ITER, SOA (Deemed to be) University, Bhubaneswar

MCA I^{st} Semester
Assignment 5, November 2025
Subject: Discrete Mathematics (MA 3001)
Sections: 25C2A1, 25C2A2, 25C2B1, & 25C2B2

Answer all questions

5.1 Mathematical Induction

1. Prove that $1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ for the positive integer n.
2. Prove that $1^2 + 3^2 + 5^2 \cdots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$ whenever n is a nonnegative integer.
3. Use mathematical induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57 for every nonnegative integer n.
4. Prove that $3 + 3 \cdot 5 + 3 \cdot 5^2 + \cdots + 3 \cdot 5^n = 3(5^{n+1} - 1)/4$ whenever n is a nonnegative integer.
5. Prove that for every positive integer n, $1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = n(n+1)(n+2)/3$.

5.2 Strong Induction and Well-Ordering

6. Show that if n is an integer greater than 1, then n can be written as the product of primes.
7. Prove that every amount of postage 8 cents or more can be formed using just 3-cent and 5-cent stamps.
8. Determine which amounts of postage can be formed using just 3-cent and 10-cent stamps.

5.3 Recursion

9. Find $f(2), f(3), f(4), \text{and}, f(5)$ if f is defined recursively by $f(0) = -1$, $f(1) = 2$, and for $n = 1, 2, \cdots$
   (a) $f(n+1) = f(n) + 3f(n-1)$.
   (b) $f(n+1) = 3^{f(n)/3}$.
10. Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \cdots$ if

(a) $a_n = 6n$

(b) $a_n = 5$

(c) $a_n = 4n - 2$

(d) $a_n = n^2$

(e) $a_n = 2n + 1$