ITER, SOA (deemed to be) University, Bhubaneswar

MCA 1^{st} Semester
Assignment 1, September 2025
Subject: Discrete Mathematics (MA 3001)
Sections: 25C2A1, 25C2A2, 25C2B1, & 25C2B2

Answer all questions

1.1 Propositional Logic

1. Which of these sentences are propositions?
   (a) $x + 2 = 11$.
   (b) Answer this question.
   (c) What time is it?
   (d) $2^n \geqslant 100$.
   (e) There are no black flies in Maine.

2. What is the negation of each of these propositions?
   (a) Abby is richer than Ricardo.
   (b) Janice has more Facebook friends than Juan.
   (c) $2 + 1 = 3$.
   (d) 121 is a perfect square.
   (e) Diane rode her bicycle 100 miles on Sunday.

3. Let p and q be the propositions
   p : The election is decided
   q : The votes have been counted.
   Express each of these compound propositions as an English sentence.
   (a) $\neg\, p$
   (b) $p \wedge q$
   (c) $p \vee q$
   (d) $\neg p \wedge q$
   (e) $q \rightarrow p$
   (f) $\neg q \rightarrow \neg p$
   (g) $\neg p \rightarrow \neg q$
   (h) $p \leftrightarrow q$
   (i) $\neg q \vee (\neg p \wedge q)$
   (j) $p \rightarrow q$

4. Determine whether these statements are true or false.
   (a) $1+1=2$ if and only if 2+3=4
   (b) If $1+1=3$, then 2+2=4
   (c) If $1+1=3$, then unicorns exist.
   (d) $0>1$ if and only if $2>1$.
   (e) If monkeys can fly, then $1+2=3$.

5. Construct a truth table for each of these compound propositions.
   (a) $(p \vee \neg q) \rightarrow q$
   (b) $p \oplus (p \vee q)$
   (c) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
   (d) $(p \vee q) \rightarrow (p \oplus q)$
   (e) $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow q)$

6. What is the value of x after each of these statements is encountered in a computer program, if $x=1$ before the statement is reached?
   (a) **if** $x+2=3$ **then** $x:=x+1$
   (b) **if** $(x+1=3)$ OR $(2x+2=3)$ **then** $x:=x+1$
   (c) **if** $(2x+3=5)$ AND $(3x+4=7)$ **then** $x:=x+1$
   (d) **if** $(x+1=2)$ XOR $(x+2=3)$ **then** $x:=x+1$
   (e) **if** $x<2$ **then** $x:=x+1$

7. Evaluate each of these expressions.
   (a) $11000 \wedge (01011 \vee 11011)$
   (b) $(01111 \wedge 10101) \vee 01000$
   (c) $(01010 \oplus 11011) \oplus 01000$

1.3 Propositional Equivalences

8. Use a truth table to verify the distributive law

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

9. Show that each of these conditional statements is a tautology by using truth tables.
   (a) $\neg p \rightarrow (p \rightarrow q)$
   (b) $\neg(p \rightarrow q) \rightarrow \neg q$
   (c) $[p \wedge (p \rightarrow q)] \rightarrow q$

10. Use the laws and show that each of the following statements are Tautology
   (a) $p \rightarrow (p \vee q)$
   (b) $\neg(p \rightarrow q) \rightarrow p$

(c) $[\neg p \wedge (p \vee q)] \rightarrow q$

11. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

12. Show that $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent.

1.4 Predicate and Quantifiers

13. Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.
    (a) $\forall x(R(x) \rightarrow H(x))$
    (b) $\forall x(R(x) \wedge H(x))$
    (c) $\exists x(R(x) \rightarrow H(x))$
    (d) $\exists x(R(x) \wedge H(x))$

14. Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.
    (a) A student in your class has a cat, a dog, and a ferret.
    (b) All student in your class has a cat, a dog, or a ferret.
    (c) Some student in your class has a cat and a ferret, but not a dog.
    (d) No student in your class has a cat, a dog, and a ferret.
    (e) For each of the three animals, cat, dogs, and ferrets, there is a student in your class who has this animal as a pet.

15. Determine the truth value of each of these statements if the domain consists of all real numbers
    (a) $\exists x(x^3 = -1)$
    (b) $\exists x(x^4 < x^2)$
    (c) $\forall x((-x)^2 = x^2)$
    (d) $\forall x(2x > x)$

16. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.
    (a) Someone in your class can speak Hindi.
    (b) Everyone in your class is friendly.
    (c) There is a person in your class who was not born in California.
    (d) A student in your class has been in a movie.
    (e) No student in your class has taken a course in logic programming.

17. Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

(a) A student in your school has lived in Vietnam.
(b) There is a student in your school who cannot speak Hindi.
(c) A student in your school knows Java, Prolog, and C++.
(d) Everyone in your class enjoys Thai food.
(e) Someone in your class does not play hockey

18. Express each of these statements using predicates, quantifiers, and logical connectives.
    (a) At least one mail message, among the non-empty set of messages, can be saved if there is a disk with more than 10 Kilobytes of free space.
    (b) Whenever there is an active alert, all queued messages are transmitted.
    (c) The diagnostic monitor tracks the status of all systems except the main console.
    (d) Each participant on the conference call whom the host of the call did not put on a special list was billed.

1.5 Nested Quantifiers

19. Let $L(x, y)$ be the statement "x loves y." where the domain of both x and y consists of all people in the world. Use quantifiers to express each of these statements.
    (a) Everybody loves Jerry.
    (b) Everybody loves somebody.
    (c) There is somebody whom everybody loves.
    (d) There is somebody whom Lina does not love.
    (e) Everyone loves himself or herself.
20. Use quantifiers and predicates with more than one variable to express these statements
    (a) Every CS student needs a course in DM.
    (b) There is a student in this class who owns a personal computer.
    (c) Every student in this class has taken at least one CS course.
    (d) Every student in this class has been in every building on campus.
    (e) There is a student in this class who has taken at least one course in CS.
21. Translate each of these nested quantifications into an English statement that expresses a mathematical fact.
    (a) $\exists x \forall y (xy = y)$
    (b) $\forall x \forall y (((x < 0) \land (y < 0)) \to (xy > 0))$
    (c) $\exists x \exists y ((x^2 > y) \land (x < y))$
    (d) $\forall x \forall y \exists z (x + y = z)$
22. Express the negation of these propositions using quantifiers, and in English.
    (a) Every student in this class likes mathematics.

(b) There is a student in this class who has never seen a computer.

(c) There is a student in this class who has taken every mathematics course offered in this school.

(d) There is a student in this class who has been in at least one room of every building on campus.

1.6 Rules of Inference

23. What rules of inferences is used in each of these arguments?

    (a) Jerry is a mathematic major and a computer science major. Therefore, Jerry is a mathematics major.

    (b) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

    (c) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be at beach.

    (d) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

    (e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the materials.

24. Use rules of inference to show that the hypotheses "Reeta works hard," "If Reeta works hard, then she is a dull girl," and "If Reeta is a dull girl, then she will not get the job" imply the conclusion "Reeta will not get the job."

25. What rules of inference are used in this argument? "All men are mortal. Socrates is a man. Therefore, Socrates is mortal."

26. For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

    (a) "I am either clever or lucky." "I am not lucky." "If I am lucky, then I will win the lottery."

    (b) "All rodents gnaw their food." "Mice are rodents." "Rabbits do not gnaw their food." "Bats are not rodents."

    (c) "I am either dreaming or hallucinating." "I am not dreaming." "If I am hallucinating, I see elephants running down the road."

27. Use rules of inference to show that if $\forall x(P(x) \to (Q(x) \land S(x)))$ and $\forall x(P(x) \land R(x))$ are true, then $\forall x(R(x) \land S(x))$ is true.

28. Use rules of inference to show that if $\forall x(P(x) \lor Q(x))$, $\forall x(\neg Q(x) \lor S(x))$, $\forall x(R(x) \to \neg S(x))$, and $\exists x(\neg P(x))$ are true, then $\exists x(\neg R(x))$ is true.

1.7 Introduction to Proofs

29. Show that the proposition $P(0)$ is true, where $P(n)$ is "If $n > 1$, then $n^2 > n$" and the domain consists of all integers.

30. Let $P(n)$ be "If a and b are positive integers with $a \geqslant b$, then $a^n \geqslant b^n$," where the domain consists of all nonnegative integers. Show that $P(0)$ is true.

31. Prove that the sum of two rational number is rational.

32. Prove that if n is an integer and n^2 is odd, then n is odd.

33. Prove that $\sqrt{3}$ is irrational by giving a proof by contradiction.

34. Give a proof by contradiction of the theorem "If $3n + 2$ is odd, then n is odd."

35. Show that these statements about the integer n are equivalent:
    $p_1 : n$ is even.
    $p_2 : n - 1$ is odd.
    $p_3 : n^2$ is even.

36. Use a direct proof to show that the sum of two odd integers is even.

37. Use a direct proof to show that every odd integer is the difference of two squares.

38. Use a proof by contradiction to prove that the sum of an irrational number and a rational number is irrational.

39. Use a proof of contraposition to show that if $x + y \geqslant 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

40. Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.