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(191725)

Probability and Queuing Theory.

1. Ramesh and suresh are asked to solve the problem. the prob. that Ramesh solving it is 2/3 and that of suresh solving it is 3/4. Find the probability that (i) Both can solve the problem. (ii) at least one of them can solve.

Ans. Here,

P(Ramesh solving) = 2/3. i.e. P(R)

P(suresh solving) = 3/4 i.e. P(S)

(i) Both solving the problem:

$$P(R) \cdot P(S) = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}.$$

(ii). At least one of them can solve.

i.e. the problem will be solved.

$$P(R \cup S) = 1 - P(\overline{R \cup S})$$

$$= 1 - [P(\overline{R} \cap \overline{S})]$$

$$= 1 - [P(\overline{R}) \cdot P(\overline{S})]$$

$$= 1 - \left[\left(1-\frac{2}{3}\right) \cdot \left(1-\frac{3}{4}\right)\right]$$

$$= 1 - \left[\frac{1}{3} \cdot \frac{1}{4}\right]$$

$$= 1 - \frac{1}{12}$$

$$= \frac{11}{12}.$$

2. What do you mean by sample space? 'A' can hit a target 4 times with 5 shots, 'B' can hit the target 3 times with 5 shots and 'C' can hit the target 3 times with 5 shots. It is a condition that two hits damage the target. Each person fires a volley.

(i). What is the prob. that target will be damaged?

(ii) What is the prob. that two shots hit the target?

Ans. Sample space: The set of possible occurrance of result for the experiment is sample space.

3. A computer supplier has tabulated the number of printers sold weekly for each of the last 80 weeks. The results are summarized in the following table.

No. of printer sold :	1	2	3	4	
No. of weeks :	36	28	12	2	2.

Find the prob. of selling (i) exactly 2 printers. (ii) at least 3 printers. (iii) more than 2 printers in any given week.

Ans. Here, Total no. of printers sold $= 1+2+3+4 = 10$.

(i). Exactly 3 printer sold $= \frac{1}{4}$.

(ii). At least 3 printers sold $= \frac{2}{4} = \frac{1}{2}$.

(iii). More than 2 printer sold $= \frac{2}{4} = \frac{1}{2}$.

4. A company has 4 electrical engineers, 4 IT officers and 2 accountants. A committee of four members ~~has~~ has to be formed. What is the probability that a committee contains (i) one IT officer and 3 electrical engineers. (ii) at least one IT officer.

Ans. Here,

Total members $= 4+4+2 = 10$.

From that we have to select four members.

$\therefore$ 4 members can be selected in $\binom{10}{4}$ ways.

(i) one IT officer and 3 electrical engineers.

From 4 IT officier, one officier can be selected in $\binom{4}{1}$ ways

$= \dfrac{4!}{3!\,1!}$

$= 4$ ways.

Similarly,

From 4 electrical engineers, 3 electrical engineer can be selected in $\binom{4}{3}$ ways $= \dfrac{4!}{1!\,3!} = 4$ ways.

$\therefore$ P (one IT officier and 3 electrical engineer)

$= \dfrac{4 \times 4}{\binom{10}{4}}$

$= \dfrac{16}{\dfrac{10!}{6!\,4!}} = \dfrac{\overset{8}{\cancel{16}} \times \cancel{4} \times \cancel{3} \times \cancel{2}}{\underset{5}{\cancel{10}} \times \underset{3}{\cancel{9}} \times \cancel{8} \times 7}$

$= \dfrac{8}{105}$

(ii) at least one IT officier.

$\therefore$ Probability of not seleting IT officier $= \dfrac{\binom{6}{4}}{\binom{10}{4}}$

$= \dfrac{\dfrac{6!}{2!\,\cancel{4!}}}{\dfrac{10!}{6!\,\cancel{4!}}} = \dfrac{6!}{2!} \times \dfrac{6!}{10!}$

$$= \frac{6!}{2 \times 10 \times 9 \times 8 \times 7}$$

$$= \frac{\cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}}{2 \times \cancel{10} \times \cancel{9} \times \cancel{8} \times 7}$$

$$= \frac{1}{14}.$$

$$\therefore P(\text{at least one IT officer}) = 1 - \frac{1}{14}$$

$$= \frac{13}{14}.$$

5. If the probability that a communication system will have high fidelity is 0.81 and the probability that it will have high fidelity and high selectivity is 0.18; what is the probability that a system will have high selectivity given high fidelity?

Ans. Here,

$P(F)$ = Prob. that communication system will have high fidelity

$= 0.81.$

$P(F \cap S)$ = Prob. that communication system will have high fiedllity and high selectivity.

$= 0.18$

Now,

$$P(S|F) = \frac{P(F \cap S)}{P(F)} = \frac{0.18}{0.81} = 0.22$$

6. There are three switches in a college network namely A, B and C working independently. These switches are configured on series so that all switches should be 'on' to have successful transmission of data. The individual probability of being switch 'on' for these are 1/2, 3/4 and 1/4 respectively. Find the prob. that.

a. There will be a successful data transfer.

b. There will not be a successful data transfer.

Ans. a. Probability for successful data transfer

$= P(A) \cdot P(B) \cdot P(C)$ [∵ they are working independently]

$= \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4}$

$= \frac{3}{32}$

b. Probability for not successful data transfer is,

$1 - \frac{3}{32}$

$= \frac{32-3}{32}$

$= \frac{29}{32}$

7. Four items are taken at random from a box of 12 items and inspected. The box is rejected if more than 1 item is found to be faulty. If there are 3 faulty items in the box, find the prob. that the box is accepted.

Ans. From 12 items, 4 items can be taken in $\binom{12}{4}$ ways.

8. A coin is tossed until two heads and two tails are obtained in succession. Find the prob. that no more than three tossing will be needed.

Ans. Here, the sample space of getting two heads and two tails in succession be $w_1 = \{HH\}$, $w_2 = \{TT\}$, $w_3 = \{HTT\}$, $w_4 = \{THH\}$, $w_5 = \{HTHH\}$, $w_6 = \{THTT\}$ ......

No. of events can be infinite.

But for our case; we have no more than three tossing.

$\therefore$ Our favourable event $= \{w_1, w_2, w_3, w_4\}$.

So,

$$P(w_1) = P(HH) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

$$P(w_2) = P(TT) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

$$P(w_3) = P(HTT) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}.$$

$P(W_4) = P(THH) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Since the events are mutually exclusive.

$\therefore P(A) = P(W_1) + P(W_2) + P(W_3) + P(W_4)$

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$.

$= \frac{3}{4}$.

9. A bag contains 8 white counters and 3 black counters. Two counters are drawn one after another: (a) with replacement b) without replacement. Find the prob. of drawing (i) two white counters, (ii) two black counters. (iii) one white and one black counter.

Ans. Let W_1 = event that white counter is drawn first.

W_2 = event that white counter is drawn second.

B_1 = event that black counter is drawn first.

B_2 = event that black counter is drawn second.

(a) With replacement:

(i) Two white counters $= P(W_1) \times P(W_2)$

$= \frac{8}{11} \times \frac{8}{11}$

$= \frac{64}{121}$.

(ii) Two Black counters $= P(B_1) \times P(B_2)$

$= \frac{3}{11} \times \frac{3}{11}$

$= \frac{9}{121}$.

(iii). One white and one Black counter

(b) Without replacement.

(i) two white counters

$= P(W_1) \times P(W_2)$

$= \frac{8}{11} \times \frac{7}{10}$.

$= \frac{56}{110}$.

(ii) Two Black counters $= P(B_1) \times P(B_2)$

$= \frac{3}{11} \times \frac{2}{10}$

$= \frac{6}{110}$.

(iii). One white and one Black counter :

10. Anisha wants one white, one blue, one red and two yellow beads to be threaded on a ring to make a bracelet. Suppose that she wishes a red and a white bead to be next to each other. Find the prob. that the red and white beads are next to each other.

Ans. Let E be the event 'the red and the white beads are next to each other', then $\left[\text{Here, } n(S) = \frac{4!}{2!\,2!} = 6\right]$

$n(E) = \frac{2!\,3!}{2!\,2} = 3.$

where, $2! \Rightarrow$ Red & white can be arranged in $2!$ ways.

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}.$

$3! \Rightarrow$ No of ways of arranging 4 objects in a ring.

$2! \Rightarrow$ Two yellows.

$2 \Rightarrow$ anti-clockwise & clockwise arrangment are the same.

11 The 2x2 contingency table are as follows:

Capacitor value in μF	Box I	Box II	Total
0.1	60	120	180
0.01	70	90	160
Total	130	210	340

i). Find the prob. that it is $0.1\mu F$ capacitor.

ii). Find the prob. that it is capacitor from box II.

iii). Find the prob. that it is $0.1\mu F$ capacitor and box I.

iv). Find the prob. that it is $0.01\mu F$ capacitor and box II.

v). Find the prob. that it is capacitor from box II given that it is $0.01\mu F$.

vi). Find the prob. that the capacitor is either $0.01\mu F$ or from box I.

Ans (i). $P(0.1 \mu F \text{ capacitor}) = \frac{180}{340} = \frac{9}{17}$.

(ii). $P(\text{Capacitor from box I}) = \frac{210}{340} = \frac{21}{34}$.

(iii). $P(0.1 \mu F \text{ capacitor } \cancel{\text{of each}} \text{ box I}) = \frac{60}{340} = \frac{6}{34} = \frac{3}{17}$

(iv). $P(0.01 \mu F \text{ Capacitor from box II}) = \frac{90}{340} = \frac{9}{34}$

(v) $P(\text{Box II} / 0.01 \mu F) = \frac{90}{160} = \frac{9}{16}$.

(vi). $P(0.01 \mu F \text{ from box I}) = \frac{70}{340} = \frac{7}{34}$

12. The following table gives the details of consumer preference for a new product to be introduced in the market.

	Number of consumers			
	Like	Dislike	Neutral	Total
Male	500	250	125	875
Female	200	350	75	625
Total	700	600	200	1500

What is the probability that a consumer selected at random from the group will be:

i. a male who dislike the product? $\Rightarrow \frac{250}{1500} = \frac{5}{30} = \frac{1}{6}$.

ii. one who like the product, given that person is female?

$\Rightarrow \frac{200}{1500} = \frac{2}{15}$.

13. In a certain group of engineers 60% have insufficient background of information theory, 50% have inadequate knowledge of probability and 80% are either in one or both of two categories. Find the prob. of engineers who know prob. given that they have a sufficient background of information theory.

Ans. Here,

Let A = Group of engineers having insufficient knowledge in IT.

B = Group of engineers having insufficient knowledge in probability.

$\therefore P(A) = 60\% = 0.6$ $\qquad P(\bar{A}) = 1 - P(A) = 0.4$

$\therefore P(B) = 50\% = 0.5$ $\qquad P(\bar{B}) = 0.5$

and, $P(A \cup B) = 80\% = 0.8$

According to question,

P(Engineers who know probability, given they have sufficient background of Information theory)

$$\Rightarrow P(\bar{B}/\bar{A}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})}$$

$$= \frac{P(\overline{A \cup B})}{P(\bar{A})}$$

$$= \frac{1 - 0.8}{0.4}$$

$$= \frac{0.2}{0.4}$$

$$= \frac{1}{2}$$

$\therefore$ 50% of engineers.

14. The odds against student X solving a PQT problem are 8:6 and odds in favor of students Y solving the same problem are 14:16.

i. What is the chance that the problem will be solved if they both try independently of each other?

ii. What is the probability that neither solves the problem.

Ans. Here,

Prob. of solving by X = $\frac{8}{6} = \frac{4}{3}$

Ily, Odds of solving by Y = $\frac{14}{16} = \frac{7}{8}$.

i). Chance that the problem will be solved if they try independently = $\frac{4}{3} \times \frac{7}{8_2} = \frac{7}{6}$.

ii): Prob. that neither solves the problem

= 1 - P(solving the problem)

= 1 - $\frac{7}{6}$

15. Two weak students in a programming wrote a program and they obtain the same answer. Their chances of writing a program correctly are 1/8 and 1/12. From the past experience, teacher knows the prob. of making a common error is 1/10001. Find the chances that their program is correct. Also find the probability that the program is correct given that they get the same answers.

Ans. Let A and B denote those two weak students in a programming who writes the program correctly.

$\therefore P(A) = 1/8$ and, $P(B) = 1/12$.

Since, A and B may get the same answer when they are both correct or when they both are incorrect.

The first of these contingencies have the prob. $\frac{1}{8} \times \frac{1}{12}$ while the later has the prob.

$$\left(1-\frac{1}{8}\right)\left(1-\frac{1}{12}\right)\left(\frac{1}{10001}\right).$$

$\therefore P(\text{Both getting same answer}) = \frac{1}{8} \times \frac{1}{12} + \left(1-\frac{1}{8}\right)\left(1-\frac{1}{12}\right)\left(\frac{1}{10001}\right)$

Therefore, the required probability that the program is correct given that they get the same answer is,

$$= \frac{\frac{1}{8} \times \frac{1}{12}}{\frac{1}{8} \times \frac{1}{12} + \left(1-\frac{1}{8}\right)\left(1-\frac{1}{12}\right)\left(\frac{1}{10001}\right)}$$

$= 0.9924.$

16. Given that a binary communication channels, where A = input and B = Output, let $P(A) = 0.4$, $P(B/A) = 0.9$, and $P(\bar{B}/\bar{A}) = 0.6$.
Find $P(A \cup B)$, $P(A \cap B)$, $P(A/B) = 0.9$ and $P(A/\bar{B})$.

Ans. Here,

$$P(A) = 0.4$$

$$P(B/A) = 0.9$$

and,

$$P(\bar{B}/\bar{A}) = 0.6.$$

i.e.
$$\frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})} = 0.6$$

or,
$$\frac{P(\overline{A \cup B})}{1 - P(A)} = 0.6$$

or,
$$\frac{1 - P(A \cup B)}{1 - 0.4} = 0.6$$

or, $1 - P(A \cup B) = 0.36$

$\therefore P(A \cup B) = 0.64$

Also, $P(A \cap B) = P(B/A) \cdot P(A)$

$$= 0.9 \times 0.4$$

$$= 0.36$$

Using the relation,

$$P(A) + P(B) - P(A \cap B) = P(A \cup B)$$

or, $P(B) = P(A \cup B) - P(A) + P(A \cap B)$

or, $P(B) = 0.64 - 0.4 + 0.36$

$= 0.6$

Now,

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B/A) \cdot P(A)}{P(B)} = \frac{0.9 \times 0.4}{0.6} = 0.6$

$P(A/\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{P(A) - P(A \cap B)}{1 - P(B)} = \frac{0.4 - 0.36}{1 - 0.6} = 0.1$

17. The probability that a TV station will receive 0, 1, 2 .... 8 and 9 complaints after showing a controversial program are respectively 0.01, 0.03, 0.07, 0.15, 0.19, 0.18, 0.14, 0.12, 0.09 and 0.02. What are the probabilities that after showing such a program the station will receive:

a) At most 4 complaints.

b) At least 6 complaints.

c) From 5 to 8 complaints.

18. If the probabilities are 0.58, 0.25, and 0.19 that a person in a certain income bracket will invest in money market funds, common stocks, or both; find the probabilities that a person in that income bracket.

i. Who invest in money markets funds will also invest in common stocks

ii. Who invest in common stock will also invest in money market funds.

Ans. Here,

Let A & B represent the person of income bracket who invests in market funds and common stock respectively.

$\therefore P(A) = 0.58, \quad P(B) = 0.25, \quad P(A \cap B) = 0.19.$

i>. $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.19}{0.58} = 0.33$

ii> $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.19}{0.25} = 0.76$

19. The first of three urn contains 3 white and 6 black balls, the second contains 4 white and 5 black balls and the third contains 2 white and 4 black balls. A person choses an urn at random, and draws a ball from it. What is the prob. that it is white? (Baye's theorem).

Ans. Let's define the event.

E_1 = Ball drawn from first urn.

E_2 = Ball drawn from second urn.

E_3 = Ball drawn from third urn.

20. There are two boxes with parts of the same type. The first contains 90 sound parts and 10 defective parts. The second contains 80 sound parts and 20 defective parts. A box is selected at random and one part is drawn from it. Find the prob. that :

i) the first box was selected.

ii) the sound part was drawn given that it is from box 2.

iii) a defective part was drawn.

Ans. Let

A_1 = {the first box was selected}

A_2 = { the second box was selected }

B = {a sound part was drawn}

C = { a defective part was drawn }.

i). the first box was selected = $P(A_1) = \frac{1}{2}$.

ii). Sound part was drawn given that it is from box 2.

$= P(B/A_2)$

$= \frac{80}{80+20}$

$= \frac{80}{100}$

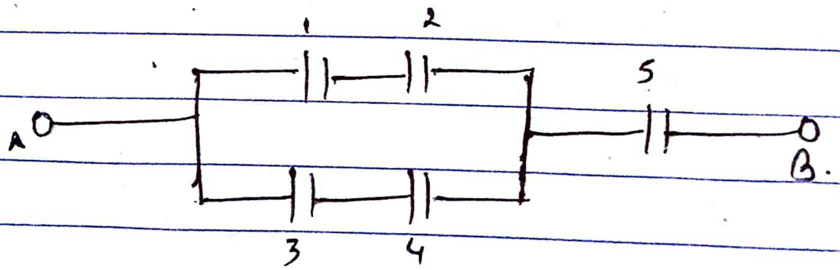
$= \frac{4}{5}$

(iii). a defective part was drawn,

$P(C/A_1) = \frac{10}{90+10} = \frac{10}{100} = 0.1$ [from Box 1].

$P(C/A_2) = \frac{20}{80+20} = \frac{20}{100} = \frac{1}{5} = 0.2$ [from Box 2]

21. For the circuit given below, the probability of closing each relay of the circuit is known to be p. Assume that the relays act independently. What is the probability that a current will exist betn the terminals A and B.



Ans.

Let the event of closing each relay 1, 2, 3, 4 and 5 be E_1, E_2, E_3, E_4 and E_5 respectively. Since the events are independent

So, the event that the current exist betn terminal A and B is,

$$E = E_1 \cdot E_2 + E_3 E_4 + E_5$$

$\therefore$ The probability of a current existing betn terminals A and B is,

$$P(E) = P[E_1 E_2 + E_3 E_4 + E_5]$$

$$= P(E_1 E_2) + P(E_3 E_4) + P(E_5) - P(E_1 E_2 E_3 E_4 E_5)$$

$$= P(E_1) \cdot P(E_2) + P(E_3) \cdot P(E_4) + P(E_5) - P(E_1) \cdot P(E_2) \cdot P(E_3) \cdot P(E_4) \cdot P(E_5).$$

$$= p^2 + p^2 + p - p^5$$

$$= 2p^2 + p - p^5.$$

22. A binary communication channel carries data as one of two types of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometime received as a 1 and a transmitted 1 is sometime received as a 0. For a given channel, assume a prob. of 0.94 that a transmitted 0 is correctly received as a 0 and a probability of 0.91 that a transmitted 1 is received as a 1. Further assume a probability of 0.45 of transmitting a 0. If a signal is send, find the probability that;

i. 1 is received.

ii. 0 is received.

iii. 1 was transmitted given that 1 was received.

iv. 0 was transmitted given that 0 was received.

v. an error.

Ans. Here,

lets define the event.

Let. T_0 = "A 0 is transmitted".

R_0 = "A 0 is received".

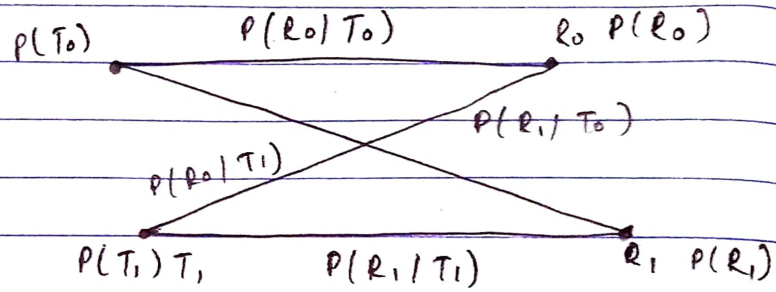
Let $T_1 = \overline{T_0}$ = "A 1 is transmitted".

$R_1 = \overline{R_0}$ = "A 1 is received".

Here, $P(R_0 | T_0) = 0.94$

$P(R_1 | T_1) = 0.91$

$P(T_0) = 0.45$



i> Prob. that 1 is received,

$= P(R_1) = P(\overline{R_0})$.

$= 1 - P(R_0)$

$= 1 - 0.4725 = 0.5275$.

ii> Prob. that 0 is received,

$P(R_0) = P(R_0|T_0)P(T_0) + P(R_0|T_1)P(T_1)$

$= 0.94 \times 0.45 + 0.09 \times 0.55$

$= 0.4725$.

Here,

$P(R_1|T_0) = P(\overline{R_0}/T_0)$

$= 1 - P(R_0|T_0)$

$= 0.06$

$P(R_0|T_1) = P(\overline{R_1}/T_1)$

$= 1 - P(R_1|T_1)$

$= 0.09$

$P(T_1) = P(\overline{T_0})$

$= 1 - P(T_0)$

$= 0.55$

iii> $P(T_1/R_1)$

$= \frac{P(R_1|T_1)P(T_1)}{P(R_1)}$

$= 0.9488$

$\therefore P(T_0|R_0) = \dfrac{P(R_0|T_0)\,P(T_0)}{P(R_0)} = \dfrac{0.94 \times 0.45}{0.4725} = 0.8925.$

Now,

$P(T_1/R_0) = P(\bar{T_0}/R_0) = 1 - P(T_0|R_0) = 0.1048.$

$P(T_0/R_1) = 1 - P(T_1/R_1) = 0.0512.$

and,

$P(\text{error}) = P(T_1 \cap R_0) + P(T_0 \cap R_1)$

$= P(T_1/R_0)\,P(R_0) + P(T_0/R_1)\,P(R_1)$

$= 0.0765.$

23. There are three candidates Dr pradhan, Mr shrestha and Mr sharma for the post of campus chief of CC. Their chances of being appointed to the post are in the proportion 5/10, 2/10 and 3/10 respectively. The chances that Dr pradhan will launch BBA programme is 0.7. The same for Mr shrestha and Mr sharma are 0.4 and 0.4 respectively. If BBA programme is launched, what is the prob. that the campus chief appointed was Dr. pradhan.

Ans. Here,

Let E_1 = ~~chances of~~ Dr pradhan appointed for campus chief post.

E_2 = Mr shrestha appointed for campus chief post.

E_3 = Mr sharma appointed for campus chief post.

Also, Let D be the arbitary event.

$\therefore P(E_1) = 5/10 = 1/2$ and, $P(D/E_1) = 0.7$

$P(E_2) = 2/10 = 1/5.$ $P(D/E_2) = 0.4$

$P(E_3) = 3/10$ $P(D/E_3) = 0.4$

→ The prob. that the campus chief appointed was Dr. Pradhan is,

$$P(E_1/D) = \frac{P(E_1) \cdot P(D/E_1)}{P(E_1) \cdot P(D/E_1) + P(E_2) \cdot P(D/E_2) + P(E_3) \cdot P(D/E_3)}$$

$$= \frac{1/2 \times 0.7}{1/2 \times 0.7 + 1/5 \times 0.4 + 3/10 \times 0.4}$$

$$= \frac{0.35}{0.35 + 0.08 + 0.12}$$

$$= \frac{0.35}{0.55}$$

$$= 0.63$$

24. A box containing 5000 IC chips, of which 1000 are manufactured by company X and rest by company Y. It is known that 10% of chips made by company X and 5% of chips made by company Y are defective. A chip is selected at random and found to be defective. Find the prob. that it come from Company X. (use Baye's theorem).

Ans. Let E_1 = Chips manufactured by Company X.
E_2 = Chips manufactured by Company Y.

$\therefore P(E_1) = \frac{1000}{5000} = \frac{1}{5}$. $P(E_2) = \frac{4000}{5000} = \frac{4}{5}$.

Let D be the arbitary event.

$\therefore P(D/E_1) = 10\% = 0.1$, $P(D/E_2) = 5\% = 0.05$

$$\rightarrow P(E_1/D) = \frac{P(E_1)\cdot P(D/E_1)}{P(E_1)\cdot P(D/E_1) + P(E_2)\cdot P(D/E_2)} = \frac{\frac{1}{5}\times 0.1}{\frac{1}{5}\times 0.1 + \frac{4}{5}\times 0.05}$$

$$= \frac{0.02}{0.06} = 0.33$$

25. In a factory, machine A produces 30% of the total output, machine B produces 25% and machine C produce the remaining 45% of the output. From the past experience, it has been found that 1%, 2% and 3% of the output from machine A, B and C respectively. Suppose an item was drawn from the day's production and was found to be defective. Find the prob. that a defective item chosen at random was produced by machine A, B and C respectively.

Ans. Let

E_1 = Items manufactured by machine A.

E_2 = Items manufactured by machine B.

E_3 = Items manufactured by machine C.

$\therefore P(E_1) = 30\% = 0.3$, $P(E_2) = 25\% = 0.25$, $P(E_3) = 45\% = 0.45$.

Let D be the aubitrary event.

$\therefore P(D/E_1) = 1\% = 0.01$, $P(D/E_2) = 2\% = 0.02$ & $P(D/E_3) = 3\%$
$= 0.03$

→ Produced by Machine A,

$$P(E_1/D) = \frac{P(E_1)\cdot P(D/E_1)}{P(E_1)\cdot P(D/E_1) + P(E_2)\cdot P(D/E_2) + P(E_3)\cdot P(D/E_3)}$$

$$= \frac{0.3\times 0.01}{0.3\times 0.01 + 0.25\times 0.02 + 0.45\times 0.03}$$

$$= \frac{0.003}{0.0215} = 0.1395$$

→ Produced by machine B.

$$P(E_2/D) = \frac{P(E_2) \cdot P(D/E_2)}{P(E_1) \cdot P(D/E_1) + P(E_2) \cdot P(D/E_2) + P(E_3) \cdot P(D/E_3)}$$

$$= \frac{0.25 \times 0.02}{0.0215}$$

$$= 0.2326$$

→ Produced by machine C.

$$P(E_3/D) = \frac{P(E_3) \cdot P(D/E_3)}{\sum_{i=1}^{3} P(E_i) \cdot P(D/E_i)}$$

$$= \frac{0.45 \times 0.03}{0.0215}$$

$$= 0.6279$$

26. A consulting firm rents cars from three agencies, 20% from agency D, 20% from agency E, 60% from agency F. If 10% of the car from D, 12% of the cars from E and 4% of the cars from F have had tires, what is the prob. that the firm will get car with bad tires? Also, what is the prob. that a car with bad tires are rented by agency F?

Ans. Let

E_1 = car rented from agency D.

E_2 = Car rented from agency E.

E_3 = Car rented from agency F.

$\therefore P(E_1) = 20\% = 0.2, \quad P(E_2) = 20\% = 0.2$ and $P(E_3) = 60\% = 0.6$

Let D be the arbitary event.

$\therefore P(D/E_1) = 10\% = 0.1. \quad P(D/E_2) = 12\% = 0.12$

$P(D/E_3) = 4\% = 0.04$

Here, prob. that the firm will get car with bad tires is,

$= (10 + 12 + 4)\%$

$= 26\%$

$= 0.26$

And,

$$P(E_1/D) = \frac{P(E_1) \cdot P(D/E_1)}{\sum_{i=1}^{3} P(E_i) \cdot P(D/E_i)}$$

$$= \frac{0.2 \times 0.1}{0.2 \times 0.1 + 0.2 \times 0.12 + 0.6 \times 0.04}$$

$$= \frac{0.02}{0.068}$$

$= 0.2941$

27. Three children A, B and C have equal plots in a circular patch of garden. Pebbles mark out the boundaries. A has 80 red, and 20 white flowers in her patch. B has 30 red and 40 white flowers and C has 10 red and 60 white flowers. Their sister D wants to pick a flower for her teacher.

i. Find the prob. that she picks a red flower if she chooses a flower at random from the garden, ignoring the boundaries

ii. Find the probability that she picks a red flower if she chooses a plot at random.

iii. If she picks a red flower at random, what is the prob. that it come from A's plot? B's plot? C's plot?

Ans

In A Garden,	In B Garden,	In C's Garden,
Red flower = 80.	Red flower = 30	Red flower = 10
White flower = 20	White flower = 40	White flower = 60

∴ Total Red flower $= 80 + 30 + 10 = 120$

∴ Total white flower $= 20 + 40 + 60 = 120$.

∴ Total flowers $= 120 + 120 = 240$.

i). Total ways of picking flower $= {}^{240}C_1$

Total ways of picking a red flower $= {}^{120}C_1$

$$\therefore \text{Prob.} = \frac{{}^{120}C_1}{{}^{240}C_1}$$

ii>.

For A Garden,

$$Prob = \frac{{}^{80}C_1}{{}^{100}C_1}$$

For B Garden,

$$Prob. = \frac{{}^{30}C_1}{{}^{70}C_1}$$

For C Garden,

$$Prob. = \frac{{}^{10}C_1}{{}^{70}C_1}.$$

iii>.

28. Four Technicians regularly make repairs when breakdown occurs on an automated production line. Janet, who services 20 % of the breakdown make an incomplete repair of 1 time in 10; Georgia, who services 15 % of the breakdown, make an incomplete repair of 1 time in 10; and peter, who services 5 % of the breakdown, make an incomplete repair of 1 time in 20. For the next problem with the production line diagnosed as being due to an initial repair that was incomplete, what is the prob. that this initial repair was made by Janet?

Ans. Here,

Let E_1 = Breakdown serviced done by Janet.

E_2 = Breakdown service done by Georgia.

E_3 = Breakdown service done by Peter.

$\therefore P(E_1) = 20\% = 0.2$, $P(E_2) = 15\% = 0.15$, $P(E_3) = 5\% = 0.05$

Let D be the arbitary event.

$P(D/E_1) = 1/10$, $P(D/E_2) = 1/10$, $P(D/E_3) = 1/20$.

Now,

$$P(E_1/D) = \frac{P(E_1) \cdot P(D/E_1)}{P(E_1) \cdot P(D/E_1) + P(E_2) \cdot P(D/E_2) + P(E_3) \cdot P(D/E_3)}$$

$$= \frac{0.2 \times 1/10}{0.2 \times 1/10 + 0.15 \times 1/10 + 0.05 \times 1/20}$$

$$= \frac{0.02}{0.0375}$$

$$= 0.533$$