BOARD EXAMINATION SOLVED QUESTIONS

1. **Evaluate the integral $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, dx$. Compare the result in both conditions for Simpson $\frac{1}{3}$ and $\frac{3}{8}$ rule.** **[2013/Fall]**

Solution:

Given that;

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, dx$$

$$a = 0,\quad b = \frac{\pi}{2}$$

Taking n = 6,

$$h = \frac{b-a}{n} = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12}$$

Now, table is created at the interval of $\frac{\pi}{12}$ from 0 to $\frac{\pi}{2}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
y	0	0.508	0.707	0.840	0.930	0.982	1
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Now, by Simpson's $\frac{1}{3}$ rule

$$I = \frac{h}{3}[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{\pi}{3 \times 12}[0 + 1 + 4\,(0.508 + 0.840 + 0.982) + 2\,(0.707 + 0.930)]$$

$$= 1.186$$

Again, by Simpson's $\frac{3}{8}$ rule

$$I = \frac{3h}{8}[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3\pi}{8 \times 12}[0 + 1 + 3\,(0.508 + 0.707 + 0.930 + 0.982) + 2\,(0.840)]$$

$$= 1.184$$

and, Absolute value of I

$$I_{abs} = \int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, dx = 1.198$$

NOTE: Use calculator to directly obtain the absolute value in radian mode.

Now,

Error by Simpson $\frac{1}{3}$ rule = |1.186 − 1.198| = 0.012

Error by Simpson $\frac{3}{8}$ rule = |1.184 − 1.198| = 0.014

Here, the error by Simpson $\frac{1}{3}$ rule is less than Simpson $\frac{3}{8}$ rule.

2. **Evaluate the integral $I = \int_0^6 \frac{1}{1+x^2}\, dx$. Compare the absolute error in both conditions for Simpson $\frac{1}{3}$ rule and Simpson $\frac{3}{8}$ rule.** **[2013/Spring]**

Solution:

Given that;

$$I = \int_0^6 \frac{1}{1+x^2}\, dx$$

$$a = 0,\ b = 6$$

Let, n = 6 then

$$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$$

Now, Table is created at the interval of 1 from 0 to 6

Formulating the table,

x	0	1	2	3	4	5	6
y	1	0.5	0.2	0.1	0.058	0.038	0.027
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By Simpson's $\frac{1}{3}$ rule,

$$I = \frac{h}{3}[(y_0 + y_6) + 4\,(y_1 + y_3 + y_5) + 2\,(y_2 + y_4)]$$

$$= \frac{1}{3}[1 + 0.027 + 4\,(0.5 + 0.1 + 0.038) + 2\,(0.2 + 0.058)]$$

$$= 1.365$$

By Simpson's $\frac{3}{8}$ rule,

$$I = \frac{3h}{8}[y_0 + y_6 + 3\,(y_1 + y_2 + y_4 + y_5) + 2\,(y_3)]$$

$$= \frac{3}{8}[1 + 0.027 + 3\,(0.5 + 0.2 + 0.058 + 0.038) + 2\,(0.1)]$$

$$= 1.355$$

Now, Absolute value of I,

$$I = \int_0^6 \frac{1}{1+x^2}\, dx = \tan^{-1}(x)\Big|_0^6 = 1.405$$

Now,

Error by Simpson $\frac{1}{3}$ rule = |1.405 − 1.365| = 0.04

Error by Simpson $\frac{3}{8}$ rule = |1.405 − 1.355| = 0.05

Here, the error by Simpson $\frac{1}{3}$ rule is less than Simpson $\frac{3}{8}$ rule.

3. **Find the integral value $I = \int_0^1 \frac{dx}{1+x^2}$ correct to three decimal place by using Romberg integration.** **[2013/Spring, 2018/Spring]**

Solution:

Given that:

$$I = \int_0^1 \frac{dx}{1+x^2}$$

Here, $a = 0, b = 1$

i) Taking h = 0.5 and creating interval of 0.5 from 0 to 1.

x	0	0.5	1
y = f(x)	1	0.8	0.5
	y_0	y_1	y_2

Now, using trapezoidal rule,

$$I(0.5) = \frac{h}{2}[y_0 + y_2 + 2y_1]$$

$$= \frac{0.5}{2}[1 + 0.5 + 2(0.8)]$$

$$= 0.775$$

ii) Taking h = 0.25 and creating interval of 0.25 from 0 to 1.

x	0	0.25	0.5	0.75	1
y	1	0.9411	0.8	0.64	0.5
	y_0	y_1	y_2	y_3	y_4

Now, using trapezoidal rule,

$$I(0.25) = \frac{h}{2}[y_0 + y_4 + 2(y_1 + y_2 + y_3)]$$

$$= \frac{0.25}{2}[1 + 0.5 + 2(0.9411 + 0.8 + 0.64)]$$

$$= 0.7827$$

iii) Taking h = 0.125

x	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1
y	1	0.9846	0.9411	0.8767	0.8	0.7191	0.64	0.5663	0.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

Now, using Trapezoidal rule,

$$I(0.125) = \frac{h}{2}[y_0 + y_8 + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)]$$

$$= \frac{0.125}{2}[1 + 0.5 + 2(0.9846 + 0.9411 + 0.8767 + 0.8 + 0.7191 + 0.64 + 0.5663)]$$

$$= 0.7847$$

Now, optimizing values by Romberg integration,

$$I(0.5, 0.25) = \frac{1}{3}[4I(0.25) - I(0.5)]$$

$$= \frac{1}{3}[4 \times 0.7827 - 0.775]$$

$$= 0.7852$$

$$I(0.25, 0.125) = \frac{1}{3}[4I(0.125) - I(0.25)]$$

$$= \frac{1}{3}[4 \times 0.7847 - 0.7827]$$

$$= 0.7853$$

$$I(0.5, 0.25, 0.125) = \frac{1}{3}[4I(0.25, 0.125) - I(0.5, 0.25)]$$

$$= 0.7853$$

Hence the value of integral $\int_0^1 \frac{dx}{1+x^2} = 0.7853$

Also, $\int_0^1 \frac{dx}{1+x^2} = \tan^{-1}(x)\Big|_0^1 = 0.7853$

Table of obtained values;

$$\left.\begin{matrix} \left.\begin{matrix} I(0.5) \\ I(0.25) \end{matrix}\right\} I(0.5, 0.25) \\ \left.\begin{matrix} I(0.25) \\ I(0.125) \end{matrix}\right\} I(0.25, 0.125) \end{matrix}\right\} I(0.5, 0.25, 0.125)$$

4. **The following table gives the displacement, x (cms) of an object at various of time, t(seconds). Find the velocity and acceleration of the object at t = 1.6 sec. Using suitable interpolation method. [2014/Fall]**

T	1.0	1.2	1.4	1.6	1.8
X	9.0	9.5	10.2	11.0	13.2

Solution:

Creating the difference table from given data

x = T	y = x'	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1.0	9.0				
		0.5			
			0.2		
1.2	9.5			−0.1	
		0.7			
			0.1		
1.4	10.2			1.3	1.4
		0.8			
			1.4		
1.6	11.0				
		2.2			
1.8	13.2				

Here the data of T is equispaced and t = 1.6 sec is near the end of the table, so using Newton's backward formula for numerical differentiation.

$$h = 1.8 - 1.6 = 0.2$$

Now, at t = 1.6 sec

From numerical differentiation, using Newton's backward formula ,

$$\left(\frac{dy}{dx}\right)_{1.6} = y' = \frac{1}{h}\left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3}\right]$$

$$= \frac{1}{0.2}\left[0.8 + \frac{0.1}{2} + \frac{-0.1}{3}\right]$$

$= 4.083$ cm/s is the required velocity of an object

Now, for acceleration

$$y'' = \frac{1}{h^2}[\nabla^2 y_n + \nabla^3 y_n] = \frac{1}{0.22}[0.1 + -0.1]$$

$\therefore$ $y'' = 0$ cm/s² is the required acceleration of an object.

5. **Evaluate the integral $\int_0^{\pi} (1 + 3\cos^2 x)\,dx$ by,**
   i) **Trapezoidal rule**
   ii) **Simpson's $\frac{3}{8}$ rule, taking number of intervals (n) = 6**

[2014/Spring]

Solution:

Given that;

$$I = \int_0^{\pi} (1 + 3\cos^2 x)\,dx$$

$$n = 6$$

Also,

$$a = 0, b = \pi$$

Then,

$$h = \frac{b-a}{n} = \frac{\pi - 0}{6} = \frac{\pi}{6}$$

Now, table is created at the interval of $\frac{\pi}{6}$ from 0 to π

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	4	3.25	1.75	1	1.75	3.25	4
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

i) By trapezoidal rule,

$$I = \frac{h}{2}[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$$

$$= \frac{\pi}{2 \times 6}[4 + 4 + 2(3.25 + 1.75 + 1 + 1.75 + 3.25)]$$

$\therefore$ $I = 7.8539$

ii) By Simpson's $\frac{3}{8}$ rule,

$$I = \frac{3h}{8}[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3\pi}{8 \times 6}[4 + 4 + 3(3.25 + 1.75 + 1.75 + 3.25) + 2(1)]$$

$$= 7.8539$$

Also,

$$I_{abs} = \int_0^{\pi} (1 + 3\cos^2 x)\,dx = \int_0^{\pi} 1 + \frac{3}{2}(\cos 2x + 1) = 7.8539$$

6. **Evaluate the integral $I = \int_0^{\frac{\pi}{2}} \sin x\,dx$ for n = 6 and compare the result in both conditions for Simpson $\frac{1}{3}$ and $\frac{3}{8}$ rule.** **[2015/Fall]**

Solution:

Given that;

$$I = \int_0^{\frac{\pi}{2}} \sin x\,dx$$

$$a = 0,\ b = \frac{\pi}{2},\ n = 6$$

$$h = \frac{b-a}{n} = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12}$$

Now, creating table at the interval of $\frac{\pi}{12}$ from 0 to $\frac{\pi}{2}$

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
y	0	0.258	0.5	0.707	0.866	0.965	1
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Now, By Simpson's $\frac{1}{3}$ rule

$$I = \frac{h}{3}[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{\pi}{3 \times 12}[0 + 1 + 4(0.258 + 0.707 + 0.965) + 2(0.5 + 0.866)]$$

$$= 0.9993$$

Again, by Simpson's $\frac{3}{8}$ rule

$$I = \frac{3h}{8}[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3\pi}{8 \times 12}[0 + 1 + 3(0.258 + 0.5 + 0.866 + 0.965) + 2(0.707)]$$

$$= 0.9995$$

and, $I_{abs} = \int_0^{\frac{\pi}{2}} \sin x \cdot dx = [-\cos x]_0^{\pi/2} = -\cos\frac{\pi}{2} + \cos 0 = 1$

Now, Error by Simpson $\frac{1}{3}$ rule $= |1 - 0.9993| = 0.0007$

Error by Simpson $\frac{3}{8}$ rule $= |1 - 0.9995| = 0.0005$

Here, the error by Simpson $\frac{1}{3}$ rule is more than Simpson $\frac{3}{8}$ rule, so Simpson $\frac{3}{8}$ rule is more accurate.

7. Use following table of data to estimate velocity at t = 7 sec

Time, t(s)	5	6	7	8	9
Distance Travelled, s(t) (km)	10.0	14.5	19.5	25.5	32.0

Hint: velocity is first derivative of s(t). **[2015/Spring]**

Solution:

Creating difference table

$t = x$	$y = s(t)$	1st diff	2nd diff	3rd diff	4th diff
5	10.0				
		4.5			
6	14.5		0.5		
		5		0.5	
7	19.5		1		−1
		6		−0.5	
8	25.5		0.5		
		6.5			
9	32.0				

Now, to estimate velocity at t = 7 sec which lies at the mid of table.

Using Stirling's central difference formula,

We have,

$$y_p = y_0 + \frac{p}{1!}\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{p^2}{2!}\Delta^2 y_{-1} + \frac{p(p^2 - 1^2)}{3!}\ \frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2} + \ldots\ldots$$

$$= x_0 + ph$$

Differentiating with respect to p, we get,

$$\frac{dy_p}{dx} = \frac{\Delta y_0 + \Delta y_{-1}}{2} + p\Delta^2 y_{-1} + \frac{3p^2 - 1}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \ldots\ldots$$

and, $\frac{dx}{dp} = h$

Then,

$$\frac{dy_p}{dx} = \frac{dy_p}{dp} \times \frac{dp}{dx}$$

$$= \frac{1}{h}\left[\frac{\Delta y_0 + \Delta y_{-1}}{2} + p\Delta^2 y_{-1} + \frac{3p^2 - 1}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \ldots\ldots\right]$$

At $x = x_0$, $p = 0$,

so, $$\left(\frac{dy}{dx}\right)_{x_0} = \frac{1}{h}\left[\frac{\Delta y_0 + \Delta y_{-1}}{2} - \frac{1}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \ldots\ldots\right]$$

Now,

$$s'(t) = \frac{d(s(t))}{dt} = \left(\frac{dy}{dx}\right)_7 = \frac{1}{h}\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right]$$

NOTE:
Formula is placed according to the data available in difference table *i.e.*, Δy_0 and Δy_{-1} are present but not for other $\Delta^3 y_{-1}$, $\Delta^3 y_{-2}$ etc for t = 7.

or, $$s'(t) = \frac{1}{1}\left[\frac{6 + 5}{2}\right]$$

$\therefore$ $s'(t) = 5.5$ km/s is the required velocity

8. Evaluate the integral $I = \int_0^{10} \exp\left(\frac{-1}{1 + x^2}\right) dx$, using gauss quadrature formula with n = 2 and n = 3. **[2016/Fall]**

Solution:

Given that;

$$I = \int_0^{10} f(x)\, dx$$

where, $f(x) = \exp\left(\frac{-1}{1 + x^2}\right)$

Using gauss quadrature formula with n = 2 and n = 3 since limit a = 0 and b = 10 is not from −1 to 1, so using,

$$x = \frac{1}{2}(b - a)u + \frac{1}{2}(b + a)$$

or, $x = \frac{1}{2}(10-0)u + \frac{1}{2}(10+0)$

$\therefore$ $x = 5u + 5$ ..... (1)

Differentiating on both sides

$dx = 5\,du$ ..... (2)

Then, substituting the values form (1) and (2) to I,

$$I = \int_{-1}^{1} \exp\left(\frac{-1}{1+(5u+5)^2}\right) 5du$$

Now,

i) Gauss formula for n = 2 is

$$I = \int_{-1}^{1} f(x)\,dx = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$= 5\exp\left[\frac{-1}{1+\left(5\left(\frac{-1}{\sqrt{3}}\right)+5\right)^2}\right] + 5\exp\left[\frac{-1}{1+\left(\frac{5}{\sqrt{3}}+5\right)^2}\right]$$

$$= 4.164 + 4.921 = 9.085$$

Then,

ii) Gauss formula for n = 3 is,

$$I = \frac{8}{9}f(0) + \frac{5}{9}\left[f\left(-\sqrt{\frac{3}{5}}\right) + f\left(\sqrt{\frac{3}{5}}\right)\right]$$

$$= \frac{8}{9}\left(5\exp\left(\frac{-1}{1+(0+5)^2}\right)\right)$$

$$+ \frac{5}{9}\left[5\exp\left(\frac{-1}{1+\left(5\left(-\sqrt{\frac{3}{5}}\right)+5\right)^2}\right)\right]$$

$$+ 5\exp\left[5\exp\left(\frac{-1}{1+\left(5\left(\sqrt{\frac{3}{5}}\right)+5\right)^2}\right)\right]$$

$$= 4.276 + 4.531 = 8.807$$

9. **Evaluate the integral $\int_0^{0.6} e^{x^2}dx$, using Simpson $\frac{1}{3}$ rule and Simpson $\frac{1}{8}$ rule, dividing the interval into six parts.** **[2016/Spring]**

Solution:

Given that;

$$I = \int_0^{0.6} e^{x^2}dx,$$

$a = 0, b = 0.6$ and $n = 6$

Then,

$$h = \frac{b-a}{n} = \frac{0.6-0}{6} = 0.1$$

Now, table is created at the interval of 0.1 from 0 to 0.6.

x	0	0.1	0.2	0.3	0.4	0.5	0.6
y	1	1.010	1.040	1.094	1.173	1.284	1.433
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Now, by Simpson's $\frac{1}{3}$ rule,

$$I = \frac{h}{3}[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{0.1}{3}[1 + 1.433 + 4(1.010 + 1.094 + 1.284) + 2(1.04 + 1.173)]$$

$$= 0.68036$$

Again, by Simpson's $\frac{3}{8}$ rule,

$$I = \frac{3h}{8}[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3\times 0.1}{8}[1 + 1.433 + 3(1.010 + 1.040 + 1.173 + 1.284) + 2(1.094)]$$

$$= 0.68032$$

Also, $I_{abs} = \int_0^{0.6} e^{x^2}dx = 0.68049$

10. **Estimate the following integrals by,**

i) **Simpson's $\frac{3}{8}$ method**

ii) **Simpson's $\frac{1}{3}$ method and compare the result**

$$\int_2^1 \frac{e^x\,dx}{x} \text{ (Assume n = 4)}$$ **[2017/Fall]**

Solution:

Given that;

$$I = \int_2^1 \frac{e^x}{x}dx$$

$a = 2, b = 1, n = 4$

Then,

$$h = \frac{b-a}{n} = \frac{1-2}{4} = -0.25$$

Now, creating table at the interval of (−0.25) from 2 to 1.

x	2	1.75	1.5	1.25	1
y	3.694	3.288	2.987	2.792	2.718
	y_0	y_1	y_2	y_3	y_4

Now, by Simpson's $\frac{1}{3}$ rule,

$$I = \frac{h}{3}[y_0 + y_4 + 4(y_1 + y_3) + 2(y_2)]$$

$$= \frac{-0.25}{3}[3.694 + 2.718 + 4(3.288 + 2.792) + 2(2.987)]$$

$$= -3.0588$$

And by Simpson's $\frac{3}{8}$ rule,

$$I = \frac{3h}{8}[y_0 + y_4 + 3(y_1 + y_2) + 2y_3)$$

$$= \frac{3 - 0.25}{8}[3.694 + 2.718 + 3(3.288 + 2.987) + 2(2.792)]$$

$$= -2.8894$$

Then, $I_{ex} = \int_2^1 \frac{e^x}{x} dx = -3.0591$

Now, Error by Simpson $\frac{1}{3}$ rule $= |-3.0591 + 3.0588| = 0.0003$

Error by Simpson $\frac{3}{8}$ rule $= |-3.0591 + 2.8894| = 0.1697$

So, Simpson's $\frac{1}{3}$ rule is more accurate.

11. **Apply Romberg's method to evaluate**

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1 + \sin x}} dx$$ **[2017/Fall]**

Solution:

Given that;

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1 + \sin x}} dx$$

$a = 0, b = \frac{\pi}{2}$

i) Taking $h = \frac{\pi}{4}$ and creating interval of $\frac{\pi}{4}$ from 0 to $\frac{\pi}{2}$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
y	1	0.541	0
	y_0	y_1	y_2

Now, using trapezoidal rule

$$I\left(\frac{\pi}{4}\right) = \frac{h}{2}[y_0 + y_2 + 2y_1]$$

$$= \frac{\pi}{2 \times 4}[1 + 0 + 2(0.541)] = 0.8175$$

ii) Taking $h = \frac{\pi}{8}$

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	1	0.785	0.541	0.275	0
	y_0	y_1	y_2	y_3	y_4

$$I\left(\frac{\pi}{8}\right) = \frac{h}{2}[y_0 + y_4 + 2(y_1 + y_2 + y_3)]$$

$$= \frac{\pi}{2 \times 16}[1 + 0 + 2(0.785 + 0.541 + 0.275)]$$

$$= 0.8250$$

iii) Taking $h = \frac{\pi}{16}$

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$	$\frac{5\pi}{16}$	$\frac{3\pi}{8}$	$\frac{7\pi}{16}$	$\frac{\pi}{2}$
y	1	0.897	0.785	0.667	0.541	0.410	0.275	0.138	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

$$I = \left(\frac{\pi}{16}\right) = \frac{h}{2}[y_0 + y_8 + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)]$$

$$= \frac{\pi}{2 \times 16}[1 + 0 + 2(0.897 + 0.785 + 0.667 + 0.541 + 0.410 + 0.275 + 0.138)]$$

$$= 0.8272$$

Now, optimizing values by Romberg Integration

$$I\left(\frac{\pi}{4}, \frac{\pi}{8}\right) = \frac{1}{3}\left[4I\left(\frac{\pi}{8}\right) - I\left(\frac{\pi}{4}\right)\right]$$

$$= \frac{1}{3}[4 \times 0.8250 - 0.8175] = 0.8275$$

$$I\left(\frac{\pi}{8}, \frac{\pi}{16}\right) = \frac{1}{3}\left[4I\left(\frac{\pi}{16}\right) - I\left(\frac{\pi}{8}\right)\right]$$

$$= \frac{1}{3}[4(0.8279) - (0.8250)] = 0.8279$$

$$I\left(\frac{\pi}{4}, \frac{\pi}{8}, \frac{\pi}{16}\right) = \frac{1}{3}\left[4I\left(\frac{\pi}{8}, \frac{\pi}{16}\right) - I\left(\frac{\pi}{4}, \frac{\pi}{8}\right)\right]$$

$$= \frac{1}{3}[4 \times 0.8272 - 0.8275] = 0.8280$$

Hence the value of integral $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1 + \sin x}} dx = 0.8280$

Also, $I_{abs} = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1 + \sin x}} dx = 0.8284$

12. **A slider in a machine moves along a fixed straight rod 9 + s distance x (cm) along the rod is given below for various values of time t seconds. Find the velocity and the acceleration of the slider when t = 0.2.** **[2017/Spring]**

t	0	0.1	0.2	0.3
x	30.13	31.62	32.87	33.95

Solution:

Creating difference table from given data

x = t	y = x	1st diff	2nd diff	3rd diff
0	30.13			
		1.49		
			-0.24	
0.1	31.62			
		1.25		0.07
			-0.17	
0.2	32.87			
		1.08		
0.3	33.95			

Here, the data of t is equispaced and t = 0.2 lies near the end of the table so using Newton's backward formula for numerical differentiation.

$$h = 0.3 - 0.2 = 0.1$$

Now, at t = 0.2

From, numerical differentiation using Newton's backward formula

$$y' = \frac{1}{h}\left[\nabla y_n + \frac{\nabla^2 y_n}{2}\right] = \frac{1}{0.1}\left[1.25 + \frac{-0.24}{2}\right]$$

$\therefore$ $y' = 11.3$ *cm/s is the required velocity of an object.*

Now, for acceleration

$$y'' = \frac{1}{h^2}[\nabla^2 y_n] = \frac{1}{0.1^2} \times -0.24$$

$\therefore$ $y'' = -24 \text{ cm/s}^2$

is the required acceleration of an object

13. **The velocity 'v' of a particle at a distance 's' from a point on its path is given is given by the following table.**

s(m)	0	10	20	30	40	50	60
v(m/s)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 metres by using Simpson's $\frac{1}{3}$ rule and Simpson's $\frac{3}{8}$ rule. **[2017/ Spring]**

Solution:

We have,

$$v = \frac{ds}{dt}$$

$$dt = \frac{1}{v} ds = y \cdot ds \boxed{\Rightarrow y = \frac{1}{v}}$$

On integration,

$$t = \int_0^{60} y \cdot ds$$

Here; $a = 0, b = 60, n = 6$

so, $h = \frac{60 - 0}{6} = 10$

Creating table at the interval of 10 from 0 to 60.

x = s	0	10	20	30	40	50	60
$y = \frac{1}{v}$	$\frac{1}{47}$	$\frac{1}{58}$	$\frac{1}{64}$	$\frac{1}{65}$	$\frac{1}{61}$	$\frac{1}{52}$	$\frac{1}{38}$
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Now, by Simpson's $\frac{1}{3}$ rule,

$$\therefore \quad I = \frac{h}{3}[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{10}{3}\left[\frac{1}{47} + \frac{1}{38} + 4\left(\frac{1}{58} + \frac{1}{65} + \frac{1}{52}\right) + 2\left(\frac{1}{64} + \frac{1}{61}\right)\right] = 1.063$$

Again, by Simpson's $\frac{3}{8}$ rule

$$\therefore \quad I = \frac{3h}{8}[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3 \times 10}{8}\left[\frac{1}{47} + \frac{1}{38} + 3\left(\frac{1}{58} + \frac{1}{64} + \frac{1}{61} + \frac{1}{52}\right) + 2\left(\frac{1}{65}\right)\right] = 1.064 \text{ s}$$

14. Evaluate the integral $I = \int_0^{\frac{\pi}{2}} (1 + 3\cos 2x)\,dx$. **Compare the result in both conditions for Simpson $\frac{1}{3}$ and $\frac{3}{8}$ rule.** **[2018/Fall]**

Solution:

Given that:

$$I = \int_0^{\frac{\pi}{2}} (1 + 3\cos 2x)\,dx$$

$$a = 0, b = \frac{\pi}{2}, n = 6$$

$$h = \frac{b - a}{n} = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12}$$

Now, table is created at the interval of $\frac{\pi}{12}$

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
y	4	3.598	2.5	1	-0.5	-1.598	-2
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Now, by Simpson's $\frac{1}{3}$ rule

$$I = \frac{h}{3}[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{\pi}{3 \times 12}[4 + (-2) + 4(3.598 + 1 - 1.598) + 2(2.5 - 0.5)]$$

$$= 1.57079$$

Again, by Simpson's $\frac{3}{8}$ rule

$$I = \frac{3h}{8}[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3)]$$

$$= \frac{3\pi}{8 \times 12}[4 + (-2) + 3(3.598 + 2.5 - 0.5 - 1.598) + 2(1)]$$

$$= 1.57079$$

Also, $I_{exact} = \int_0^{\frac{\pi}{2}} (1 + 3\cos 2x)\,dx = 1.57079$

Now, Error by Simpson $\frac{1}{3}$ rule $= |1.57079 - 1.57079| = 0$

Error by Simpson $\frac{3}{8}$ rule $= |1.57079 - 1.57079| = 0$

Hence, the Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule is accurate with zero error.

15. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.2$.

x	1.0	1.2	1.4	1.6	1.8
y	2.7183	3.3201	4.0552	4.9530	6.0496

[2018/Spring]

Solution:

Creating difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.0	2.7183				
		0.6018			
1.2	3.3201		0.1333		
		0.7351		0.0294	
1.4	4.0552		0.1627		0.0067
		0.8978		0.0361	
1.6	4.9530		0.1988		
		1.0966			
1.8	6.0496				

Here, the data of x is equispaced and x = 1.2 lies near the starting of table so using Newton's forward formula for numerical differentiation.

$h = 1.2 - 1.0 = 0.2$

Now, at $x = 1.2$.

From numerical differentiation, using Newton's forward formula

$$\frac{dy}{dx} = y' = \frac{1}{h}\left[\Delta y_n - \frac{\Delta^2 y_n}{2} + \frac{\Delta^3 y_n}{3}\right]$$

$$= \frac{1}{0.2}\left[0.7351 - \frac{0.1627}{2} + \frac{0.0361}{3}\right]$$

$\therefore$ $y' = 3.328$

Again, for $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = y'' = \frac{1}{h^2}[\Delta^2 y_n - \Delta^3 y_n]$$

$$= \frac{1}{0.2^2}[0.1627 - 0.0361]$$

$\therefore$ $y'' = 3.165$

16. The following data gives corresponding values of pressure 'p' and specific volume 'v' of steam.

p	105	42.7	25.3	16.7	13
v	2	4	6	8	10

Find the rate of change of volume when pressure is 105 and 13.

[2018/Fall]

Solution:

As the values of p are not equispaced, we use Newton's divided difference formula.

The divided difference table is

	x = p	y = v	1st diff	2nd diff	3rd diff	4th diff
x_0	105	2				
			−0.0321			
x_1	42.7	4		**0.0010**		
			−0.1149		**-3.96×10^{-5}**	
x_2	25.3	6		0.0045		**7.06×10^{-6}**
			−0.2325		-6.90×10^{-4}	
x_3	16.7	8		0.0250		
			−0.5405			
x_4	13	10				

Now, Newton's divided formula for the 1st derivative.

We get

$$f'(x) = \frac{dv}{dp} = [x_0, x_1] + (2x - x_0 - x_1)[x_0, x_1, x_2]$$
$$+ [3x^2 - 2x(x_0 + x_1 + x_2) + x_0x_1 + x_1x_2 + x_2x_0][x_0, x_1, x_2, x_3]$$
$$+ [4x^3 - 3x^2(x_0 + x_1 + x_2 + x_3)$$
$$+ 2x(x_0x_1 + x_1x_2 + x_2x_3 + x_3x_0 + x_1x_3 + x_0x_2)$$
$$- (x_0x_1x_2 + x_1x_2x_3 + x_2x_3x_0 + x_0x_1x_3)][x_0, x_1, x_2, x_3, x_4]$$

Now, when pressure is 105

$$\frac{dv}{dp_{at\ p=105}} = -0.0321 + (2(105) - 105 - 42.7)(0.0010)$$
$$+ [3(105)^2 - 2(105)(105 + 42.7 + 25.3) + (105 \times 42.7)$$
$$+ (42.7 \times 25.3) + (25.3 \times 105)](-3.96 \times 10^{-5})$$
$$+ [4(105)^3 - 3(105)^2(105 + 42.7 + 25.3 + 16.7)$$
$$+ 2(105)(105 \times 42.7 + 42.7 \times 25.3 + 25.3 \times 16.7$$
$$+ 16.7 \times 105 + 42.7 \times 16.7 + 105 \times 25.3)$$
$$- (105 \times 42.7 \times 25.3 + 42.7 \times 25.3 \times 16.7$$
$$+ 25.3 \times 16.7 \times 105 + 105 \times 42.7 \times 16.7)](7.06 \times 10^{-6})$$
$$= 2.9289$$

Similarly when pressure is 13, using x = 13 in the formula, we get,

$$\frac{dV}{dp_{at\ p=13}} = -0.6689$$

17. Evaluate $\int_{-2}^{2} \frac{x}{x + 2e^x} dx$ by using trapezoidal, Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule with n = 6. **[2019/Fall]**

Solution:

Given that;

$$I = \int_{-2}^{2} \frac{x\,dx}{x + 2e^x}$$

$$a = -2, b = 2, n = 6$$

Then,

$$h = \frac{b-a}{n} = \frac{2+2}{6} = \frac{2}{3}$$

Now, table is created at the interval of $\frac{2}{3}$ from −2 to 2.

x	−2	$\frac{-4}{3}$	$\frac{-2}{3}$	0	$\frac{2}{3}$	$\frac{4}{3}$	2
y	1.156	1.653	−1.850	0	0.146	0.149	0.119
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Now, by trapezoidal rule,

$$I = \frac{h}{2}[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$$
$$= \frac{2}{2 \times 3}[1.156 + 0.119 + 2(1.653 - 1.850 + 0 + 0.146 + 0.149)] = 0.490$$

Again, by Simpson's $\frac{1}{3}$ rule,

$$I = \frac{h}{3}[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$
$$= \frac{2}{3 \times 3}[1.56 + 0.119 + 4(1.653 + 0 + 0.149) + 2(-1.850 + 0.146)]$$
$$= 1.1277$$

And, by Simpson's $\frac{3}{8}$ rule,

$$I = \frac{3h}{8}[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$
$$= \frac{3 \times 2}{8 \times 3}[1.156 + 0.119 + 3(1.653 - 1.850 + 0.146 + 0.149) + 2 \times 0]$$
$$= 0.3922$$

18. Using three-point Gaussian Quadrature formula, evaluate,

$$\int_0^1 \frac{dx}{1+x}$$

[2019/Fall]

Solution:

Given that;

$$I = \int_0^1 \frac{dx}{1+x}$$

Using gauss quadrature formula with n = 3.

Since limit a = 0 and b = 1 is not from −1 to 1 so using,

$$x = \frac{1}{2}(b-a)u + \frac{1}{2}(b+a)$$

or, $$x = \frac{1}{2}(1-0)u + \frac{1}{2}(1+0)$$

∴ $$x = \frac{u}{2} + \frac{1}{2} \quad \text{..... (1)}$$

Differentiating on both sides

$$dx = \frac{du}{2} \quad \text{..... (2)}$$

Now, substituting the values from (1) and (2) to I,

$$I = \int_{-1}^{1} \frac{\frac{du}{2}}{1 + \left(\frac{u}{2} + \frac{1}{2}\right)} = \int_{-1}^{1} \frac{du}{3+u}$$

Now, Gauss formula for n = 3 is

$$I = \frac{8}{9}f(0) + \frac{5}{9}\left[f\left(-\sqrt{\frac{5}{3}}\right) + f\left(\sqrt{\frac{3}{5}}\right)\right]$$

$$= \frac{8}{9} \times \frac{1}{3} + \frac{5}{9}\left[\frac{1}{3-\sqrt{\frac{3}{5}}} + \frac{1}{3+\sqrt{\frac{3}{5}}}\right]$$

$\therefore$ $I = 0.69312$

19. The following table gives the velocity of a velocity at various points of time.

Time, t(seconds)	1	2	4	5
Velocity, v(m/s)	0.25	1	2.2	4

Find the acceleration of the vehicle at t = 1.1 second and t = 2.5 second using any suitable differential formula. **[2019/Spring]**

Solution:

As the values of time are not equispaced, we use Newton's divided difference formula.

The divided difference table is

	x = t	y = v	1^{st} diff	2^{nd} diff	3^{rd} diff
x_0	1	0.25			
			0.75		
				-0.05	
x_1	2	1			
			0.6		**0.1125**
				0.4	
x_2	4	2.2			
			1.8		
x_3	5	4			

From Newton's divided formula for the 1^{st} derivative, we get,

$$f'(x) = [x_0, x_1] + (2x - x_0 - x_1)[x_0, x_1, x_2] + [3x^2 - 2x(x_0 + x_1 + x_2) + x_0x_1 + x_1x_2 + x_2x_0][x_0, x_1, x_2, x_3]$$

Now, when t = 1.1

$$f'(x)_{1.1} = 0.75 + [2(1.1) - 1 - 2](-0.05) + [3(1.1)^2 - 2(1.1)(1 + 2 + 4) + (1)(2) + (2)(4) + (1)(4)(0.1125)$$

$$= 0.75 + 0.04 + 0.2508$$

$\therefore$ $f'(x)_{1.1} = 1.0408$ is the required acceleration in m/s^2

Again, when t = 2.5

$$f'(x)_{2.5} = 0.75 + 2(2.5) - 1 - 2)(-0.05) + [3(2.5)^2 - 2(2.5)(1 + 2 + 4) + (1)(2) + (2)(4) + (1)(4)](0.1125)$$

$$= 0.75 - 0.1 - 0.2531$$

$= 0.3969\ m/s^2$ is the required acceleration.

20. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin u}{u}\,du$ by using trapezoidal, Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule with n = 6. **[2019/Spring]**

Solution:

Given that;

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin u}{u}\,du$$

$$a = 0, b = \frac{\pi}{2}, n = 6$$

$$h = \frac{b-a}{n} = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12}$$

Now, table is created at the interval of $\frac{\pi}{12}$ from 0 to $\frac{\pi}{2}$

x = u	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
y	1	0.988	0.954	0.9	0.826	0.737	0.636
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

NOTE:

At $x = u = 0$, $\frac{\sin u}{u} = \frac{0}{0}$, so we use L-Hopital's rule for 0.

Rest of the values are normally calculated.

Now, by trapezoidal rule,

$$I = \frac{h}{2}[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$$

$$= \frac{\pi}{24}[1 + 0.636 + 2(0.988 + 0.954 + 0.9 + 0.826 + 0.737)]$$

$$= 1.367$$

Again, by Simpson's $\frac{1}{3}$ rule,

$$I = \frac{h}{3}[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{\pi}{36}[1 + 0.636 + 4(0.988 + 0.9 + 0.737) + 2(0.954 + 0.826)]$$

$$= 1.369$$

And, by Simpson's $\frac{3}{8}$ rule,

$$I = \frac{3h}{8}[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3\pi}{96}[1 + 0.636 + 3(0.988 + 0.954 + 0.826 + 0.737) + 2(0.9)]$$

$$= 1.369$$

21. **Use Gauss-Legendre 2-point and 3 point formula to evaluate;** **[2019/Spring]**

$$\int_{0.5}^{1.5} e^{x^2} dx$$

Solution:

Given that:

$$I = \int_{0.5}^{1.5} e^{x^2} dx$$

Since limit a = 0.5 and b = 1.5 is not from −1 to 1

so, $x = \frac{1}{2}(b-a)u + \frac{1}{2}(b+a)$

or, $x = \frac{1}{2}(1.5-0.5)u + \frac{1}{2}(1.5+0.5)$

or, $x = \frac{u}{2} + 1$ ..... (1)

Differentiating on both sides

$dx = \frac{du}{2}$ ..... (2)

Then, substituting the values from (1) and (2) to I,

$$I = \int_{-1}^{1} \frac{e^{(\frac{u}{2}+1)^2}}{2} du$$

Now,

i) Gauss formula for n = 2 is

$$I = \int_{-1}^{1} f(x)\,dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{e^{(-\frac{1}{2\sqrt{3}}+1)^2}}{2} + \frac{e^{(\frac{1}{2\sqrt{3}}+1)^2}}{2}$$

$= 0.829 + 2.631$

$= 3.46$

ii) Gauss formula for n = 3 is

$$I = \frac{8}{9}f(0) + \frac{5}{9}\left[f\left(-\sqrt{\frac{3}{5}}\right) + f\left(\sqrt{\frac{3}{5}}\right)\right]$$

$$= \frac{8}{9}\left(\frac{e^{(0+1)^2}}{2}\right) + \frac{5}{9}\left[\frac{e^{(-\frac{1}{2}\sqrt{3/5}+1)^2}}{2} + \frac{e^{(\frac{1}{2}\sqrt{3/5}+1)^2}}{2}\right]$$

$= 1.208 + 2.307$

$\therefore \quad I = 3.515$

22. **Obtain divided difference table for the given data set** **[2019/Fall]**

e	−1	2	5	7
y	−8	3	1	12

Solution:

Creating the divided difference table

x	y	1st diff	2nd diff	3rd diff
−1	−8			
		$\frac{3+8}{2+1} = 3.667$		
			$\frac{-0.667-3.667}{5+1} = -0.722$	
2	3			
		$\frac{1-3}{5-2} = -0.667$		$\frac{1.233+0.722}{7+1} = 0.244$
			$\frac{5.5+0.667}{7-2} = 1.233$	
5	1			
		$\frac{12-1}{7-5} = 5.5$		
7	12			

23. **Integrate the given integral using Romberg integration,** **[2020/Fall]**

$$\int_1^2 \frac{1}{1+x^3} dx$$

Solution:

Given that;

$$I = \int_1^2 \frac{1}{1+x^3} dx$$

Here, a = 1, b = 2

i) Taking h = 0.5

x	1	1.5	2
y	0.5	0.228	0.111
	y_0	y_1	y_2

Now using Trapezoidal rule

$$I(0.5) = \frac{h}{2}[y_0 + y_2 + 2y_1]$$

$$= \frac{0.5}{2}[0.5 + 0.111 + 2(0.228)] = 0.266$$

ii) Taking h = 0.25

x	1	1.25	1.5	1.75	2
y	0.5	0.338	0.228	0.157	0.111
	y_0	y_1	y_2	y_3	y_4

Now, using Trapezoidal rule

$$I(0.25) = \frac{h}{2}[y_0 + y_4 + 2(y_1 + y_2 + y_3)]$$

$$= \frac{0.25}{2}[0.5 + 0.111 + 2(0.338 + 0.228 + 0.157)] = 0.257$$

(iii) Taking h = 0.125

x	1	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2
y	0.5	0.412	0.338	0.277	0.228	0.188	0.157	0.131	0.111
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

Now, using Trapezoidal rule

$$I(0.125) = \frac{h}{2}[y_0 + y_8 + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)]$$

$$= \frac{0.125}{2}[0.5 + 0.111 + 2(0.412 + 0.338 + 0.277 + 0.228 + 0.118 + 0.157 + 0.131)]$$

$$= 0.254$$

Now, optimizing values by Romberg Integration

$$I(0.5, 0.25) = \frac{1}{3}[4\,I(0.25) - I(0.5)]$$

$$= \frac{1}{3}[4(0.257) - 0.266]$$

$$= 0.254$$

$$I(0.25, 0.125) = \frac{1}{3}[4I(0.125) - I(0.25)]$$

$$= \frac{1}{3}[4(0.254) - 0.257]$$

$$= 0.253$$

$$I(0.5, 0.25, 0.125) = \frac{1}{3}[4I(0.25, 0.125) - I(0.5, 0.25)]$$

$$= \frac{1}{3}[4(0.253) - 0.254]$$

$$= 0.252$$

Hence the value of integral $\int_1^2 \frac{1}{1+x^3}dx = 0.252$

Also, $I_{abs} = \int_1^2 \frac{1}{1+x^3}dx = 0.2543$

24. Compute the integral using Gaussian 3-point formula.

$$\int_2^5 \frac{e^x + \sin x}{1+x^2}dx$$

[2020/Fall]

Solution:

Given that;

$$I = \int_2^5 \frac{e^x + \sin x}{1+x^2}dx$$

ince limit a = 2 and b = 5 is not from −1 to 1,

$$x = \frac{1}{2}(b-a)u + \frac{1}{2}(b+a)$$

or, $x = \frac{1}{2}(5-2)u + \frac{1}{2}(5+2)$

or, $x = \frac{3}{2}u + \frac{7}{2}$ ..... (1)

or, Differentiating on both sides, we get,

$$dx = \frac{3}{2}du \quad \text{..... (2)}$$

Then, substituting the values from (1) and (2) to I,

$$I = \int_{-1}^{1} \frac{e^{\frac{3u+7}{2}} + \sin\left(\frac{3u+7}{2}\right)}{1 + \left(\frac{3u+7}{2}\right)^2} \cdot \frac{3}{2}du$$

Now, using Gaussian 3-point formula,

$$I = \frac{8}{9}f(0) + \frac{5}{9}\left[f\left(-\sqrt{\frac{3}{5}}\right) + f\left(\sqrt{\frac{3}{5}}\right)\right]$$

$$= \frac{8}{9}\left[\frac{e^{(7/2)} + \sin^{(7/2)}}{1 + \left(\frac{7}{2}\right)^2} \cdot \frac{3}{2}\right] + \frac{5}{9}\left[\left(\frac{3}{2} \cdot \frac{e^{\frac{-3\sqrt{3/5}+7}{2}} + \sin\left(\frac{-3\sqrt{3/5}+7}{2}\right)}{1 + \left(\frac{-3\sqrt{3/5}+7}{2}\right)^2}\right)\right.$$

$$\left. + \left(\frac{3}{2} \cdot \frac{e^{\frac{3\sqrt{3/5}+7}{2}} + \sin\left(\frac{3\sqrt{3/5}+7}{2}\right)}{1 + \left(\frac{3\sqrt{3/5}+7}{2}\right)^2}\right)\right]$$

$$= 3.297 + 5.271$$

$\therefore$ $I = 8.568$

25. Write short notes on Romberg integration.

[2013/Fall, 2015/Fall, 2015/Spring]

Solution: See the topic 3.6.