Drawbacks of Newton Raphson Method:

1. It's convergence is not guaranteed. So, sometimes, for a given equation and for given guess we may not get solution.
2. Division by zero problem can occur.
3. Inflection point issue might occur.
4. In case of multiple roots, this method converges slowly.

Application of NM in Science and Engineering.

1. Numerical methods provide a way to solve problems quickly and easily compared to analytic solutions.
2. Numerical methods are algorithms used for computing numeric data. They are used to provide ‘approximate’ results for the problems being dealt with and their necessity is felt when it becomes impossible or extremely difficult to solve a given problem analytically.
3.

Interpolation :
Interpolation is a statistical method by which related known values are used to estimate an unknown value.

The difference between the Gauss–Seidel and Jacobi methods is that the Jacobi method uses the values obtained from the previous step while the Gauss–Seidel method always applies the latest updated values during the iterative procedures.

Il Conditioned and well conditioned systems;
Il-conditioned system : if a small relative error in data can cause a large relative error in the computed solution, regardless of the method of solution.

Formulas to remember :

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Simpson's 1/3 rule,

$$I = \frac{h}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) \right].$$

Simpson's 8/3 rule,

$$I = \frac{8h}{3}\left[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3 \right].$$

Trapezoidal rule,

$$I = \frac{h}{2}\left[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5) \right].$$

Gauss Legendre;

for limit not from -1 to 1,

$$x = \frac{1}{2}(b-a)u + \frac{1}{2}(b+a).$$

for $n = 2$, $I = \int_{-1}^{1} f(x)\,dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$.

for $n = 3$, $I = \frac{8}{9} f(0) + \frac{5}{9}\left[f\left(-\sqrt{\frac{3}{5}}\right) + f\left(\sqrt{\frac{3}{5}}\right) \right]$.

Runge Kutta method of order 4.

$K_1 = hf(x_0, y_0)$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$K_4 = hf(x_0 + h, y_0 + k_3)$.

$K = \frac{1}{6}\left[K_1 + K_4 + 2(k_3 + k_2) \right]$.

for second order, $K = \frac{1}{2}(K_1 + K_2)$

Picard's method;

$$y = y_0 + \int_{x_0}^{x} f(x, y)\, dx.$$

Taylor's series:

$$y(x) = y(0) + x\,y'(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y'''(0) + \cdots$$

Newton's divided difference:

$$f(x) = f(x_0) + (x - x_0)\,\Delta f(x_0) + (x - x_0)(x - x_1)\,\Delta^2 f(x_0)$$

$$\boxed{\text{or } y(x)} \quad + (x - x_0)(x - x_1)(x - x_2)\,\Delta^3 f(x_0) + \cdots$$

Approximate method:

↳ Euler method

Numerical differentiation:

1. Forward Newton:

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \cdots\right]$$

$$\frac{d^2 y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0\right] \cdots$$

2. Backward Newton:

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \cdots\right]$$

$$\frac{d^2 y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n\right] - \cdots$$

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① Dolittle method: $A = LU$.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

② Crout's factorization; $A = LU$.

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \qquad U = \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

③ Cholesky method; $A = LL^T$. $[U = L^T]$.

$$L = \begin{bmatrix} {}^{a}l_{11} & 0 & 0 \\ {}^{b}l_{21} & {}^{c}l_{22} & 0 \\ {}^{d}l_{31} & {}^{e}l_{32} & {}^{f}l_{33} \end{bmatrix} \qquad L^T(U) = \begin{bmatrix} a & b & d \\ 0 & c & e \\ 0 & 0 & f \end{bmatrix}$$

Heun's Method = Modified Euler's method.

Newton's Backward Interpolation formula;

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n.$$

$p = \frac{x - x_n}{h}$

Newton's Forward Interpolation formula;

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_n$$

$p = \frac{x - x_0}{h}$.

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eq^n:

line: $y = a + bx$

parabola: $y = a + bx + cx^2$

power eq^n: $y = ax^b$

Exponential eq^n: $y = ae^{bx}$

Gas eq^n: $x \cdot y^a = b$ / $PV^r = K$

Let U_1, U_2, U_3, U_4 be internal nodes of poisson equation and replacing $\Delta^2 f$ by difference eq^n with $x = ih$, $y = jk$ (where $h = k = 1$).

Then,

$$U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + U_{i,j-1} - 4U_{i,j} = (h)^2 \cdot f(x).$$

Romberg Integration,

$I(a) =$ $1/2$

$I(b) =$ $1/4$

$I(c) =$ $1/8$

$$I(a,b) = \frac{1}{3}[4I(b) - I(a)]$$

$$I(b,c) = \frac{1}{3}[4I(c) - I(b)]$$

$$\therefore I(a,b,c) = \frac{1}{3}[4I\{b,c) - I(a,b)]$$