$$\begin{pmatrix} F_2 & W_2F_2 \\ F_2 & -W_2F_2 \end{pmatrix} \tag{1}$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \\ 1 & \omega^2 & 1 & \omega^2 \\ 1 & \omega^3 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

$$F_n = (F_2 \otimes I_m)T_m^{2m}(I_2 \otimes F_m)L_m^{2m} \tag{3}$$

As $(L_m^{2m})^{-1}$ is the inverse

$$F_nL_m^{2m} = (F_2 \otimes I_m)T_m^{2m}(I_2 \otimes F_m) \tag{4}$$

L_m^{2m} takes stride 2(just the columns). What is this?

$$W_m = \begin{pmatrix} 1 & .... \\ 0 & \omega \end{pmatrix} \tag{5}$$

etc.

$$\begin{pmatrix} \omega^{i2j} & \omega^{i(2j+1)} \\ \omega^{(i+m)2j} & \omega^{(i+m)(2j+1)} \end{pmatrix} = \begin{pmatrix} \omega^{i2j} & \omega^{2ij}\omega^{i} \\ \omega^{2ij}\omega^{2mj} & \omega^{2ij}\omega^{2mj}\omega^{m}\omega^{i} \end{pmatrix} \tag{6}$$

This is

$$\begin{pmatrix} F_m & W_mF_m \\ F_m & -W_mF_m \end{pmatrix} = \begin{pmatrix} I_m & I_m \\ I_m & -I_m \end{pmatrix} \begin{pmatrix} I_m & 0 \\ 0 & W_m \end{pmatrix} \begin{pmatrix} F_m & 0 \\ 0 & F_m \end{pmatrix} \tag{7}$$

Thus proven

Notes

$$\frac{C[x]}{X^{rs}-1} \tag{8}$$

with fft $F_r \otimes I_s$

$$\prod_{i=0}^{r-1} \frac{C[x]}{X^s - \omega_r^i} \tag{9}$$

example

$$x^5 + x^4 + x^3 + x^2 + x + 1 \tag{10}$$

mod $x^3 - \alpha$

$$\alpha x^2 + \alpha x + \alpha + x^2 + x_1 \tag{11}$$

We are grouping together chunks of size 3.

When we do dft, we evaluate the rs size vector at each ω^i for each ith power.

Then we make $x \to \omega_R^i X$

$r = 3, s = 2.$

$$x^6 - 1 = (x^2-1)(x^2-\omega)(x^2-\omega^2) \tag{12}$$

These polynomials are

$$f_0 + f_x ... + f_5 x^5 \tag{13}$$

this mod $x^2 - \omega^i$ then we get

$$f_0 + f_2 + f_4 + (f_1 + f_3 + f_5)x \tag{14}$$

if we have the coefficients as vectors

$$f_0, f_2, f_4, f_1, f_3, f_5 \tag{15}$$

Then we group together as

$$f_0 + f_2 + f_4, f_1 + f_3 + f_5 \tag{16}$$

This is $1, 1, 1 \otimes I_2$. This works as when multiplied with $(f_0, f_1, ...f_5)$ we get the vector of size 2 which is the If ω we get

$$f_0 + f_2\omega + f_4\omega^2 + (f_1 + f_3\omega + f_5\omega^2)x \tag{17}$$

This is the same as

$$F_r \otimes I_s \tag{18}$$

$$(x^2 - \omega^i) \tag{19}$$

is

$$\begin{pmatrix} 1 & 0 \\ 0 & \omega^i \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \end{pmatrix} \tag{20}$$

since after reduction we just have $f_0 + f_1\omega^i x$.

to get rid of ω

$$\prod_{i=0}^{r-1} \frac{C[x]}{X^s - \omega_r^i} \tag{21}$$

we take it out using the matrix in dfft. Then we can make a matrix

$$\begin{pmatrix} 1 & 0 & .... & \\ 0 & 1 & .... & \\ .. & .. & 1 & 0 \\ .. & .. & 0 & \omega \\ ... & & & \end{pmatrix} \tag{22}$$

Which is triangle matrix! Once we project this out we get

$$\prod_{i=0}^{r-1} \frac{C[x]}{X^s - 1} \tag{23}$$

which is

$$I_r \otimes F_s \tag{24}$$