

σ -Algebras

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Definition 3.4 (σ -Algebra). A family \mathcal{A} of subsets of X with:

- (i) $X \in \mathcal{A}$,
- (ii) $A \in \mathcal{A} \Rightarrow A^c \in \mathcal{A}$,
- (iii) $(A_n)_{n \in \mathbb{N}} \in \mathcal{A} \Rightarrow \bigcup_{n \in \mathbb{N}} A_n \in \mathcal{A}$

Theorem 3.5 (and Definition).

(i) *The intersection of arbitrarily many σ -algebras in X is again a σ -algebra in X .*

(ii) *For every system of sets $p \subset \mathcal{P}(X)$ there exists a smallest σ -algebra containing p . This is the σ -algebra generated by p , denoted $\sigma(p)$, and $\sigma(p)$ is called its generator.*

Definition 3.6 (Borel). The σ -algebra $\sigma(\mathcal{O})$ generated by the open sets $\mathcal{O} = \mathcal{O}_{\mathbb{R}^n}$ of \mathbb{R}^n is called **Borel σ -algebra**, and its members are called **Borel sets** or **Borel measurable sets**.