NATCAR – Background Information
Lecture #1 – Operational Amplifiers

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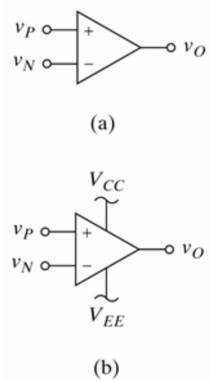
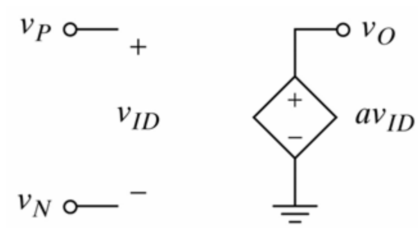
Op Amp Circuits

- Review of basic op amp operation
- Negative feedback & virtual short circuit
- Gain circuits
- Frequency-dependent circuits
- Offsets
- Slew rate and full-power bandwidth

Basic Op Amp Review



The power supply connections are not usually shown (but should be when you build and debug your circuits).

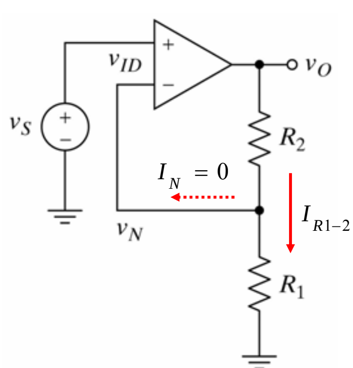


The simplest circuit model is shown above and the output voltage is given by:

$$v_O = a\left(v_P - v_N\right)$$

$$v_O = av_{ID}$$

Non-Inverting Amplifier



Assuming the op amp input currents are zero, R_1 and R_2 have the same current and are in series. Therefore,

$$v_N = \frac{R_1}{R_1 + R_2} v_O \equiv b v_O$$

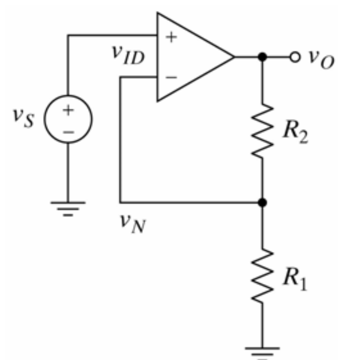
$$v_O = av_{ID} = a\left(v_S - bv_O\right)$$

$$v_O\left(1 + ab\right) = av_S$$

$$v_O = \frac{a}{1+ab} v_S$$

$$\frac{v_O}{v_S} = \frac{a}{1+ab}$$

Non-Inverting Amplifier



Assuming the op amp input currents are zero;

$$v_N = \frac{R_1}{R_1 + R_2} v_O \equiv b v_O$$

feedback factor

$$v_O = a\left(v_S - b v_O\right)$$

$$\frac{v_O}{v_S} = \frac{a}{1 + ab}$$

closed-loop gain

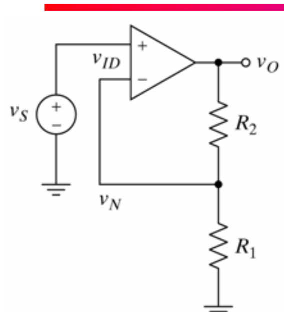
ab = loop gain

Note negative feedback - key element

With an ideal op amp, the gain is infinite, so →

$$\lim_{a \to \infty}\left(\frac{a}{1+ab}\right) = \frac{1}{b} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

Virtual Short Circuit



We had;

1) $v_N = \frac{R_1}{R_1 + R_2} v_O \equiv b v_O$

2) $v_O = a v_{ID} = a\left(v_S - b v_O\right)$

3) $v_O = \frac{a}{1+ab} v_S$

Therefore;

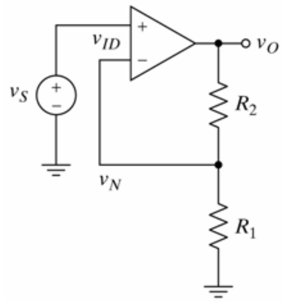
$$v_{ID} = v_S - v_N = v_S - b v_O = v_S\left(1 - \frac{ab}{1+ab}\right) = v_S\left(\frac{1}{1+ab}\right)$$

NOTE:

as $a \to \infty$, $v_{ID} \to 0$ $\Rightarrow$ VIRTUAL SHORT CIRCUIT

A virtual short requires negative feedback and large loop gain.

Virtual Short Circuit



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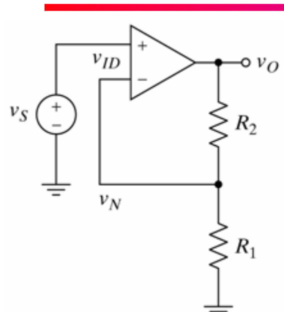
It is a "virtual" short because no current can flow through it.

Virtual Short Circuit



The justification often given for the virtual short circuit (or virtual ground) is:

1) $v_O = a v_{ID}$

2) $a \to \infty$

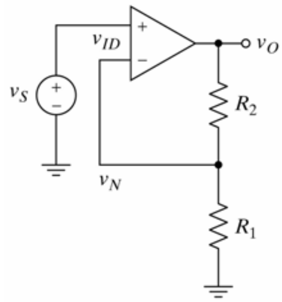
3) v_O must be finite

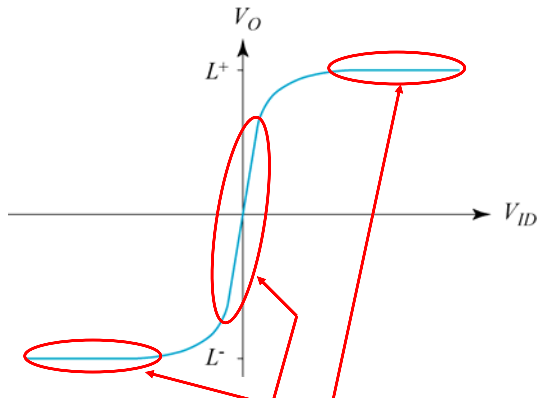
4) $\therefore, v_{ID} \to 0$

This syllogism is not valid! Note that reversing the signs on the opamp inputs would not affect the syllogism, but the result is clearly wrong in that case.

The problem is equivocation; $a \to \infty$ assumes that the opamp operates in its normal region, while saying v_o is finite requires that the output be saturated, and *a* approaches zero there!

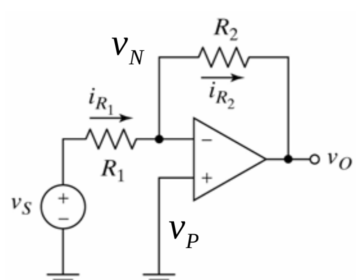
Virtual Short Circuit





The problem is equivocation;
$a \rightarrow \infty$ assumes that the opamp operates here,
saying v_o is finite requires that the output be saturated, and a approaches zero there! So, there is equivocation about the meaning of a.

Inverting Amplifier



Negative feedback and large loop gain imply a virtual short (or virtual ground here), so

$$v_N = v_P = 0 \quad \text{and} \quad i_{R_1} = \frac{v_S}{R_1}$$

No input current implies that

$$i_{R_1} = i_{R_2}$$

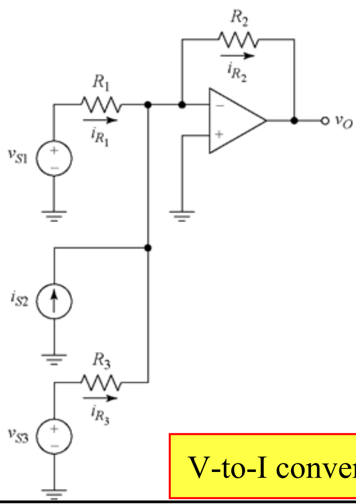
so we obtain,

$$v_O = v_N - i_{R_2} R_2 = \frac{-R_2}{R_1} v_S$$

Note that the virtual ground allows us to think of the operation as a V-to-I conversion followed by an I-to-V conversion.

A Summing Amplifier

- Using this alternate view of the operation allows us to see how to make summing amplifiers:



$$i_{R2} = i_{R1} + i_{S2} + i_{R3}$$

$$v_O = -i_{R2} R_2$$

← I-to-V conversion

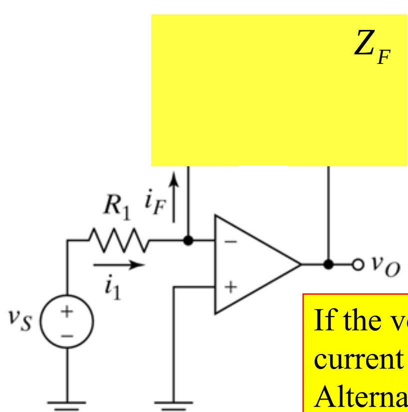
$$v_O = -\left(i_{R1} + i_{S2} + i_{R3}\right) R_2$$

$$v_O = -\left(\frac{v_{S1}}{R_1} + i_{S2} + \frac{v_{S3}}{R_3}\right) R_2$$

V-to-I conversions

A Low-Pass Filter (Integrator)

- The alternate view of the operation also allows us to see how to make low-pass filters (integrators):



$$v_O = -i_F Z_F = -i_1 Z_F$$

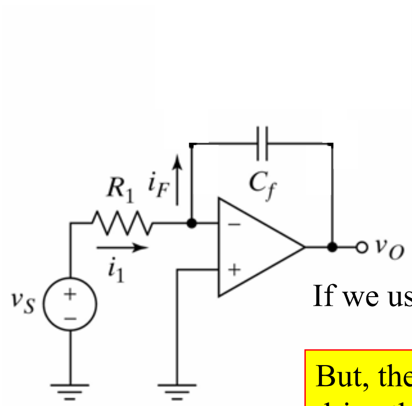
$$i_1 = \frac{v_S}{R_1}$$

Therefore, $$v_O = \frac{-Z_F}{R_1} v_S$$

If the voltage across Z_f is the integral of the current through it, we get an integrator. Alternatively, if we make Z_f have a LP xfer function, we get a LPF. So, what do we use?

A Low-Pass Filter (Integrator)

- The alternate view of the operation also allows us to see how to make low-pass filters (integrators):



$$v_O = -i_F Z_F = -i_1 Z_F$$

$$i_1 = \frac{v_S}{R_1}$$

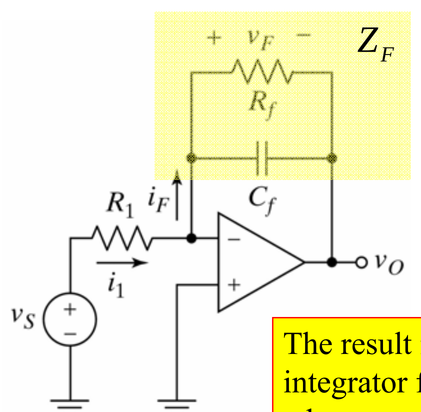
Therefore, $$v_O = \frac{-Z_F}{R_1} v_S$$

If we use a capacitor, $$v_O(t) = \frac{-1}{R_1 C_f} \int_{-\infty}^{t} v_S(\tau) d\tau$$

But, the DC gain is infinite and any offset will drive the amplifier into saturation!

A Low-Pass Filter (Integrator)

- Therefore, we use a resistor in parallel with the capacitor to bring the DC gain down:



$$\frac{v_O}{v_S} = \frac{-Z_F}{R_1} = \frac{-R_f}{R_1\left(1 + j\omega R_f C_f\right)}$$

$$= \frac{-R_f / R_1}{\left(1 + j\omega R_f C_f\right)} = \frac{A_0}{1 + j\,\omega/\omega_p}$$

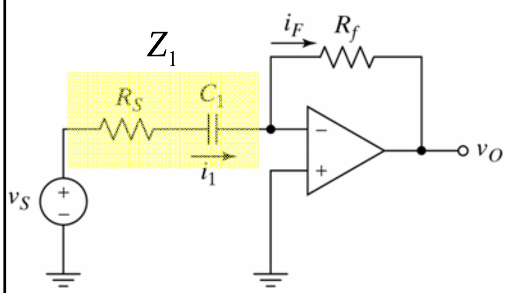
where $A_0 = -R_f/R_1$ and $\omega_p = \dfrac{1}{R_f C_f}$

The result is a LPF, but a LPF looks like an integrator for frequencies well above the pole where you can ignore the '1' in the denominator.

A High-Pass Filter (Differentiator)

- Similarly, we see how to make high-pass filters (differentiators):

$$v_O = -i_F R_f = -i_1 R_f$$



$$i_1 = \frac{v_S}{Z_1} = \frac{v_S}{R_S + \dfrac{1}{j\omega C_1}}$$

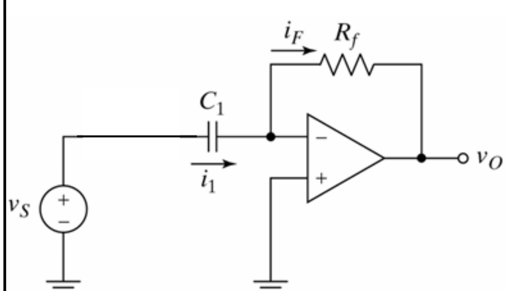
$$i_1 = \frac{j\omega C_1}{1 + j\omega R_S C_1} v_S$$

Therefore,

$$\frac{v_O}{v_S} = \frac{-j\omega R_f C_1}{1 + j\omega R_S C_1}$$

The HPF as a Differentiator

- We can see how this circuit functions as a differentiator (approximately) by assuming that R_S is negligible (i.e., we can ignore the voltage drop across it):



Then $\quad i_1 \approx C_1 \dfrac{dv_S}{dt}$

and

$$v_O = -i_1 R_f \approx -R_f C_1 \frac{dv_S}{dt}$$

Note that a perfect differentiator does not have any poles, but we can't avoid having one since $R_S = 0$ is impossible.

The HPF as a Differentiator

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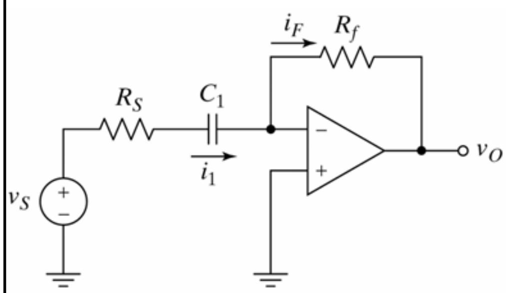

Then $i_1 \approx C_1 \dfrac{dv_S}{dt}$

and

$$v_O = -i_1 R_f \approx -R_f C_1 \frac{dv_S}{dt}$$

In addition, we *want* the pole to be there – increasing gain at high frequencies without limit is *not* desirable; we would just be amplifying noise.

The HPF as a Differentiator

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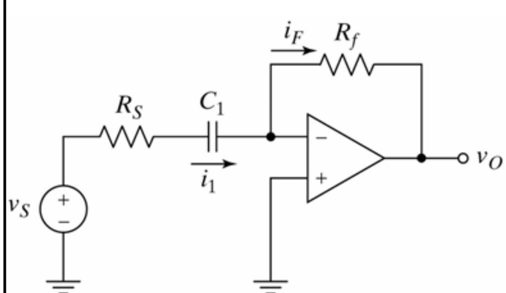


Then $i_1 \approx C_1 \dfrac{dv_S}{dt}$

and

$$v_O = -i_1 R_f \approx -R_f C_1 \frac{dv_S}{dt}$$

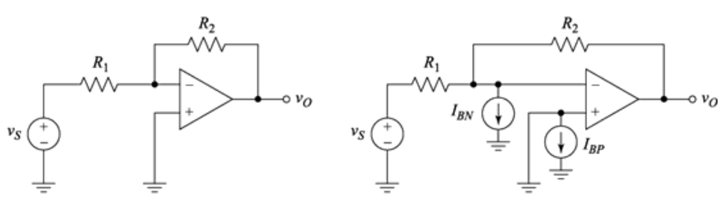
With the pole, the gain of this circuit asymptotically approaches $-R_f / R_S$ as the frequency approaches infinity. With small R_S, you probably want to roll that off again at higher frequencies!

Real Integrators & Differentiators

- To summarize about integrators & differentiators:
- We cannot build a real integrator, it would have infinite DC gain and would saturate. But, a single-pole low-pass filter (LPF) is indistinguishable from an integrator when used at frequencies well above the pole.
- Similarly, a single-pole high-pass filter (HPF) is indistinguishable from a differentiator when used at frequencies well below the pole frequency. Real differentiation is undesirable because the continually increasing gain at high frequencies amplifies noise.

Input Bias Currents

- All real op amps have non-zero input bias currents
- We model these as current sources as shown in part (b) of the figure below

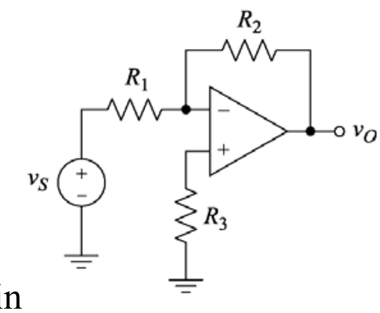


(a) (b)

- Note that if $v_S = v_O = 0$, I_{BN} "sees" R_1 in parallel with R_2 to ground. Therefore, the bias currents will generate an input offset voltage.

Input Bias Current Correction

- We can cancel the offset voltage produced by the input bias currents by adding a resistor, R_3, set equal to R_1 in parallel with R_2 as shown below.
- In practice, the two bias currents will not be equal, nor will $v_S = v_O = 0$, so there will still be some error (this is the input *offset* current), but we can only correct for the random difference in the bias currents by individually trimming our circuits.

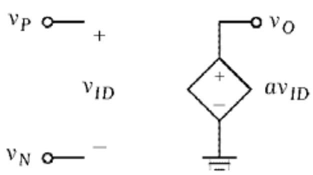


Finite Bandwidth

All real op amps have a finite bandwidth. This limitation is a small-signal, linear phenomenon and is not at all likely to affect the operation of any circuits you build for this project.

Finite Bandwidth

A simple way to model this limitation is to let the controlled source in the op amp circuit model have a single-pole low-pass transfer function. Note that all op amps have low-pass transfer functions and internally compensated op amps can be well modeled by *single-pole* low-pass transfer functions:



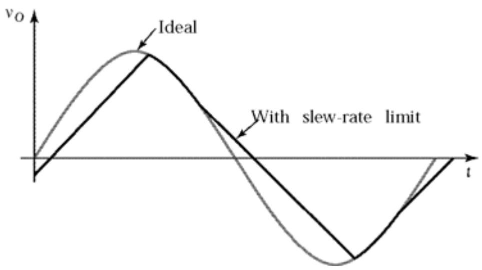
$$a(j\omega) = \frac{a_0}{1 + j\omega/\omega_0}$$

Slew Rate & Full-Power Bandwidth

All real op amps also have a large-signal, nonlinear, limitation on how fast the output voltage can change. This is called the slew rate.

Slew Rate & Full-Power Bandwidth

The slew rate is the maximum rate of change of the output voltage and is caused by having a limited amount of current available to charge some capacitor inside the op amp. When the output is too large at a given frequency, the desired output slope exceeds the slew rate and causes distortion. The frequency can be well below the small-signal bandwidth. If the output signal is a sine wave with the maximum possible peak-to-peak value, the frequency at which the slew rate limit first causes distortion is called the full-power bandwidth.



When is Slew Rate Important?

- Whenever you require the output of an op amp to swing over a large range quickly you should check the slew rate – it is common to have a slew rate limitation affect the operation of a circuit at frequencies well below the small-signal bandwidth if the circuit has large signals.