

## 3.0 Relational Model

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### 3.0 Relational Model

Relational database model is a primary data model for commercial data-processing applications. It is popular because of its simplicity; it provides simple but powerful way of representing data. It also supports complex query. Database in a relational model is simply a collection of one or more relations, where each relation is represented by table with rows and columns. It allows simple high level languages to query data.

#### 3.1 Structure of Relational Databases

In relational model, table is a major construct for representing data. Relational database consist set of tables. A row in a table represents a relationship among set of values. So table can be refers as a collection of such relationship.

##### 3.1.1 Basic Construct

To illustrate the basic structure of database, let us consider a table "account"

account_number	branch_name	balance
A-1	Kathmandu	500
A-2	Lalitpur	300
A-3	Kathmandu	700
A-4	Bhaktpur	600

Figure: The account table

The columns of table "account" are: account\_number, brance\_name and balance refer attributes. The set of permitted values for each attribute known as domain of that attribute. For example: set of all accounts numbers is a domain of attribute account\_number.

Let  $D_1$  denotes set of all account number,  $D_2$  denotes set of all branch names and  $D_3$  denotes set of all balances. Any row of table "account" must consist 3-tuple  $(v_1, v_2, v_3)$ , where  $v_1$  is account number (i.e.  $v_1$  is a domain of  $D_1$ ),  $v_2$  is branch name (i.e.  $v_2$  is a domain of  $D_2$ ), and  $v_3$  is account balance (i.e.  $v_3$  is a domain of  $D_3$ ). In general, we can express table "account" will contain only subset of all possible rows. That is, "account" is a subset of

$$D_1 \times D_2 \times D_3$$

In general, a table of n attributes must be subset of

$$D_1 \times D_2 \times D_3 \times \dots \times D_{n-1} \times D_n$$

This implies that definition of table is almost similar to the definition of relation in mathematics. In mathematics, relationship is a subset of Cartesian product of a list of domains. Therefore, in relational model, table can be refer as a relation and row of table can be refers as tuple. We can define tuple variable to represent a tuple. The above account relation consist seven tuples. Assume that tuple variable t represents first tuple of the relation. Then, the notation  $t[\text{account\_number}]$  indicates value of t on account\_number attribute. That is,  $t[\text{account\_number}] = \text{"A-1"}$ . Similarly  $t[\text{branch}] = \text{"Kathmandu"}$ , and  $t[\text{balance}] = 500$ . We can also represent it as follow:  $t[1]$  where 1 indicated first attribute of relation; that is "account\_number". Therefore,  $t[1] = \text{"A-1"}$ . In relational model, we can also express relation as a set of tuples. So definitely,  $t \in r$ .

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For each relations  $r$ , domains of all attributes of  $r$  must be atomic. The domain is said to be **atomic** if elements of domain is indivisible unit. Domains of multivalued and composite attributes are **nonatomic**.

Several attributes may have same domain. Suppose we have two relation customer with customer\_name as one of its attribute and employee is another relation with one of its attribute as employee\_name. It is possible that attribute customer\_name and employee\_name may have same domain. If we look attributes: customer\_name and branch\_name of relations customer and account respectively, at physical level, their domain may be same, both are defined by set of character string. But at logical level, customer\_name and branch\_name must have distinct domain. Null value is a member of any possible domain. It signifies value is unknown or does not exist. Domain for customer\_phoneno of customer entity set may null, meaning is that particular customer does not have any phone number or phone no. is not available.

### 3.1.2 Database Schema

A relation schema is a list of attributes and their corresponding domains. For example, the relation schema for relation customer is express as

Customer-schema = (customer\_id, customer\_name, customer\_city)

We may also specify domains of attributes as

Customer-schema = (customer\_id: integer, customer\_name: string,  
customer\_city:string)

We may state customer is a relation on Customer-schema by

customer(Customer-schema)

Values or data contain in relation change when it is updated. Relation instance is a snapshot of data in relation at particular time. But, in general, we simply say relation even it is actually relation instance.

In terms of relation, relational database is a collection of relations and relational database schema is a collection of schemas for relations in database. It describes logical design of database. The instance of relational database is collection of relation instances. It is actually a snapshot of data in database at a particular time.

Database schemas for banking enterprise

Branch-schema = (branch\_name,branch\_city,assets)  
Account\_schema = (account\_number,branch\_name,balance)  
Customer-schema = (customer\_id,customer\_name,customer\_street,customer\_city)  
Depositor-schema = (customer\_id,account\_number)  
Loan-schema = (loan\_number,branch\_name,amount)  
Borrower-schema = (customer\_id,loan\_no)

### 3.1.3 Keys

In relational model, keys (superkey, candidate key and primary key) play important roles. For example: in Branch-schema, {branch\_name} and {branch\_name, branch\_city} are superkey. Since {branch\_name} itself is a superkey, {branch\_name, branch\_city} can not

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candidate key in Branch-schema. In Branch-schema, {branch\_name} is a single candidate key so ultimately it is a primary key of Branch-schema.

Let  $R$  be a relation schema and  $K \subseteq R$ . If  $K$  is superkey for  $R$  then it restricts relations  $r(R)$  in which no two distinct tuples have same values on all attribute in  $K$ . That is, if  $t_1$  and  $t_2$  are in  $r$  and  $t_1 \neq t_2$  then  $t_1[K] \neq t_2[K]$ .

Relational database schema can be derive from an E-R schema where primary key for relation schema is primary key of entity or relationship set from which relation schema is derived. If relation is derive from strong entity set then primary key for relation is primary key of that strong entity set. Similarly, if relation is derived from weak entity set then primary key for relation is union of primary key of strong entity set and discriminator of weak entity set. And the relation consist

- all attributes of weak entity set
- primary key of the strong entity set on which weak entity set depends

In relational model, relationship set is also represented by a relation. Its primary key is depends up on mapping cardinalities of relationship set. If relationship is many to many then primary key of relation representing relationship set is a union of primary keys of related entity sets. If relationship is one to one then primary key of relation representing relationship set is primary key of any one related entity set. Similarly, if relationship is many to one then primary key of relation representing relationship set is the primary key of the "many" entity set.

In relational model, one relational schema may contain primary key of another relation schema. If relation schema  $r_1$  contains primary key of another relation schema  $r_2$  then this attribute (i.e. PK in  $r_2$ ) in  $r_1$  called *foreign key*. The relation  $r_1$  called referencing relation (detail table) of the foreign key dependency and  $r_2$  called referenced relation (master table).

Example:

In Branch-schema, branch\_name is a primary key. In Account-schema, branch\_name is a foreign key referencing Branch-schema. This implies, in any database instance, any tuple  $t_a$  in account relation, there must be some tuple,  $t_b$  in branch relation such that the value of the branch\_name attribute of  $t_a$  is same as the values of the primary key, branch\_name of  $t_b$ .

### 3.1.4 Schema Diagram

Schema diagram is a graphical representation of database schema along with primary key and foreign key dependencies.

In schema diagram, each relation is represented by box where attributes are listed inside box and relation name is specified above it. Primary key in relation is place above the horizontal line that crosses the box. Foreign key in schema diagram appear as arrow from the foreign key attributes of the referencing relation to the primary key of the referenced relation.

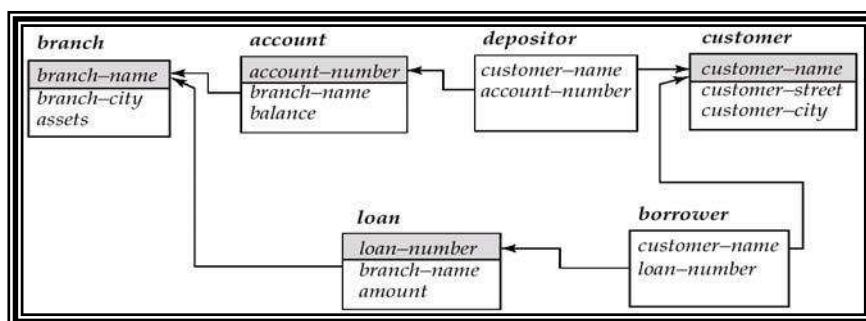


Figure: Schema diagram

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**Note:** The difference between E-R diagram and schema diagram is, E-R diagram do not shows the foreign key but schema diagram shows it explicitly.

### 3.1.5 Query Languages

Query language is a language through which user request information from database. Query languages can be categories in procedural and non procedural. In procedural query language, user required to specify sequence of operations to system to compute desired information where as in nonprocedural query language, user need to specify required information without specifying special procedure for obtaining that information.

Most commercial relational database system offers query language, both procedural and non procedural. SQL is most popular nonprocedural query language.

In this section we discuss pure query language: relational algebra, tuple relational calculus and domain relational calculus. These query languages can not commercially use by people but it describes fundamental techniques for extracting data from database and provides basis for commercial query language.

## 3.2 Relational algebra

The relational algebra is a procedural query language. It consist set of operation that takes one or more relations as inputs and produce a new relation as output. The fundamental operations in relational algebra are selection, projection, union, set difference, Cartesian product, and rename. Set intersection, natural join, division and assignments other operations of relational algebra which can be define in terms of fundamental operations.

### 3.2.1 Fundamental Operations

The fundamental operations selection, projection and rename on one relation so they called unary operations. Others operations union, set difference and Cartesian product operates on pairs of relations and so called binary operations.

#### *The Selection Operation*

The Select Operation selects tuples that satisfy a given predicate. Select is denoted by a lowercase Greek letter sigma ( $\sigma$ ), with the predicate appearing as a subscript. The relation is specifying within parentheses after  $\sigma$ . That is, general structure of selection is

$$\sigma_p(r)$$

where p is selection predicate.

Formally, selection operation define as

$$\sigma_p(r) = \{t | t \in r \text{ and } p(t)\}$$

where p is formula in propositional calculus consisting terms connected by connectives:  $\wedge$  (and),  $\vee$  (or),  $\neg$  (not). Each term is in the format

$$\langle \text{attribute} \rangle \text{op} \langle \text{attribute} \rangle \text{or} \langle \text{constant} \rangle$$

where op is one of the comparison operators:  $=, \neq, <, \leq, >, \geq$

Examples:

1. Select those tuples of loan relation where the branch is Kathmandu.

$$\sigma_{\text{branch\_name}=\text{"Kathmandu"}}(\text{loan})$$

$$\text{output} = \{t \mid t[\text{branch\_name}] = \text{Kathmandu}\}$$

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2. Find all tuples in loan relation in which amount loan is more than 5000

$$\sigma_{\text{amount} > 5000}(\text{loan})$$

3. Find all tuples in loan relation where amount is more than 5000 and branch is Kathmandu.

$$\sigma_{\text{branch\_name} = \text{"Kathmandu"} \wedge \text{amount} > 5000}(\text{loan})$$

### *The projection Operation*

The projection operation retrieves tuples for specified attributes of relation. It eliminates duplicate tuples in relation. The projection is denoted by uppercase Greek letter pi ( $\Pi$ ). We need to specify attributes that we wish to appear in the result as a subscript to  $\Pi$ .

The general structure of projection is

$$\Pi_{A_1, A_2, \dots, A_k}(r)$$

where  $A_1, A_2, \dots, A_k$  are attributes of relation  $r$ .

Example: Find account number and their balance from account relation

$$\Pi_{\text{account\_number}, \text{balance}}(\text{account})$$

$$\text{output} = \{t \mid t[\text{account\_number}, \text{balance}]\}$$

### Composition of relational operations

Relational algebra operations can be composed together into relational-algebra expression. This required for complicated query.

Example: Find those customers who say in Kathmandu.

$$\Pi_{\text{customer\_name}}(\sigma_{\text{customer\_city} = \text{"Kathmandu"}}(\text{customer}))$$

### *Union Operation*

Let  $r$  and  $s$  are two relations then their union defines as

$$r \cup s = \{t \mid t \in r \text{ or } t \in s\}$$

For  $r \cup s$  to be valid, it must hold

- $r, s$  must have same arity (same number of attributes)
- The attribute domain must be compatible (e.g. domain of  $i^{\text{th}}$  column of  $r$  must deals with same type of domain of  $i^{\text{th}}$  column of  $s$ )

Example: Find all customers with either account or loan.

$$\Pi_{\text{customer\_name}}(\text{depositor}) \cup \Pi_{\text{customer\_name}}(\text{borrower})$$

### *Set difference Operation*

The set difference allows us to find tuples that are in one relation but not in another relation. The expression  $r - s$  produces a relation containing those tuples in  $r$  but not in  $s$ .

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Formally, let r and s are two relations then their difference r-s define as

$$r-s=\{t \mid t\in r \text{ and } t\notin s\}$$

The set difference must be taken between compatible relations. For r-s to be valid, it must hold

- R and s must have the same arity
- Attribute domains of r and s must be compatible

Example: Find all customer of the bank who have account but not loan

$$\Pi_{customer\_name}(depositor) - \Pi_{customer\_name}(borrower)$$

#### Cartesian Product Operation

The Cartesian product operation denoted by cross ( $\times$ ). It allows us to combine information from any two relations. Cartesian product of two relations r and s, denoted by  $r \times s$  returns a relation instance whose schema contains all the fields of r (in same order as they appear in r) followed all field of s (in the same order as they appear in s). The result of  $r \times s$  contains one tuples  $\langle r,s \rangle$  (concatenation of tuples of r and s) for each pair tuples  $t \in r, q \in s$ . Formally,

$$r \times s = \{ \langle t,q \rangle \mid t \in r \text{ and } q \in s \}$$

Example 1:

A	B
$\alpha$	1
$\beta$	2

C	D	E
$\alpha$	10	a
$\beta$	10	a
$\beta$	20	b
$\gamma$	10	b

Relation r

Relation s

$r \times s$ :

A	B	C	D	E
$\alpha$	1	$\alpha$	10	a
$\alpha$	1	$\beta$	10	a
$\alpha$	1	$\beta$	20	b
$\alpha$	1	$\gamma$	10	b
$\beta$	2	$\alpha$	10	a
$\beta$	2	$\beta$	10	a
$\beta$	2	$\beta$	20	b
$\beta$	2	$\gamma$	10	b

$\sigma_{A=a}(r \times s)$ :

A	B	C	D	E
$\alpha$	1	$\alpha$	10	a
$\alpha$	1	$\beta$	10	a
$\alpha$	1	$\beta$	20	b
$\alpha$	1	$\gamma$	10	b

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Example 2:

customer_name	loan_number
X	L01
Y	L02

Relation borrower

loan_number	branch_name	amount
L01	B1	5000
L02	B2	6000

Relation loan

Query: Find all customer who taken loan from branch "B1".

$\Pi_{\text{customer\_name}}(\sigma_{\text{borrower.loan\_number}=\text{loan.loan\_number}}(\sigma_{\text{branch\_name}=\text{"B1"}}(\text{borrower} \times \text{loan})))$

Process:

$\text{borrower} \times \text{loan}$

customer_name	borrower.loan_number	loan.loan_number	branch_name	amount
X	L01	L01	B1	5000
X	L01	L02	B2	6000
Y	L02	L01	B1	5000
Y	L02	L02	B2	6000

$\sigma_{\text{branch\_name}=\text{"B1"}}(\text{borrower} \times \text{loan})$

customer_name	borrower.loan_number	loan.loan_number	branch_name	amount
X	L01	L01	B1	5000
Y	L02	L01	B1	5000

$\sigma_{\text{borrower.loan\_number}=\text{loan.loan\_number}}(\sigma_{\text{branch\_name}=\text{"B1"}}(\text{borrower} \times \text{loan}))$

customer_name	borrower.loan_number	loan.loan_number	branch_name	amount
X	L01	L01	B1	5000

$\Pi_{\text{customer\_name}}(\sigma_{\text{borrower.loan\_number}=\text{loan.loan\_number}}(\sigma_{\text{branch\_name}=\text{"B1"}}(\text{borrower} \times \text{loan})))$

customer_name
X

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## The Rename Operation

The result of relational-algebra expression does not have a name to refer it. It is better to give name to result relation. The rename operator is denoted by lower case Greek letter rho ( $\rho$ ). Rename operation in relation-algebra expressed as

$$\rho_x(E)$$

where E is a relational algebra expression and x is name for result relation. It returns the result of expression E under the name x.

Since a relation r is itself a relational-algebra expression thus, the rename operation can also apply to rename the relation r (i.e. to get same relation under a new name). Rename operation can also used to rename attributes of relation. Assume a relational algebra expression E has arity n. Then expression

$$\rho_{x(A1,A2, \dots, An)}(E)$$

returns the result of expression E under the name x and it renames attributes to A1,A2, ...,An.

Example 1: Find the largest account balance in the bank.

$$\Pi_{balance}(account) - \Pi_{account.balance}(\sigma_{account.balance < d.balance}(account \times \rho_d(account)))$$

Process:

account_number	balance
A1	500
A2	600
A3	700

Relation account

Account_number	balance
A1	500
A2	600
A3	700

Relation d

$$account \times \rho_d(account)$$

account.account_number	account.balance	d.balance
A1	500	500
A1	500	600
A1	500	700
A2	600	500
A2	600	600
A2	600	700
A3	700	500
A3	700	600
A3	700	700

$$\Pi_{account.balance}(\sigma_{account.balance < d.balance}(account \times \rho_d(account)))$$

Account.balance
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500
600

$$\Pi_{\text{balance}}(\text{account}) - \Pi_{\text{account.balance}}(\sigma_{\text{account.balance} < \text{d.balance}}(\text{account} \times \rho_{\text{d}}(\text{account})))$$

balance
500
600
700

account.balance
500
600

Output:

balance
700

Example 2: Find the names of all customers who live on the same street and in the same city as smith.

$$\Pi_{\text{customer.customer}}(\sigma_{\text{customer.customer\_street}=\text{smith\_add.street} \wedge \text{customer.customer\_city}=\text{smith\_add.city}}(\text{customer} \times \rho_{\text{smith\_add}(\text{street}, \text{city})}(\Pi_{\text{customer\_street}, \text{customer\_city}}(\sigma_{\text{customer\_name}=\text{"smith"}}(\text{customer}))))))$$

### 3.2.2 Formal Definition of Relational Algebra

Basic expressions in relational algebra are

- Relation in a database
- A constant relation
  - A constant relation is expression by listing its tuples
  - $\{(A01, "B1", 500)(A02, "B2", 600)\}$

From the basic expression we can construct other expression. Let E1 and E2 be relational algebra expression, then following are also relational-algebra expression.

$$\begin{aligned} &E_1 \cup E_2 \\ &E_1 - E_2 \\ &E_1 \times E_2 \\ &\sigma_p(E_1), \text{ p is a predicate on attributes in } E_1. \\ &\Pi_s(E_1), \text{ s is a list of some attributes in } E_1 \\ &\rho_x(E_1), \text{ x is the new name for the result of } E_1 \end{aligned}$$

### 3.2.3 Additional Operations

The fundamental operations of the relational algebra are sufficient to express any relational algebra query. But for complex query it is difficult. Additional operations (set intersection, natural join, division, assignment) simplify the common queries.

#### Set-intersection operation

Let r and s are two relation having same arity and attributes of r and s are compatible then their intersection  $r \cap s$  define as

$$r \cap s = \{t \mid t \in r \text{ and } t \in s\}$$

In terms of fundamental operation of relational algebra it can express as

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$$r \cap s = r - (r - s)$$

Example 1:

A	B
$\alpha$	1
$\alpha$	2
$\beta$	1

Relation r

A	B
$\alpha$	2
$\beta$	3

relation s

A	B
$\alpha$	2

$r \cap s$ :

Example 2: Find all customer who have both loan and account

$$\Pi_{\text{customer\_name}}(\text{borrower}) \cap \Pi_{\text{customer\_name}}(\text{depositor})$$

#### *The natural join operation*

The natural join operation generally needs to simplify queries that required a Cartesian product. The natural join allow to combine certain selections and a Cartesian product into one operation. It is denoted by symbol  $\bowtie$ .

The natural join operation forms a Cartesian product of its two arguments, performs a selection forcing equality on those attributes that appear in both relation schema and finally removes duplicate attributes.

Formally, let r and s are two relations on schema R and S respectively then  $r \bowtie s$  is a relation on schema  $R \cup S$ . That is,

$$r \bowtie s = \Pi_{R \cup S}(\sigma_{r.A_1=s.A_1 \wedge r.A_2=s.A_2 \wedge \dots \wedge r.A_n=s.A_n}(r \times s))$$

where  $\{A_1, A_2, \dots, A_n\}$  are common attributes in R and S.

Example 1:

Let

$$R = (A, B, C, D)$$

$$S = (E, B, D)$$

Now,

$$\text{Result schema} = (A, B, C, D, E)$$

$r \bowtie s$  is define as

$$\Pi_{r.A, r.B, r.C, r.D, s.E}(\sigma_{r.B=s.B \wedge r.D=s.D}(r \times s))$$

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Suppose r and s are two relation as follow

A	B	C	D
$\alpha$	1	$\alpha$	a
$\beta$	2	$\gamma$	a
$\gamma$	4	$\beta$	b
$\alpha$	1	$\gamma$	a
$\delta$	2	$\beta$	b

Relation r

B	D	E
1	a	$\alpha$
3	a	$\beta$
1	a	$\gamma$
2	b	$\delta$
3	b	$\epsilon$

relation s

$r \bowtie s$ :

A	B	C	D	E
$\alpha$	1	$\alpha$	a	$\alpha$
$\alpha$	1	$\alpha$	a	$\gamma$
$\alpha$	1	$\gamma$	a	$\alpha$
$\alpha$	1	$\gamma$	a	$\gamma$
$\delta$	2	$\beta$	b	$\delta$

Example 2: Find the names of all customer who have a loan at the bank

$$\Pi_{\text{customer\_name}}(\text{borrower} \bowtie \text{loan})$$

Same query is express by fundamental operation as follow

$$\Pi_{\text{customer\_name}}(\sigma_{\text{borrower.loan\_number}=\text{loan.loan\_number}}((\text{borrower} \times \text{loan})))$$

Example 3: Find names of all branches with customer who have account in the bank and who live in Kathmandu.

$$\Pi_{\text{customer\_name}}(\sigma_{\text{customer\_city}=\text{"Kathmandu"}}(\text{customer} \bowtie \text{account} \bowtie \text{depositor}))$$

Example 4: Find all customers who have both loan and account at the bank.

$$\Pi_{\text{customer\_name}}(\text{borrower} \bowtie \text{depositor})$$

In the set intersection form this can be express as

$$\Pi_{\text{customer\_name}}(\text{borrower}) \cap \Pi_{\text{customer\_name}}(\text{depositor})$$

**Note 1:** Let  $r(R)$  and  $s(S)$  e relations without any attributes in common. That is  $R \cap S = \Phi$  then

$$r \bowtie s = r \times s$$

#### Note 2: Theta Join

The theta join is an extension to the natural join operations that allow us to combine a selection and a Cartesian product into a single operation with predicate on attributes.

Let relation  $r(R)$  and  $s(S)$ , and  $\Theta$  be a predicate on attributes in the schema  $R \cup S$  then theta join operation  $r \bowtie_{\Theta} s$  is defined as

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$r \bowtie_{\Theta} s = \sigma_{\Theta}(r \times s)$

Example: Find all customers who have loan and stay in Kathmandu.

$\Pi_{customer\_name}(borrower \bowtie_{customer\_city="Kathmandu"} loan)$

## Division Operation

Let  $r$  and  $s$  be the relations on schemas  $R$  and  $S$  respectively where  $S \subseteq R$  (i.e. every attributes of schema  $S$  is also in schema  $R$ ) then  $r \div s$  is a relation on schema  $(R-S)$ , define as

$r \div s = \{t | t \in \Pi_{R-S}(r) \wedge \forall u \in s (tu \in r)\}$

Example:

A	B
$\alpha$	1
$\alpha$	2
$\alpha$	3
$\beta$	1
$\gamma$	1
$\delta$	1
$\delta$	3
$\delta$	4
$\epsilon$	6
$\epsilon$	1
$\beta$	2

Relation  $r$

B
1
2

Relation  $s$

$r \div s :$

A
$\alpha$
$\beta$

Example 2:

A	B	C	D	E
$\alpha$	a	$\alpha$	a	1
$\alpha$	a	$\gamma$	a	1
$\alpha$	a	$\gamma$	b	1
$\beta$	a	$\gamma$	a	1
$\beta$	a	$\gamma$	b	3
$\gamma$	a	$\gamma$	a	1
$\gamma$	a	$\gamma$	b	1
$\gamma$	a	$\beta$	b	1

Relation  $r$

D	E
a	1
b	1

Relation  $s$

Example: Find all customers who have an account at all the branches located in Kathmandu.

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We can obtain all branches in Kathmandu by expression

$$r1 = \Pi_{\text{branch\_name}}(\sigma_{\text{cbranch\_city} = \text{"Kathmandu"}}(\text{branch}))$$

We can obtain all (customer\_name, branch\_name) pair for the customer who have account.

$$r2 = \Pi_{\text{customer\_name}, \text{branch\_name}}(\text{depositor} \bowtie \text{account})$$

The required customer can obtain from

$$r2 \div r1$$

That is,

$$\Pi_{\text{customer\_name}, \text{branch\_name}}(\text{depositor} \bowtie \text{account}) \div \Pi_{\text{branch\_name}}(\sigma_{\text{cbranch\_city} = \text{"Kathmandu"}}(\text{branch}))$$

### *The assignment operations*

The assignment operation provides convenient way to express complex query. The assignment operation denoted by  $\leftarrow$ , works like assignment in programming language. The evaluation of an assignment does not result any relation being displayed to the user. But the result of the expression to the right of the  $\leftarrow$  is assigned to the relation variable. This relation variable may used in subsequent expressions. With the assignment expression, a query can be written as a sequential program consisting a series of assignments followed by an expression whose value is displayed as the result of the query.

Example: Find all customer who taken loan from bank as well as he/she has bank account.

$$\text{Temp1} \leftarrow \Pi_{\text{customer\_name}}(\text{borrower})$$

$$\text{Temp2} \leftarrow \Pi_{\text{customer\_name}}(\text{depositor})$$

$$\text{result} \leftarrow \text{Temp1} \cap \text{temp2}$$

### 3.2.4 Extended Relational-Algebra operations

Generalized projection, outer join and aggregation function are extension on basic relational algebra operation.

#### *Generalized Projection*

Generalized projection operation allows arithmetic functions in the projection list. The general structure is

$$\Pi_{F1, F2, \dots, Fn}(E)$$

where E is any relational algebra expression. Each F1, F2, . . . Fn are arithmetic expression involving constraints and attributes in the schema of E.

Example 1:

Suppose a relation

$$\text{credit\_info}(\text{customer\_name}, \text{credit\_limit}, \text{credit\_balance})$$

Query: Find how much more each person can spend.

$$\Pi_{\text{customer\_name}, \text{credit\_limit} - \text{credit\_balance}}(\text{credit\_info})$$

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The resulting attribute from credit\_limit – credit\_balance does not have name; its name can be specify as below

$$\Pi_{customer\_name, credit\_limit - credit\_balance \text{ as } credit\_available}(credit\_info)$$

Example 2:

Suppose relation

$$employee(employee\_id,ename,salary)$$

Find employee and their corresponding bonus, assume that bonus for each employee is 10% of his/her salary.

$$\Pi_{ename,salary*1.10 \text{ as } bonus}(employee)$$

#### Aggregate function and operations

Aggregate function takes a collection of values and return as a single value as a result. Some aggregate functions are

- AVG: average value
- MIN: minimum value
- SUM: sum of values
- Count: number of values

The aggregate operation in relational algebra denoted by the symbol *g* (i.e. *g* is the letter G in calligraphic font). The general structure is

$$G_1,G_2, \dots ,G_n \text{ } g \text{ } F_1(A_1),F_2(A_2), \dots ,F_n(A_n)(E)$$

where E is any relational algebra expression.

- G1,G2, . . .,Gn is a list of attributes on which to group (it could be empty)
- Each Fi is aggregate function.
- Each Ai is an attribute name

Example:

A	B	C
$\alpha$	$\alpha$	7
$\alpha$	$\beta$	7
$\alpha$	$\beta$	3
$\beta$	$\beta$	10

Relation r

*g*<sub>sum(C)(r):</sub>

Sum-C
27

Example 2: Find the balance to each branch.

$$branch\_name \text{ } g \text{ } sum(blance) \text{ as } sum-blance(account)$$

Example 3: Find no. of account in each branch

## 3.0 Relational Model

branch\_name *g* count(account\_number) (account)

### Outer join

The outer-join operation is extension to natural join. It has a capability to deal with missing information. There are three form of outer-join operation

(a) Left outer-join ( $\bowtie\leftarrow$ )

- Takes all tuples in the left relation. If there are any tuples in right relation that does not match with tuple in left relation, simply pad these right relation tuples with null.
- Add them to the result of the left outer-join.

(b) Right outer-join ( $\bowtie\rightarrow$ )

- Takes all tuples in the right relation. If there are any tuples in the left relation that does not match with tuple in right relation, simply pad left relation tuples with null.
- Add them to the result of the left outer-join.

(c) Full outer-join ( $\bowtie\leftrightarrow$ )

- Pad tuples from the left relation that that did not match any from the right relation
- Pad tuples from the right relation that that did not match any from the left relation
- Add them to the result of full outer-join.

Example:

Consider relations

loan_number	branch_name	amount
L01	B1	500
L02	B2	600
L05	B1	700

Relation loan

customer_name	loan_number
X	L01
Y	L02
Z	L07

Relation borrower

### Natural join (Inner join)

Loan  $\bowtie$  borrower

loan_number	branch_name	amount	customer_name
L01	B1	500	X
L02	B2	600	Y

## 3.0 Relational Model

### Left outer-join

loan  $\bowtie$  borrower

loan_number	branch_name	amount	customer_name
L01	B1	500	X
L02	B2	600	Y
L05	B1	700	null

### Right outer-join

Loan  $\bowtie$  borrower

loan_number	branch_name	amount	customer_name
L01	B1	500	X
L02	B2	600	Y
L07	null	null	Z

### Full outer-join

Loan  $\bowtie$  borrower

loan_number	branch_name	amount	customer_name
L01	B1	500	X
L02	B2	600	Y
L05	B1	700	null
L07	null	null	Z

### 3.2.5 Null Values

Tuples may not have any values for some of the attributes. At that time the attribute is said to have null value and is denoted by null. It simplifies an unknown value or a value does not exist. The result of any arithmetic or comparison involving null is null. There are often more than one possible way of dealing with null values, as a result our definition can sometimes be arbitrary. Therefore arithmetic operations and comparison on null values should avoid if possible.

Comparison involving nulls may occur inside Boolean expression: AND, OR and NOT operations. Boolean operation deal with null value is as follow.

AND: (true and null)=null  
(false and null)=false  
(null and null)=null

OR: (null or true)=true  
(null or false)=null  
(null or null)=null

NOT: (not null)=null



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### How different relation operations deal with null values ?

#### Select

The select operation evaluates predicate  $p$  in  $\sigma_p(E)$  on each tuple  $t$  in  $E$ . If predicate returns true value then  $t$  is added to the result. Otherwise predicate returns null or false and  $t$  is not added to the result.

#### Join

In natural join,  $r \bowtie s$ , if two tuples  $t_r \in r$  and  $t_s \in s$ , both have a null values in common attributes then tuples do not match.

#### Projection

The projection treats null just like any other value when eliminating duplicates. If two tuples in the projection result are exactly the same, and both have nulls in the same fields, they are treated as duplicated. For example

A	B	C
$\alpha$	null	$\gamma$
$\alpha$	null	$\gamma$

Here, projection assumes these two tuples are duplicates.

This is arbitrary decision since without knowing the actual value, we can not tell two instances of null are duplicates or not.

#### Union, intersection and difference

These operation also treats null value as any other values when eliminating duplicates. These operations treat tuples that have same values on all fields as duplicates even if some of the fields have null values in both tuples. This decision is arbitrary, especially in the case of intersection and difference since the actual values (if any) represented by nulls are same.

#### Generalized projection

It treats null value same as projection.

#### Outer join

Outer join operation behaves null value just like natural join operation. In outer join, tuples that do not occur in the natural join result may be added to the result of outer join padded with nulls.

### 3.2.6 Modification of the database

Insertion, deletion and updating operations are responsible for database modification.

#### Deletion

A delete request is expressed similarity to the query, except instead of displaying tuples, the selected tuples are removed from the databases. Delete request can delete only whole tuples, can not delete values on only particular attributes. Deletion is expressed as

$$r \leftarrow r - E$$

where  $r$  is a relation and  $E$  is a relational algebra query.

Example 1: Delete all account in the "B1" branch.

account  $\leftarrow$  account -  $\sigma_{\text{branch\_name}="B1"}(\text{account})$

Example 2: Delete all records with account in the range of 0 to 5.

loan  $\leftarrow$  loan -  $\sigma_{\text{amount} > 0 \text{ and } \text{amount} \leq 50}(\text{loan})$

*insertion*

## 3.0 Relational Model

to insert data into relation we can either specify a tuple to be inserted or write a query whose result is a set of tuples to be inserted. In relational-algebra, an insertion is expressed by

$$r \leftarrow r \cup E$$

where  $r$  is a relation and  $E$  is a relational algebra expression.

Example: insert information in the database specifying customer "X" has 5000 in account A1 at the kathmandu city.

account  $\leftarrow$  account  $\cup \{(A1, "kathmandu", 5000)\}$   
depositor  $\leftarrow$  depositor  $\cup \{("x", A1)\}$

### Updating

Updating allows to change a value in a tuple without changing all values in tuple. In relational algebra, updating is expressed by

$$r \leftarrow \Pi_{F_1, F_2, \dots, F_n}(r)$$

where each  $F_i$  is either

- $i^{\text{th}}$  attributes of  $r$ , if the  $i^{\text{th}}$  attribute is not updated or
- expression involving only constant and attributes of  $r$ , if the attribute is to be updated. It gives the new value for the attribute.

Example 1: increase balance by 5% to all branches.

account  $\leftarrow \Pi_{\text{account\_number}, \text{branch\_name}, \text{balance} * 1.05}(\text{account})$

Example 2: Increase the balance by 6% for those accounts which balance is over 5000 and for the rest of accounts increase balance by 5%.

account  $\leftarrow \Pi_{\text{account\_number}, \text{branch\_name}, \text{balance} * 1.06}(\sigma_{\text{balance} > 5000}(\text{account}))$   
 $\cup \Pi_{\text{account\_number}, \text{branch\_name}, \text{balance} * 1.05}(\sigma_{\text{balance} \leq 5000}(\text{account}))$

### 3.2.7 Views

For a security reason, it is not desirable for all users to see the entire logical model of database (i.e. all actual relations stored in database). We may require certain data to be hidden from users. We may wish to create a personalized collection of relations that is better matched to certain user's intuition than whole database. For example, an employee in advertising department might see a relation consisting of the customers who have either an account or loan at the bank and the branches with which they do business. In relational algebra it is expressed by

$\Pi_{\text{branch\_name}, \text{customer\_name}}(\text{depositor} \bowtie \text{account}) \cup \Pi_{\text{branch\_name}, \text{customer\_name}}(\text{borrower} \bowtie \text{loan})$

Any relation that is not the part of the logical model, but it is made visible to a user as a virtual relation called a view. There would be no. of views for any given set of actual relations.

### 3.2.8 View Definition

View is defined by using the create view statement. The general structure is

Create view <view name> as <query expression>

where <query expression> is any legal relational-algebra query expression. Once a view is defined, it can refer by its virtual name, called view name.

Example: Create a view "all-customer" consisting of branches and their customers.

Create view all-customer as

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$$\Pi_{\text{branch\_name}, \text{customer\_name}}(\text{depositor} \bowtie \text{account}) \cup \Pi_{\text{branch\_name}, \text{customer\_name}}(\text{borrower} \bowtie \text{loan})$$

We can query on this view as in other relation. For example,

Query: find all customer of "B1" branch.

$$\Pi_{\text{customer\_name}}(\sigma_{\text{branch\_name}="B1"}(\text{all-customer}))$$

View definition is not same as creating a new relation evaluating the query express. It is actually substitution to query expression from which it is defined. If the view is stored, it may become out of date. If the relations used to be define it are modified. To avoid this, database system stores the definition of view itself, rather than the result of evaluation of relational-algebra expression that defines the view. Whenever we evaluate the query, the view relation is recomputed.

Certain database system stores result of evaluation of the relational algebra expression that defines view. If the actual relation used in the view definition change, the view need to kept up to date; such view called *materialized view*. The process of keeping the view up to date called *view maintenance*.

### 3.2.9 Updates through view and null values

Although views are useful tool for queries, it gives serious problem if we allow insertion, deletion and updates from view. Any modification made by view must be translated to actual relations in database. It is difficult task. In some case, it is not possible Let us consider example to illustrate this.

Consider a view loan-branch which is provided to clerk to see all loan data in loan relation, except loan amount.

Create view loan-branch as

$$\Pi_{\text{loan\_number}, \text{branch\_name}}(\text{loan})$$

Assume that clerk try to insert loan information and write

$$\text{loan-branch} \leftarrow \text{loan-branch} \cup \{(L01, "B1")\}$$

In fact, this insertion must made to the relation loan. However to insert a tuple into loan, we must have value for account. To deal with this problem we may choose to options

- Reject insertion and return an error message to the user
- Insert tuple (L01, "B1", null) into a loan relation.

Let us consider another view to illustrate another problem with modification of the database through view.

Consider a view loan-info providing loan amount for each loan of customer

Create view loan-info as

$$\Pi_{\text{customer\_name}, \text{amount}}(\text{borrower} \bowtie \text{loan})$$

Assume that following insertion is perform to view

$$\text{loan-info} \leftarrow \text{loan-info} \cup \{("X", 5000)\}$$

Since view loan-info is created from multiple relation (i.e customer\_nae is taken from borrower, account is taken from loan). So we have to insert tuple ("X", null) into borrower and (null, null, 5000) into loan. This insertion may leads several complication, loan can be taken with out loan number so it is impossible to map loan and their corresponding customer in actual relation. If loan\_number is define as primary key in loan relation then this insertion is not possible. Because of these problems, modification on view is not permitted generally.

## 3.0 Relational Model

### 3.2.10 Tuple relational calculus

Tuple relational calculus is nonprocedural query language. It describes desired information without specifying procedure for obtaining that information. The general structure of query in relational calculus is express as

$\{t | p(t)\}$

read as; set of all tuples  $t$  such that predicate  $p$  is true for  $t$ .

General notation

- $t$  is a tuple variables.
- $T[A]$  denotes the value of tuple  $t$  on attribute  $A$ .
- $t \in r$  denotes the tuple  $t$  in relation  $r$ .
- $p$  is a formula similar to the predicate calculus. Predicate calculus formula consist
  - set of attributes and constant
  - set of comparison operator (e.g.  $<, \leq, =, \neq, >, \geq$ )
  - set of connectives: and ( $\wedge$ ), or ( $\vee$ ), not ( $\neg$ )
  - implication ( $\Rightarrow$ ):  $X \Rightarrow Y$  (i.e. if  $X$  is true, then  $Y$  is true)
  - set of quantifiers
    - $\exists t \in r(Q(t))$  : "there exist" a tuple  $t$  in relation  $r$  such that predicate  $Q(t)$  is true.
    - $\forall \exists t \in r(Q(t))$  :  $Q$  is true "for all" tuples  $t$  in relation  $r$

Example queries

1. Find the loan number, branch name and amount for loan of over 5000  
 $\{t | t \in \text{loan} \wedge t[\text{amount}] > 5000\}$
2. Find the loan number for each loan of account greater than 1200.  
 $\{t | \exists s \in \text{loan}(t[\text{loan\_number}] = s[\text{loan\_number}] \wedge s[\text{amount}] > 1200)\}$
3. Find the names of all customer having a loan, an account or n=both at the bank.  
 $\{t | \exists s \in \text{borrower}(t[\text{customer\_name}] = s[\text{customer\_name}] \vee$   
 $\exists u \in \text{depositor}(t[\text{customer\_name}] = u[\text{customer\_name}])\}$
4. Find the names of all customers who have a loan and account at the bank.  
 $\{t | \exists s \in \text{borrower}(t[\text{customer\_name}] = s[\text{customer\_name}] \wedge$   
 $\exists u \in \text{depositor}(t[\text{customer\_name}] = u[\text{customer\_name}])\}$
5. Find the names of all customers having a loan at the "B1" branch.  
 $\{t | \exists s \in \text{borrower}(t[\text{customer\_name}] = s[\text{customer\_name}]$   
 $\wedge \exists u \in \text{loan}(u[\text{branch\_name}] = "B1" \wedge u[\text{loan\_number}] = s[\text{loan\_number}]))\}$
6. Find the names of all customers who have a loan at the "B1" branch, bt no. amount at any branch of the bank.  
 $\{t | \exists s \in \text{borrower}(t[\text{customer\_name}] = s[\text{customer\_name}]$   
 $\exists u \in \text{loan}(u[\text{branch\_name}] = "B1" \wedge u[\text{loan\_number}] = s[\text{loan\_number}]$   
 $\wedge \text{not } \exists v \in \text{depositor}(v[\text{customer\_name}] = t[\text{customer\_name}]))\}$
7. Find the names of all customer and their city they stay having a loan from the branch "b1"  
 $\{t | \exists s \in \text{loan}(s[\text{branch\_name}] = "B1"$   
 $\wedge \exists u \in \text{borrower}(u[\text{loan\_number}] = s[\text{loan\_number}]$   
 $\wedge t[\text{customer\_name}] = u[\text{customer\_name}])$   
 $\wedge \exists v \in \text{customer}(u[\text{customer\_name}] = v[\text{customer\_name}]$   
 $\wedge t[\text{customer\_city}] = v[\text{customer\_city}] = v[\text{customer\_city}]))\}$
8. Find the names of all customers who have an account at all branch located in "Kathmandu".  
 $\{t | \exists c \in \text{customer}(t[\text{customer\_name}] = c[\text{customer\_name}]$   
 $\wedge \forall s \in \text{branch}(s[\text{branch\_city}] = "Kathmandu" \Rightarrow$   
 $\exists u \in \text{account}(s[\text{branch\_name}] = u[\text{branch\_name}]$   
 $\wedge \exists s \in \text{depositor}(t[\text{customer\_name}] = s[\text{customer\_name}]$   
 $\wedge s[\text{account\_number}] = u[\text{account\_number}]))\}$