MID-SEMESTER EXAMINATION, December-2023
Discrete Mathematics (MA 3001)

Programme: MCA **Semester:1st**
Full Marks: 30 **Time: 2 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
To get introduced to the basic concepts of Discrete Mathematics such as set theory, relations and functions, propositional logic and Decision problems of propositional logic. Ability to apply the knowledge for modelling and solving the problem in computing sciences.	L1, L2, L3, L4	1, 2, 3, 5	24
To get introduced to Induction and recursion	L1, L2, L5	4	6
To get introduced to Algorithms, The Growth of Functions, Complexity of Algorithms, Divisibility and Modular Arithmetic, Integer Representations and Algorithms, Primes and Greatest Common Divisors, Solving Congruences, Applications of Congruences, Cryptography and their applications	L2		
To understand the principle of counting techniques such as mutual inclusive exclusive principle, pigeonhole principle, permutations and combinations and summation. Ability to apply them to real life counting problems. Ability to write recurrence relations and generating functions.	L2, L3		

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

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Answer all questions. Each question carries equal mark.

1. (a) Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent. 2

(b) Show that the premises "A student in this class has not read the book", and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book". 2

(c) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction. 2

2. (a) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ 2

(b) Prove that if x is any real number, then $\lfloor 2x \rfloor = \lfloor x \rfloor + \left\lfloor x + \frac{1}{2} \right\rfloor$ 2

(c) Prove or disprove that for all sets A, B and C, we have 2

I. $A \cup (B \cup C) = (A \cap B) \cup (A \cap C)$

II. $\bar{A} \times (\overline{B \cup C}) = (A \times (B \cup C))$

3. (a) Determine whether the function $f: Z \times Z \to Z$ is bijective if 2

I. $f(m, n) = m^2 - 4$

II. $f(m, n) = |n|$

III. $f(m, n) = |m| - |n|$

IV. $f(m, n) = m^2 + n^2$

(b) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find the 2

I. join of A and B

II. meet of A and B

III. Boolean product of A and B

(c) Suppose that $A_i = \{1, 2, 3, \dots, i\}$ for $i = 1, 2, 3, \dots$. Then, find 2

I. $\bigcup_{i=1}^{\infty} A_i$

II. $\bigcap_{i=1}^{\infty} A_i$

4. (a) Prove that $\displaystyle\sum_{i=1}^{n} \left(-\frac{1}{2}\right)^i = \frac{2^{n+1} + (-1)^n}{3.2^n}$ 2

(b) I. Give the recursive definition of the sequence $\{a_n\}$ for n = 1, 2, 3, … if $a_n = 1 + (-1)^n$ 2

II. Give a recursive definition of the set of positive integers not divisible by 11

(c) Suppose that $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$. Prove by mathematical induction that $A^n = \begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix}$ 2

5. (a) Find all solutions, if exists, of the following system of linear congruence equations by The Chinese Remainder theorem: 2

$x \equiv 5 (mod\ 8)$

$x \equiv 7\ (mod\ 13)$

$x \equiv 3\ (mod\ 11)$

(b) Show that 1729 is a Carmichael number. 2

(c) Show that if m is an integer greater than 1 and $ac \equiv bc (mod\ m)$, then $a \equiv b \left(mod \frac{m}{\gcd(c, m)}\right)$ 2

End of Questions