual clauses
- Problems of **masking**, **logical overlap** and **side-effects** (e.g. **variable synonyms**) between clauses
- Consider e.g.  $p = (x > 10) \rightarrow (x > 0)$
- If the first clause is set to true the second can never be false.

# Different Assignments

- **3.15 General Active Clause Coverage (GACC)** For each predicate  $p \in P$  and each clause  $c \in C_p$  which determines  $p$ , the set  $TR$  contains two requirements for  $c$ : (1)  $c$  is reached and evaluates to **true**, and (2)  $c$  is reached and evaluates to **false**. The values chosen for the other clauses  $d \in C_p$ ,  $d \neq c$ , **need not be the same** in both cases.

**Example:** For GACC of clause  $D$  above there are 4 possible pairs of test cases

(G1,G2), (R1,R2), (G1,R2), (G2,R1)

# GACC Problem: Clause Correlation

- One problem is that **GACC** does not imply **PC**.

Consider the predicate  $p = a \leftrightarrow b$

For some  $\alpha$ ,  $a$  determines  $p$  (so does  $b$ ) so let:

TC1:  $a = T$  ,  $b = T$  so  $p = \text{true}$

TC2:  $a = F$  ,  $b = F$  so  $p = \text{true}$

For this test suite  $p$  never becomes **false** so **PC** is not satisfied although **GACC** is!

- Here the correlation between  $a$  and  $b$  is **explicit** in the condition, but it may be **implicit** in the code.

# Same Assignments

## 3.15 Restricted Active Clause Coverage (RACC)

For each predicate  $p \in P$  and each clause  $c \in C_p$  which determines  $p$ , the set  $TR$  contains two requirements for  $c$ : (1)  $c$  is reached and evaluates to **true**, and (2)  $c$  is reached and evaluates to **false**. The values chosen for the other clauses  $d \in C_p$ ,  $d \neq c$ , **must be the same** in both cases.

**Example:** For RACC of clause D above there are 2 possible test suites (G1,G2), (R1,R2).

# Problem: Determination is a global property!

Consider the code

```
x := y;  
...  
if (x>0 or y>0) then ...
```

For predicate  $p = (x > 0 \mid y > 0)$  it looks like PC using RACC is possible from a determinacy analysis of  $(a \mid b)$  where  $a = x > 0$  and  $b = y > 0$ .

But this is misleading, since here  $x$  and  $y$  are effectively synonyms and RACC is not satisfiable at all.

Amman and Offut give a different kind of example, based (essentially) on logical overlap of clauses.

# Combining GACC and PC

(Assignments are not the same,  
but not too different either)

## 3.16 Correlated Active Clause Coverage (CACC)

For each predicate  $p \in P$  and each clause  $c \in C_p$  which determines  $p$ , the set  $TR$  contains two requirements for  $c$ : (1)  $c$  is reached and evaluates to **true**, and (2)  $c$  is reached and evaluates to **false**. The values chosen for the other clauses  $d \in C_p$ ,  $d \neq c$ , must cause  $p$  to be **true** in one case and **false** in the other.

**Example:** For CACC of clause D above there are 2 possible test suites (G1,G2), (R1,R2).

# Safety Critical Testing: MCDC

- In DO-178B, the Federal Aviation Authority (FAA) has mandated a minimum level of logic coverage for level A (highest safety) avionic software.
- “*Modified Condition Decision Coverage*” (MCDC)
- Has been some confusion about this definition
- “*Unique Cause MCDC*” (original definition) is RACC
- “*Masking MCDC*” (new definition) is CACC

# Type 3: Data Flow Coverage

- A **definition** of a variable  $v$  is any statement that writes to  $v$  in memory
- A **use** of  $v$  is any statement that reads  $v$  from memory.
- A path  $p = (n_1, \dots, n_k)$  from a node  $n_1$  to a node  $n_k$  is **def-clear** for  $v$  if for each  $1 < j < k$  node  $n_j$  has no statements which write to  $v$

# Definition/Use Paths (du-paths)

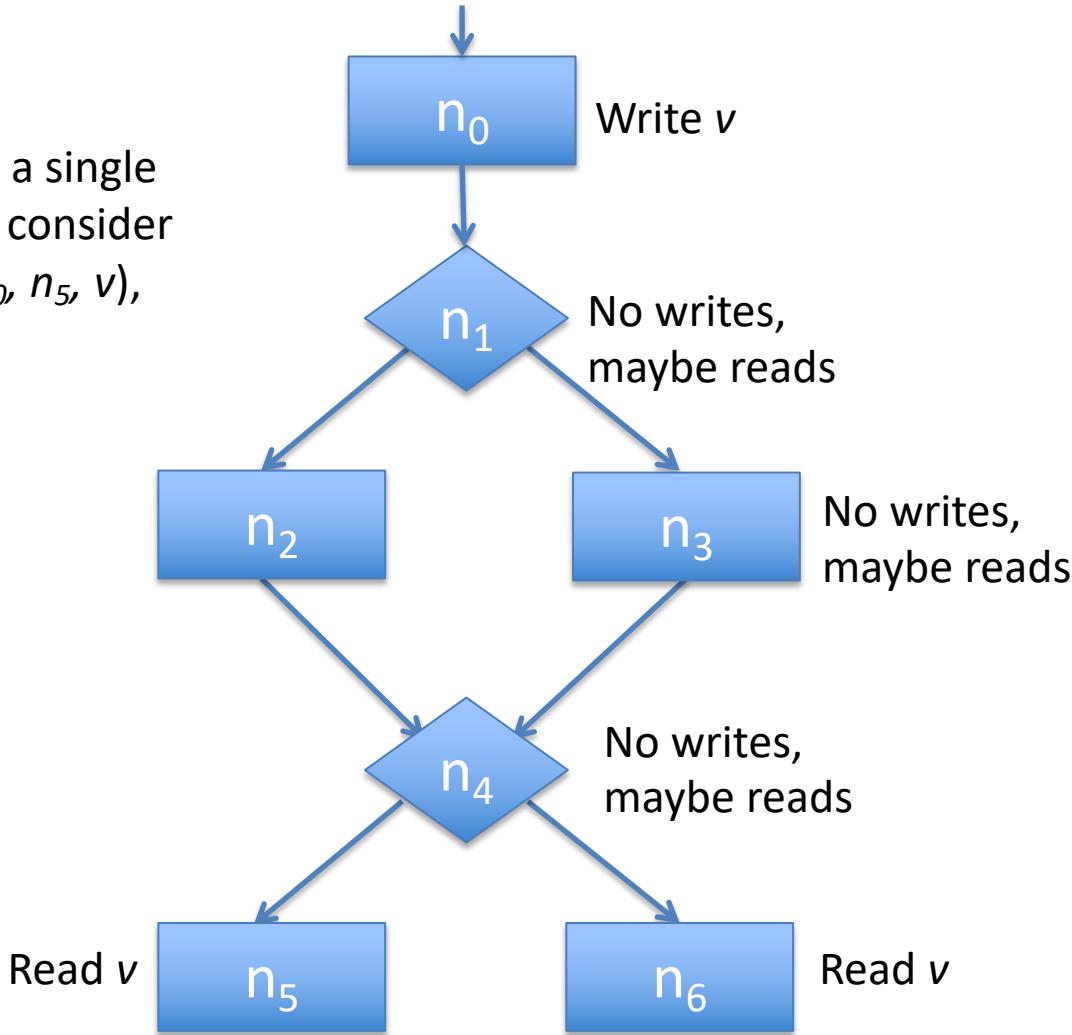
- A **du-path** w.r.t.  $v$  is a simple path
$$p = (n_1, \dots, n_k)$$
- Such that:
  1. A statement in  $n_1$  writes to  $v$
  2. Path  $p$  is def-clear for  $v$
  3. A statement in  $n_k$  reads  $v$
- $du(n, v) =$  set of all du-paths wrt  $v$  starting at  $n$
- $du(m, n, v) =$  set of all du-paths wrt  $v$  starting at  $m$  and ending at  $n$

# Data flow Coverage

## Models

- **2.9. All-defs Coverage (ADC)** For each def-path set  $S = du(n, v)$  the set  $TR$  contains at least one path  $d$  in  $S$ .
- **2.10 All-uses Coverage (AUC)** For each def-pair set  $S = du(m, n, v)$  the set  $TR$  contains at least one path  $d$  in  $S$ .
- **2.11 All-du-paths Coverage (ADUPC)** For each def-pair set  $S = du(m, n, v)$  the set  $TR$  contains every path  $d$  in  $S$ .

Suppose  $G$  has a single variable  $v$ , and consider  $du(n_0, v)$ ,  $du(n_0, n_5, v)$ ,  $du(n_0, n_6, v)$



$$\text{All-defs} = \{(n_0, n_1, n_2, n_4, n_5)\}$$

$$\begin{aligned} \text{All-uses} = & \{(n_0, n_1, n_2, n_4, n_5), \\ & (n_0, n_1, n_2, n_4, n_6)\} \end{aligned}$$

$$\begin{aligned} \text{All-du-paths} = & \{(n_0, n_1, n_2, n_4, n_5), \\ & (n_0, n_1, n_2, n_4, n_6), \\ & (n_0, n_1, n_3, n_4, n_5), \\ & (n_0, n_1, n_3, n_4, n_6)\} \end{aligned}$$

# Lecture Summary

- We have looked at Glass-box testing
- Most commonly used approach in industry
- Compared glass and black-box testing
- Looked at 3 types of coverage
  - Control flow
  - Logic
  - Data flow
- For each type we have seen several models
- Compared model advantages and disadvantages
- Studied in more detail in Lab 1
- Learn which to choose in a given situation.