BOARD EXAMINATION SOLVED QUESTIONS

1. **Find the inverse of the given matrix by applying Gauss Elimination Method (GEM) with partial pivoting technique.**

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$

[2013/Fall]

Solution: Given that;

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$

Using partial pivoting technique so, arranging the matrix as

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 4 & 1 & 2 \\ 2 & 3 & -1 \end{bmatrix}$$

Now, the augmented matrix is given by

$$[A : I] = \begin{bmatrix} 1 & -2 & 2 & : & 1 & 0 & 0 \\ 4 & 1 & 2 & : & 0 & 1 & 0 \\ 2 & 3 & -1 & : & 0 & 0 & 1 \end{bmatrix}$$

Operate $R_2 \to R_2 - 4R_1$ and $R_3 \to R_3 - 2R_1$

$$[A : I] = \begin{bmatrix} 1 & -2 & 2 & : & 1 & 0 & 0 \\ 0 & 9 & -6 & : & -4 & 1 & 0 \\ 0 & 7 & -5 & : & -2 & 0 & 1 \end{bmatrix}$$

Operate $R_3 \to R_3 - \frac{7}{9} R_2$

$$[A : I] = \begin{bmatrix} 1 & -2 & 2 & : & 1 & 0 & 0 \\ 0 & 9 & -6 & : & -4 & 1 & 0 \\ 0 & 0 & -1/3 & : & 10/9 & -7/9 & 1 \end{bmatrix}$$

Now, $\begin{bmatrix} 1 & -2 & 2 \\ 0 & 9 & -6 \\ 0 & 0 & -1/3 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 10/9 \end{bmatrix}$

$$\begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix} = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 9 & -6 \\ 0 & 0 & -1/3 \end{bmatrix} \begin{bmatrix} 1 \\ -4 \\ 10/9 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix} = \begin{bmatrix} 2.33 \\ -2.66 \\ -3.33 \end{bmatrix}$$

Also, $\begin{bmatrix} 1 & -2 & 2 \\ 0 & 9 & -6 \\ 0 & 0 & -1/3 \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \\ x_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -7/9 \end{bmatrix}$

$$\therefore \begin{bmatrix} x_{12} \\ x_{22} \\ x_{32} \end{bmatrix} = \begin{bmatrix} -1.33 \\ 1.66 \\ 2.33 \end{bmatrix}$$

And, $\begin{bmatrix} 1 & -2 & 2 \\ 0 & 9 & -6 \\ 0 & 0 & -1/3 \end{bmatrix}\begin{bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\therefore \begin{bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -3 \end{bmatrix}$

Hence, the inverse of matrix is

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 2.33 & -1.33 & 2 \\ -2.66 & 1.66 & -2 \\ -3.33 & 2.33 & -3 \end{bmatrix}$$

2. Solve the following system of equations by applying Gauss-Seidal iterative method. Carry out the iterations upto 6th stage

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

[2013/Fall]

Solution:

Arranging the equations such that magnitude of all the diagonal element is greater than the sum of magnitude of other two elements in the row i.e.,

$$\begin{aligned} 28x + 4y - z &= 32 \\ 2x + 17y + 4z &= 35 \\ x + 3y + 10z &= 24 \end{aligned} \qquad \begin{bmatrix} |28| > |4| + |-1| \\ |17| > |2| + |4| \\ |10| > |1| + |3| \end{bmatrix}$$

Forming the equations as

$$x = \frac{32 - 4y + z}{28}$$

$$y = \frac{35 - 2x - 4z}{17}$$

$$z = \frac{24 - x - 3y}{10}$$

Let initial guess be 0 for x, y and z.

Solving the iterations in tabular form.

NOTE: Use the most recent values obtained to find the next one in this method.

Iteration	$x = \frac{32 - 4y + z}{28}$	$y = \frac{35 - 2x - 4z}{17}$	$z = \frac{24 - x - 3y}{10}$
Guess	0	0	0
1	$\frac{32 - 4(0) + 0}{28}$ = 1.142	$\frac{35 - 2(1.142) - 4(0)}{17}$ = 1.924	$\frac{24 - 1.142 - 3(1.924)}{10}$ = 1.708
2	0.929	1.547	1.843
3	1.130	1.492	1.839
4	0.995	1.509	1.847
5	0.993	1.507	1.848
6	1.136	1.490	1.839

NOTE:

Procedure to iterate in programmable calculator

Let, A = x, B = y, C = z

Step 1: Set the following in calculator

$$A = \frac{31 - 4B + C}{28} : B = \frac{35 - 2A - 4C}{17} : C = \frac{24 - A - 3B}{10}$$

Step 2: Press CALC then

enter the value of B? then press =

enter the value of C? then press =

Step 3: Now press = only, again and again to get the values for respective row for each column.

Step 4: The values are updated automatically so continue pressing = till the required number of iterations.

3. Solve the following system of equations using Gauss elimination method.

$$10x_1 - 7x_2 + 3x_3 + 5x_4 = 6$$
$$-6x_1 + 8x_2 - x_3 + 4x_4 = 5$$
$$3x_1 + x_2 + 4x_3 + 11x_4 = 2$$
$$5x_1 - 9x_2 - 2x_3 + 4x_4 = 7$$

[2013/Spring]

Solution:

Writing the given system of equations in matrix form,

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ -6 & 8 & -1 & -4 \\ 3 & 1 & 4 & 11 \\ 5 & -9 & -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 2 \\ 7 \end{bmatrix}$$

Operate $R_2 \to R_2 - \left(-\frac{6}{10}\right) R_1$, $R_3 \to R_3 - \frac{3}{10} R_1$, $R_4 \to R_4 - \frac{5}{10} R_1$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ 0 & 3.8 & 0.8 & -1 \\ 0 & 3.1 & 3.1 & 9.5 \\ 0 & -5.5 & -3.5 & 1.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8.6 \\ 0.2 \\ 4 \end{bmatrix}$$

Operate $R_3 \to R_3 - \frac{3.1}{3.8} R_2$, $R_4 \to R_4 - \frac{-5.5}{3.8} R_2$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ 0 & 3.8 & 0.8 & -1 \\ 0 & 0 & 2.44 & 10.31 \\ 0 & 0 & -2.34 & 0.05 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8.6 \\ -6.81 \\ 16.44 \end{bmatrix}$$

Operate $R_4 \to R_4 - \left(-\frac{2.34}{2.44}\right) R_3$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ 0 & 3.8 & 0.8 & -1 \\ 0 & 0 & 2.44 & 10.31 \\ 0 & 0 & 0 & 0.993 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \end{bmatrix} = \begin{bmatrix} 6 \\ 8.6 \\ -6.81 \\ \end{bmatrix}$$

Now, performing back substitution,

$9.93x_4 = 9.90$

$\therefore \quad x_4 = 0.99 \approx 1$

$2.44x_3 + 10.31x_4 = -6.81$

or, $\quad 2.44x_3 = -6.81 - 10.31 \times -1$

$\therefore \quad x_3 = -7.01 \approx -7$

$3.8x_2 + 0.8x_3 - x_4 = 8.6$

or, $\quad 3.8x_2 + 0.8(-7) + 1 = 8.6$

$\therefore \quad x_2 = 3.47 \approx 3.5$

$10x_1 - 7x_2 + 3x_3 + 5x_4 = 6$

or, $\quad 10x_1 - 7(3.5) + 3(-7) + 5(1) = 6$

$\therefore \quad x_1 = 4.65$

4. Determine the highest even value and its corresponding eigen vector for the following matrix using power method.

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

[2013/Spring]

Solution:

Let the vector be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Then, the iterations are carried out as,

$$AX_0 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 13 \end{bmatrix}$$

The highest value in AX_0 is 13 so dividing each element by 13.

$$AX_0 = 13\begin{bmatrix} 0.2307 \\ 0.6923 \\ 1 \end{bmatrix}$$

Again,

$$AX_1 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.2307 \\ 0.6923 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.3076 \\ 6.0767 \\ 12.5385 \end{bmatrix} = 12.5385\begin{bmatrix} 0.1042 \\ 0.4864 \\ 1 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.1042 \\ 0.4864 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5634 \\ 5.2854 \\ 11.8414 \end{bmatrix} = 11.8414\begin{bmatrix} 0.0475 \\ 0.4463 \\ 1 \end{bmatrix}$$

$$AX_3 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}\begin{bmatrix} 0.0475 \\ 0.4463 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.3864 \\ 5.0351 \\ 11.7377 \end{bmatrix} = 11.7377\begin{bmatrix} 0.0329 \\ 0.4289 \\ 1 \end{bmatrix}$$

$$AX_4 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 0.0329 \\ 0.4289 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.3196 \\ 4.9565 \\ 11.6827 \end{bmatrix} = 11.6827 \begin{bmatrix} 0.0273 \\ 0.4242 \\ 1 \end{bmatrix}$$

$$AX_5 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 0.0273 \\ 0.4242 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2999 \\ 4.9303 \\ 11.6695 \end{bmatrix} = 11.6695 \begin{bmatrix} 0.0256 \\ 0.4224 \\ 1 \end{bmatrix}$$

Hence, the required eigen value $11.6695 \approx 12$.

And, required eigen vector $= \begin{bmatrix} 0.0256 \\ 0.4224 \\ 1 \end{bmatrix}$.

NOTE:
Procedure to solve in programmable calculator

Step 1: Press MODE then select MATRIX by pressing 6.

Step 2: Select MatA by pressing 1 and select 3 × 3 by pressing 1.

Step 3: Initialize the given matrix from the question.

Step 4: Press SHIFT then 4(MATRIX) and select Dim by pressing 1.

Step 5: Select MatB by pressing 2 and select 3 × 1 by pressing 3.

Step 6: Initialize the initial vector value and press AC.

Step 7: Press SHIFT then 4(MATRIX) and select MatA by pressing 3 and then press Multiply (×).

Step 8: Press SHIFT then 4(MATRIX) and select MatB by pressing 4 and then press =

Step 9: Now find the largest value in matrix and then press Divide (÷) and enter the largest value and then press =

Step 10: Now for next iteration press AC

Step 11: Press SHIFT then 4(MATRIX) and select MatA by pressing 3 then Multiply(×).

Step 12: Press SHIFT then 4(MATRIX) and select MatAns by pressing 6 and then press =

Step 13: Go to step 9.

5. Using Factorization method, solve the following system of linear equations:

$$3x + 2y + 7z = 4$$
$$2x + 3y + z = 5$$
$$3x + 4y + z = 7$$

Solution: **[2013/Spring]**

In matrix form

$$\begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix} \quad i.e., AX = B$$

In factorization method, we represent A as

$$\begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Solving for unknown values

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{12}u_{11} & u_{12}l_{21} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

$u_{11} = 3$	$u_{12} = 2$	$u_{13} = 7$
$l_{21} = \frac{2}{3} = 0.667$	$2 \times 0.667 + u_{22} = 3$ $\therefore u_{22} = 1.666$	$0.667 \times 7 + u_{23} = 1$ $\therefore u_{23} = -3.669$
$l_{31} = \frac{3}{3} = 1$	$l \times 2 + l_{32}(1.666) = 4$ $\therefore l_{32} = 1.2$	$1 \times 7 + 1.2(-3.669) + u_{33} = 1$ $\therefore u_{33} = -1.597$

Now, substituting obtained coefficients, we have overall system of

$LUX = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.667 & 1 & 0 \\ 1 & 1.2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 7 \\ 0 & 1.666 & -3.669 \\ 0 & 0 & -1.597 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

Let $UX = V$ then

$LV = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.667 & 1 & 0 \\ 1 & 1.2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

Using forward substitution

$\therefore$ $v_1 = 4$

or, $0.667v_1 + v_2 = 5$

$\therefore$ $v_2 = 2.332$

or, $1v_1 + 1.2v_2 + v_3 = 7$

$\therefore$ $v_3 = 0.201$

Using the obtained values at $UX = V$

$$\begin{bmatrix} 3 & 2 & 7 \\ 0 & 1.666 & -3.669 \\ 0 & 0 & -1.597 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2.332 \\ 0.201 \end{bmatrix}$$

Using backward substitution

$\therefore$ $z = \frac{0.201}{-1.597} = -0.125$

or, $1.666y - 3.669z = 2.332$

$\therefore$ $y = 1.174$

or, $3x + 2y + 7z = 4$

$\therefore$ $x = 0.842$

6. **Solve the following system of equations by applying Gauss Elimination Method (GEM) with partial pivoting technique. And also determine the determinant value.**

$$2x + 2y + z = 6$$
$$4x + 2y + 3z = 4$$
$$x - y + z = 0$$

[2014/Fall]

Solution:

By partial pivoting technique, the system of linear equation can be arranged as,

$$4x + 2y + 3z = 4$$
$$2x + 2y + z = 6$$
$$x - y + z = 0$$

The augmented matrix can be written as

$$[A : B] = \begin{bmatrix} 4 & 2 & 3 & : & 4 \\ 2 & 2 & 1 & : & 6 \\ 1 & -1 & 1 & : & 0 \end{bmatrix}$$

Operate $R_1 \to R_1 - 3R_3$, $R_2 \to R_2 - 2R_3$

$$= \begin{bmatrix} 1 & 5 & 0 & : & 4 \\ 0 & 4 & -1 & : & 6 \\ 1 & -1 & 1 & : & 0 \end{bmatrix}$$

Operate $R_3 \to R_3 - R_1$

$$= \begin{bmatrix} 1 & 5 & 0 & : & 4 \\ 0 & 4 & -1 & : & 6 \\ 0 & -6 & 1 & : & -4 \end{bmatrix}$$

Operate $R_2 \to \frac{R_2}{4}$

$$= \begin{bmatrix} 1 & 5 & 0 & : & 4 \\ 0 & 1 & -1/4 & : & 3/2 \\ 0 & -6 & 1 & : & -4 \end{bmatrix}$$

Operate $R_3 \to R_3 + 6R_2$

$$[A : B] = \begin{bmatrix} 1 & 5 & 0 & : & 4 \\ 0 & 1 & -1/4 & : & 3/2 \\ 0 & 0 & -1/2 & : & 5 \end{bmatrix}$$

Performing back substitution,

$$-\frac{1}{2}z = 5$$

$\therefore \quad z = -10$

Then, $y - \frac{1}{4}z = \frac{3}{2}$

or, $y + \frac{10}{4} = \frac{3}{2}$

$\therefore \quad y = -1$

and, $x + 5y + 0 = 4$

or, $x + 5(-1) = 4$

$\therefore \quad x = 9$

Also, determinant value

$$\begin{vmatrix} 4 & 2 & 3 \\ 2 & 2 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= 4\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= 4(2+1) - 2(2-1) + 3(-2-2)$$

$$= -2$$

7. **Find the largest eigen value and the corresponding eigen vector correct upto 3 decimal places using power method for the matrix**

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

[2014/Fall, 2017/Fall, 2019/Spring]

Solution:

Let initial eigen vector be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Then the iterations are carried out as

$$AX_0 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Again,

$$AX_1 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 3 \end{bmatrix} = 4\begin{bmatrix} 0.75 \\ -1 \\ 0.75 \end{bmatrix}$$

$$AX_3 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0.75 \\ -1 \\ 0.75 \end{bmatrix} = \begin{bmatrix} 2.5 \\ -3.5 \\ 2.5 \end{bmatrix} = 3.5\begin{bmatrix} 0.7142 \\ -1 \\ 0.7142 \end{bmatrix}$$

$$AX_4 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0.7142 \\ -1 \\ 0.7142 \end{bmatrix} = \begin{bmatrix} 2.4284 \\ -3.4284 \\ 2.4284 \end{bmatrix} = 3.4284\begin{bmatrix} 0.7083 \\ -1 \\ 0.7083 \end{bmatrix}$$

$$AX_5 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0.7083 \\ -1 \\ 0.7083 \end{bmatrix} = \begin{bmatrix} 2.4166 \\ -3.4166 \\ 2.4166 \end{bmatrix} = 3.4166\begin{bmatrix} 0.7073 \\ -1 \\ 0.7073 \end{bmatrix}$$

$$AX_6 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0.7073 \\ -1 \\ 0.7073 \end{bmatrix} = \begin{bmatrix} 2.4146 \\ -3.4146 \\ 2.4146 \end{bmatrix} = 3.4146\begin{bmatrix} 0.707 \\ -1 \\ 0.707 \end{bmatrix}$$

$$AX_7 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0.707 \\ -1 \\ 0.707 \end{bmatrix} = \begin{bmatrix} 2.414 \\ -3.414 \\ 2.414 \end{bmatrix} = 3.414\begin{bmatrix} 0.707 \\ -1 \\ 0.707 \end{bmatrix}$$

Hence the required eigen value is 3.414 correct upto 3 decimal places.

And required eigen vector $= \begin{bmatrix} 0.707 \\ -1 \\ 0.707 \end{bmatrix}$

8. Solve the following system of by using Gauss Seidal method.

$$10x - 5y - 2z = 3$$
$$x + 6y - 10z = -3$$
$$4x - 10y + 3z = -3$$

[2014/Fall]

Solution:

$$10x - 5y - 2z = 3$$
$$x + 6y - 10z = -3$$
$$4x - 10y + 3z = -3$$

Arranging the equations such that magnitude of all the diagonal element is greater than the sum of magnitude of other two elements in the row.

$$10x - 5y - 2z = 3$$
$$4x - 10y + 3z = -3$$
$$x + 6y - 10z = -3$$

Now, forming the equations as,

$$x = \frac{3 + 5y + 2z}{10}$$

$$y = \frac{-3 - 3z - 4x}{-10} = \frac{3 + 3z + 4x}{10}$$

$$z = \frac{-3 - x - 6y}{-10} = \frac{3 + x + 6y}{10}$$

Let initial guess be 0 for x, y and z.

Solving the iterations in tabular form.

Iteration	$x = \frac{3 + 5y + 2z}{10}$	$y = \frac{3 + 3z + 4x}{10}$	$z = \frac{3 + x + 6y}{10}$
Guess	0	0	0
1	0.3	0.42	0.582
2	0.6264	0.7251	0.7977
3	0.8220	0.8681	0.9030
4	0.9146	0.9367	0.9534
5	0.9590	0.9696	0.9776
6	0.9803	0.9854	0.9892
7	0.9905	0.9929	0.9947
8	0.9953	0.9965	0.9974
9	0.9977	0.9983	0.9987
10	0.9988	0.9991	0.9993
11	0.9994	0.999[illegible]	[illegible]

Hence the approximated values of x, y, and z is $0.999 \approx 1$.

NOTE:
Procedure to iterate in programmable calculator:
Let $A = x, B = y, C = z$
Set the following in calculator:

$$A = \frac{3 + 5B + 2C}{10} : B = \frac{3 + 3C + 4A}{10} : C = \frac{3 + A + 6B}{10}$$

Now press CALC and enter the initial value of B and C and continue pressing = only for the required no. of iterations.

9. Use Gauss Elimination Method to solve the equation. Use partial pivoting method where necessary.

$$4x_1 + 5x_2 - 6x_3 = 28$$
$$2x_1 - 7x_3 = 29$$
$$-5x_1 - 8x_2 = -64$$

[2014/Spring]

Solution:

Writing the given system of equation in matrix form,

$$\begin{bmatrix} 4 & 5 & -6 \\ 2 & 0 & -7 \\ -5 & -8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 28 \\ 29 \\ -64 \end{bmatrix}$$

Operate $R_2 \to R_2 - \left(\frac{2}{4}\right) R_1$ and $R_3 \to R_3 - \frac{(-5)}{4} R_1$

$$\begin{bmatrix} 4 & 5 & -6 \\ 0 & -2.5 & -4 \\ 0 & -1.75 & -7.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 28 \\ 15 \\ -29 \end{bmatrix}$$

Operate $R_3 \to R_3 - \frac{-1.75}{-2.5} R_2$

$$\begin{bmatrix} 4 & 5 & -6 \\ 0 & -2.5 & -4 \\ 0 & 0 & -4.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 28 \\ 15 \\ -39.5 \end{bmatrix}$$

Now, performing back substitution

$-4.7x_3 = -39.5$

$\therefore$ $x_3 = 8.404$

$-2.5x_2 - 4x_3 = 15$

or, $-2.5x_2 - 4(8.404) = 15$

$\therefore$ $x_2 = -19.446$

$4x_1 + 5x_2 - 6x_3 = 28$

or, $4x_1 + 5(-19.446) - 6(8.404) = 28$

$\therefore$ $x_1 = 43.913$

10. Find the largest eigen value λ and the corresponding eigen vector X of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

[2014/Spring]

Solution:

Let initial eigen vector be $\begin{bmatrix}1\\1\\1\end{bmatrix}$.

Then the iterations are carried out as

$$AX_0 = \begin{bmatrix}1 & 2 & 0\\2 & 1 & 0\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}3\\3\\-1\end{bmatrix} = 3\begin{bmatrix}1\\1\\-0.333\end{bmatrix}$$

Again,

$$AX_1 = \begin{bmatrix}1 & 2 & 0\\2 & 1 & 0\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}1\\1\\-0.333\end{bmatrix} = \begin{bmatrix}3\\3\\0.333\end{bmatrix} = 3\begin{bmatrix}1\\1\\0.111\end{bmatrix}$$

$$AX_2 = \begin{bmatrix}1 & 2 & 0\\2 & 1 & 0\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}1\\1\\0.111\end{bmatrix} = \begin{bmatrix}3\\3\\-0.111\end{bmatrix} = 3\begin{bmatrix}1\\1\\-0.037\end{bmatrix}$$

$$AX_3 = \begin{bmatrix}1 & 2 & 0\\2 & 1 & 0\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}1\\1\\-0.037\end{bmatrix} = \begin{bmatrix}3\\3\\0.037\end{bmatrix} = 3\begin{bmatrix}1\\1\\0.012\end{bmatrix}$$

$$AX_4 = \begin{bmatrix}1 & 2 & 0\\2 & 1 & 0\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}1\\1\\0.012\end{bmatrix} = \begin{bmatrix}3\\3\\-0.012\end{bmatrix} = 3\begin{bmatrix}1\\1\\-0.004\end{bmatrix}$$

Hence the largest eigen value λ is 3 and largest eigen vector is $\begin{bmatrix}1\\1\\0\end{bmatrix}$.

11. Solve the following by Gauss-Siedal Method

$$b + 3c + 2d = 19$$
$$3b + 2c + 2d = 20$$
$$a + 4b + 2d = 17$$
$$-2a + 2b + c + d = 9$$

Solution: **[2014/Spring]**

Here, the provided system is not diagonally dominant as the magnitude of all the diagonal element is not greater than the sum of magnitude of other elements in the row.

i.e., |coefficient of a| $\ngtr$ |sum coefficient of b, c and d|.

Hence we cannot solve for the convergence from this method.

If it is to be solved from other methods the acquired values a, b, c and d are;

a = 1
b = 2
c = 3
d = 4

12. Solve the following set of equation using LU factorization method.

$$3x + 2y + z = 10$$
$$2x + 3y + 2z = 14$$
$$x + 2y + 3z = 14$$

[2015/Fall, 2017/Fall, 2019/Spring]

Solution:

Writing the equation in matrix from AX = B

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \\ 14 \end{bmatrix}$$

Here, we represent A as

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Solving for unknown values,

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & u_{12}l_{21} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$u_{11} = 3$	$u_{12} = 2$	$u_{13} = 1$
$l_{21} = \frac{2}{3} = 0.667$	$u_{12}l_{21} + u_{22} = 3$ $\therefore u_{22} = 1.666$	$0.667 \times 1 + u_{23} = 2$ $\therefore u_{23} = 1.333$
$l_{31} = \frac{1}{3} = 0.333$	$0.333\times2+l_{32}(1.666) = 2$ $\therefore l_{32} = 0.8$	$0.333\times1+0.8\times1.333+u_{33} = 1$ $\therefore u_{33} = 1.6$

Substituting the values,

$$\underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 0.667 & 1 & 0 \\ 0.333 & 0.8 & 1 \end{bmatrix}}\underset{U}{\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1.666 & 1.333 \\ 0 & 0 & 1.6 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 10 \\ 14 \\ 14 \end{bmatrix}}$$

Let LUX = B

$\Rightarrow$ UX = V

so, LV = B

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.667 & 1 & 0 \\ 0.333 & 0.8 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \\ 14 \end{bmatrix}$$

Now, performing forward substitution,

$\therefore$ $v_1 = 10$

$\Rightarrow$ $0.667v_1 + v_2 = 14$

$\therefore$ $v_2 = 7.33$

$\Rightarrow$ $0.333v_1 + 0.8v_2 + v_3 = 14$

$\therefore$ $v_3 = 4.80$

Then, UX = V becomes

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1.666 & 1.333 \\ 0 & 0 & 1.6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 7.33 \\ 4.80 \end{bmatrix}$$

Performing backward substitution,

$$z = \frac{4.8}{1.6} = 3$$

$\Rightarrow \quad 1.666y - 1.333z = 7.33$

$\therefore \quad y = 1.99 \approx 2$

$\Rightarrow \quad 3x + 2y + z = 10$

$\therefore \quad x = \frac{3.02}{3} = 1.02 \approx 1$

Hence, $x = 1$;

$y = 2$;

and, $z = 3$.

13. Use Gauss-Seidal iterative method to solve given equations.

$$40x - 20y - 10z = 390$$
$$10x - 60y + 20z = -280$$
$$10x - 30y + 120z = -860$$

[2015/Fall]

Solution:

Here the equations have the dominance of diagonal element so forming the equations as

$$x = \frac{390 + 20y + 10z}{40}$$

$$y = \frac{-280 - 10x - 20z}{-60}$$

$$z = \frac{-860 - 10x - 30y}{120}$$

Let the initial guess be 0 for x, y and z.

Now, solving the iteration in tabular form

Iteration	$x = \frac{390 + 20y + 10z}{40}$	$y = \frac{-280 - 10x - 20z}{-60}$	$z = \frac{-860 - 10x - 30y}{120}$
Guess	0	0	0
1	9.75	6.291	-9.551
2	10.507	3.234	-8.850
3	9.154	3.242	-8.74
4	9.186	3.284	-8.753
5	9.203	3.282	-8.754
6	9.202	.3282	-8.754
7	9.202	3.282	-8.754

Here, the values of x, y and z are correct upto 3 decimal places.

So the approximate values of $x = 9.202$, $y = 3.282$ and $z = -8.754$

NOTE:
Procedure to iterate in programmable calculator
Let A = x, B = y, C = z
Set the following in calculator

$$A = \frac{390 + 20B + 10C}{40};\ B = \frac{280 + 10A + 20C}{60};\ C = \frac{-860 - 10A - 30B}{120}$$

Now press CALC and enter the initial value of B and C and continue pressing = only for the required no. of iterations.

14. Find the eigen vlaue and corresponding eigen vector of given matrix

$$\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

[2015/Fall]

Solution:

Let the initial vector be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Then the iterations are carried out as

$$AX_0 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ 6 \end{bmatrix} = 6\begin{bmatrix} 1 \\ -0.333 \\ 1 \end{bmatrix}$$

Again,

$$AX_1 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ -0.333 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0.666 \\ 6 \end{bmatrix} = 6\begin{bmatrix} 1 \\ 0.111 \\ 1 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 0.111 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -0.222 \\ 6 \end{bmatrix} = 6\begin{bmatrix} 1 \\ -0.037 \\ 1 \end{bmatrix}$$

$$AX_3 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ -0.037 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0.074 \\ 6 \end{bmatrix} = 6\begin{bmatrix} 1 \\ 0.012 \\ 1 \end{bmatrix}$$

$$AX_4 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 0.012 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -0.024 \\ 6 \end{bmatrix} = 6\begin{bmatrix} 1 \\ -0.004 \\ 1 \end{bmatrix}$$

$$AX_5 = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ -0.004 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0.008 \\ 6 \end{bmatrix} = 6\begin{bmatrix} 1 \\ 0.001 \\ 1 \end{bmatrix}$$

Hence the required eigen value = 6.

And the required eigen vector is $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

15. Find the largest eigen value and corresponding eigen vector of the following square matrix using power method.

$$\begin{bmatrix} 3 & 1 & 4 \\ 1 & 2 & 3 \\ 4 & 3 & 5 \end{bmatrix}$$

[2015/Spring]

Solution:

Let the initial vector be $\begin{bmatrix}1\\1\\1\end{bmatrix}$.

Then the iterations are carried out as

$$AX_0 = \begin{bmatrix}3 & 1 & 4\\1 & 2 & 3\\4 & 3 & 5\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}8\\6\\12\end{bmatrix} = 12\begin{bmatrix}0.667\\0.5\\1\end{bmatrix}$$

Again,

$$AX_1 = \begin{bmatrix}3 & 1 & 4\\1 & 2 & 3\\4 & 3 & 5\end{bmatrix}\begin{bmatrix}0.667\\0.5\\1\end{bmatrix} = \begin{bmatrix}6.501\\4.667\\9.168\end{bmatrix} = 9.168\begin{bmatrix}0.709\\0.509\\1\end{bmatrix}$$

$$AX_2 = \begin{bmatrix}3 & 1 & 4\\1 & 2 & 3\\4 & 3 & 5\end{bmatrix}\begin{bmatrix}0.709\\0.509\\1\end{bmatrix} = \begin{bmatrix}6.636\\4.727\\9.363\end{bmatrix} = 9.363\begin{bmatrix}0.708\\0.504\\1\end{bmatrix}$$

$$AX_3 = \begin{bmatrix}3 & 1 & 4\\1 & 2 & 3\\4 & 3 & 5\end{bmatrix}\begin{bmatrix}0.708\\0.504\\1\end{bmatrix} = \begin{bmatrix}6.628\\4.716\\9.344\end{bmatrix} = 9.344\begin{bmatrix}0.709\\0.504\\1\end{bmatrix}$$

$$AX_4 = \begin{bmatrix}3 & 1 & 4\\1 & 2 & 3\\4 & 3 & 5\end{bmatrix}\begin{bmatrix}0.709\\0.504\\1\end{bmatrix} = \begin{bmatrix}6.631\\4.717\\9.348\end{bmatrix} = 9.348\begin{bmatrix}0.709\\0.504\\1\end{bmatrix}$$

$$AX_5 = \begin{bmatrix}3 & 1 & 4\\1 & 2 & 3\\4 & 3 & 5\end{bmatrix}\begin{bmatrix}0.709\\0.504\\1\end{bmatrix} = \begin{bmatrix}6.631\\4.717\\9.348\end{bmatrix} = 9.348\begin{bmatrix}0.709\\0.504\\1\end{bmatrix}$$

Hence the required eigen value = 9.348.

And the required eigen vector is $\begin{bmatrix}0.709\\0.504\\1\end{bmatrix}$.

16. Solve the following system of equation by the process of Gauss elimination. (Use partial pivoting if necessary)

$$3x + 2y + z = 10$$
$$2x + 3y + 2z = 14$$
$$x + 2y + 3z = 14$$

[2015/Spring]

Solution:

Writing given equations in matrix form

$$\begin{bmatrix}3 & 2 & 1\\2 & 3 & 2\\1 & 2 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}10\\14\\14\end{bmatrix}$$

Operate $R_2 \to R_2 - \frac{2}{3}R_1$ and $R_3 \to R_3 - \frac{1}{3}R_1$

$$\begin{bmatrix}3 & 2 & 1\\0 & 5/3 & 4/3\\0 & 4/3 & 8/3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}10\\22/3\\32/3\end{bmatrix}$$

Operate $R_3 \to R_3 - \frac{4}{5}R_2$

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 5/3 & 4/3 \\ 0 & 0 & 8/5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 22/3 \\ 24/5 \end{bmatrix}$$

Now, performing backward substitution,

or, $\frac{8}{5}z = \frac{24}{5}$

$\therefore$ $z = 3$

or, $\frac{5}{3}y + \frac{4}{3}z = \frac{22}{3}$

$\therefore$ $y = 2$

or, $3x + 2y + z = 10$

$x = \frac{10 - 2y - z}{3}$

$\therefore$ $x = 1$

17. Use Gauss Seidal iteration method to solve

$2x + y + z = 5$

$3x + 5y + 2z = 15$

$2x + y + 4z = 8$ **[2015/Spring]**

Solution:

Here the equations are in diagonally dominant form.

Forming the equations as

$$x = \frac{5 - y - z}{2}$$

$$y = \frac{15 - 3x - 2z}{5}$$

$$x = \frac{8 - 2x - y}{4}$$

Let initial guess be 0 for x, y and z.

Solving the iterations in tabular form.

Iteration	$x = \frac{5-y-z}{2}$	$y = \frac{15-3x-2z}{5}$	$z = \frac{8-2x-y}{4}$
Guess	0	0	0
1	2.5	1.5	0.375
2	1.562	1.912	0.741
3	1.173	1.999	0.931
4	1.044	2.008	0.976
5	1.008	2.004	0.995
6	1.0005	2.0017	0.9993
7	0.999	2.0008	1.0003
8	0.999	2.0008	1.0003

Hence the required valued of x, y and z are 1, 2 and 1 respectively.

NOTE:
Procedure to iterate in programmable calculator
Let $A = x, B = y, C = z$
Set the following in calculator

$$A = \frac{5 - B - C}{2};\ B = \frac{15 - 3A - 2C}{5};\ C = \frac{8 - 2A - B}{4}$$

Now press CALC and enter the initial value of B and C and continue pressing = only for the required no. of iterations.

18. Solve the following system of equations by using Gauss elimination method with partial pivoting technique.

$$x + y + z + w = 2$$
$$x + y + 3z - 2w = -6$$
$$2x + 3y - z + 2w = 7$$
$$x + 2y + z - w = -2$$

[2016/Fall]

Solution:

Writing the system of equations in matrix form,

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & -2 \\ 2 & 3 & -1 & 2 \\ 1 & 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 7 \\ -2 \end{bmatrix}$$

Interchanging R_1 and R_3 but not variable x and z as partial pivoting

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 1 & 1 & 3 & -2 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \\ 2 \\ -2 \end{bmatrix}$$

Operate $R_2 \to R_2 - \frac{1}{2}R_1$, $R_3 \to R_3 - \frac{1}{2}R_1$, $R_4 \to R_4 - \frac{1}{2}R_1$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -0.5 & 3.5 & -3 \\ 0 & -0.5 & 1.5 & 0 \\ 0 & 0.5 & 1.5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 7 \\ -9.5 \\ -1.5 \\ -5.5 \end{bmatrix}$$

Operate $R_3 \to R_3 - R_2$, $R_4 \to R_4 + R_2$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -0.5 & 3.5 & -3 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & 5 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 7 \\ -9.5 \\ 8 \\ -15 \end{bmatrix}$$

Interchanging R_3 and R_4 but not the variable z and w as partial pivoting.

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -0.5 & 3.5 & -3 \\ 0 & 0 & 5 & -5 \\ 0 & 0 & -2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 7 \\ -9.5 \\ -15 \\ 8 \end{bmatrix}$$

Operate $R_4 \rightarrow R_4 - \frac{(-2)}{5} R_3$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -0.5 & 3.5 & -3 \\ 0 & 0 & 5 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 7 \\ -9.5 \\ -15 \\ 2 \end{bmatrix}$$

Performing backward substitution

or, $1w = 2$

$\therefore$ $w = 2$

or, $5z - 5w = -15$

$\therefore$ $z = -1$

or, $-0.5y + 3.5yz - 3w = -9.5$

$\therefore$ $y = 0$

or, $2x + 3y - z + 2w = 7$

$\therefore$ $x = 1$

19. **Solve the following system of equations by using Crout's algorithm.**

$$2x - 3y + 10z = 3$$
$$-x + 4y + 2z = 20$$
$$5x + 2y + z = -12$$

[2016/Fall]

Solution:

Writing the system of equations in matrix form,

$$\underset{A}{\begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}}$$

Now, using Crout's algorithm, we represent A as

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{22} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix} = \begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix}$$

Solving for unknown values,

$l_{11} = 2$	$l_{11}u_{12} = -3$ $\therefore u_{12} = -1.5$	$l_{11}u_{13} = 10$ $\therefore u_{13} = 5$
$l_{21} = -1$	$l_{21}u_{12} + l_{22} = 4$ $\therefore l_{22} = 2.5$	$l_{21}u_{13} + l_{22}u_{23} = 2$ $\therefore u_{23} = 2.8$
$l_{31} = 5$	$l_{31}u_{12} + l_{32} = 2$ $\therefore l_{32} = 9.5$	$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 1$ $\therefore l_{33} = -50.6$

Now, substituting obtained coefficients as LUX = B

$$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 2.5 & 0 \\ 5 & 9.5 & -50.6 \end{bmatrix} \begin{bmatrix} 1 & -1.5 & 5 \\ 0 & 1 & 2.8 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}$$

Let UX = V, so LV = B then,

$$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 2.5 & 0 \\ 5 & 9.5 & -50.6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}$$

Using forward substitution,

$\therefore$ $v_1 = 1.5$

or $-1v_1 + 2.5v_2 = 20$

$\therefore$ $v_2 = 8.6$

or, $5v_1 + 9.5v_2 - 50.6v_3 = -12$

$\therefore$ $v_3 = 2$

Then, UX = V

$$\begin{bmatrix} 1 & -1.5 & 5 \\ 0 & 1 & 2.8 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1.5 \\ 8.6 \\ 2 \end{bmatrix}$$

Performing backward substitution,

$\therefore$ $z = 2$

or, $y + 2.8z = 8.6$

$\therefore$ $y = 3$

or, $x - 1.5y + 5z = 1.5$

$\therefore$ $x = -4$

20. Find the largest eigen value and corresponding eigen vector of given matrix using power method.

$$\begin{bmatrix} 4 & 6 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 3 \end{bmatrix}$$

[2016/Fall]

Solution:

Let the initial vector be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Then performing the iterations as follows,

$$AX_0 = \begin{bmatrix} 4 & 6 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 5 \end{bmatrix} = 10 \begin{bmatrix} 1 \\ 0.8 \\ 0.5 \end{bmatrix}$$

Again,

$$AX_1 = \begin{bmatrix} 4 & 6 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.8 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 8.8 \\ 5.5 \\ 3.5 \end{bmatrix} = 8.8 \begin{bmatrix} 1 \\ 0.625 \\ 0.397 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} 4 & 6 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.625 \\ 0.397 \end{bmatrix} = \begin{bmatrix} 7.75 \\ 4.316 \\ 3.191 \end{bmatrix} = 7.75 \begin{bmatrix} 1 \\ 0.556 \\ 0.411 \end{bmatrix}$$

$$AX_3 = \begin{bmatrix} 4 & 6 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.556 \\ 0.411 \end{bmatrix} = \begin{bmatrix} 7.336 \\ 4.013 \\ 3.233 \end{bmatrix} = 7.336 \begin{bmatrix} 1 \\ 0.547 \\ 0.440 \end{bmatrix}$$

$$AX_4 = \begin{bmatrix} 4 & 6 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.547 \\ 0.440 \end{bmatrix} = \begin{bmatrix} 7.282 \\ 4.055 \\ 3.320 \end{bmatrix} = 7.282 \begin{bmatrix} 1 \\ 0.556 \\ 0.455 \end{bmatrix}$$

$$AX_5 = \begin{bmatrix} 4 & 6 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.556 \\ 0.455 \end{bmatrix} = \begin{bmatrix} 7.336 \\ 4.145 \\ 3.365 \end{bmatrix} = 7.336 \begin{bmatrix} 1 \\ 0.565 \\ 0.458 \end{bmatrix}$$

$$AX_6 = \begin{bmatrix} 4 & 6 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.565 \\ 0.458 \end{bmatrix} = \begin{bmatrix} 7.390 \\ 4.199 \\ 3.374 \end{bmatrix} = 7.390 \begin{bmatrix} 1 \\ 0.568 \\ 0.456 \end{bmatrix}$$

$$AX_7 = \begin{bmatrix} 4 & 6 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.568 \\ 0.456 \end{bmatrix} = \begin{bmatrix} 7.408 \\ 4.208 \\ 3.368 \end{bmatrix} = 7.408 \begin{bmatrix} 1 \\ 0.568 \\ 0.454 \end{bmatrix}$$

$$AX_8 = \begin{bmatrix} 4 & 6 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.568 \\ 0.454 \end{bmatrix} = \begin{bmatrix} 7.408 \\ 4.202 \\ 3.362 \end{bmatrix} = 7.408 \begin{bmatrix} 1 \\ 0.567 \\ 0.453 \end{bmatrix}$$

Hence the required eigen vector is $\begin{bmatrix} 1 \\ 0.567 \\ 0.453 \end{bmatrix}$.

And the required eigen value 7.408.

21. **Using Gauss Seidal method solve the following system of liner equations.**

$$10x_1 + 6x_2 - 5x_3 = 27$$
$$3x_1 + 8x_2 + 10x_3 = 27$$
$$4x_1 + 10x_2 + 3x_3 = 27$$

[2016/Spring]

Solution:

Arranging the system of liner equations in diagonally dominant forms,

$$10x_1 + 6x_2 - 5x_3 = 27$$
$$4x_1 + 10x_2 + 3x_3 = 27$$
$$3x_1 + 8x_2 + 10x_3 = 27$$

Forming the equations as,

$$x_1 = \frac{27 - 6x_2 + 5x_3}{10}$$

$$x_2 = \frac{27 - 4x_1 - 3x_3}{10}$$

$$x_3 = \frac{27 - 3x_1 - 8x_2}{10}$$

Solving the

Iteration	$x_1 = \frac{27 - 6x_2 + 5x_3}{10}$	$x_2 = \frac{27 - 4x_1 - 3x_3}{10}$	$x_3 = \frac{27 - 3x_1 - 8x_2}{10}$
Guess	0	0	0
1	2.7	1.62	0.594
2	2.025	1.711	0.723
3	2.034	1.669	0.754
4	2.075	1.643	0.763
5	2.095	1.633	0.765
6	2.102	1.629	0.766
7	2.105	1.628	0.766
8	2.106	1.627	0.766
9	2.106	1.627	0.766

Hence the required valued of x_1, x_2 and x_3 are 2.106, 1.627 and 0.766 respectively which are correct upto 3 decimal places.

> **NOTE:**
> Procedure to iterate in programmable calculator
> Let $A = x_1$, $B = x_2$, $C = x_3$
> Set the following in calculator
> $$A = \frac{27 - 6B + 5C}{10} : B = \frac{27 - 4A - 3C}{10} : C = \frac{27 - 3A - 8B}{10}$$
> Now press CALC and enter the initial value of B and C and continue pressing = only for the required no. of iterations.

22. Find the largest eigen value and corresponding eigen vector of the matrix

$$\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

[2016/Spring, 2018/Spring]

Solution:

Let the initial vector be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Then performing the iterations as

$$AX_0 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 13 \end{bmatrix} = 13 \begin{bmatrix} 0.230 \\ 0.692 \\ 1 \end{bmatrix}$$

Again,

$$AX_1 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 0.230 \\ 0.692 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.306 \\ 6.074 \\ 12.5\ldots \end{bmatrix} = 12.538 \begin{bmatrix} 0.104 \\ 0.484 \\ 1 \end{bmatrix}$$

$$AX_2=\begin{bmatrix}1&3&-1\\3&2&4\\-1&4&10\end{bmatrix}\begin{bmatrix}0.104\\0.484\\1\end{bmatrix}=\begin{bmatrix}0.556\\5.28\\11.832\end{bmatrix}=11.832\begin{bmatrix}0.046\\0.446\\1\end{bmatrix}$$

$$AX_3=\begin{bmatrix}1&3&-1\\3&2&4\\-1&4&10\end{bmatrix}\begin{bmatrix}0.046\\0.446\\1\end{bmatrix}=\begin{bmatrix}0.384\\5.03\\11.738\end{bmatrix}=11.738\begin{bmatrix}0.032\\0.428\\1\end{bmatrix}$$

$$AX_4=\begin{bmatrix}1&3&-1\\3&2&4\\-1&4&10\end{bmatrix}\begin{bmatrix}0.032\\0.428\\1\end{bmatrix}=\begin{bmatrix}0.316\\4.952\\11.68\end{bmatrix}=11.68\begin{bmatrix}0.027\\0.423\\1\end{bmatrix}$$

$$AX_5=\begin{bmatrix}1&3&-1\\3&2&4\\-1&4&10\end{bmatrix}\begin{bmatrix}0.027\\0.423\\1\end{bmatrix}=\begin{bmatrix}0.296\\4.927\\11.665\end{bmatrix}=11.668\begin{bmatrix}0.025\\0.422\\1\end{bmatrix}$$

$$AX_6=\begin{bmatrix}1&3&-1\\3&2&4\\-1&4&10\end{bmatrix}\begin{bmatrix}0.025\\0.422\\1\end{bmatrix}=\begin{bmatrix}0.291\\4.919\\11.663\end{bmatrix}=11.663\begin{bmatrix}0.024\\0.421\\1\end{bmatrix}$$

Hence the required eigen value 11.663.

And the required eigen vector is $\begin{bmatrix}0.024\\0.421\\1\end{bmatrix}$.

23. Find the inverse of the matrix by using Gauss Jordan Method.

$$A=\begin{bmatrix}1&-1&2\\3&0&1\\1&0&2\end{bmatrix}$$ **[2017/Fall]**

Solution:

The augmented matrix can be written as

$$[A:I]=\begin{bmatrix}1&-1&2&:&1&0&0\\3&0&1&:&0&1&0\\1&0&2&:&0&0&1\end{bmatrix}$$

Operate $R_2\to R_2-3R_1$ and $R_3\to R_3-R_1$

$$[A:I]=\begin{bmatrix}1&-1&2&:&1&0&0\\0&3&-5&:&-3&1&0\\0&1&0&:&-1&0&1\end{bmatrix}$$

Operate $R_2\to R_2-3R_3$

$$[A:I]=\begin{bmatrix}1&-1&2&:&1&0&0\\0&1&-5&:&-1&1&-2\\0&1&0&:&-1&0&1\end{bmatrix}$$

Operate $R_3\to R_3-R_2$

$$[A:I]=\begin{bmatrix}1&-1&2&:&1&0&0\\0&1&-5&:&-1&1&-2\\0&0&5&:&0&-1&3\end{bmatrix}$$

Operate $R_3 \to \frac{1}{5} R_3$

$$[A : I] = \left[\begin{array}{ccc:ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -5 & -1 & 1 & -2 \\ 0 & 0 & 1 & 0 & -0.2 & 0.6 \end{array}\right]$$

Operate $R_2 \to R_2 + 5R_3$

$$[A : I] = \left[\begin{array}{ccc:ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -0.2 & 0.6 \end{array}\right]$$

Operate $R_1 \to R_1 + R_2$

$$[A : I] = \left[\begin{array}{ccc:ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -0.2 & 0.6 \end{array}\right]$$

Operate $R_1 \to R_1 - 2R_3$

$$[A : I] = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & 0.4 & -0.2 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -0.2 & 0.6 \end{array}\right]$$

Now, inverse of matrix,

$[A : I] = [I : A]$

$[A = A^{-1}]$

$$\text{Hence, } A^{-1} = \begin{bmatrix} 0 & 0.4 & -0.2 \\ -1 & 0 & 1 \\ 1 & -0.2 & 0.6 \end{bmatrix}$$

24. Solve the following set of equation using LU factorization method.

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

[2017/Spring]

Solution:

Writing the system of equations in matrix form.

$$\begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$$

In LU factorization method, we represent A as

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix} = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$$

Solving for unknown values,

$u_{11} = 5$	$u_{12} = -2$	$u_{13} = 1$
$l_{21}u_{11} = 7$ $\therefore l_{21} = 1.4$	$l_{21}u_{12} + u_{22} = 1$ $\therefore u_{22} = 3.8$	$l_{21}u_{13} + u_{23} = -5$ $\therefore u_{23} = -6.4$
$l_{31}u_{11} = 3$ $\therefore l_{31} = 0.6$	$l_{31}u_{12} + l_{32}u_{22} = 7$ $\therefore l_{32} = 2.15$	$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 4$ $\therefore u_{33} = 17.16$

Now, substituting obtained coefficient and we have overall system of

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1.4 & 1 & 0 \\ 0.6 & 2.15 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 5 & -2 & 1 \\ 0 & 3.8 & -6.4 \\ 0 & 0 & 17.16 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}}_{B}$$

Let, $LUX = B$

$UX = V$

$LV = B$, then,

$$\begin{bmatrix} 1 & 0 & 0 \\ 1.4 & 1 & 0 \\ 0.6 & 2.15 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$$

Now, performing forward substitution,

$\therefore$ $v_1 = 4$

or, $1.4v_1 + v_2 = 8$

$\therefore$ $v_2 = 2.4$

or, $0.6v_1 + 2.15v_2 + v_3 = 10$

$\therefore$ $v_3 = 2.44$

Now,

$UX = V$

$$\begin{bmatrix} 5 & -2 & 1 \\ 0 & 3.8 & -6.4 \\ 0 & 0 & 17.16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2.4 \\ 2.44 \end{bmatrix}$$

Performing backward substitution,

or, $17.16z = 2.44$

$\therefore$ $z = 0.142$

or, $3.8y - 6.4z = 2.4$

$\therefore$ $y = 0.870$

or, $5x - 2y + z = 4$

$\therefore$ $x = \frac{5.598}{5} = 1.119$

25. Solve the equation by Gauss-Jacobi method.

$20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$ **[2017/Spring]**

Solution:

Given that;

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

The given equations are in diagonally dominant form.

Now, forming the equations as,

$$x = \frac{1}{20}[17 - y + 2z]$$

$$y = \frac{1}{20}[-18 + z - 3x]$$

$$z = \frac{1}{20}[25 + 3y - 2x]$$

Let $x_0 = 0$, $y_0 = 0$ and $z_0 = 0$ be initial guesses.

And solving the iterations in tabular form

Iteration	$x = \frac{1}{20}[17 - y + 2z]$	$y = \frac{1}{20}[-18 + z - 3x]$	$z = \frac{1}{20}[25 + 3y - 2x]$
Guess	0	0	0
1	0.85	−0.9	1.25
2	1.02	−0.965	1.03
3	1.00125	−1.0015	1.00325
4	1.0004	−1.000025	0.99965
5	0.99996	−1.00007	0.99995

Hence the required values of x, y and z are 1, −1 and 1 respectively.

NOTE:

Procedure to iterate in programmable calculator

Let, A = x, B = y, C = z

Step 1: Set the following in calculator

$$A : B : C : D = \frac{17 - B + 2C}{20} : E = \frac{-18 + C - 3A}{20} : F = \frac{25 + 3B - 2A}{20}$$

Step 2: Press CALC then

enter the value of A? then press =

enter the value of B? then press =

enter the value of C? then press =

Step 3: Now press = only, again and again to get the values for respective row for each column.

Step 4: Update the values of A?, B? and C? when asked again.

Step 5: Got to step 3.

26. **Determine the largest eigen value and the corresponding eigen vector of the matrix using power method.**

$$A = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}$$

[2017/Spring, 2018/Fall]

Solution:

Let the initial vector be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Now using power method, the iterations are carried out as

$$AX_0 = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ -18 \end{bmatrix} = -18\begin{bmatrix} -0.444 \\ 0.222 \\ 1 \end{bmatrix}$$

NOTE: Here $|-18| > 8$ and $|-4|$

Again,

$$AX_1 = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}\begin{bmatrix} -0.444 \\ 0.222 \\ 1 \end{bmatrix} = \begin{bmatrix} -10.548 \\ 1.104 \\ 7.768 \end{bmatrix} = -10.548\begin{bmatrix} 1 \\ -0.105 \\ -0.736 \end{bmatrix}.$$

$$AX_2 = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -0.105 \\ -0.736 \end{bmatrix} = -18.948\begin{bmatrix} -0.930 \\ 0.361 \\ 1 \end{bmatrix}$$

$$AX_3 = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}\begin{bmatrix} -0.930 \\ 0.361 \\ 1 \end{bmatrix} = -18.394\begin{bmatrix} 1 \\ -0.415 \\ -0.981 \end{bmatrix}$$

$$AX_4 = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -0.415 \\ -0.981 \end{bmatrix} = -19.698\begin{bmatrix} -0.995 \\ 0.462 \\ 1 \end{bmatrix}$$

$$AX_5 = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}\begin{bmatrix} -0.995 \\ 0.462 \\ 1 \end{bmatrix} = -19.773\begin{bmatrix} 1 \\ -0.480 \\ -0.999 \end{bmatrix}$$

$$AX_6 = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -0.480 \\ -0.999 \end{bmatrix} = -19.922\begin{bmatrix} -0.997 \\ 0.490 \\ 1 \end{bmatrix}$$

$$AX_7 = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}\begin{bmatrix} -0.997 \\ 0.490 \\ 1 \end{bmatrix} = -19.956\begin{bmatrix} 1 \\ -0.495 \\ -0.999 \end{bmatrix}$$

$$AX_8 = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0.495 \\ 0.999 \end{bmatrix} = 19.956\begin{bmatrix} 1 \\ -0.495 \\ -0.999 \end{bmatrix} \approx 20\begin{bmatrix} -1 \\ 0.5 \\ 1 \end{bmatrix}$$

Hence the dominant eigen value is 20 and eigen vector is $\begin{bmatrix} -1 \\ 0.5 \\ 1 \end{bmatrix}$.

27. Find the inverse of matrix using Gauss Jordan method [2018/Fall]

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 3 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

Solution:

The augmented matrix can be written as,

$$[A:I] = \begin{bmatrix} 1 & 1 & 3 & : & 1 & 0 & 0 \\ 3 & 3 & -3 & : & 0 & 1 & 0 \\ -2 & -4 & -4 & : & 0 & 0 & 1 \end{bmatrix}$$

Operate $R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 + 2R_1$,

$$[A:I] = \begin{bmatrix} 1 & 1 & 3 & : & 1 & 0 & 0 \\ 0 & 0 & -12 & : & -3 & 1 & 0 \\ 0 & -2 & 2 & : & 2 & 0 & 1 \end{bmatrix}$$

Interchanging R_2 and R_3,

$$[A:I] = \begin{bmatrix} 1 & 1 & 3 & : & 1 & 0 & 0 \\ 0 & -2 & 2 & : & 2 & 0 & 1 \\ 0 & 0 & -12 & : & -3 & 1 & 0 \end{bmatrix}$$

Operate $R_2 \to \frac{R_2}{-2}$ and $R_3 \to \frac{R_3}{-12}$,

$$[A:I] = \begin{bmatrix} 1 & 1 & 3 & : & 1 & 0 & 0 \\ 0 & 1 & -1 & : & -1 & 0 & -0.5 \\ 0 & 0 & 1 & : & 0.25 & -0.083 & 0 \end{bmatrix}$$

Operate $R_2 \to R_2 + R_3$,

$$[A:I] = \begin{bmatrix} 1 & 1 & 3 & : & 1 & 0 & 0 \\ 0 & 1 & 0 & : & -0.75 & -0.083 & -0.5 \\ 0 & 0 & 1 & : & 0.25 & -0.083 & 0 \end{bmatrix}$$

Operate $R_1 \to R_1 - R_2$,

$$[A:I] = \begin{bmatrix} 1 & 0 & 3 & : & 1.75 & 0.083 & 0.5 \\ 0 & 1 & 0 & : & -0.75 & -0.083 & -0.5 \\ 0 & 0 & 1 & : & 0.25 & -0.083 & 0 \end{bmatrix}$$

Operate $R_1 \to R_1 - 3R_3$,

$$[A:I] = \begin{bmatrix} 1 & 0 & 0 & : & 1 & 0.332 & 0.5 \\ 0 & 1 & 0 & : & -0.075 & -0.083 & -0.5 \\ 0 & 0 & 1 & : & 0.25 & -0.083 & 0 \end{bmatrix}$$

For inverse of matrix,

$[A:I] = [I:A]$

$[A = A^{-1}]$

Hence,

$$A^{-1} = \begin{bmatrix} 1 & 0.332 & 0.5 \\ -0.75 & -0.083 & -0.5 \\ 0.25 & -0.083 & 0 \end{bmatrix}$$

28. Solve the following system of equation

$$6x_1 - 2x_2 + x_3 = 4$$
$$-2x_1 + 7x_2 + 2x_3 = 5$$
$$x_1 + 2x_2 - 5x_3 = -1$$

Using Gauss factorization method. **[2018/Fall]**

Solution:

Writing the given system of equation in matrix from AX = B

$$\begin{bmatrix} 6 & -2 & 1 \\ -2 & 7 & 2 \\ 1 & 2 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$$

In Gauss factorization method, we decompose matrix A in the following form,

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{12} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 6 & -2 & 1 \\ -2 & 7 & 2 \\ 1 & 2 & -5 \end{bmatrix}$$

Here, A = LU

Solving for unknown values,

$u_{11} = 6$	$u_{12} = -2$	$u_{13} = 1$
$l_{21}u_{11} = -2$ $\therefore l_{21} = -0.333$	$u_{12}l_{21} + u_{22} = 7$ $\therefore u_{22} = 6.334$	$l_{21}u_{13} + u_{23} = 2$ $\therefore u_{23} = 2.333$
$l_{31}u_{11} = 1$ $\therefore l_{31} = 0.167$	$l_{31}u_{12} + l_{32}u_{22} = 2$ $\therefore l_{32} = 0.368$	$l_{31}u_{13} + l_{32}u_{23} + u_{33} = -5$ $\therefore u_{33} = -6.025$

Now, substituting obtained coefficients, we have overall system as,

$$\underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ -0.333 & 1 & 0 \\ 0.167 & 0.368 & 1 \end{bmatrix}}\underset{U}{\begin{bmatrix} 6 & -2 & 1 \\ 0 & 6.334 & 2.333 \\ 0 & 0 & -6.025 \end{bmatrix}}\underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}}$$

Let UX = V,

so, LV = B then

$$\begin{bmatrix} 1 & 0 & 0 \\ -0.333 & 1 & 0 \\ 0.167 & 0.368 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$$

Using forward substitution

$\therefore$ $v_1 = 4$

or, $-0.333v_1 + v_2 = 5$

$\therefore$ $v_2 = 6.332$

or, $0.167v_1 + 0.368v_2 + v_3 = -1$

$\therefore$ $v_3 = -3.998$

Now,

UX = V

$$\begin{bmatrix} 6 & -2 & 1 \\ 0 & 6.334 & 2.333 \\ 0 & 0 & -6.025 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6.332 \\ -3.998 \end{bmatrix}$$

Using backward substitution

or, $-6.025x_3 = -3.998$

$\therefore$ $x_3 = 0.663$

or, $6.33x_2 + 2.333x_3 = 6.332$

$\therefore$ $x_2 = 0.755$

or, $6x_1 - 2x_2 + x_3 = 4$

$\therefore$ $x_1 = 0.807$

29. Solve the following system of equations using factorization method.

$2x + 3y + z = 9$

$x + 2y + 3z = 6$

$3x + y + 2z = 8$ **[2018/Spring]**

Solution:

Writing the system of equations in matrix form,

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

In factorization method, we decompose matrix in the following form A = LU

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

Solving for unknown values

$u_{11} = 2$	$u_{12} = 3$	$u_{13} = 1$
$l_{21}u_{11} = 1$ $\therefore l_{21} = 0.5$	$l_{21}u_{12} + u_{22} = 2$ $\therefore u_{22} = 0.5$	$l_{21}u_{13} + u_{23} = 3$ $\therefore u_{23} = 2.5$
$l_{31}u_{11} = 3$ $\therefore l_{31} = 1.5$	$l_{31}u_{12} + l_{32}u_{22} = 1$ $\therefore l_{32} = -7$	$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 5$ $\therefore u_{33} = 21$

Now, substituting obtained coefficient, we have overall system of LUX = B as

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 1.5 & -7 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ 0 & 0.5 & 2.5 \\ 0 & 0 & 21 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

UX = V

Then, LV = B

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 1.5 & -7 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

Now, performing forward substitution,

$\therefore$ $v_1 = 9$

$\rightarrow$ $0.5v_1 + v_2 = 6$

$v_2 = 1.5$

$\therefore$ $1.5v_1 - 7v_2 + v_3 = 8$

$\therefore$ $v_3 = 5$

Then, $UX = V$

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 0.5 & 2.5 \\ 0 & 0 & 21 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 1.5 \\ 5 \end{bmatrix}$$

Now, performing backward substitution

or, $12z = 5$

$\therefore$ $z = 0.238$

or, $05y + 2.5z = 1.5$

$\therefore$ $y = 1.81$

or, $2x + 3y + 1z = 9$

$\therefore$ $x = 1.66$

30. Find inverse of the matrix, using Gauss Jordan method

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$ **[2019/Fall]**

Solution:

The augumented matrix can be written as

$$[A:I] = \left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$$

Operate $R_2 \to R_2 - R_1$ and $R_3 \to R_3 + 2R_1$

$$[A:I] = \left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right]$$

Operate $R_2 \to \frac{R_2}{2}$

$$[A:I] = \left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -1/2 & -1/2 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right]$$

Operate $R_3 \to R_3 + 2R_2$

$$[A:I] = \left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right]$$

Operate $R_3 \to \frac{R_3}{-4}$

$$[A:I] = \left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$$

Operate $R_1 \rightarrow R_1 - R_2$

$$[A:I] = \begin{bmatrix} 1 & 0 & 6 & : & 1.5 & -0.5 & 0 \\ 1 & 1 & -3 & : & -0.5 & +0.5 & 0 \\ 0 & 0 & 1 & : & -0.25 & -0.25 & -0.25 \end{bmatrix}$$

Operate $R_1 \rightarrow R_1 - 6R_3$

$$[A:I] = \begin{bmatrix} 1 & 0 & 0 & : & 3 & 1 & 1.5 \\ 0 & 1 & -3 & : & -0.5 & +0.5 & 0 \\ 0 & 0 & 1 & : & -0.25 & -0.25 & -0.25 \end{bmatrix}$$

Operate $R_2 \rightarrow R_2 + 3R_3$

$$[A:I] = \begin{bmatrix} 1 & 0 & 0 & : & 3 & 1 & 1.5 \\ 0 & 1 & 0 & : & -1.25 & -0.25 & -0.75 \\ 0 & 0 & 1 & : & -0.25 & -0.25 & -0.25 \end{bmatrix}$$

Now, for inversion of matrix

$$[A:I] = [I:A]$$

$$[A = A^{-1}]$$

Hence,

$$A^{-1} = \begin{bmatrix} 3 & 1 & 1.5 \\ -1.25 & -0.25 & -0.75 \\ -0.25 & -0.25 & -0.25 \end{bmatrix}$$

31. Determine the largest eigen value and the corresponding eigen vector of the matrix using power method.

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$$

[2019/Fall]

Solution:

Let the initial vector be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

$$AX_0 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \\ 14 \end{bmatrix} = 14\begin{bmatrix} 0 \\ 0.5 \\ 1 \end{bmatrix}$$

Again,

$$AX_1 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0 \\ 0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1 \\ 6.5 \end{bmatrix} = 6.5\begin{bmatrix} 0.076 \\ 0.153 \\ 1 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0.076 \\ 0.153 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.617 \\ -0.084 \\ 5.915 \end{bmatrix} = 5.915\begin{bmatrix} 0.273 \\ -0.014 \\ 1 \end{bmatrix}$$

$$AX_3 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0.273 \\ -0.014 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.429 \\ -0.116 \\ 6.482 \end{bmatrix} = 6.482\begin{bmatrix} 0.374 \\ -0.017 \\ 1 \end{bmatrix}$$

$$AX_4 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.374 \\ -0.017 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.425 \\ 0.428 \\ 7.193 \end{bmatrix} = 7.193 \begin{bmatrix} 0.337 \\ 0.059 \\ 1 \end{bmatrix}$$

$$AX_5 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.337 \\ 0.059 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.160 \\ 0.584 \\ 7.199 \end{bmatrix} = 7.199 \begin{bmatrix} 0.300 \\ 0.081 \\ 1 \end{bmatrix}$$

$$AX_6 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.300 \\ 0.081 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.057 \\ 0.524 \\ 7.043 \end{bmatrix} = 7.043 \begin{bmatrix} 0.292 \\ 0.074 \\ 1 \end{bmatrix}$$

$$AX_7 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.292 \\ 0.074 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.07 \\ 0.464 \\ 6.974 \end{bmatrix} = 6.974 \begin{bmatrix} 0.296 \\ 0.066 \\ 1 \end{bmatrix}$$

$$AX_8 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.296 \\ 0.066 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.098 \\ 0.066 \\ 1 \end{bmatrix} = 6.974 \begin{bmatrix} 0.3 \\ 0.064 \\ 1 \end{bmatrix}$$

Hence the required largest eigen value is $6.974 \approx 7$

And corresponding eigen vector is $\begin{bmatrix} 0.3 \\ 0.064 \\ 1 \end{bmatrix}$.

32. Use relaxation method to solve the given systems of equations.

$20x + y - 2z = 17$

$3x + 20y - z = 18$

$2x - 3y + 20z = 25$ **[2019/Fall]**

Solution:

The diagonal elements of the coefficient matrix dominate the other coefficients in the corresponding row.

i.e., $|20| \geq |1| + |-2|$

$|20| \geq |3| + |-1|$

$|20| \geq |2| + |-3|$

Now, using relaxation method.

The residuals are given by

$R_x = 17 - 20x - y + 2z$

$R_y = 18 - 3x - 20y + z$

$R_z = 25 - 2x + 3y - 20z$

The operation table is

	δR_x	δR_y	δR_z
$\delta x = 1$	-20	-3	-2
$\delta y = 1$	-1	-20	3
$\delta z = 1$	2	1	-20

Now, the relaxation table is shown below.

Taking $z = y = z = 0$ as initial assumption

	R_x	R_y	R_z	
x = y = z = 0	17	18	25	
$\delta z = 1$	17+(1×2)=19	18+(1×1)=19	25−(20×1)=5	i
$\delta x = 0.5$	19−(20×0.5)=9	19+(−3×0.5)=17.5	5−(5×0.5)=4	ii
$\delta y = 0.5$	9+(−1×0.5)=8.5	17.5−(20×0.5)=7.5	4+3(0.5)=5.5	iii
$\delta x = 0.5$	8.5+(−20×0.5)=−1.5	7.5−(3×0.5)=6	5.5−2×0.5= 4.5	
$\delta y = 0.33$	−1.83	−0.6	5.49	
$\delta z = 0.28$	−1.27	−0.32	−0.11	
$\delta x = -0.06$	−0.07	−0.14	0.010	
$\delta y = -0.007$	−0.063	0.00	−0.010	
$\delta x = -0.003$	−0.003	0.009	0.006	

Now,

$$\Sigma\delta x = 0.5 + 0.5 - 0.06 - 0.003 = 0.937$$

$$\Sigma\delta y = 0.5 + 0.33 - 0.007 = 0.823$$

$$\Sigma\delta z = 1 + 0.28 = 1.28$$

Thus, $x = 0.937$, $y = 0.823$ and $z = 1.28$

NOTE:
In (i) in the table, the largest residual is 25 so to reduce it, we give an increment in δz at $\delta z = 1$ and the resulting residuals are shown in (ii).
i.e., larger residuals are reduced by assuming suitable increment values. Similarly the steps are carried out.
Also when increment is done in either δx or δy or δz, use the operation table respectively.

33. Solve the equation by relaxation method

$$9x - y + 2z = 9$$
$$x + 2y - 2z = 15$$
$$2x - 2y - 13z = -17$$

[2020/Fall]

Solution:

$$9x - y + 2z = 9$$
$$x + 2y - 2z = 15$$
$$2x - 2y - 13z = -17$$

Using relaxation method,
The residuals are given by,

$$R_x = 9 - 9x + y - 2z$$
$$R_y = 15 - x - 2y + 2z$$
$$R_z = -17 - 2x + 2y + 13z$$

The operation table is

	δR_x	δR_y	δR_z
$\delta x = 1$	−9	−1	−2
$\delta y = 1$	1	−2	2
$\delta z = 1$	−2	2	13

Taking initial guess of x = y = z = 0.

Now, the relaxation table is,

	R_x	R_y	R_z
0	9	15	-17
$\delta z = 1$	7	17	-4
$\delta y = 8$	15	1	12
$\delta z = 2$	-3	-1	8
$\delta z = -0.615$	-1.77	-2.23	0.005
$\delta y = -1.115$	-2.885	0	-0.225
$\delta x = -0.32$	-0.005	0.32	0.415
$\delta z = -0.031$	0.057	0.25	0.012
$\delta y = 0.125$	0.182	0	0.262

Now,

$$\Sigma\delta x = 2 - 0.32 = 1.68$$

$$\Sigma\delta y = 8 - 1.115 + 0.125 = 7.01$$

$$\Sigma\delta z = 1 - 0.615 - 0.031 = 0.354$$

Thus, x = 1.68, y = 7.01 and z = 0.354

34. Determine the largest eigen value and the corresponding eigen vector of the matrix using the power method.

$$A = \begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 8 \\ 4 & 8 & 1 \end{bmatrix}$$ **[2020/Fall]**

Solution:

$$A = \begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 8 \\ 4 & 8 & 1 \end{bmatrix}$$

Let the initial vector be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Then using power method, performing the iterations as,

$$AX_0 = \begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 8 \\ 4 & 8 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ 13 \end{bmatrix} = 13\begin{bmatrix} 0.692 \\ 1 \\ 1 \end{bmatrix}$$

Again,

$$AX_1 = \begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 8 \\ 4 & 8 & 1 \end{bmatrix}\begin{bmatrix} 0.692 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8.692 \\ 11.768 \\ 11.768 \end{bmatrix} = 11.768\begin{bmatrix} 0.738 \\ 1 \\ 1 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 8 \\ 4 & 8 & 1 \end{bmatrix}\begin{bmatrix} 0.738 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8.738 \\ 11.952 \\ 11.952 \end{bmatrix} = 11.952\begin{bmatrix} 0.731 \\ 1 \\ 1 \end{bmatrix}$$

$$AX_3 = \begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 8 \\ 4 & 8 & 1 \end{bmatrix}\begin{bmatrix} 0.731 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8.731 \\ 11.924 \\ 11.924 \end{bmatrix} = 11.924\begin{bmatrix} 0.732 \\ 1 \\ 1 \end{bmatrix}$$

$$AX_4 = \begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 8 \\ 4 & 8 & 1 \end{bmatrix}\begin{bmatrix} 0.732 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8.732 \\ 11.928 \\ 11.928 \end{bmatrix} = 11.928\begin{bmatrix} 0.732 \\ 1 \\ 1 \end{bmatrix}$$

$$AX_5 = \begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 8 \\ 4 & 8 & 1 \end{bmatrix}\begin{bmatrix} 0.732 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8.732 \\ 11.928 \\ 11.928 \end{bmatrix} = 11.928\begin{bmatrix} 0.732 \\ 1 \\ 1 \end{bmatrix}.$$

Hence the required eigen value is 11.928.

and the eigen vector is $\begin{bmatrix} 0.732 \\ 1 \\ 1 \end{bmatrix}$.

35. Solve the following set of equations by using LU decomposition method.

$$3x + 2y + 7z = 32$$
$$2x + 3y + z = 40$$
$$3x + 4y + z = 56$$

[2020/Fall]

Solution:

Writing the system of equations in matrix form AX = B

$$\begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 32 \\ 40 \\ 56 \end{bmatrix}$$

In LU factorization method, we represent A as

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

Solving for unknown values

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & u_{12}l_{21} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

Solving for unknown values,

$u_{11} = 3$	$u_{12} = 2$	$u_{13} = 7$
$l_{21}u_{11} = 2$ $\therefore l_{21} = 0.667$	$l_{21}u_{12} + u_{22} = 3$ $\therefore u_{22} = 1.666$	$l_{21}u_{13} + u_{23} = 1$ $\therefore u_{23} = -3.669$
$l_{31}u_{11} = 3$ $\therefore l_{31} = 1$	$l_{31}u_{12} + l_{32}u_{22} = 4$ $\therefore l_{32} = 1.2$	$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 1$ $\therefore u_{33} = -1.597$

Substituting the values

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0.667 & 1 & 0 \\ 1 & 1.2 & 1 \end{bmatrix}}_{L}\underbrace{\begin{bmatrix} 3 & 2 & 7 \\ 0 & 1.666 & -3.669 \\ 0 & 0 & -1.597 \end{bmatrix}}_{U}\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 32 \\ 40 \\ 56 \end{bmatrix}}_{B}$$

Here, LUX = B

Let UX = V

so, LV = B then,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.667 & 1 & 0 \\ 1 & 1.2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 32 \\ 40 \\ 56 \end{bmatrix}$$

Performing forward substitution

$\therefore$ $v_1 = 32$

or, $0.667 v_1 + v_2 = 40$

$\therefore$ $v_2 = 18.656$

or, $v_1 + 1.2 v_2 + v_3 = 56$

$\therefore$ $v_3 = 1.612$

Now, UX = V

$$\begin{bmatrix} 3 & 2 & 7 \\ 0 & 1.666 & -3.669 \\ 0 & 0 & -1.597 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 32 \\ 18.656 \\ 1.612 \end{bmatrix}$$

Again, performing backward substitution

or, $-1.597z = 1.612$

$\therefore$ $z = -1.009 \approx -1$

or, $1.666y - 3.669z = 18.656$

$\therefore$ $y = \dfrac{14.953}{1.666} = 8.975 \approx 9$

or, $3x + 2y + 7z = 32$

$\therefore$ $x = 7.037 \approx 7$

36. **Write short notes on: Relaxation method.** **[2014/Fall]**

Solution: See the topic 4.6.3.

37. **Write short notes on Ill conditioned system.**

[2014/Spring, 2016/Spring, 2019/Spring]

Solution: See the topic 4.5.

38. **Write short notes on: Gauss Seidel method of iteration.** **[2017/Fall]**

Solution: See the topic 4.6.2.

39. **Write a program in any high level language C or C++ to solve a system of linear equation, using gauss elimination method.**

[2016/Spring]

Solution: See the "Appendix", program number 11.

40. **Write a program to solve a system of linear equations by Gauss Seidal method.** **[2018/Spring]**

Solution: See the "Appendix", program number 16.