Given recursive definition

$$F_{2^t} = (\prod_{c=1}^{t} (I_{2^{c-1}} \otimes F_2 \otimes I_{2^{t-c}})(I_{2^{c-1}} \otimes T_{2^{t-c}}^{2^{t-c+1}}))R_{2^t} \tag{1}$$

Where

$$T_n^{mn}(e_i^m \otimes e_j^n) = \omega^{ij}(e_i^m \otimes e_j^n) \tag{2}$$

and

$$L_n^{mn}(e_i^m \otimes e_j^n) = (e_j^n \otimes e_i^m) \tag{3}$$

From this we can get

$$F_{2^t} = (\prod_{c=1}^{t} L_2^{2^t}(I_{2^{t-1}} \otimes F_2)T_c)R_{2^t} \tag{4}$$

where

$$T_c = L_{2^{t-c}}^{2^t}(I_{2^{c-1}} \otimes T_{2^{t-c}}^{2^{t-c+1}})L_{2^c}^{2^t} \tag{5}$$

and

$$A^{m\times m} \otimes B^{n\times n} = L_m^{mn}(B^{n\times n} \otimes A^{m\times m})L_n^{mn} \tag{6}$$

1.

$$T_n^{mn}(e_i^m \otimes e_j^n) = \omega^{ij}(e_i^m \otimes e_j^n) \tag{7}$$

while

$$W_n e_i^n = \omega^i e_i^n \tag{8}$$

So for $T_n^{mn}(e_i^m \otimes e_j^n) = \omega^{ij}(e_i^m \otimes e_j^n)$, if we have $T_n^{'mn}$ just be a diagonal of W_ns repeating we get

$$T_n^{'mn}(e_i^m \otimes e_j^n) = \omega^i(e_i^m \otimes e_j^n) \tag{9}$$

Thus as

$$(W_n)^j e_i^n = \omega^{ij} e_i^n \tag{10}$$

$$T_n^{mn}(e_i^m \otimes e_j^n) = \omega^{ij}(e_i^m \otimes e_j^n) \tag{11}$$

$T_n^{'mn}$ just be a diagonal of W_n^j from j 0 to m. So

$$T_n^{mn} = \oplus_{i=0}^{m-1} W_n^i(\omega) \tag{12}$$

2.

$$\begin{align} L_n^{mn}T_n^{mn}L_m^{mn}(e_i^n \otimes e_j^m) &= L_n^{mn}T_n^{mn}(e_j^m \otimes e_i^n) \tag{13}\\ &= \omega^{ij}L_n^{mn}(e_j^m \otimes e_i^n) \tag{14}\\ &= \omega^{ij}(e_i^n \otimes e_j^m) \tag{15}\\ &= T_m^{mn}(e_i^n \otimes e_j^m) \tag{16}\end{align}$$

3. With $m = 2^c$, $n = 2^{t-c}$

$$T_c = L^{2^t}_{2^{t-c}}(I_{2^{c-1}} \otimes T^{2^{t-c+1}}_{2^{t-c}})L^{2^t}_{2^c} \tag{17}$$
$$= (L^{2^t}_{2^{t-c}} I_{2^{c-1}} L^{2^t}_{2^c} \otimes L^{2^t}_{2^{t-c}} T^{2^{t-c+1}}_{2^{t-c}} L^{2^t}_{2^c}) \tag{18}$$
$$= (I_{2^{c-1}} \otimes T^{2^{t-c+1}}_{2}) \tag{19}$$

As

$$T^{mn}_n = \oplus^{m-1}_{i=0} W^i_n(\omega) \tag{20}$$

$$= I_{2^{c-1}} \otimes \oplus^{2^{t-c}-1}_{i=0} W^i_2(\omega) \tag{21}$$

how where did the 2^c come from?

4.

$$F_{2^t} = (\prod_{c=1}^{t}(I_{2^{c-1}} \otimes F_2 \otimes I_{2^{t-c}})(I_{2^{c-1}} \otimes T^{2^{t-c+1}}_{2^{t-c}}))R_{2^t} \tag{22}$$
$$= (\prod_{c=1}^{t} L^{2^t}_{2^{c-1}}(F_2 \otimes I_{2^{t-c}} \otimes I_{2^{c-1}})L^{2^t}_{2^{t-c+1}} L^{2^t}_{2^{c-1}}(T^{2^{t-c+1}}_{2^{t-c}} \otimes I_{2^{c-1}})L^{2^t}_2 L^{2^t}_{2^{t-c}})R_{2^t} \tag{23}$$
$$= (\prod_{c=1}^{t} L^{2^t}_{2^{c-1}}(F_2 \otimes I_{2^{t-1}})(T^{2^{t-c+1}}_{2^{t-c}} \otimes I_{2^{c-1}})L^{2^t}_{2^{t-c+1}})R_{2^t} \tag{24}$$

final step hmm

$$(F_2 \otimes I_{2^t})T^{2^{t+1}}_{2^t}(F_{2^t} \otimes I_2)L^{2^{t+1}}_2 \tag{25}$$

5.

$$F_{2^t} = (\prod_{c=1}^{t} L^{2^t}_2(I_{2^{t-1}} \otimes F_2)T_c)R_{2^t} \tag{26}$$
$$= (\prod_{c=1}^{t} L^{2^t}_2(I_{2^{t-1}} \otimes F_2)L^{2^t}_{2^{t-c}}(I_{2^{c-1}} \otimes T^{2^{t-c+1}}_{2^{t-c}})L^{2^t}_{2^c})R_{2^t} \tag{27}$$
$$= (\prod_{c=1}^{t} L^{2^t}_2 L^{2^t}_{2^{t-1}}(F_2 \otimes I_{2^{t-1}})L^{2^t}_2 L^{2^t}_{2^{t-c}} L^{2^t}_{2^{c-1}}(T^{2^{t-c+1}}_{2^{t-c}} \otimes I_{2^{c-1}})L^{2^t}_{2^{t-c+1}} L^{2^t}_{2^c})R_{2^t} \tag{28}$$
$$= (\prod_{c=1}^{t}(F_2 \otimes I_{2^{t-1}})L^{2^t}_{2^c}(T^{2^{t-c+1}}_2 \otimes I_{2^{c-1}})L^{2^t}_2)R_{2^t} \tag{29}$$
$$\tag{30}$$