Paper review of

Game Tree Searching by Min / Max Approximation

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Introduction

This paper introduces a new technique for searching min/max game trees, based on the idea of approximating the min and max operators by generalized mean-value operators. The goal of this method is to always expand the node that is expected to have the largest effect on the value, focusing computational resources on the important lines of play.

Implementation

One of the citations [1] enunciates: "A method is needed which will always expand the node that is expected to have the largest effect on the value". This min/max approximation tries to answer this request with a heuristic that will allocate resources in a sensible manner, searching shallowly in unpromising parts of the tree and deeper in promising sections.

The technique implemented in this paper is a penalty-based iterative search method. As an iterative heuristic, it grows the game search tree one step at a time and, on every step, new backed-up values are provided to the leaves' ancestors. As a penalty-based method, a weight is assigned so that edges representing bad moves are penalized more than edges representing good moves. These penalties are defined in terms of the derivatives of the approximating functions, which are based on generalized means instead of the min and max functions.

Results

The presented novel approach to game tree searching based on a min/max approximation by mean-value functions has been compared to alpha-beta with iterative deepening, restricting the algorithms either by number of moves explored or CPU time. Experimental results from almost 1000 games of Connect-Four show that, as the computational cost of this new approach is much higher, if the resource bound is time alpha-beta, able of exploring a much higher number of moves, wins. On the other side, in terms of accuracy, when the resource limit is move-based, this min/max approximation seems to be superior.

Discussion and open problems

Penalty-based schemes, like all iterative schemes, require a large amount of memory. Also, as their aim is to improve the value of the heuristic function at the root rather than to make the best choice for the next move, they evaluate all successors and may waste time evaluating non-optimal lines of play or even exploring deeper the game tree even when there is only one choice available.

How to choose the optimal values for the penalty function and how to combine these ideas with more traditional approaches in order to improve efficiency are some of the ideas worth exploring that arise from this paper.

Conclusion

The scheme presented has been successful outplaying alpha-beta with iterative deepening when both schemes are restricted to the same number of calls to the move operator. Due to the computational cost of this algorithm, it is still defeated when the resource limitation is time-based. Thus, further work can still be done to optimize this approach.

References

[1] Nau, D.S., Purdom, P. and Tzeng, C.-H., An evaluation of two alternatives to minimax, in: *Proceedings on Uncertainty and Probability in Artificial Intelligence*, University of California, Los Angeles, CA (1985) 232-235.