NATCAR – Background Information

Pulsewidth Modulation (PWM) and DC Motors

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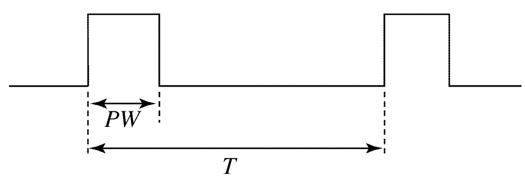
Outline

- Pulse-Width Modulation (PWM)
- Comparator with Hysteresis
- Relaxation Oscillator
- The 555 Timer
- Review of basic electromagnetics
- DC motor operation & modeling

Pulsewidth Modulation (PWM)

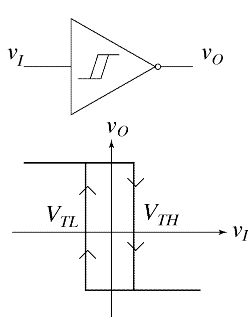
- A PWM signal is frequently used for transmitting information and for motor speed control.
- PWM signals are characterized by their period, *T*, which may or may not vary, their pulsewidth, *PW*, which does vary, and their voltage levels.

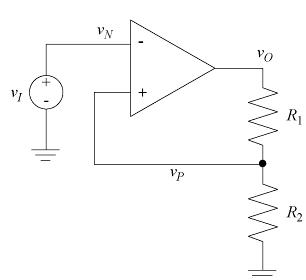


Comparator with Hysteresis - Circuit

To make a relaxation oscillator we need a comparator with hysteresis, as shown below.

This circuit can be made with an opamp:





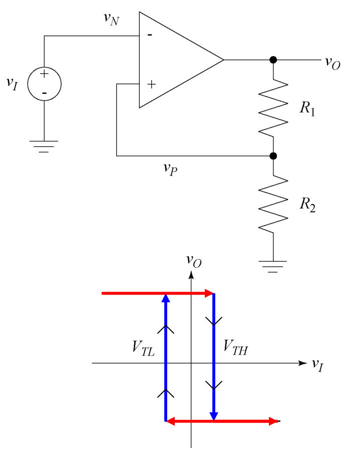
$$v_P = \frac{R_2}{R_1 + R_2} v_O$$
$$= b v_O$$

$$v_O = \begin{cases} V^+ : v_P > v_N \\ V^- : v_P < v_N \end{cases}$$

Comparator with hysteresis

Positive feedback produces hysteresis

Comparator with Hysteresis - Operation



Comparator with hysteresis

To see the hysteresis in this circuit, start with v_O high (so $V_P = bv_O = V_{TH}$) and v_I low, then increase v_I.

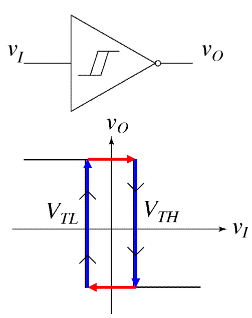
When v_I reaches V_{TH}, the output goes low and V_P changes to V_{TL}.

Further increasing v_I doesn't change v_O.

Now reduce v_I; this time, the threshold is at V_{TL}, and v_O goes high when v_I drops below V_{TL}.

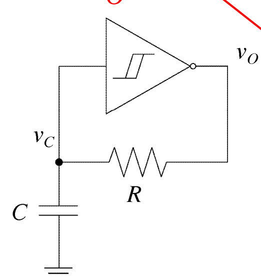
Relaxation Oscillator

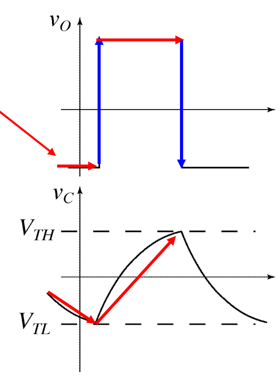
The basic principle of a relaxation oscillator is shown here. A capacitor is alternately charged and discharged between two levels.



Comparator with hysteresis

Start at t = 0- with v_O low:

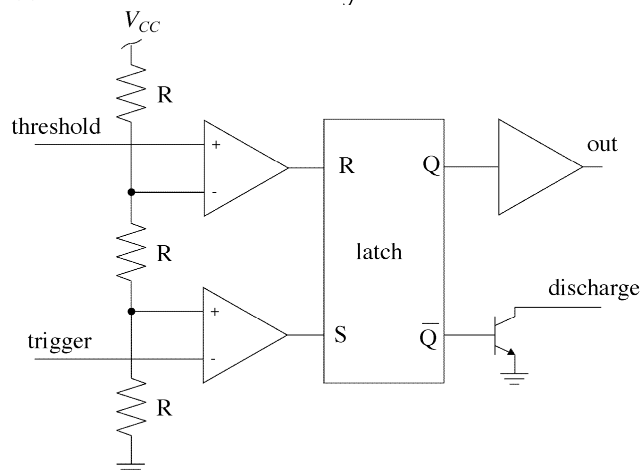




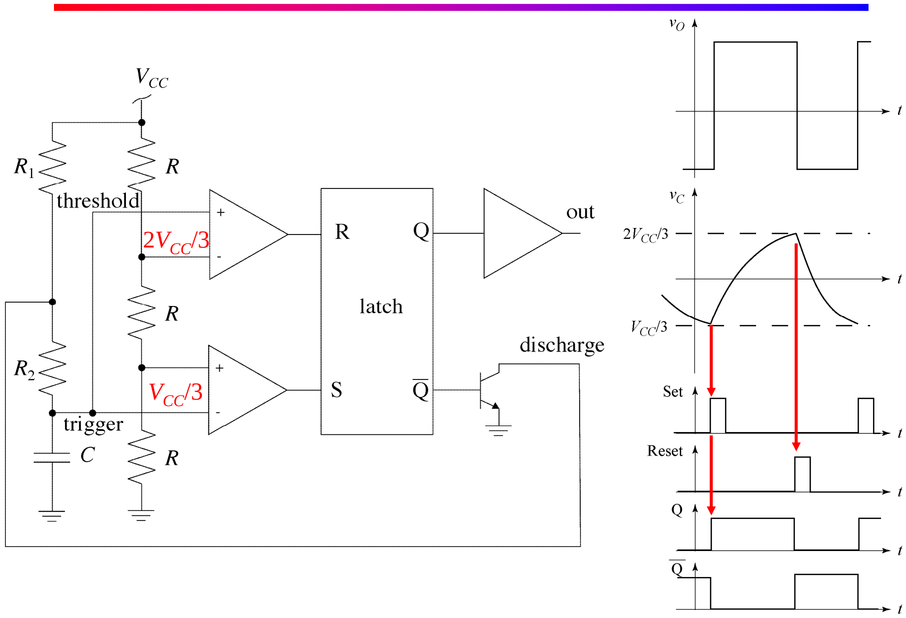
Relaxation oscillator

The 555 Timer

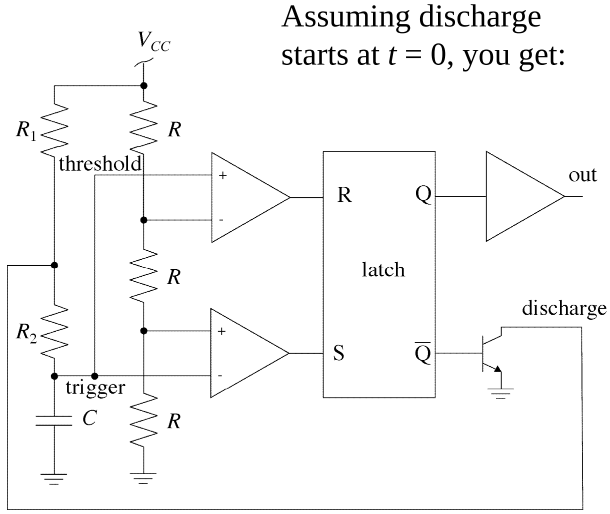
The 555 IC can be used to make an astable multivibrator (oscillator), a monostable (one-shot), a PW modulator and many other useful circuits.



Astable Oscillator with a 555



Astable Oscillator with a 555



Assuming discharge starts at $t = 0$, you get:

$$\frac{2V_{CC}}{3} e^{\frac{-t_L}{R_2 C}} = \frac{V_{CC}}{3}$$

$$t_L = -R_2 C \ln \frac{1}{2}$$

$$t_L = 0.7 R_2 C$$

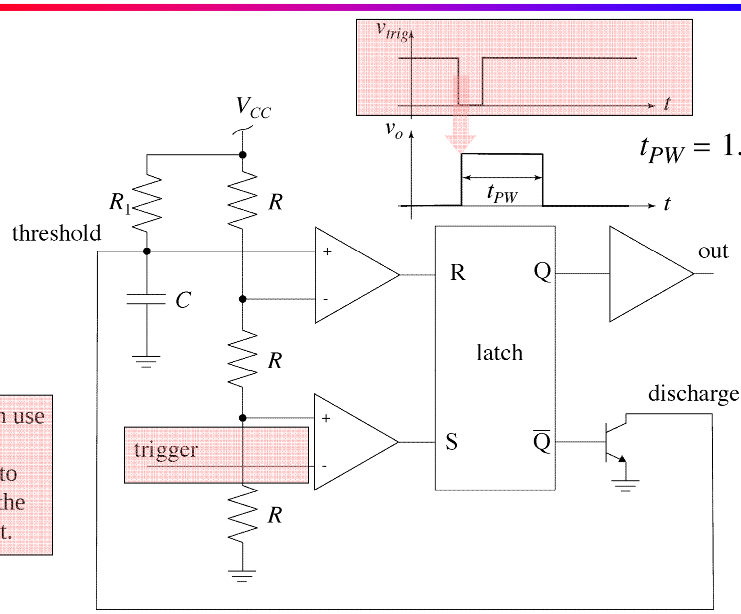
You find t_H in the same way, so we get:

$$t_L = 0.7 R_2 C$$

$$t_H = 0.7 \left(R_1 + R_2 \right) C$$

$$f = \frac{1}{T} = \frac{1}{0.7 \left(R_1 + 2R_2 \right) C}$$

One-Shot with 555



$t_{PW} = 1.1 R_1 C$

You can use a 555 astable to trigger the one shot.

PWM with 555 Timer

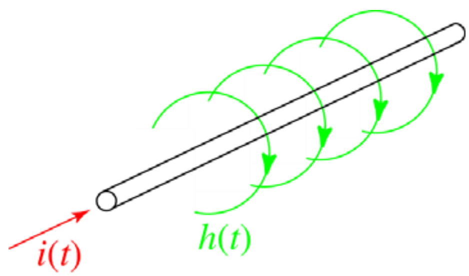
- You can use the 555 one-shot circuit and vary the control voltage, the resistor, or the capacitor to change the pulse width.
- You can also build your own PWM circuit with a linear ramp and a variable threshold comparator. Custom PWM circuits may be more linear and have larger ranges.

PWM for the Steering Servo

- Standard radio-control (RC) car servos use a PWM control signal.
- You can drive them with a fixed or variable period PWM signal, all that matters is the pulse width and the voltage levels.
- Your servos can be supplied by a 5 or 6 volt supply (6 volts makes the servo faster, but > 6 volts will damage it).
- The PWM input signal is 0 to 5 volts. Typically, varying the *PW* from 0.5 ms to 2.5 ms causes the servo to turn lock-to-lock.

Basic Electromagnetics

- We will be dealing with motors and magnetic sensing. Therefore, we need to go over very basic electromagnetics.
- An electric current, *I*, produces a circular magnetic field with a magnetic field intensity, *H*, given by the law of Biot-Savart and the right-hand rule:



$$dH = \frac{\vec{I} d\vec{L} \times \vec{a}_R}{4\pi R^2}$$

Electromagnetics Continued

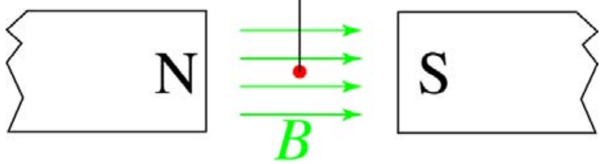
- The magnetic field intensity is analogous to the electric field intensity, *E*. The magnetic flux, Φ, is analogous to the electric current, *I*, and the flux density, *B*, is analogous to current density, *J*. The flux density is $B = \mu H$, where μ is the permeability (similar to conductivity). The flux is $\Phi = B \cdot area$
- The electromotive force, *emf*, is given by Faraday's law and opposes the change that produced it: $emf = - d\Phi/dt$
- The Lorentz force on a charge is

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

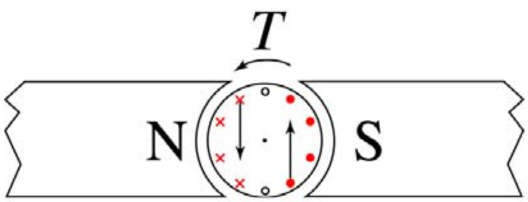
DC Motor Operation

- DC motors use the Lorentz force on a moving charge in a magnetic field. Assume no electric field.

$$\vec{F} = q\vec{v} \times \vec{B}$$



- Multiple wires mounted on a rotor will produce torque.

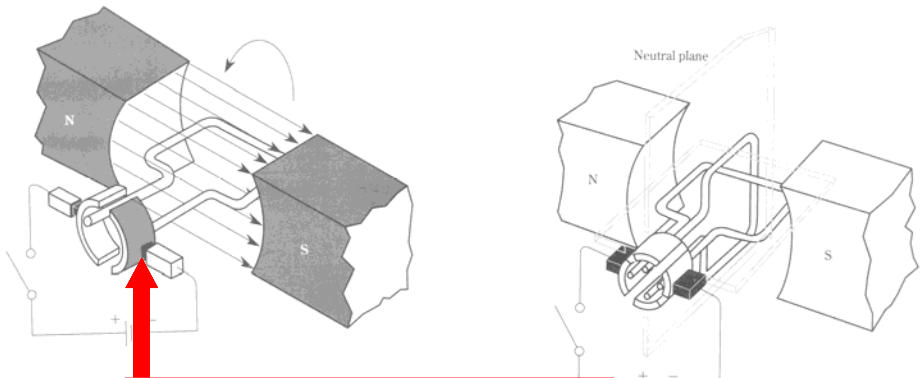


DC Motor Construction

- The magnetic field can be produced by an electromagnet called the field winding, or by a permanent magnet. Our motors are permanent-magnet DC motors.
- Brushes are used to switch the current direction in the armature windings (see picture on next slide). This operation is called commutation.
- The torque produced is proportional to the current in the armature windings, I_A, and the magnetic flux, Φ:

$$T_m = k\Phi I_A$$

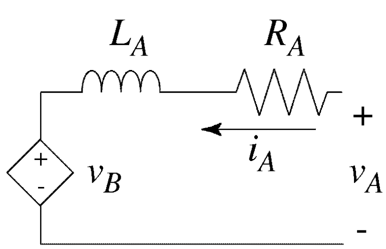
DC Motor Construction Continued



The brushes make contact with the slip ring and change the current direction as needed

Permanent Magnet DC Motor

- As the armature rotates, a back *emf* is produced that is proportional to the flux and the angular velocity. $v_B = k'\Phi\omega_m$
- The armature winding has resistance, inductance and the back *emf*, which leads to the electrical model shown:



$$v_A = v_B + L_A \frac{di_A}{dt} + i_A R_A$$

Permanent Magnet DC Motor Continued

- Newton's second law in rotational form says that the product of the moment of inertia, J_m, and the rotational acceleration, equals the net torque;

$$J_m \cdot \frac{d\omega_m}{dt} = T_m - T_L - b\omega_m$$

where T_m is the motor torque, T_L is the load torque and b is the equivalent damping constant.

- In steady-state operation, the time derivatives are zero and we get

$$V_A = V_B + I_A R_A \qquad \text{and} \qquad T_m = T_L + b\omega_m$$

Remember we had: $T_m = k\Phi I_A$ and $v_B = k'\Phi\,\omega_m$

Permanent Magnet DC Motor Continued

Substitute for the current from the torque equation and use the equation we have for the back *emf* to get

$$V_A = V_B + I_A R_A = k'\Phi\Omega_M + \frac{R_A T_m}{k\Phi}$$

which can be rewritten as $\dfrac{\Omega_M}{\Omega_O} + \dfrac{T_M}{T_S} = 1$

where the no load speed is $\Omega_O = \dfrac{V_A}{k'\Phi}$

and the stall torque is $T_S = \dfrac{V_A k\Phi}{R_A}$

Motor Torque versus Speed

Note:

- That the current is largest when the speed and, therefore, the back *emf* are zero.
- That large stall torque is achieved with small armature resistance $\Rightarrow$ high stall current.
- That both Ω_O and T_S are proportional to V_A.