$$(I_4 \otimes F_2)R_8 = R_8(F_2 \otimes I_4) \tag{1}$$

$$\begin{aligned}(I_4 \otimes F_2)R_8(e_i^2 \otimes e_j^2 \otimes e_k^2) &= (I_4 \otimes F_2)(e_{2k+j}^4 \otimes e_i^2) &(2)\\ &= (e_{2k+j}^4 \otimes F_2 e_i^2) &(3)\end{aligned}$$

$$R_8(F_2 \otimes I_4)(e_i^2 \otimes e_j^2 \otimes e_k^2) \tag{4}$$

I see

$$R_8 = (R_4 \otimes I_2)L_4^8 = (L_2^4 \otimes I_2)L_4^8 \tag{5}$$

$$F_{rs} = (F_r \otimes I_s)T_s^{rs}(I_r \otimes F_s)L_r^{rs} \tag{6}$$

so

$$F_{16} = (F_2 \otimes I_8)T_8^{16}(I_2 \otimes F_8)R_{16} \tag{7}$$

Now as

$$F_8 = (F_2 \otimes I_4)T_4^8(I_2 \otimes F_2 \otimes I_2)(I_2 \otimes T_2^4)(I_4 \otimes F_2)R_8 \tag{8}$$

We can do

$$\begin{aligned}F_8 &= (F_2 \otimes I_4)T_4^8(I_2 \otimes F_2 \otimes I_2)(I_2 \otimes T_2^4)R_8(F_2 \otimes I_4) &(9)\\ &= (F_2 \otimes I_4)T_4^8 R_8(F_2 \otimes I_4)(T_2^4 \otimes I_2)(F_2 \otimes I_4) &(10)\end{aligned}$$

$$(R_n \otimes I_2)L_n^{2n} = R_{2n} \tag{11}$$