EENS 6310 Problem Set 2 Due Date March 14th 2022

1) Experiments by Nikuradse determined the following two relationships for hydraulically smooth flow

$$\frac{\delta_v U_*}{\upsilon} = 11.6$$

And

$$z_0 = \frac{\delta_v}{104}$$

Where ν is the kinematic viscosity of the fluid, δ_v is the thickness of the viscous sublayer, u_* is the shear velocity and z_0 is the reference level, near the boundary where the velocity for a turbulent flow goes to zero. You are beginning the study of a quasi-straight and uniform reach of a lowland channel cut into mudstone and are interested in estimating the structure of the near-bed velocity profile in order to properly instrument the near-bed environment to measure flow related forces. In particular you are interested in estimating the structure of the velocity profile in the viscous sublayer and the interior flow and comparing the linkage between the two zones. The water depth in the study reach is 1.0 m, the water-surface slope is 1.00 x 10^{-4}, and the roughness parameter, k_s, for the smooth bottom composed of mudstone is 1.00x10^{-4} m. Water temperature is 10ºC and its kinematic viscosity and density are 1.22 x 10^{-6} m^2/s and 1072 kg/m^3, respectively.

   a) Assuming that for the interval of interest the flow can be characterized as steady, horizontally uniform and approximately 2D, calculate and plot the velocity profile described by the Law of the Wall.
   b) Applying the same set of assumptions used in (a) generate the expression defining the velocity profile found in the viscous boundary layer using the constitutive law for a linear viscous fluid. Hint: $\tau_{zx} \sim \tau_b$ throughout the viscous sublayer. Show your work.
   c) Plot the velocity profile within the viscous sublayer on the same graph with the profile describing the turbulent interior flow. What does the transition look like?
   d) What is the Froude number for the channel flow under the study condition?

2) In class we developed a description for the velocity profile in a turbulent flow typically referred to as the "Law of the Wall". This development had two steps. In the first step, Cauchy's 1st law of Continuum Mechanics was simplified to yield a relationship between

the body force and change in stress at every level in the fluid. The second step used a constitutive law to relate this stress to the stain rate, delivering the velocity profile.

A pressure gradient can also drive flow. This problem allows you to explore/develop equations describing a pressure-gradient driven circulation in a shallow puddle. Have fun.

If a fluid mechanical system is very broad relative to its depth and if the boundary conditions are applied uniformly across nearly parallel upper and lower surfaces, then the flow can be considered horizontally uniform over a large region away from the lateral boundaries of the fluid. This approximation can be applied to the central part of a large shallow puddle of nearly uniform depth (L_x and $L_y >> h = 8.0 \text{ x } 10^{-3}$m) being acted on by a surface wind stress of $\tau_s = (\tau_{zx})_s = 2.0 \text{ x } 10^{-5}$ kg/ms^2.

a) Write down the appropriate governing equations for this entire problem, then simplify them as much as permitted by the expected conditions at the center of the puddle. Note that the wind will blow the water to one side of the puddle where the surface elevation will rise until a pressure gradient sufficient to cause a return flow of the same discharge as the stress driven flow is set up. At this point of equilibrium (steady) flow is established.
b) Simplify the equations again for this equilibrium situation.
c) Solve the equations of part b to find the velocity and stress fields in the fluid in terms of τ_s, h, and μ by assuming the change in depth with downstream distance ($h(x)$) caused by the sloping surface is negligible ($h(x) = h_0 = 8.0\text{x}10^{-3}$m). To do this, 1) calculate the local shear stress profile in terms of the surface stress and the pressure gradient caused by the sloping surface, 2) calculate the velocity profile using Newton's viscous law, 3) calculate the fluid discharge per unit width and set it equal to zero. Step 3 permits the pressure gradient to be related to the surface stress, allowing for pressure gradient to be eliminated from the equations for the shear stress and velocity fields. Find these new shear stress and velocity profile equations (shear stress and velocity as functions of z).
d) Calculate the surface slope, boundary shear stress, surface velocity, level of zero velocity, and discharge of the lower layer.
e) Sketch the velocity and stress profiles.