END-SEMESTER EXAMINATION, February-2024
Discrete Mathematics (MA 3001)

Programme: M.C.A. **Semester:1st**
Full Marks: 60 **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
To get introduced to the basic concepts of Discrete Mathematics such as set theory, relations and functions, propositional logic and Decision problems of propositional logic. Ability to apply the knowledge for modelling and solving the problem in computing sciences.	L1, L2, L3, L4	1, 2, 7, 8	24
To get introduced to Induction and recursion	L1, L2, L5	4	6
To get introduced to Algorithms, The Growth of Functions, Complexity of Algorithms, Divisibility and Modular Arithmetic, Integer Representations and Algorithms, Primes and Greatest Common Divisors, Solving Congruences, Applications of Congruences, Cryptography and their applications	L2	3, 9, 10	18
To understand the principle of counting techniques such as mutual inclusive exclusive principle, pigeonhole principle, permutations and combinations and summation. Ability to apply them to real life counting problems. Ability to write recurrence relations and generating functions.	L2, L3	5, 6	12

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1. (a) Show that $\neg p$ is tautologically implied by $\neg(p \wedge \neg q), \neg q \vee r, \neg r$. 2

(b) Express the following statements in English 2

I. Some drivers do not obey the speed limits.
II. There is someone in the class who does not have good attitude.
III. Many students in this class can speak Hindi.

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(c) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. 2

2. (a) Draw the corresponding Venn-diagram for the following: 2
   - I. Let A and B be sets such that $A \cup B \subseteq B$ and $B \subsetneq A$.
   - II. Let A, B, and C be sets such that $A \subseteq B, A \subseteq C, B \cap C \subseteq A$, and $A \subseteq B \cap C$.

   (b) Find the power set of the following sets, where a and b are distinct elements: 2
   - I. $\mathcal{P}(\{a, b, \{a, b\}\})$
   - II. $\mathcal{P}\left(\left\{\phi, a, \{a\}, \{\{a\}\}\right\}\right)$

   (c) Let $f_1(x) = x + 4, f_2(x) = x - 4,$ and $f_3(x) = 4x$. Find $(f_1 \circ f_2) \circ f_3$, $(f_3 \circ f_2) \circ f_1$. 2

3. (a) 2
   - I. State Fermat's Little Theorem.
   - II. What is Pseudorprimes?

   (b) Let m be a positive integer and let a, b, and c be integers. If $ac \equiv bc \ (mod\ m)$ and $\gcd(c, m) = 1$, then $a \equiv b \ (mod\ m)$. 2

   (c) We receive the encrypted message 0981 0461. Find the decrypted message using the RSA decryption method with the key $(2537, 13)$. 2

4. (a) Show that for positive integer n, $11^{n+2} + 12^{2n+1}$ is divisible by 133. 2

   (b) Among integers 1 to 300, how many of them are not divisible by 3, nor by 5, nor by 7? 2

   (c) Show that if n is an integer greater than 1, then n can be written as product of primes. 2

5. (a) State the Pigeonhole Principle 2

   (b) How many numbers must be selected from the set $\{1,2,3,4,5,6\}$ to guarantee that at least one pair of these numbers add up to 7? 2

   (c) 2
   - I. How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school ?.
   - II. How many bit strings of length n contains exactly r 1s?

6. (a) 2
   - I. What is the coefficient of x^7 in $(1 + x)^{11}$?
   - II. What is the coefficient of x^9 in $(2 - x)^{19}$?

   (b) How many bit strings of length 12 contain 2
   - I. exactly four 1s?
   - II. at most four 1s?
   - III. at least four 1s?

   (c) The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain 2
   - I. exactly one vowels?
   - II. exactly two vowels?

7. (a) Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric and or transitive, where $(x, y) \in R$ if and only if 2
   - I. $x \neq y$.
   - II. $xy \geq 1$.
   - III. $x \equiv y \ (mod\ 7)$.

   (b) Show that the "divides" relation on the set of positive integers is not an equivalence relation. 2

   (c) Let R be an equivalence relation on a set A. These statements for elements a and b of A are equivalent: 2
   - I. aRb.
   - II. $[a] = [b]$.
   - III. $[a] \cap [b] \neq \emptyset$.

8. (a) Draw the Hasse diagram for divisibility on the set 2
   - I. {1,2,3,4,5,6}
   - II. {2,3,5,10,11,15,25}
   - III. {1,3,9,27,81,243}

   (b) What are the equivalence classes of 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 for congruence modulo 10? 2

   (c) How many non-zero entries does the matrix representing the relation R on A={1,2,3,...,100} consisting of the first 100 positive integers have if R is 2
   - I. $\{(a, b) | a > b\}$.
   - II. $\{(a, b) | a \neq b\}$.
   - III. $\{(a, b) | ab = 1\}$.

9. (a) List all the steps used to search for 9 in the sequence 1, 3, 4, 5, 6, 8, 9, 11 using 2
   - I. a linear search
   - II. a binary search.

   (b) Use the insertion sort to put 6, 2, 3, 1, 5, 4 into increasing order. 2

   (c) Use the bubble sort to put the elements of the list 3, 2, 4, 1, 5 in increasing order. 2

10 (a) Show that 2
   - I. $\frac{(x^2+1)}{(x+1)}$ is $O(x)$
   - II. $\frac{(x^3+2x)}{(x+1)}$ is $O(x^2)$.

   (b) Determine whether each of these functions is $\Omega(x^2)$. 2
   - I. $f(x) = x \ log x$
   - II. $f(x) = 2^x$

   (c) Describe the time complexity of the binary search algorithm. 2

End of Questions