(c) Use mathematical Induction to show that 2

$$1^3+2^3+3^3+\cdots+n^3=\frac{n^2(n+1)^2}{4}.$$

8\. (a) Give the recursive definition of the sequence 2

$a_n = 6n$

where $n \geq 1$.

(b) Prove that every amount of postage stamp of 12 or more can be formed using just 4 cents and 5 cents stamps. 2

(c) How many permutations of the letters "ABCDEFG" contain the string "BCD". 2

9\. (a) How many different strings can be made from the letters in "ABRACADABRA". 2

(b) What is the binomial expansion of $(x+y)^4$? 2

(c) Draw the Hasse diagram for the partial ordering $\{(A,B) \mid A \subseteq B\}$ on the power set $P(S)$, where $S=\{a,b,c\}$. 2

10 (a) Check whether $(\mathbb{Z}, \geq)$ is a POSET or not? 2

(b) Let m be an integer with $m>1$. Show that the relation $R=\{(a,b) \mid a \equiv b (\bmod m)\}$ is an equivalence relation on the set of integers. 2

(c) Draw the directed graph of the relation 2

$R=\{(1,1),(1,3),(2,1),(2,3),(2,4),(3,1),(3,2),(4,1)\}$

on the set $\{1,2,3,4\}$.

End of Questions

END-SEMESTER EXAMINATION, February-2025
Discrete Mathematics (MA 3001)

Programme: MCA **Semester: 1st**
Full Marks: 60 **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Understand and apply rules of logic to distinguish between valid and invalid arguments and use them to prove mathematical statements.	L2, L3	1(a, b, c) 2(a, b, c) 3(a, b)	14
Comprehend sets, their various operations and use them to analyze functions and its various concepts as well as study sequences and summations	L3, L4	3(c), 4(a, b, c) 5(a)	12
Analyze the searching and sorting algorithms and use the growth of functions to study the time complexity of algorithms as well as apply some of the important concepts of number theory to divisibility and modular arithmetic, integer representation of algorithms, congruence and cryptography	L3, L4, L5	5(b, c), 6(a, b, c), 7(a, b)	14
Construct proofs by mathematical induction and formulate recursive definitions and develop structural induction	L4, L5	7(c), 8(a,b)	6
Apply different counting techniques to solve various problems.	L3, L4	8(c), 9(a, b)	6
Implement relations and their properties to analyze equivalence relations and partial orderings	L4, L5	9(c) 10(a, b, c)	8

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1\. (a) What is the value of x after each of these statements is encountered in a computer program, if $x=1$ before 2 the statement is reached?

(i) **if** $x+2=3$ **then** $x:=x+1$

(ii) **if** $(x+1=3)$ OR $(2x+2=3)$ **then** $x:=x+1$

(b) Construct a truth table for the compound proposition $p \oplus (p \vee q)$. 2

(c) Evaluate the expressions $11000 \wedge (01011 \vee 11011)$. 2

2. (a) Prove or disprove that the given conditional statement is a tautology. 2

$$\neg p \rightarrow (p \rightarrow q)$$

(b) Determine the truth value of $\forall x((-x)^2 = x^2)$ if the domain consists of all real numbers. 2

(c) What rules of inferences is used in each of these arguments? 2
"If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the materials."

3. (a) Prove that if n is a positive integer, then n is odd if and only if $5n+6$ is odd. 2

(b) Find a matrix A such that 2

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} A = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}.$$

(c) Let $A=\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find $A \odot B$. 2

4. (a) Find $\sum_{k=99}^{200} k^2(k-3)$. 2

(b) Check whether the following sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if $a_n = 0$. 2

(c) Determine whether the function $f:\mathbb{Z}\times\mathbb{Z}\rightarrow\mathbb{Z}$ is one-one and onto if $f(m,n) = m^2 + n^2$. Justify your answer. 2

5. (a) List all the steps used to search for 9 in the sequence $1,3,4,5,6,8,9,11$ using binary search. 2

(b) Use the definition of "$f(x)$ is $O(g(x))$" to show that $x^4 + 9x^3 + 4x + 7$ is $O(x^4)$. 2

(c) What is the worst case complexity of the bubble sort in terms of the number of comparisons made? 2

6. (a) Evaluate $(32^3 \text{ mod } 13)^2 \text{ mod } 11$. 2

(b) Use extended Euclid algorithm to express the GCD of 144 and 89 as a linear combination of 144 and 89. 2

(c) Use CRT to find the solution of the system 2

$$x \equiv 1 \bmod 2$$
$$x \equiv 2 \bmod 3$$
$$x \equiv 3 \bmod 5$$
$$x \equiv 4 \bmod 11$$

7. (a) Encrypt the message "*WATCH YOUR STEP*" using shift cipher, for shift 11. 2

(b) Find $f(2), f(3), f(4), f(5)$, if f is defined recursively by $f(n+1) = f(n) + 3f(n-1)$ where $f(0) = -1$ and $f(1) = 2$. 2