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A minimax control design for nonlinear systems based on genetic programming: Jung's collective unconscious approach

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Abstract - When it comes to the minimax controller design, it would be extremely difficult to obtain such controllers in the nonlinear situations. One of the reasons is that the minimax controller should be robust against any kind of disturbances in the nonlinear situations. In this paper, we propose a difficulty-free design method of minimax control problems. First, based on the genetic programming and Jung's collective unconscious, this paper presents a very simple design technique to solve the minimax control problems, where the minimax controller may be constructed only paying attention to the minimization process. It would be surprising that the maximization process is not needed in the construction of minimax controllers. Then, some simulations are given to demonstrate the usefulness of the proposed design technique with the identification problem, and minimax control problems.

1. Introduction

We deal with the optimal controllers for the minimax problems of the nonlinear systems[7,8]. Generally speaking, there exist two ways to attack such minimax problems. One is so-called "indirect approach", where the solutions of partial differential equations associated with the problems, such as Hanilton-Jacobi-Isaacs equations, are used for obtaining such controllers "indirectly", and the other is so-called "direct approach", where both of the minimization and maximization processes are done for obtaining such controllers "directly".

Unfortunately, each approach has drawbacks. In the former, it would be very difficult to solve the nonlinear partial differential equations. In the latter, it would be very difficult to perform the minimization and maximization processes at the same time. In this paper, we focus on the direct approach and, based on genetic programming (GP), propose a new design method where no maximization process could be required. This idea comes from a fusion of GP and Jung's collective unconscious.

Our approach mainly based on GP technique. What is the GP technique? In 1990's, J.R.Koza extended the J.Holland's genetic algorithm (GA) to the genetic programming in which the population consists of computer programs of varying size and shapes. The individual in GP has the structure of the tree. In the evolution process, mutation, inversion, and crossover are used as the genetic operators, which are called Gmutation, Ginversion, and Gcrossover, respectively. The GP can generate computer programs by itself, that is to say, the GP has the emergent property. The proposed approach is mainly based on the emergent property of the GP, and lets the computers generate the optimal/robust controllers of nonlinear systems.

Of course, GP-based methods are already reported as one of the promising approaches in a field of nonlinear control problems [3, 4, 5, 6]. However, it should be noted that our approach is completely different from the existing GP methods in the minimax situations, because of no maximization process.

The paper is organized as follows. In Section 2, the minimax control problems are formulated. In Section 3, the detailed description of the GP is given and also what is Jung's idea of the collective unconscious is described in detail. Because they play key roles in constructing minimax controllers. In Section 4 the new design method is proposed using such a fusion approach. We here describe the design procedure in details, focusing on the minimax control problems. In Section 5, some numerical examples such as the identification problem and minimax robust problems, are given to show the effectiveness of our approach.

2. Problem formulation

2.1 Minimax control problem

One of the important control design problems is as follows. Consider the dynamical system described by the ordinary differential equation with the performance equation.

$$\begin{aligned} \dot{x} &= f(x,u,w) \\ z &= h(x,u,w) \end{aligned} \qquad (2.1)$$

Here, "x" is the state, "u" is the control, and "w" is a disturbance. We here assume that the design specifications can be written by the to-be-minimized performance index

$$J = \max_{w} \tilde{J} \tag{2.2}$$

$$\tilde{J} = \frac{\int_{t_0}^{t_1} z^T z dt}{\int_{t_0}^{t_1} w^T w dt} \tag{2.3}$$

where "t_0" and "t_1" are the initial and terminal time, respectively. Then, the control design problem is to find the optimal feedback controller minimizing the performance index mentioned above.

Those problems are called "robust control problems" because, once the controllers are obtained, the resultant control systems are expected to be strong against any type of disturbances given to the systems. Therefore, the minimax problem is considered to be one of the important issues to solve in a control engineering field. However, those problems seem to be extremely difficult to solve, because all kinds of disturbances must be taken for each admissible control in the minimax situations. Is it possible? How do we give all the disturbances to the systems? One of the answers could be given in this paper.

3. Preliminaries

In this section, we describe about the genetic programming and the Jung's idea of the collective unconsciousness.

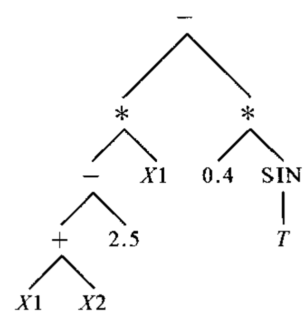
3.1 Genetic Programming[1]

Genetic Algorithm has been extended to the so-called Genetic Programming, where the population consists of computer programs of varying sizes and shapes. The individual in GP has the tree structure. See Fig.1, for example. The tree structure in Fig.1 (a) is equivalent to the symbolic expression Fig.1 (b), which means the function ($X1$+$X2$-2.5) $X1$- 0.4 sin(T). For more details, see the reference [1].

In the evolutionary process of the GP, the genetic operators, such as Gmutation, Ginversion, and Gcrossover, are used. See Fig.2, for example. More exactly, Gmutation consists of four types, such as terminal-to-function type, function-to-terminal type, terminal-to-terminal type, and function-to-function type. In Fig.2, F_i (i=1, 2,$\cdots$) are called functions, and T_i (i=1, 2, $\cdots$) are called terminals.

Here is a standard procedure of the GP algorithm, which consists of an iterative process.

<step 0> Select GP parameters.
<step 1> Generate the initial population.
<step 2> Evaluate the fitness of individuals.
<step 3> Execute the criterion on the convergence.
<step 4> Based on the fitness, perform the operations of Gcrossover, Ginversion, and Gmutation.
<step 5> Go back to step2.

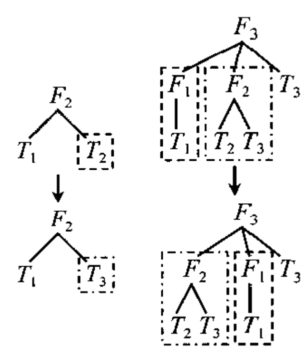


(a) Tree structure

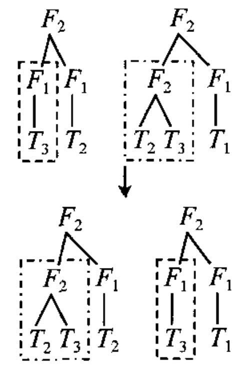
```
(-
  (*
    (-
      (+
        X1
        X2)
      2.5)
    X1)
  (*
    0.4
    (SIN
      T)))
```

(b) Symbolic expression

Fig.1 Individual in GP for "(X1+X2-2.5)X1-0.4 sin(T)"



(a) Gmutation (b) Ginversion



(c) Gcrossover

Fig.2 GP operators

3.2 The collective unconscious

When it comes to the minimax problems, both of the minimization and maximization processes are usually needed in obtaining the minimax controllers. From a computational point of view, one of the easy ways to obtain the minimax solution is to proceed the minimization and maximization processes alternately. In this paper, we propose another easy way to tackle such minimax problems, based on the Jung's idea of the collective unconscious. Due to such an idea, the proposed approach depends only on the minimization process, that is, it does not need the maximization process. We describe what is about the Jung's idea of the collective unconscious, as follows.

Carl Jung was a student of Freud, and he accepted the existence of a conscious and unconscious mind. However, Jung gave a unique interpretation of the unconscious. He insisted that the unconscious consists of two parts, the personal unconscious and the collective unconscious. Although the personal unconscious is the almost same as Freud's and it depends mainly on environments, the collective unconscious is beyond Freud's. It does not depend on environments, and it would rather depend on the evolutionary process of human beings. We should note that the collective unconscious exists within the evolutionary process. If we pay attention to this idea, we could omit the maximization process in the minimax approach of the design problems. Because, if we regard disturbances as materials of the collective unconscious, we could deal with any disturbance 'unconsciously' in the evolutionary process. This gives us such a possibility as we need not pay attention to all disturbances in minimax process in the evolutionary process.

4. Design method

According to the usual procedure of GP, we decide the terminals, functions, and fitness.

1) Terminals: The input of the controller is the state of the plant system. So, we use the state variable x as the terminal. The terminal set also contains real value $R(-5.0 \leq R \leq 5.0)$.
2) Functions: This set is the non-terminal set. A lot of functions are proposed so far. For the purpose of the simplicity in control design, we choose the usual operators such as '+', '-', '*', '/'.
3) Fitness: The fitness function is determined minimizing the performance index (2.3). It should be noted that no maximization operation is required.

The followings are the procedure of GP based design method, not including the maximization process.

[GP design method]

<step0> Select GP parameters.
<step1> Generate randomly an initial population, consisting of functions and terminals, with the size N of the population.
<step2> Evaluate the fitness of the individual, calculating the performance index (2.3) subject to the dynamical equation (2.1). Then, due to Jung's idea of the collective unconscious, only one arbitrary disturbance is adopted.
<step3> Based on the fitness, select the best individual of the population (say, best-of-generation individual).
<step4> Preserve the best individual over all the best-of-generation individuals (say, best-so-far individual). Here, we adopt another criterion to choose the best-so-far individual. For more details, see Remark 4.
<step5> Perform genetic operations, such as Gcrossover, Gmutation, and Ginversion. And create a new population.
<step6> Go back to Step2.

Remark 1. It should be noted that no maximization process is given in Step2.
Remark 2. GP parameters in Step0 are as follows. The deepness of the tree structure is 6 in the first generation and 4 in Gmutation. The rate of Gcrossover is 0.1 in functions, and 0.7 in terminals, respectively. The rate of Gmutation or Gcopy is 0.1. The size of the population is 256.
Remark 3. In Step1, we generate two intitial populations; one is for the minimax controller and the other is for the disturbance. However, the population for the disturbance

does not evolve itself. The individual of the disturbance population is used just in the evaluation of the fitness of Step3.

Remark 4. In choosing the best-so-far individual of Step4, we again evaluate only the best-of-generation individual by applying the 256 kinds of the disturbances to that individual, and compare it with the best-so-far individual of the one-generation-before.

5. Simulations

We tackle three control design problems, where the identification problem is in Example 5.1, and minimax problems are in Examples 5.2-5.4. Details are omitted here.

5.1 Identification problem

Consider the following function to be identified.

$$y = 3x^2 + 4x + 6$$

The identification problem seems to be a kind of the minimization problem. In this paper, however, for a check-out of the possibility of our new approach, this identification problem is converted into the minimax one, as follows: Find a function $\hat{y}$ with the index

$$\min_{\hat{y}} \max_{x} J_1$$

where $J_1 = (\hat{y}/y - 1)^2$, and $\hat{y}$ is the identified equation. It should be noted that a minimization problem could be formulated as follows: Find a function $\hat{y}$ with the index

$$\min_{\hat{y}} \int (\hat{y}/y - 1)^2 dx$$

The proposed method is applied with 1000 iterations. After 555 generations, we obtained the best-so-far individual. The GP solution is

$$\hat{y} = 3.1682x^2 + 2.9679x$$

See Fig.3 for the optimal and GP solutions. In simulations the range of the variable x is given with $1.0 \le x \le 20.0$.

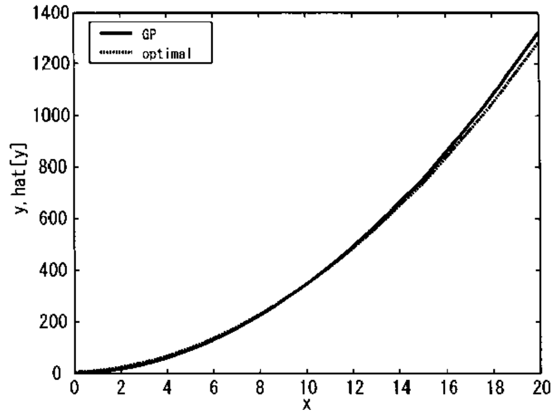


Fig.3 Identification results with optimal and GP ones

5.2 Minimax control problem (Linear one-dimensional system)

Consider the equations.

$$\dot{x} = -x + u + w$$

$$z = [x \quad u \quad w]^T$$

The performance index is given as follows.

$$\tilde{J} = \frac{\int_0^\infty z^T z \, dt}{\int_0^\infty w^T w \, dt}$$

So, we have

$$\tilde{J} = \frac{\int_0^\infty (x^2 + u^2) dt}{\int_0^\infty w^2 dt}$$

A simple calculation leads to analytical solutions. The optimal solution is obtained as

$$u_{op} = -x$$

The problem is tackled in 200 iterations. The range of the disturbance is set to $|w| \le 1$. After 147 generations, we obtained the best-so-far individual. The minimization value of the GP solution is 0.466048, which is close to the optimal value 0.5. The GP controller is given like this.

$$u_{GP} = \frac{\begin{array}{l} x(0.023944 + 0.011391x + 0.9475x^2 \\ + 1.2971x^3 + 2.4468x^4 + 1.7909x^5 \\ - 0.99182x^6 - 0.95245x^7 + 0.15544\dot{x}^8 \\ + 0.15037x^9 - 0.0026144x^{10} \\ - 0.004175x^{11} \end{array}}{\begin{array}{l} (0.22063 + 0.04398x)(-1.5015 - 1.5983x \\ - 4.1924x^2 - 2.2558x^3 + 0.54897x^4) \\ (0.072277 - 0.05696x + 2.7248x^2 \\ + 0.5026x^3 - 1.3195x^4 - 0.04797x^5 + 0.1722x^6) \end{array}}$$

For a comparison with the optimal solution, see Fig.4. In simulations the control time interval is given with $0 \le t \le 10$. It would be surprising that the GP solution is extremely close to the optimal one.

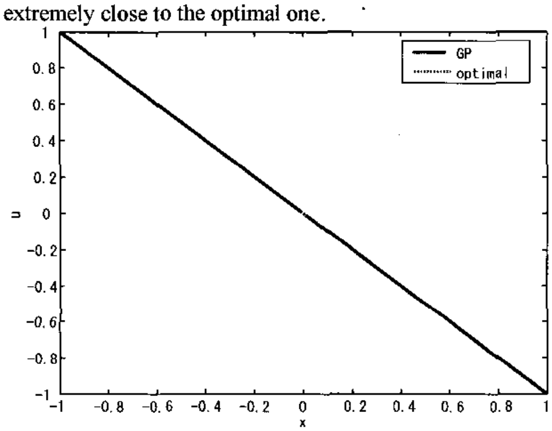


Fig.4 Optimal and GP solutions for Example 5.2

5.3 Minimax control problem (Linear two-dimensional system)

Consider the following 2-dimensional dynamical system.

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$z = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

In this case, the performance index is given as follows.

$$\tilde{J} = \frac{\int_0^\infty (x_2^2 + u^2) dt}{\int_0^\infty w^2 dt}$$

An analytical solution is given like this.

$$u_{op} = x_1 - x_2$$

In simulations, the range of the disturbance is constrained to $|w| \le 5.0$. We proceeded the calculations with 200 generations. After 19 generations, we obtained the GP solution, where the performance index of the best-so-far individual is 0.474868. Note that the optimal one is 0.5.

$$u_{GP} = x_1 / (2x_1 - 3.4305) - x_2$$

For a comparison with the optimal solution, see Fig.5 and 6.

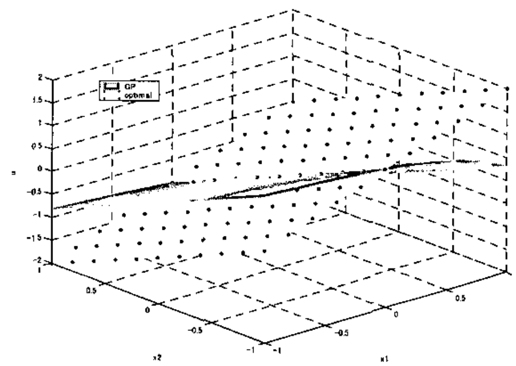


Fig.5 Optimal and GP solutions for Example 5.3

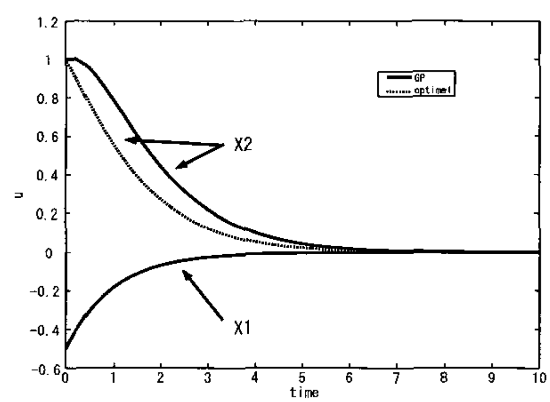


Fig.6 State trajectories with the optimal/GP controllers and worst disturbance

5.4 Minmax control problem (Nonlinear one-dimensional system)[2]

We have the following system.

$$\dot{x} = -x\sqrt{2x^4 + 4x^2 + 1} + (1 + x^2)w + u$$
$$z = \begin{bmatrix} 1 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

We tackle the nonlinear problem. Generally speaking, nonlinear problems are difficult to obtain the analytical solutions. However, in this problem the solution is given in the reference [2]. The optimal solution is

$$u_{op} = -\frac{x}{\sqrt{2x^4 + 4x^2 + 1}}$$

The proposed method is applied to this nonlinear problem with 3000 generations. The range of the disturbance is set to $|w| \leq 5.0$. Genetic operations are restricted to '+', '-', and '*', because the operation '/' did not work well.

After 2273 generations, we obtained the best-so-far individual. The obtained GP solution is given as follows.

$$u_{GP} = \frac{\begin{array}{l} 0.00974x - 3.1958x^2 + 3.3365x^3 - 12.377x^4 \\ +19.500x^5 + 26.046x^6 - 19.685x^7 - 26.742x^8 \\ +4.1978x^9 + 4.5621x^{10} + 2.8893x^{11} + 3.0472x^{12} \\ -1.2695x^{13} - 0.32606x^{14} + 0.02926x^{15} + 0.00612x^{16} \\ +0.00389x^{17} - 0.00208x^{18} - 0.00008x^{19} \end{array}}{\begin{array}{l} 4.2278x - 5.3132x^2 + 20.307x^3 - 33.068x^4 \\ -18.936x^5 + 17.100x^6 + 11.285x^7 + 4.8124x^8 \\ +14.461x^9 - 8.4170x^{10} - 6.2108x^{11} + 0.62400x^{12} \\ -2.1413x^{13} + 1.1665x^{14} + 0.10492x^{15} - 0.04125x^{16} \\ +0.00125x^{17} - 0.00302x^{18} + 0.00050x^{19} - 0.01289 \end{array}}$$

See Fig.7. The GP-solution is close to the analytical one. This demonstrates that the proposed method works well even with nonlinear problems.

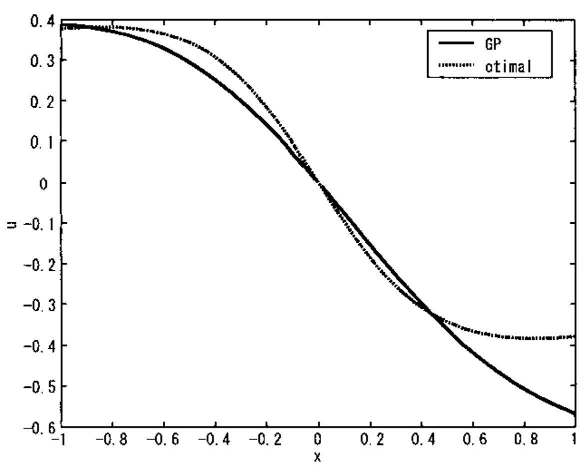


Fig.7 Optimal and GP solutions for Example 5.4

6. Conclusions

In a field of the minimax control problems, a new type of GP based design method is proposed, where the maximization process is not required in minimax calculation process. It is due to Jung's idea of the collective unconscious. Some simulations are given to demonstrate the powerfulness of the proposed GP based method.

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