CSO → Computer System Organization. dt-16-09-25

Transister is the smallest unit.
↳ semiconductor device.

dt-17-09-25:

Number System

1- Decimal - base 10, 0-9
10 distinct symbol

$(45)_{10} = \underbrace{4\times10^1}_{\text{weight}} + \underbrace{5\times10^0}_{\text{weight}}$

$(527)_{10} = 5\times10^2 + 2\times10^1 + 7\times10^0$

2. Binary number system: 0, 1 (base 2)

0 - 0
1 - 1
10 - 2
11 - 3
100 - 4
101 - 5
110 - 6
111 - 7
1000 - 8
1001 - 9
|
|
|
|

$2^{10} \to K$
$2^{20} \to M$
$2^{30} \to G$

Decimal	Binary (4bit)
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

[2 bit → $2^2 = 4$ → 0 – 3]
3bit → $2^3 = 8$ – 0 – 7
4bit – 2^4 – 16 → 0 – 15
5bit – 2^5 – 32 → 0 – 31

8 bit → 1byte.

Conversion Dec to Bi & Binary to Decimal.

$(45)_{10} \rightarrow (101101)_2$

$-(97)_{10} \rightarrow (100001)_2$

1 0 0 0 0 1
32 16 8 4 2 1

$(97)_{10} \rightarrow (1100001)_2$

$(97.25)_{10} = (1100001.01)_2$

$2^0 \quad 2^{-1} \quad 2^{-2}$

$$\begin{array}{rl} 0.25 & \\ \times\, 2 & \\ \hline 0.5 & 0 \\ \times\, 2 & \\ \hline 1.0 & 1 \end{array}$$

$0 \times 2^{-1} + 1$

$(0.32)_{10} = 0.32$

Octal number system: 0, 1, 2, 3, 4, 5, 6, 7 dt-22-09-25

Base → 8

Dec to oct

$(923)_{10} = (1633)_8$

$$\begin{array}{r|l} 8 & 923 \\ \hline 8 & 115 \rightarrow 3 \\ \hline 8 & 14 \rightarrow 3 \\ \hline & 1 \rightarrow 6 \end{array}$$

Oct to Dec.

$(1633)_8 =$

$= 1\times8^3 + 6\times8^2 + 3\times8^1 + 3\times8^0$

$= (923)_{10}$

- Decimal to any number system devide by it's base or radix number.
- Any number system to decimal multiply by it's weight.

Dec to oct

$(923.25)_{10} =$

$$\begin{array}{r} 0.25 \\ \times\ 8 \\ \hline 2.00 \end{array} \rightarrow 2.$$

$= (1633.2)_8$.

Dec to oct

$(496.22)_{10} =$

$$\begin{array}{r|l} 8 & 496 \\ \hline 8 & 62 \rightarrow 0 \\ \hline & 7 \rightarrow 6 \end{array}$$

$$\begin{array}{r} .22 \\ \times 8 \\ \hline 1.76 \end{array} \quad 1$$

$$\begin{array}{r} .76 \\ \times 8 \\ \hline 6.08 \end{array} \quad 6$$

$$\begin{array}{r} .08 \\ \times 8 \\ \hline 0.64 \end{array} \quad 0$$

$= (760.160)_8$

$(3570.45)_8 =$

$= 3\times 8^3 + 5\times 8^2 + 7\times 8^1 + 0\times 8^0 + 4\times 8^{-1} + 0\times 8^{-2}$

$(1912.578)_{10}.$

Octal to Binary :-

$(1\ 2\ 6)_8 = 1\times 8^2 + 2\times 8^1 + 6\times 8^0$

$= 64 + 16 + 6$

$= (86)_{10}$

$$\begin{array}{r|l} 2 & 86 \\ \hline 2 & 43 - 0 \\ \hline 2 & 21 - 1 \\ \hline 2 & 10 \rightarrow 1 \\ \hline 2 & 5 \rightarrow 0 \\ \hline 2 & 2 \rightarrow 1 \\ \hline & 1 \rightarrow 0 \end{array}$$

$= (1010\ 110)_2$

(ii) Octal → Binary directly by representing each octal digit to it's corresponding $\underline{\underline{3\text{bit}}}$ Binary number.

$$\begin{array}{ccc} (1 & 2 & 6)_8 \\ \downarrow & \downarrow & \downarrow \\ (001 & 010 & 110)_2 \end{array}$$

3bit - representation.

$0 \rightarrow 000$

$1 \rightarrow 001$

$2 \rightarrow 010$

$3 \rightarrow 011$

$4 \rightarrow 100$

$5 \rightarrow 101$

$6 \rightarrow 110$

$7 \rightarrow 111$

$(7\ 0\ 3)_8 =$

$(111\ 000\ 011)_2$

$(4\ 0\ 5\ .\ 2\ 3)_8$

$(100\ 000\ 101\ .\ 010\ 011)_2$

Binary $\rightarrow$ octal - directly by representing each 3-bit binary number to its corresponding octal digit.

e.g. $(10110101)_2$

$= (2\ 6\ 5)_8$

(1011101.0101)

$= (1\ 3\ 5\ .\ 2\ 4)_8$

10101100.10101

$= (2\ 5\ 4\ .\ 5\ 2)_8$

Hexadecimal numbers:

decimal

	0	0000
	1	0001
	2	0010
	3	0011
	4	0100
	5	0101
	6	0110
	7	0111
	8	1000
	9	1001
10	A	1100
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

- decimal → Hexa

$(1493)_{10} = (5D5)_{16}$

$$\begin{array}{r|l}
16 & 1493 \\
\hline
16 & 93 \rightarrow 5 \\
\hline
 & 5 \rightarrow 13 - D
\end{array}$$

$(5D5)_{16} = (1493)_{10}$

$5 \times 16^2 + D \times 16^1 + 5 \times 16^0$

$= (1493)_{10}$

- $(2763.22)_{10} = (ACB.38)_{16}$

$$\begin{array}{r|l}
16 & 2763 \\ \hline
16 & 172 \rightarrow B \\ \hline
 & 10 \rightarrow C \\
 & A
\end{array}$$

~~2763~~

$0.22 \times 16 = \cancel{1.32}$ $3.52 \rightarrow 3$

$0.52 \times 16 = 8.32 \rightarrow 8$

Hexa $\longrightarrow$ Decimal.

- $(1A2.C3)_{16} =$

$(418.761)_{10}$

- $(1A2.1B)_{16} = 1 \times 16^2 + 10 \times 16^1 + 2 \times$

Hexa $\longrightarrow$ Binary .

$(A2)_{16} = (1010\ 0010)_2$

$A \times 16^1 + 2 \times 16^0$

$= (160 + 2)_{10} = (162)_{10}$

$$\begin{array}{r|l l}
2 & 162 & \\ \hline
2 & 81 & 0 \\ \hline
2 & 40 & 1 \\ \hline
2 & 20 & 0 \\ \hline
2 & 10 & 0 \\ \hline
2 & 5 & 0 \\ \hline
2 & 2 & 1
\end{array}$$

Hexa $\longrightarrow$ Binary $\longrightarrow$ Directly
Representing each Hexa digit to its 4 bit binary number.

A 2
↓
$(1010\ 0010)_2$.

$(10B.2C)_{16}$ =

= $(100001011.00101100)_2$

- Binary to Hexa → directly by representing each 4 bit binary number to Hexa digit.

1 0 1 0 1 1 0 0 1 . 1 1 1 0 1

$(159.E8)_{16}$.

$(1011010.01011)_2$

$(5A.58)_{16}$.

Hexa $\longrightarrow$ octal
(1) Hexa $\longrightarrow$ decimal $\longrightarrow$ octal.
(2) Hexa $\longrightarrow$ Binary $\longrightarrow$ octal.

$(193A.12)_{16}$ =

$(1100100111010.00010010)_2$

$(14472.044)_8$

$(AF.0B)_{16} =$

$= (10101111.00001011)_2$

$\cancel{(537.026)_8}$

$(257.026)_8$.

<u>Octal → Hexa</u>

(1) Octal → Decimal → Hexa.

(2) Octal → Binary → Hexa

$(127.15)_8 =$

$= (001010111.001101)_2$

$= (57.34)_{16}$

$(5432.75)_8 =$

$(101100011010.111101)_2$

$\cancel{(16320.F4)_{16}}$

$(B1A.F4)_{16}$

$\underline{\underline{Ex}}$. $\frac{40}{3} = 13$ if it is correct then find the base.

→ Let the base is 'b'

$$\frac{4\times b^1 + 0\times b^0}{3\times b^0} = 1\times b^1 + 3b^0$$

$$\Rightarrow \frac{4b+0}{3} = b+3$$

$$\Rightarrow 4b = 3b+9$$

$$\Rightarrow \boxed{b=9}$$

$12 + 21 = 33$

- Let the base is 'b'.

$$1\times b^1 + 2\times b^0 + 2\times b^1 + 1\times b^0 = 3\times b^1 + 3\times b^0$$

$$\Rightarrow b+2+2b+1 = 3b+3$$

$$\Rightarrow 3b+3 = 3b+3$$

∴ valid for any value of b.

→ b can be greater than 4 ∴

$$\boxed{b \geq 4.}$$

Binary Addition

dt - 24-09-25

$$\begin{array}{r} 1101 \\ 1111 \\ \hline \boxed{1}1100 \end{array}$$

0 + 1 = 1
1 + 0 = 1
0 + 0 = 0
1 + 1 = 10

$$\begin{array}{r} 1011 \\ 1001 \\ \hline \cancel{10000} \\ \boxed{1}0100 \end{array}$$

$$\begin{array}{r} 1001 \\ -011 \\ \hline 0110 \end{array}$$

Representation of -ve numbers.

① **Signed magnitude representation.**

→ In this representation an extra bit is added at the MSB to represent sign.

0 → +ve
1 → -ve

ex 0 101 = +5
sign, 101 magnitude.

1 101 = -5
sign, magnitude

* 1111 → -7.

. 1001 = -1

② complement system

2's complement representation.

→ In 1's complement representation change all 0's to 1's and all 1's to 0's.

— 110010

1's — 001101

→ 1 is added to it's LSB 1's complement to get 2's complement

ex.

```
     0110
1's  1001
       +1
2's comp  1010
```

```
     0000 0110
1's  1111 1001
            +1
2's  1111 1110
```

```
  6
 -3
  3
```

6 + (-3)

```
+3 → 0011
1's comp → 1100
             +1
2's com.   1101
```

```
6 → 0110
   +1101
  1 0011
```

Carry.

```
  3      011
 -6     +010
         101
```

```
+6 → 110
     001
      +1
     010
```

$$\begin{array}{r} 3 \\ -6 \end{array} = \begin{array}{r} 3 \\ +(-6) \end{array} \rightarrow \begin{array}{r} 0011 \\ +1010 \\ \hline 1101 \end{array}$$

$$\begin{array}{r} +6 \rightarrow 0110 \\ 1's \rightarrow 1001 \\ +1 \\ \hline 2's \rightarrow 1010. \end{array}$$

→ If there is no carry then no result is negative and is in 2's complement.

– If there is carry then neglect the carry & remaining is the result.

$$\begin{array}{r} 5-3 \\ \hline 0101 \\ +1101 \\ \hline 10010 \end{array}$$

carry

$$\begin{array}{r} 3-5 \\ \hline 0011 \\ +1011 \\ \hline 1110 \end{array}$$

in 2's comp. method.

Result But in –ve & 2's complement.

$$\begin{array}{r} 0010 \\ 1101 \\ +1 \\ \hline 1101 \end{array}$$

$$\begin{array}{r} 0101 \\ 1010 \\ +1 \\ \hline 1011 \end{array}$$

LOGIC GATES:

dt-26-09-25

Basic Gate:- AND gate :- The O/P is '1' when all the inputs are '1'. otherwise the O/P is '0'.

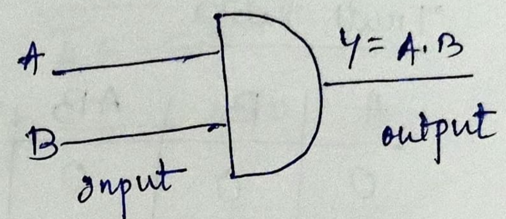
0- False
1- True.

Truth Table.

I/P

A	B	Y = A.B (O/P)
0	0	0
0	1	0
1	0	0
1	1	1

Symbol:

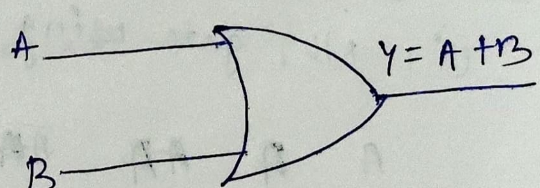


OR GATE - The O/P is '1' when any one input is '1'. The O/P is '0' when all the inputs are '0'.

Truth Table:

A	B	Y = A+B
0	0	0
0	1	1
1	0	1
1	1	1

Symbol.



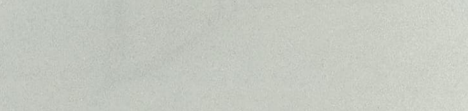
NOT GATE:

Truth Table.

A	$\bar{A}$ or A'
0	1
1	0

$\bar{A}$ or A' → A bar or A complement.

Symbol:



Universal Gate :-

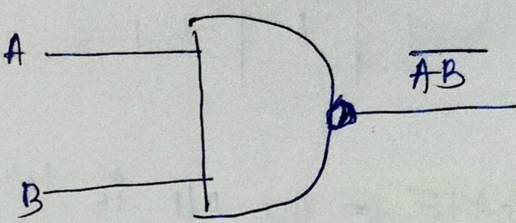
→ NAND and NOR are known as universal gates.
why? All the gates can be designed by using NAND or NOR gates only.

1. NAND gate (AND → complement → NAND)

Truth Table

A	B	AB	$\overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Symbol :-

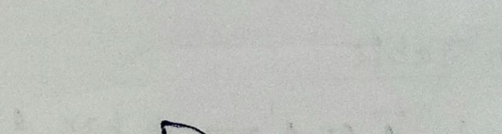


~~2. NOR gate~~

(i) NOT gate using NAND.

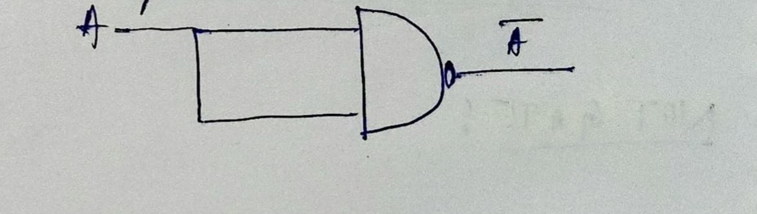
A ~~A~~ ~~AB~~ ~~$\overline{AA}$~~

Symbol:

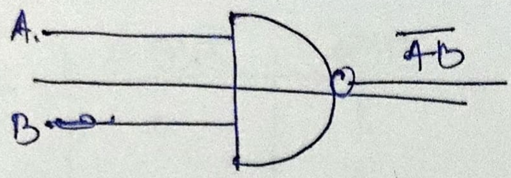


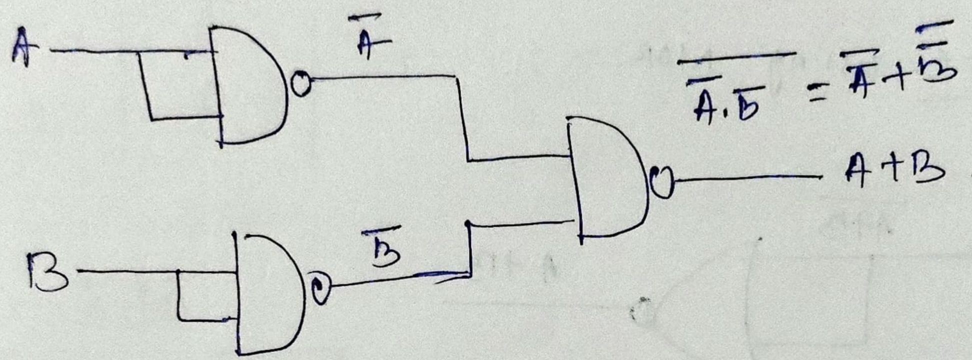
AND gate using NAND.

~~A~~ ~~B~~ ~~AB~~'



OR gate using NAND.





Demorgan's Law.

$\overline{A+B} = \overline{A}\cdot\overline{B}$

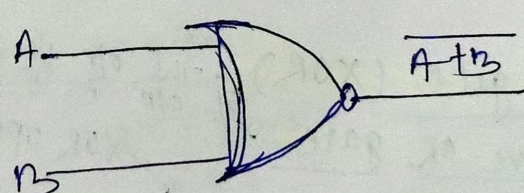
$\overline{A.B} = \overline{A}+\overline{B}$

NOR gate :- OR ⟶ complement → NOR.

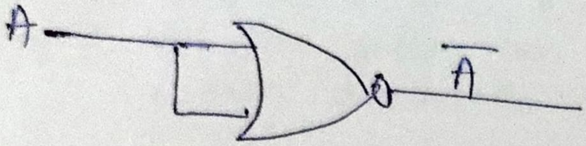
Truth Table.

A	B	A+B	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

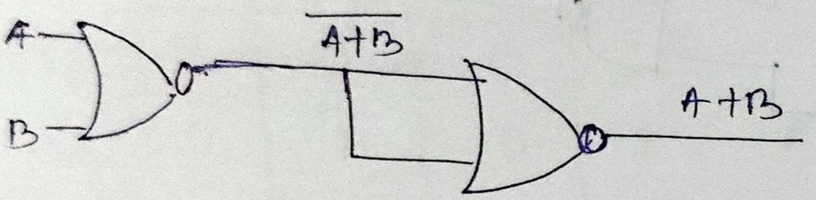
Symbol.



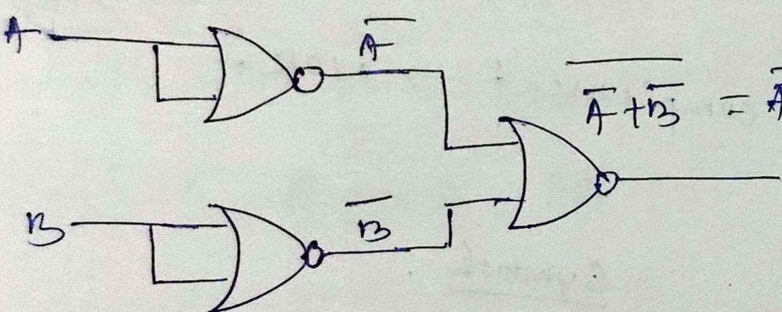
NOT gate using NOR



OR gate using NOR



AND gate using NOR.



$\overline{\overline{A}+\overline{B}} = \overline{\overline{A}}.\overline{\overline{B}} = A.B.$

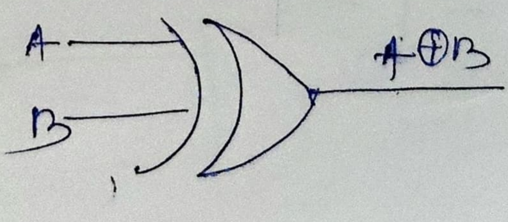
EXOR gate (XOR) ; Exclusive OR gate

The o/p is '1' when iputs are different. the o/p is '0' when inputs are same.

XOR operation

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

symbol :



dt - 06/10/25

EXNOR → XNOR

A	B	$A \oplus B$	$A \odot B$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

$A \odot B = \overline{A \oplus B}$

$$\boxed{\begin{aligned} A \oplus B &= \bar{A}B + A\bar{B} \\ A \odot B &= AB + \bar{A}\bar{B} \end{aligned}}$$

Boolean Algebra:

In boolean algebra the variables doesnot have any numerical significance.

- It has only logical values.

Ex $A + A = A$ $(0+0=0 / 1+1=1)$
$A \cdot A = A$ $(0.0=0 / 1.1=1)$

Laws in Boolean Algebra (and axioms.)

AND LAWS

(i) $A \cdot 0 = 0$
(ii) $A \cdot 1 = A$
(iii) $\boxed{A \cdot A = A}$
(iv) $\boxed{A \cdot \bar{A} = 0}$

OR LAWS

$A + 0 = A$

$\boxed{A + 1 = 1}$
$\boxed{A + \bar{A} = 1}$
$\boxed{A + A = A}$

NOT LAWS

$\bar{1} = 0$
$\bar{0} = 1$
$\bar{\bar{A}} = A$

Imp

Demorgan's Laws

$$(\overline{A+B}) = \bar{A} \cdot \bar{B}$$
$$(\overline{A \cdot B}) = \bar{A} + \bar{B}$$

Ex $\overline{A \oplus B}$ $= \overline{A \oplus B}$

$= \overline{\bar{A}B + A\bar{B}}$ $= A$.

$= \overline{\bar{A}B} \cdot \overline{A\bar{B}}$

$=$

Ex $\overline{A \oplus B}$

$= \overline{\underbrace{\bar{A}B}_{x} + \underbrace{A\bar{B}}_{y}}$

$= \overline{\bar{A}B} \cdot \overline{A\bar{B}}$

$= (\bar{\bar{A}} + \bar{B})(\bar{A} + \bar{\bar{B}})$

$= (A + \bar{B})(\bar{A} + B)$

$= 0 + AB + \bar{A}\bar{B} + 0$

$\overline{A \oplus B} = AB + \bar{A}\bar{B}$

$\Rightarrow A \odot B = AB + \bar{A}\bar{B}$

Ex $AB + BC + A\bar{B}C \rightarrow$ Minimize

Minimization

{minimum number of terms with minimum number of variable}

dt-07/10/25

Digital Mantra

(1) L^3H → Low power
→ Less space
→ Low cost

dt - 08/10/25

Q) $ABC + \bar{A} + A\bar{B}C$

$= ABC + A\bar{B}C + \bar{A}$

$= AC(B+\bar{B}) + \bar{A}$

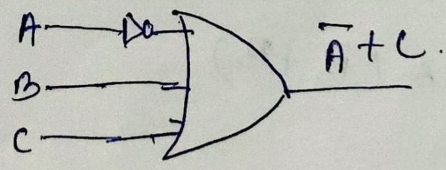
$= AC.1 + \bar{A}$

$= AC + \bar{A}$

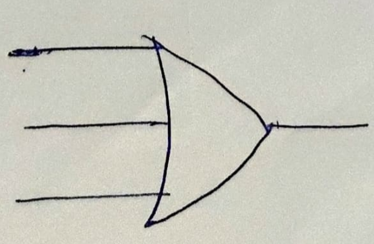
$= (\bar{A} + A).(\bar{A} + C)$

$= 1.(\bar{A} + C)$

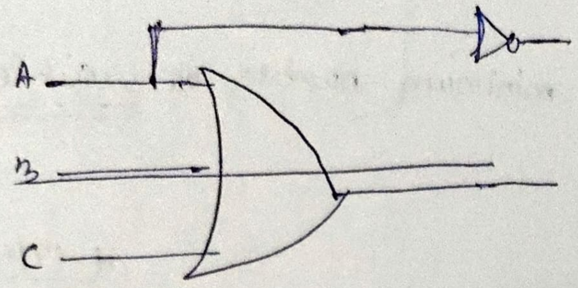
$= \bar{A} + C$

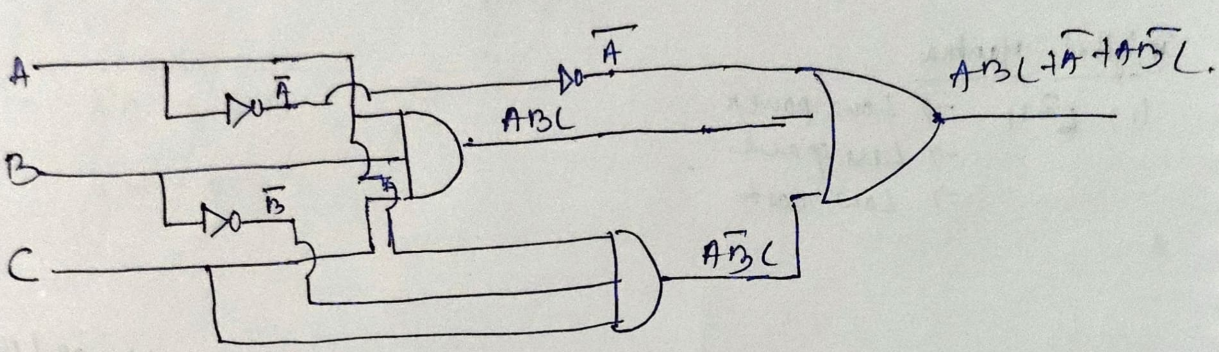


$ABC + \bar{A} + A\bar{B}C$



$ABC + \bar{A} + A\bar{B}C$





$A\bar{B} + \overline{(\bar{A}+\bar{B}+C\cdot\bar{C})}$

$\Rightarrow \cancel{A\bar{B} + \bar{\bar{A}} + \bar{\bar{B}}}$

$\Rightarrow A\bar{B} + \overline{(\bar{A}+\bar{B}+0)}$

$\Rightarrow A\bar{B} + \overline{(\bar{A}+\bar{B})}$

$\Rightarrow A\bar{B} + \bar{\bar{A}}\cdot\bar{\bar{B}}$

$\Rightarrow A\bar{B} + AB$

$= \cancel{A+A}\; A(\bar{B}+B)$

$= A\cdot 1$

$= A.$

Truth table.

A	B	C	$\bar{A}$	$\bar{B}$	$\bar{C}$	$C.\bar{C}$	$A.\bar{B}$
T	T	T	F	F	F	F	F
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	T	F	
F	T	T	T	F	F	F	
F	T	F	T	F	T	F	
F	F	T	T	T	F	F	
F	F	F	T	T	T	F	

Truth table.

A	B	C	$\bar{A}$	$\bar{B}$
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	0	
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	1	

A	B	C	$\bar{A}$	$\bar{B}$	$\bar{C}$	$A \cdot \bar{B}$	$C \cdot \bar{C}$	$\bar{A}+\bar{B}$	$\bar{A}+\bar{B}+C \cdot \bar{C}$
0	0	0	1	1	1	0	0	1	1
0	0	1	1	1	0	0	0	1	1
0	1	0	1	0	1	0	0	1	1
0	1	1	1	0	0	0	0	1	1
1	0	0	0	1	1	1	0	1	1
1	0	1	0	1	0	1	0	1	1
1	1	0	0	0	1	0	0	0	0
1	1	1	0	0	0	0	0	0	0

$\overline{\bar{A}+\bar{B}+C \cdot \bar{C}}$	$A\bar{B}+\overline{(\bar{A}+\bar{B}+C \cdot \bar{C})}$
0	0
0	0
0	0
0	0
0	1
0	1
1	1
1	1

Q. $A(A+\bar{B}C) + A(\bar{B}+C)$.

$= A \cdot A + A\bar{B}C + A\bar{B} + AC$

$= A + A\bar{B}C + A\bar{B} + AC$

$= A(1+\bar{B}C+\bar{B}+C)$

$= A \cdot 1$

$= A$.

Q) $A(C+D'B)+A'$

$= AC + AD'B + A'$

$= AC + A' + AD'B$

$= (A+A')(C+A') + AD'B$

$= C + A' + \cancel{AD} AD'B$

$= C + (A'+A)(A'+D'B)$

$= C + A' + D'B$

dt-10-10-25

SOP → Sum of product terms are sum together

- when more than one product terms are sum together then it is known as SOP.

Ex. $A\bar{B}C + ABC + AB\bar{C}$ → SOP

- In case of SOP an overbar (—) can to extend to more than one variable.

$\overline{ABC}$ (not included in SOP).

$\bar{A}\bar{B}\bar{C}$ (SOP).

Standard SOP / canonical SOP :-

If in a SOP expression all the variables is present in each term (Either complemented or non complemented) then it is known as standard SOP.

→ $Y_1 = A\bar{B}\bar{C} + AB\bar{C} + \bar{A}\bar{B}C$ → standard SOP.

→ $Y_2 = AB + AB\bar{C} + \bar{B}\bar{C}$ → Non standard SOP.

Ex: $AB + AB\bar{C} + \bar{B}\bar{C}$ → convert to standard SOP.

$= AB(C+\bar{C}) + AB\bar{C} + (A+\bar{A})\bar{B}\bar{C}$

$= ABC + AB\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$ $(AB\bar{C} + AB\bar{C} = AB\bar{C})$

$= ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

Standard SOP.

A non standard SOP can be converted to standard SOP by multiplying missing variable plus it's complement and rearranging it [more than one same term can be taken as one term
ex. $AB\bar{C} + AB\bar{C}$
$= AB\bar{C}$.]

Ex $A + BC$

$= A(B+\bar{B})\cdot(C+\bar{C}) + (A+\bar{A})BC$.

$= ABC + A\bar{B}\bar{C} + ABC + \bar{A}BC$

$= ABC + A\bar{B}\bar{C} + \bar{A}BC$.

$= AB + A\bar{B}(C+\bar{C}) + (A+\bar{A})BC$.

$= ABC + AB\bar{C} +$

$= A + BC \rightarrow$

$= A(B+\bar{B})(C+\bar{C}) + (A+\bar{A})BC$.

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + ABC + \bar{A}BC$.

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$

Ex. $A\bar{B}\bar{C} + ABC\bar{D} + A\bar{B}\bar{D}$

$= A\bar{B}\bar{C}(D+\bar{D}) + ABC\bar{D} + A\bar{B}(C+\bar{C})\bar{D}$

$= A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D}$

$= A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + ABC\bar{D} + A\bar{B}CD$.

$\therefore$ SSOP

Ex. $A + BCD$.

$= A(B+\bar{B})(C+\bar{C})(D+\bar{D}) + (A+\bar{A})BCD$

$= \cancel{ABC + A\bar{B}C + ABC + A\bar{B}\bar{C} + A}$

$= \cancel{ABCD + A\bar{B}\bar{C}\bar{D} + A}$

$= \cancel{ABCD + A}$

$= \{ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C}\}(D+\bar{D}) + ABCD + \bar{A}BCD$.

$= ABCD + AB\overline{CD} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}\overline{CD}$

$= \underline{ABCD} + AB\overline{CD} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + \bar{A}BCD$.

$+ A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \underline{ABCD}$

$=$ S.SOP.

Binary value assignment

In an standard SOP each variable is taken as '1' and variable complement is taken as '0'.

$A \rightarrow 1$

$\bar{A} \rightarrow 0$

Ex. $A\bar{B}C + A\bar{B}\bar{C} + ABC$ → Assign Binary values to it.

$101 \quad 100 \quad 111$

Assign Binary values:

$- A + BC$.

$= A(B+\bar{B})(C+\bar{C}) + (A+\bar{A})BC$:

$\Rightarrow A(B+\bar{B})(C+\bar{C}) + (A+\bar{A})BC$:

$\Rightarrow ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \underline{ABC} + \bar{A}BC$.

$\Rightarrow \underline{ABC} + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$

→ $AB + A\bar{B}C + \bar{C}$.

→

A	B	C
1	1	0
1	1	1

A	$\bar{B}$	C
1	0	1

A	B	$\bar{C}$
1	1	0
1	0	0
0	1	0
0	0	0

$ABC + AB\bar{C} + A\bar{B}C + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C}$
$+ \bar{A}\bar{B}\bar{C}$.

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}$

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SOP.

$F_1 = ABC + A\bar{B}C + AB\bar{C}$
111 101 110

- Minterm: Each term in a standard SOP is called a min term.
  - It is represented by 'm' with subscript decimal numbers.

$F_1 = ABC + AB\bar{C} + A\bar{B}\bar{C}$

Binary =	111	110	100
Decimal =	7	6	4
Minterm	m_7	m_6	m_4

Ex $A\bar{B} + \bar{A}\bar{B} + AB$ $\longrightarrow$ Represent in sum of minterms.

$= A\bar{B} + \bar{A}\bar{B} + AB$

Binary $= 10 \quad 00 \quad 11$

dec $= 2 \quad 0 \quad 3$

Minterm $= m_2 \quad m_0 \quad m_3$

$F_2 = \sum m(0, 2, 3)$

sum of minterms.

$F_3 = A\bar{B} + BC + AB\bar{C}$ $\rightarrow$ represent in $\sum m$.

$=$ ~~$A\bar{B}C\bar{C}\bar{A}ABC$~~ $A\bar{A}B\bar{C}$

1	0	1	0
1	0	1	0
1	0		0
1	0		0

$= A\bar{B}C\bar{C} \quad \bar{A}ABC \quad AB\bar{C}$

$1\ 0\ 0\ 0 \qquad 1\ 1\ 0$

1

$= A\bar{B}C \quad A\,B\,C \quad A\,B\,\bar{C}$

$1\ 0\ 1 \to 5 \quad 1\ 1\ 1 \to 7 \quad 1\ 1\ 0 \to 6.$

$1\ 0\ 0 \to 4 \quad 0\ 1\ 1 \to 3$

$= \cancel{A\bar{B}C + A\bar{B}\bar{C} + ABC + \bar{A}}$

$= 5 \quad 4 \quad 7 \quad 3 \quad 6$

$= m_5 \quad m_4 \quad m_7 \quad m_3 \quad m_6$

$= \sum m(3, 4, 5, 6, 7)$

$F_4 = \sum m\ (0, 1, 3).$

$= \bar{A}\bar{B} + \bar{A}B + AB$

$F_5 = \sum m\,(0, 7, 8, 12, 14)$

$= 0000 \quad 0111 \quad 1000 \quad 1100 \quad 1110$

$= \bar{A}\bar{B}\bar{C}\bar{D} \quad \bar{A}BCD \quad A\bar{B}\bar{C}\bar{D} \quad \bar{A}\bar{B}\bar{C}\bar{D} \quad \bar{A}\bar{B}C\bar{D}$

$F_6 = AB\bar{C} + B\bar{D} + BCD$ —.

$1101 \quad 1110 \quad 1\,1\,\bar{0} \quad 0\,1\,\bar{0} \quad 1111 \quad 0111$

=

= 1 1 0 1 → 13 1 1 1 0 → 14 1 1 1 1 → 15

1 1 0 0 → 12 0 1 0 0 → 4 0 1 1 1 → 7

0 1 1 0 → 6

1 1 0 0 → 12

⇒ ~~Σm (4, 7, 12, 13, 14, 15)~~

$F_6 = \Sigma m(4, 6, 7, 12, 13, 14, 15)$

$F_7 = AB\bar{C} + \bar{A}\bar{B}\bar{D} + \bar{B}\bar{D} + BC.$

$AB\bar{C}D$	$\bar{A}\bar{B}CD$	$A\bar{B}C\bar{D}$	$ABCD$
13 ← 1 1 0 1	0 0 1 0 → 2	0 0 1 0 → 2	1 1 1 1 → 15
12 ← 1 1 0 0	0 0 0 0 → 0	0 0 0 0 → 0	1 1 1 0 → 14
		1 0 1 0 → 10	0 1 1 1 → 7
		1 0 0 0 → 8	0 1 1 0 → 6.

Σ

2 variable.

A	B	Dec	Minterm	
0	0	$\rightarrow$ 0 $\rightarrow$	m_0 $\rightarrow$	$\bar{A}\bar{B}$
0	1	$\rightarrow$ 1 $\rightarrow$	m_1 $\rightarrow$	$\bar{A}B$
1	0	$\rightarrow$ 2 $\rightarrow$	m_2 $\rightarrow$	$A\bar{B}$
1	1	$\rightarrow$ 3 $\rightarrow$	m_3 $\rightarrow$	AB.

3 variable.

A	B	C	Dec	Minterm.	
0	0	0	0 $\rightarrow$	m_0 $\rightarrow$	$\bar{A}\bar{B}\bar{C}$
0	0	1	1 $\rightarrow$	m_1 $\rightarrow$	$\bar{A}\bar{B}C$
0	1	0	2 $\rightarrow$	m_2 $\rightarrow$	$\bar{A}B\bar{C}$
0	1	1	3 $\rightarrow$	m_3 $\rightarrow$	$\bar{A}BC$
1	0	0	4 $\rightarrow$	m_4 $\rightarrow$	$A\bar{B}\bar{C}$
1	0	1	5 $\rightarrow$	m_5 $\rightarrow$	$A\bar{B}C$
1	1	0	6 $\rightarrow$	m_6 $\rightarrow$	$AB\bar{C}$
1	1	1	7 $\rightarrow$	m_7 $\rightarrow$	ABC

4 variable.

A	B	C	D	Dec	Minterm
0	0	0	0	0	$m_0 \rightarrow \bar{A}\bar{B}\bar{C}\bar{D}$
0	0	0	1	1	$m_1 \rightarrow \bar{A}\bar{B}\bar{C}D$
0	0	1	0	2	$m_2 \rightarrow \bar{A}\bar{B}C\bar{D}$
0	0	1	1	3	$m_3 \rightarrow \bar{A}\bar{B}CD$
0	1	0	0	4	$m_4 \rightarrow \bar{A}B\bar{C}\bar{D}$
0	1	0	1	5	$m_5 \rightarrow \bar{A}B\bar{C}D$
0	1	1	0	6	$m_6 \rightarrow \bar{A}BC\bar{D}$
0	1	1	1	7	$m_7 \rightarrow \bar{A}BCD$
1	0	0	0	8	$m_8 \rightarrow A\bar{B}\bar{C}\bar{D}$
1	0	0	1	9	$m_9 \rightarrow A\bar{B}\bar{C}D$
1	0	1	0	10	$m_{10} \rightarrow A\bar{B}C\bar{D}$
1	0	1	1	11	$m_{11} \rightarrow A\bar{B}CD$
1	1	0	0	12	$m_{12} \rightarrow AB\bar{C}\bar{D}$
1	1	0	1	13	$m_{13} \rightarrow AB\bar{C}D$
1	1	1	0	14	$m_{14} \rightarrow ABC\bar{D}$
1	1	1	1	15	$m_{15} \rightarrow ABCD$

POS → product of Sum

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Standard POS

Eg. $(A+B)(A+\bar{B})(\bar{A}+\bar{B})$

→ when more than One sum terms are product together then it is called POS.

Eg $(A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})$ → POS.

Standard POS → If each product term contains all the variables (either complemented or non complemented).

Eg $(A+B)(A+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$ → convert it to standard POS.

$\Rightarrow (A+B+(C\cdot\bar{C}))$

$\Rightarrow (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$

→ Standard POS

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$(A+B)(A+C)$

$= A.A + A.C + B.A + B.C$

$= A + AC + AB + BC$

$= A[1 + \cancel{C+B}^{1}] + BC$

$= A + BC$

$(A+B)(A+C) = A+BC$

$A+BC = (A+B)(A+C)$

$F_3 = (A+B)(A+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$ - convert to standard pos.

$= (A+B+C\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$

$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$

Standard pos.

$F_4 = (A+B+\bar{C})(\bar{A}+\bar{C})(\quad \bar{B}+C)$.

$= (A+B+\bar{C})(\bar{A}+(B.\bar{B})+\bar{C})(A.\bar{A}+\bar{B}+C)$

$= (A+B+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})\quad(A+\bar{B}+C)(\bar{A}+\bar{B}+C)$

Standard pos

→ Binary value assignment →
In standard POS variable is taken as '0' and variable complement is taken as '1'. $A \to 0$
$\bar{A} \to 1$

Ex $(A+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C) \longrightarrow$.

0 0 1   1 1 1   1 0 0

$F_5 = (A+B)(A+\bar{B}+\bar{C})(A+\bar{C})$

A B C	A $\bar{B}$ $\bar{C}$	A B $\bar{C}$
0 0 0	0 1 1	0 0 1
0 0 1		0 1 1

$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})$

Maxterm → Each term in a standard POS is called a maxterm.

- It is represented by 'M' with subscript decimal numbers.

Ex. $F_6 = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})$.

Represent is product of Maxterms.

Binary = 0 0 1 0 1 1 1 1 1

Decimal = 1 3 7

Maxterms = $M_1, M_3 \cdot M_7$

$F_6 = \prod M(1, 3, 7)$

$F_7 = (A+\bar{B})(\bar{A}+\bar{B}+C+D)(A+\bar{D})$ → Represent in Product of max terms.

$= A\ \bar{B}\ C\ D$	$\bar{A}\ \bar{B}\ C\ D$	$A\ B\ C\ \bar{D}$
4 ← 0 1 0 0	12 ← 1 1 0 0	0 0 0 1 → 1
6 ← 0 1 1 0		0 0 1 1 → 3
5 ← 0 1 0 1		0 1 0 1 → 5
7 ← 0 1 1 1		0 1 1 1 → 7

$\prod M(1, 3, 4, 5, 6, 7, 12)$

$= (A+\bar{B}+C+D)(A+\bar{B}+\bar{C}+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+\bar{D})$
$(\bar{A}+\bar{B}+C+D)(A+B+C+\bar{D})(A+B+\bar{C}+\bar{D})$

2 variables

A	B	Dec	Maxterm	
0	0	0	$\rightarrow A+B$	M_0
0	1	1	$\rightarrow A+\overline{B}$	M_1
1	0	2	$\rightarrow \overline{A}+B$	M_2
1	1	3	$\rightarrow \overline{A}+\overline{B}$	M_3

3 variables.

A	B	C	Dec	Maxterm	Ex.
0	0	0	0	M_0	$A+B+C$
0	0	1	1	M_1	$A+B+\overline{C}$
0	1	0	2	M_2	$A+\overline{B}+C$
0	1	1	3	M_3	$A+\overline{B}+\overline{C}$
1	0	0	4	M_4	$\overline{A}+B+C$
1	0	1	5	M_5	$\overline{A}+B+\overline{C}$
1	1	0	6	M_6	$\overline{A}+\overline{B}+C$
1	1	1	7	M_7	$\overline{A}+\overline{B}+\overline{C}$

4 variables.

0
0
0
0
0
0
0
0
1
1
1
1
1
1

4 variable

A	B	C	D	Dec	
0	0	0	0	0	$A+B+C+D$
0	0	0	1	1	$A+B+C+\overline{D}$
0	0	1	0	2	$A+B+\overline{C}+D$
0	0	1	1	3	$A+B+\overline{C}+\overline{D}$
0	1	0	0	4	$A+\overline{B}+C+D$
0	1	0	1	5	$A+\overline{B}+C+\overline{D}$
0	1	1	0	6	$A+\overline{B}+\overline{C}+D$
0	1	1	1	7	$A+\overline{B}+\overline{C}+\overline{D}$
1	0	0	0	8	$\overline{A}+B+C+D$
1	0	0	1	9	$\overline{A}+B+C+\overline{D}$
1	0	1	0	10	$\overline{A}+B+\overline{C}+D$
1	0	1	1	11	$\overline{A}+B+\overline{C}+\overline{D}$
1	1	0	0	12	$\overline{A}+\overline{B}+C+D$
1	1	0	1	13	$\overline{A}+\overline{B}+C+\overline{D}$
1	1	1	0	14	$\overline{A}+\overline{B}+\overline{C}+D$
1	1	1	1	15	$\overline{A}+\overline{B}+\overline{C}+\overline{D}$

K-Map → Systematic way of minimization

→ Draw the K map ?

→ 2 variable SOP $2^2 = 4$ squares.

A \ B	0	1
0	$\bar{A}\bar{B}$ m_0	$\bar{A}B$ m_1
1	$A\bar{B}$ m_2	AB m_3

A \ B	0	1
0	$A+B$ M_0	$A+\bar{B}$ M_1
1	$\bar{A}+B$ M_2	$\bar{A}+\bar{B}$ M_3

→ For 3 variable SOP. $2^3 = 8$ terms.

A \ BC	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$ m_0	$\bar{A}\bar{B}C$ m_1	$\bar{A}BC$ m_3	$\bar{A}B\bar{C}$ m_2
1	$A\bar{B}\bar{C}$ m_4	$A\bar{B}C$ m_5	ABC m_7	$AB\bar{C}$ m_6

AB \ C	0	1
00	$\bar{A}\bar{B}\bar{C}$ m_0	$\bar{A}\bar{B}C$ m_1
01	$\bar{A}B\bar{C}$ m_2	$\bar{A}BC$ m_3
11	$AB\bar{C}$ m_7	ABC m_6
10	$A\bar{B}\bar{C}$ m_4	$A\bar{B}C$ m_5

→ For 4 variables $2^4 = 16$ square.

AB \ CD	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$ m_0	$\bar{A}\bar{B}\bar{C}D$ m_1	$\bar{A}\bar{B}CD$ m_3	$\bar{A}\bar{B}C\bar{D}$ m_2
01	$\bar{A}B\bar{C}\bar{D}$ m_4	$\bar{A}B\bar{C}D$ m_5	$\bar{A}BCD$ m_7	$\bar{A}BC\bar{D}$ m_6
11	$AB\bar{C}\bar{D}$ m_{12}	$AB\bar{C}D$ m_{13}	$ABCD$ m_{15}	$ABC\bar{D}$ m_{14}
10	$A\bar{B}\bar{C}\bar{D}$ m_8	$A\bar{B}\bar{C}D$ m_9	$A\bar{B}CD$ m_{11}	$A\bar{B}C\bar{D}$ m_{10}

→ For a SOP '1' is placed for the term which ~~are pr~~ present, a '0' or no entry for the term which are absent.

→ For a POS '0' is placed for the term which are present, a '1' or no entry for the term which are absent.

<u>Ex</u>. Draw the K-map.

(i) $F_1 = A\bar{B} + AB + \bar{A}\bar{B}$.

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	1 m_0	m_1
A 1	1 m_2	1 m_3

(ii) $F_2 = AB\bar{C} + A\bar{B}C + \bar{A}\bar{B}C + ABC + A\bar{B}\bar{C}$.

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	m_0	1 m_1	m_3	m_2
A 1	1 m_4	1 m_5	1 m_7	1 m_6

(iii) $F_3 = \bar{A}\bar{B}\bar{C}D + ABC\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} +$
$AB\bar{C}D + ABCD + \bar{A}\bar{B}\bar{C}\bar{D}$

AB \ CD	00	01	11	10
00	1	1		1
01				
11		1	1	1
10	1			

~~(v) $F_5 = A\bar{B} + BCD + \bar{A}\bar{B}C + ABCD$~~

(iv) $F_4 = \Sigma m(0, 1, 3, 9, 12, 13)$.

AB \ CD	00	01	11	10
00	1	1	1	
01				
11	1	1		
10		1		

(v) $F_5 =$

$A\bar{B}CD$	$ABCD$	$\bar{A}\bar{B}CD$	$ABCD$
1 0 00	~~0 1 11~~	0 0 1 0	1 1 1 1
1 0 01	0 1 11	0 0 1 1	
1 0 11	1 1 11	~~0 0 11~~	
1 0 10	~~1 1 11~~	~~0 0 10~~	

AB \ CD	00	01	11	10
00			1	1
01			1	
11			1	
10	1	1	1	1

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K-map

$F_1 = \Sigma m\,(0, 1, 5, 9, 13, 15)$.

AB \ CD	00	01	11	10
00	1	1		
01		1		
11		1	1	
10		1		

Adjecent squares

→ 2 variable K map

A \ B	0	1
0	1	1
1	1	1

F = 1.

- If all the square contains '1' the result is '1'.
- In minimizing we have to include all the ones '1's
- In case of two variable K map two adjecent 1's (pair) gives a product of 1 variable.
- And a single 1 gives the product of 2 variables.

A \ B	0	1
0	1	1
1		

→ pair.

Ans: $-\bar{A}$.

$\bar{A}\bar{B} + \bar{A}B$

$= \bar{A}\,(\underbrace{\bar{B} + B}_{\to 1})$.

$= \bar{A}$

1	1
	1

→ B.

A\B	0	1	
0	1		Gr 1
1		1	Gr 2.

$\bar{A}\bar{B} + AB$

→ For 3 variable K map a group of 8 1's (octate) (~~octect~~) gives a product = 1.

A\BC	00	01	11	10
0	1	1	1	1
1	1	1	1	1

$F = 1$

→ a group of four '1's (quad) gives a product of one variables.

→ a group of two ones (pairs) gives a product of two variables.

- a single one gives a product of 3 variables.

Adjecent is defined as a single variable change.

A\BC	00	01	11	10
0	1	1	1	1
1				

→ quad. $= \bar{A}$.

Ex.

Minimize the following expression.

(i) $F_1 = AB + A\bar{B} + \bar{A}\,\bar{B}$.

A \ B	0	1
0	1	
1	1	1

$\cancel{\bar{A}\,\bar{B} + AB}$

$\bar{B} + A$

(ii) $F_2 = AB\bar{C} + ABC + \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{B}C$

Gr 1 Gr 2

A \ BC	00	01	11	10
0	1	1	1	
1			1	1

— Gr 3

Gr 1 = $\bar{A}\bar{B}$

Gr 2 = $\bar{A}C$

Gr 3 = AB

$F_2 = \bar{A}\bar{B} + \bar{A}C + AB$.

or

$F_2 = \bar{A}\bar{B} + BC + AB$.

A \ BC				
	1	1	1	
			1	1

Minimum no of terms with minimum no of variable.

$f_3 = \Sigma m(0, 1, 5, 2, 6, 7)$.

Gr1

A \ BC	00	01	11	10
0	1	1		1
1		1	1	1

Gr 3.

Gr2

$F_3 = \bar{A}\bar{B} + AC + B\bar{C}$

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4 variable Kmap

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

→ If all 16 squares are '1' then the result is 1.

→ For Eight ones ~~the~~ (octate) the result is the product of one variable.

→ For four ones (quad) the result is the product of two variables.

→ For two ones (pair) the result is the product of three variables.

→ For a single one the result is the product of 4 variables.

$F_1 = ABC\bar{D} + A\bar{B}CD + \bar{A}\bar{B}\bar{C}\bar{D} + ABCD + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D$

Gr 1

AB \ CD	00	01	11	10
00	1	1	1	
01				
11			1	1
10			1	

Gr 3

Gr-2

$\bar{A}\bar{B}\bar{C} + BCD + ABC$

$f_1 = \bar{A}\bar{B}\bar{C} + ABC + \bar{B}CD$.

$F_2 = \sum m(2, 5, 6, 9, 12, 13)$

G4

AB \ CD	00	01	11	10
00			0	1
01		1		1
11	1	1		
10		1		

1

3

2

$f_2 = \bar{A}C\bar{D} + A\bar{C}D + AB\bar{C} + B\bar{C}D$.

$F_3 = \Sigma m(0, 1, 2, 3, 8, 9, 10, 11)$

AB \ CD	00	01	11	10
00	1	1	1	1
01				
11				
10	1	1	1	1

$F_3 = \bar{B}$

$F_4 = \Sigma m(4, 5, 6, 7, 12, 13, 14, 15$

AB \ CD	00	01	11	10
00				
01	1	1	1	1
11	1	1	1	1
10				

$F_4 = B$

$F_5 = \Sigma m(2, 6, 8, 9, 10, 11, 14)$

AB \ CD	00	01	11	10
00				1
01				1
11				1
10	1	1	1	1

$= A.\bar{B} + C\bar{D}$

=

$F_5 = A\bar{B} + C\bar{D}$

Dt = 24-10-25.

(i) $F_1 = \Sigma m(0,1,4,5,8,9,10,11,14,15)$

(ii) $F_2 = \Sigma m(0,2,5,7,8,10,13,15)$

(iii) $F_3 = \Sigma m(1,3,4,6,9,11,12,14)$

(iv) $F_4 = \bar{A}B + \cancel{ABD} + A\bar{B}C\bar{D} + BC$.

(v) $F_5 = BC + A\bar{C} + AB + BCD$.

(i)

AB \ CD	00	01	11	10
00	1	1		
01	1	1		
11			1	1
10	1	1	1	1

$\bar{A}\bar{C} + AC + A\bar{B}$.

(ii)

	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

$\bar{B}\bar{D} + BD$.

(iii)

	00	01	11	10
00		1	1	
01	1			1
11	1			1
10		1	1	

$F_3 = \bar{B}D + B\bar{D}$.

(iv)

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				
$\bar{A}B$	1	1	1	1
AB		1	1	1
$A\bar{B}$				1

$\bar{A}BCD$	$ABCD$	$A\bar{B}C\bar{D}$	$ABC\bar{D}$
0100	1101	1010	~~0110~~
0101	~~1111~~		~~0111~~
~~0110~~			1110
~~0111~~			~~1111~~

$\bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}BCD + AB\bar{C}D + ABCD + A\bar{B}C\bar{D} + ABC\bar{D}$.

$f_4 = \bar{A}B + BD + AC\bar{D}$.

(v)

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				
$\bar{A}B$			1	1
AB	1	1	1	1
$A\bar{B}$	1	1		

ABCD	ABCD	ABCD	ABCD
0110	1000	~~1100~~	~~0111~~
~~0111~~	1001	~~1101~~	~~1111~~
~~1110~~	~~1100~~	~~1110~~	
~~1111~~	~~1101~~	~~1111~~	

$\bar{A}BC\bar{D} + \bar{A}BCD + ABC\bar{D} + ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + AB\bar{C}\bar{D} + AB\bar{C}D$

$f_5 = A\bar{C} + BC$

Don't care

- In certain situations the output maybe '0' or '1'.
- These situations are termed as don't care.
- In Kmap we will substitute 'd' or x for don't care combination.

eg

$$F = \Sigma m(0,1,5) + d(4,7).$$

	00	01	11	10
0	1	1		
1	1	1	d	
x				

$\bar{A}\,\bar{B}$

dt-27-10-25

$F = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + A\bar{B}CD + A\bar{B}\bar{C}\bar{D} + AB\bar{C}D + ABCD$

AB\CD	00	01	11	10
00		1	1	
01			0	
11		1	1	
10	1		1	

Gr1 Gr2 Gr3 Gr4.

2

$F = A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}D + ABD + A\bar{B}CD$.

$F = \Sigma(1,5,7,11,13) + \Sigma d(0,12)$

AB\CD	00	01	11	10
00	X	1		
01		1	1	
11	X	1		
10	1		1	

$F = \bar{A}\bar{B}\bar{C} + \bar{A}BD + AB\bar{C} + A\bar{C}\bar{D} + A\bar{B}CD$

AB \ CD	00	01	11	10
00	X	1		
01		1	1	
11	X	1		
10	0	1	1	

$$F = \bar{C}D + \bar{A}BD + A\bar{B}D$$

AB \ CD	00	01	11	10
00	1	X		X
01	1	1	1	X
11		X	1	
10	1	1	1	

Q-

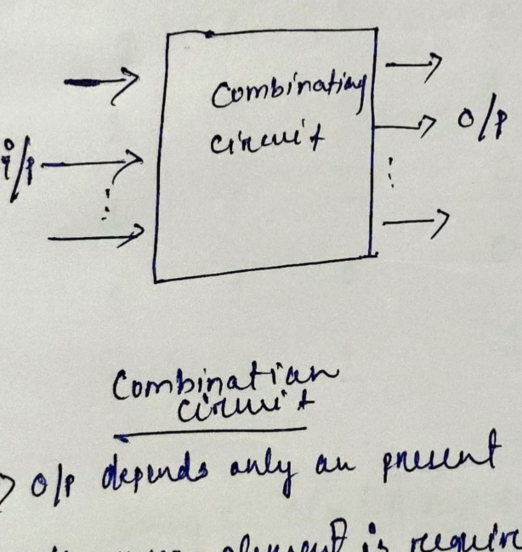
AB \ CD	00	01	11	10
00	1	X		X
01	1	1	1	X
11		X	1	
10	1	1	1	

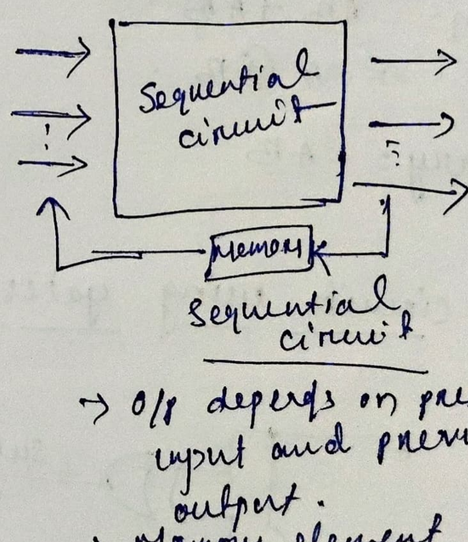
NEPI
Non-essential prime implicants.

EPI
Essential prime implicants.

dt-29-10-25

Combinational Circuit Vs Sequential Circuit





Combinational circuit

→ O/P depends only on present I/P.

→ No memory element is required.

→ There is no clock input.

eg. half adder, Full adder, multiplexer, Decoder etc.

Sequential circuit

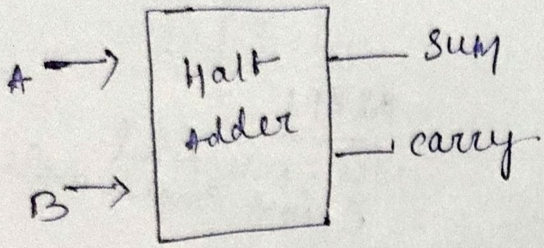
→ O/P depends on present input and previous output.

→ Memory element is required.

→ A clock input is required. (clk)

eg. counter, Flip Flop, Register etc.

Half Adder

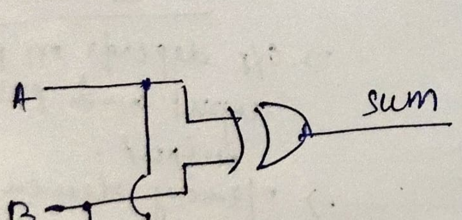


A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

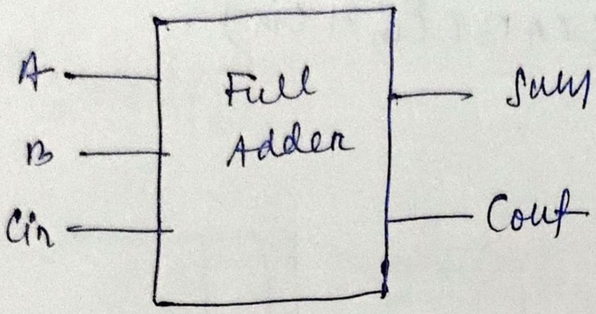
$$\text{sum} = \bar{A}B + A\bar{B}$$
$$= A \oplus B$$

$$\text{carry} = AB$$

Logic circuits using gates:



Full Adder




```
       1 - Cin
         HA HA HA
A -  1 1 0 1
B -  1 1 1 0
-------------
(1)  1 0 1 1
 ↙
Cout.
```

Truth Table

A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$Sum = \bar{A}\bar{B}C_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + ABC_{in}$

$Carry = \bar{A}BC_{in} + A\bar{B}C_{in} + AB\bar{C}_{in} + ABC_{in}$

$$Sum = C_{in}(\bar{A}\bar{B} + AB) + \bar{C}_{in}(\bar{A}B + A\bar{B})$$

$$= C_{in}(A \odot B) + \bar{C}_{in}(A \oplus B)$$

$$= C_{in}(\overline{A \oplus B}) + \bar{C}_{in}(A \oplus B)$$

$A \oplus B = X$
$C_{in} = Y$

$$= Y\bar{X} + \bar{Y}X$$

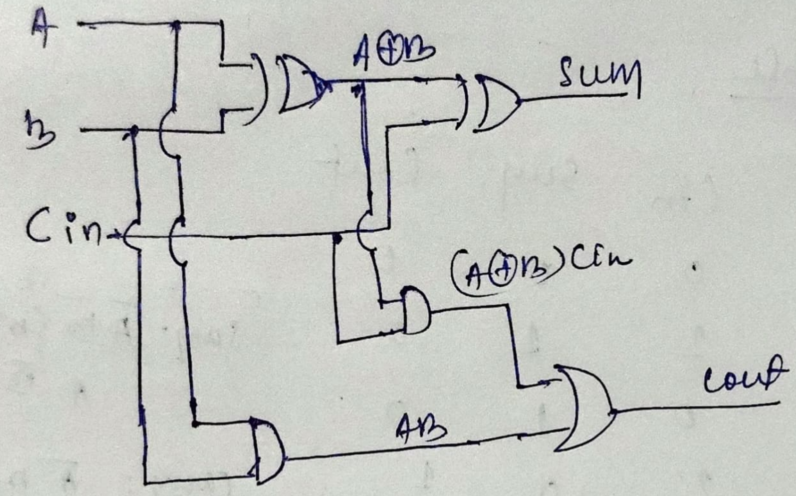
$$= X \oplus Y = A \oplus B \oplus C_{in}$$

Carry: $\bar{A}BC_{in} + A\bar{B}C_{in} + AB\bar{C}_{in} + ABC_{in}$

$= C_{in}(\bar{A}B + A\bar{B}) + AB(\bar{C}_{in} + C_{in})$

$= C_{in}(A \oplus B) + AB$

Sum:



* Design. Full Adder using half adder and any of the logic gate.

<u>HA</u>

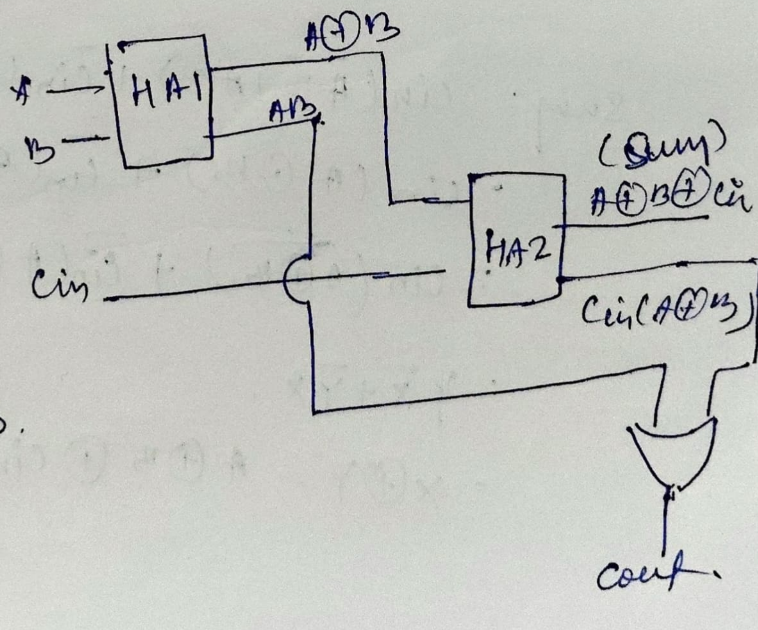
Sum = $A \oplus B$

Carry = AB

<u>FA</u>

Sum = $A \oplus B \oplus C_{in}$

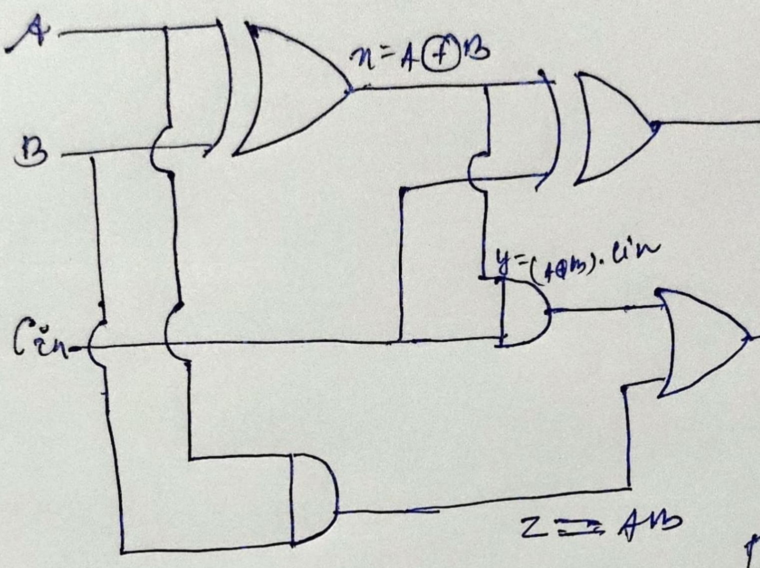
Cout = $C_{in}(A \oplus B) + AB$.



dt - 03-11-25

Combinational circuits

Analysis



[Circuit Diagram]

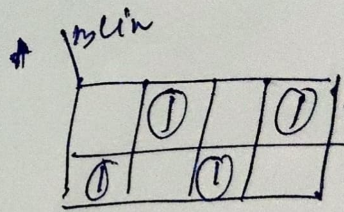
$S = A \oplus B \oplus C_{in}$

$C_{out} = AB + C_{in}(A \oplus B)$

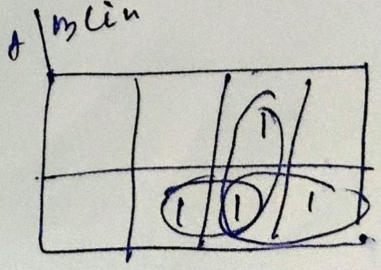
↳ Circuit function / Expression.

Truth table

A B Cin	S	Cout
0 + 0 + 0	0	0
0 + 0 + 1	1	0
0 + 1 + 0	1	0
0 + 1 + 1	0	1
1 + 0 + 0	1	0
1 + 0 + 1	0	1
1 + 1 + 0	0	1
1 + 1 + 1	1	1



$S = A \oplus B \oplus C_{in}$



$C_{out} = AB + BC_{in} + C_{in}$

7+7

$$
\begin{array}{r}
{\scriptstyle 1\,1\,\,\,\,} \\
1\,1\,1 \\
+\;1\,1\,1 \\
\hline
1\,1\,1\,0
\end{array}
$$

Cout

= 14

1 HA + 2 FA

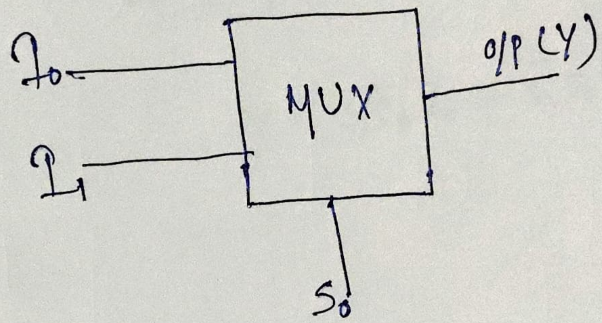
1 FA which cin = 0 ⇒ 2 HA.

Multiplexer

dt-24-11-2025

2^n input 1 output.

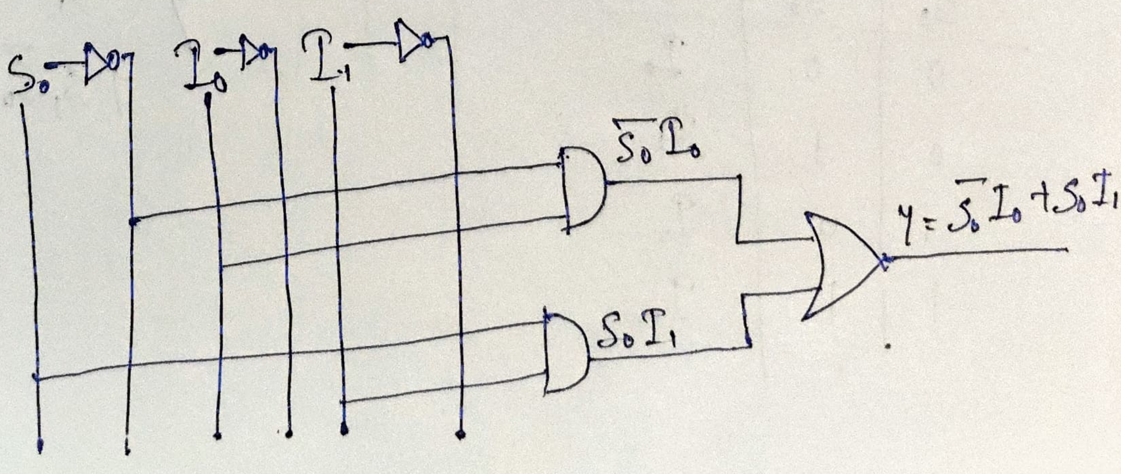
$n \rightarrow$ address lines / selection lines.



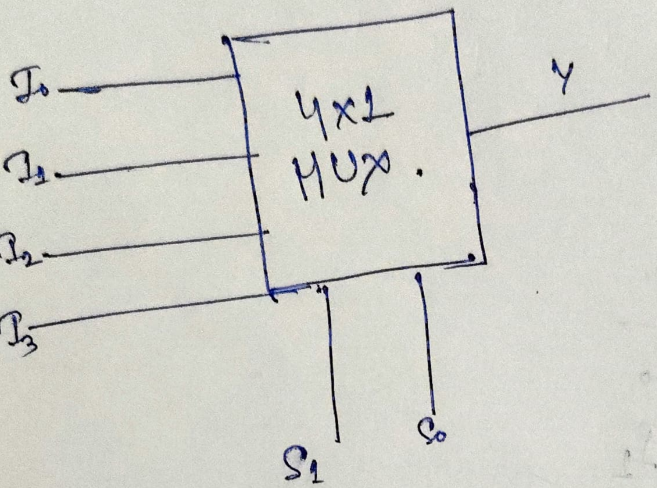
2:1 MUX

S_0	Y
0	I_0
1	I_1

$$Y = \bar{S_0} I_0 + S_0 I_1$$



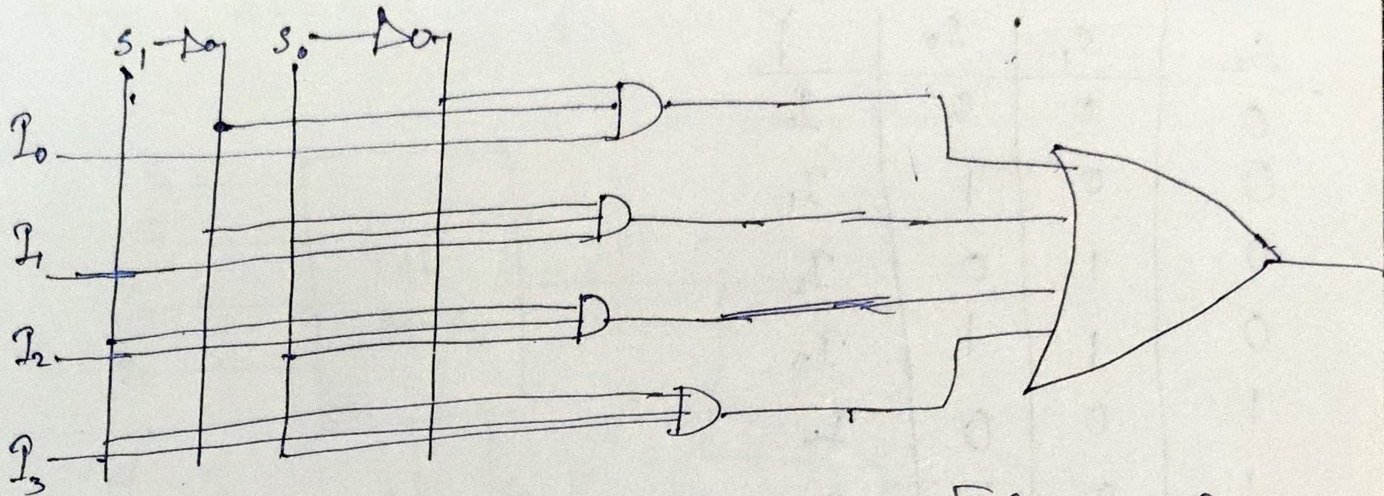
4x1 MUX.



S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

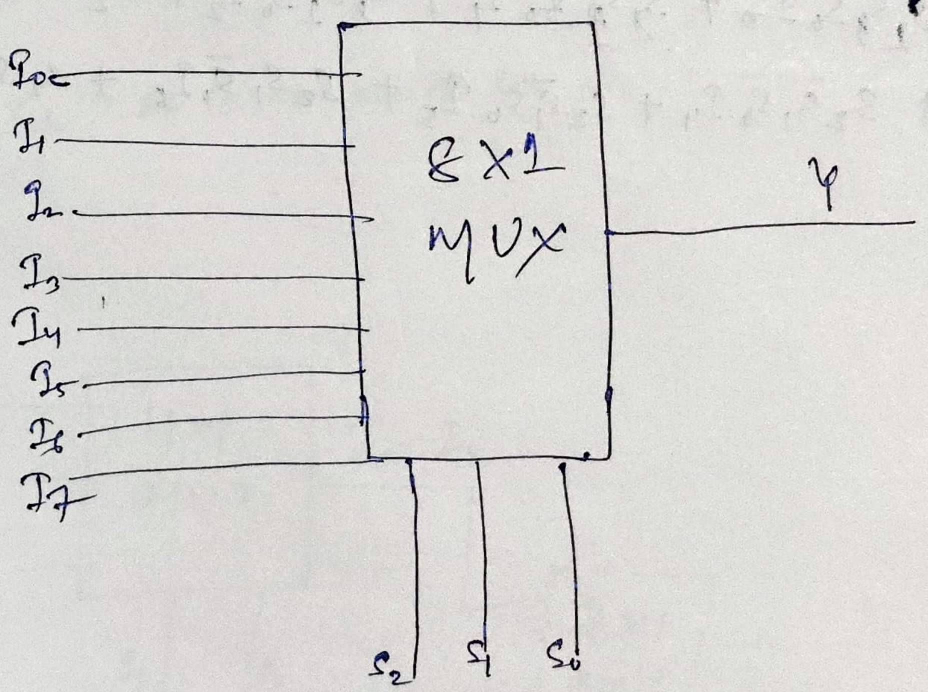
Logical Expression :-

$$Y = \bar{S_1}\bar{S_0}I_0 + \bar{S_1}S_0I_1 + S_1\bar{S_0}I_2 + S_1S_0I_3$$



$$Y = \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_1 S_0 I_1 + S_1 \bar{S}_0 I_2 + S_1 S_0 I_3$$

8x1

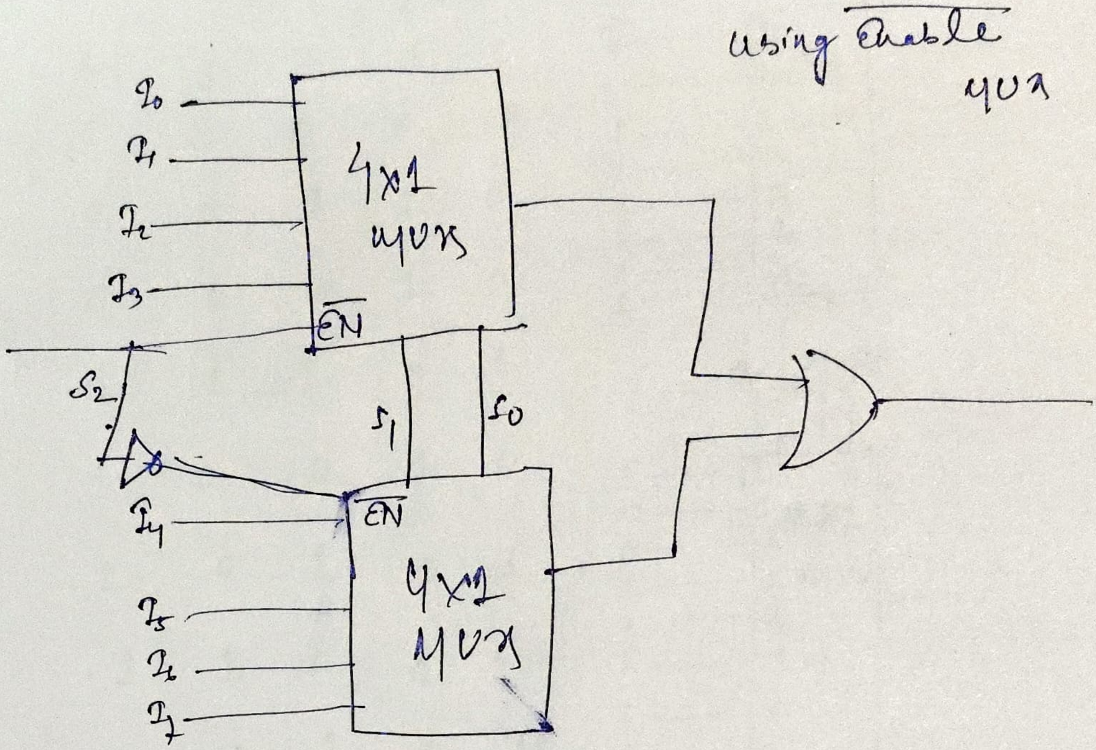


S_2	S_1	S_0	Y
0	0	0	I_0
0	0	1	I_1
0	1	0	I_2
0	1	1	I_3
1	0	0	I_4
1	0	1	I_5
1	1	0	I_6
1	1	1	I_7

Logical Expression:

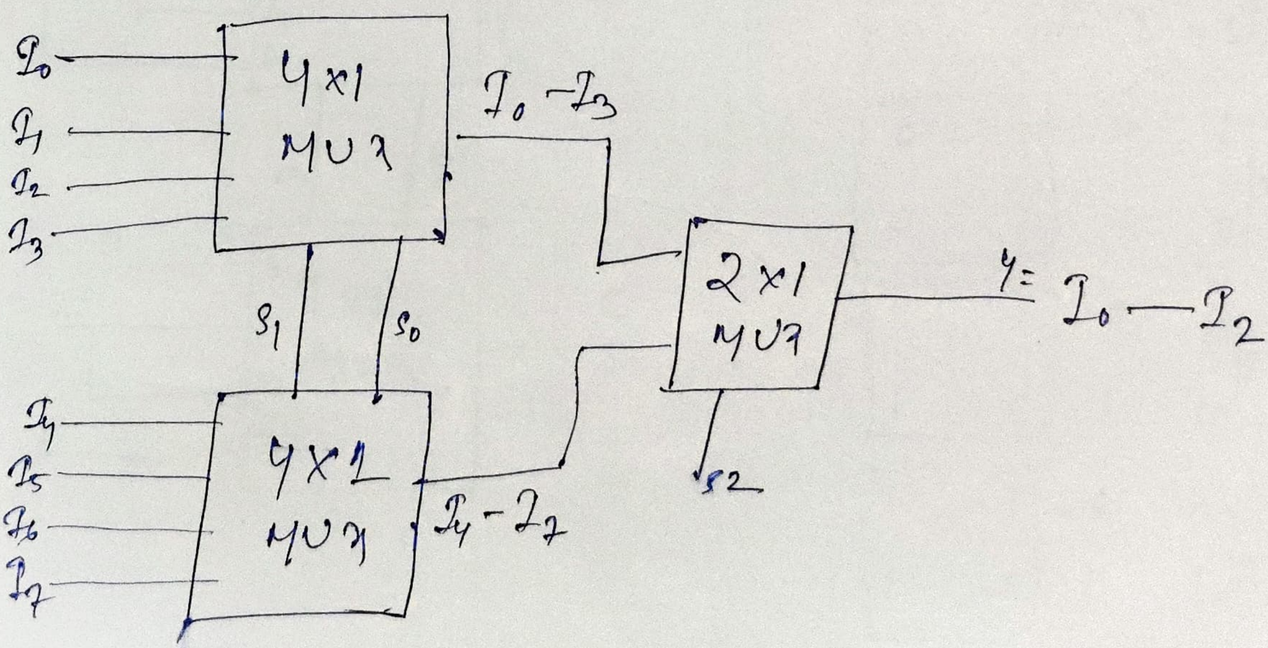
$$Y = \bar{S}_2 \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_2 \bar{S}_1 S_0 I_1 + \bar{S}_2 S_1 \bar{S}_0 I_2 + \bar{S}_2 S_1 S_0 I_3$$
$$+ S_2 \bar{S}_1 \bar{S}_0 I_4 + S_2 \bar{S}_1 S_0 I_5 + S_2 S_1 \bar{S}_0 I_6 + S_2 S_1 S_0 I_7.$$

→ Design a 8×1 mux using 4×1 mux.



Enable → It is an active low pin.

$\overline{EN}$



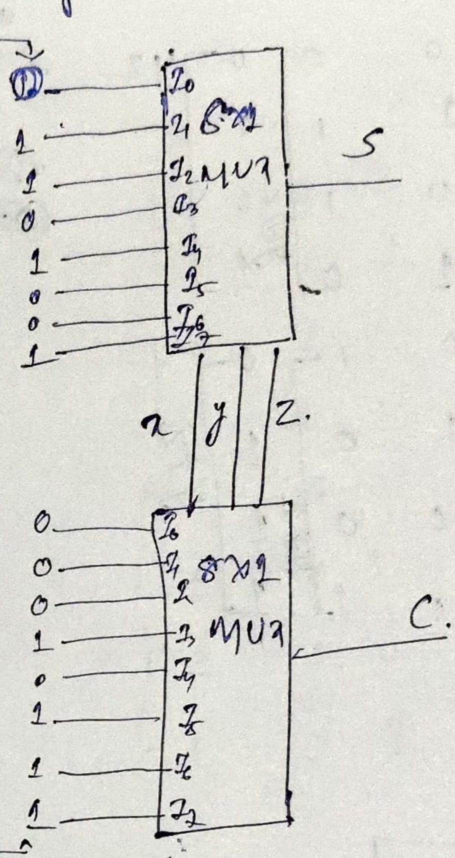
8×1 mux using 4×1 mux and 2×1 mux.

Ex Design a Full Adder using 8x1 MUX, 4x1 MUX, 2x1 MUX

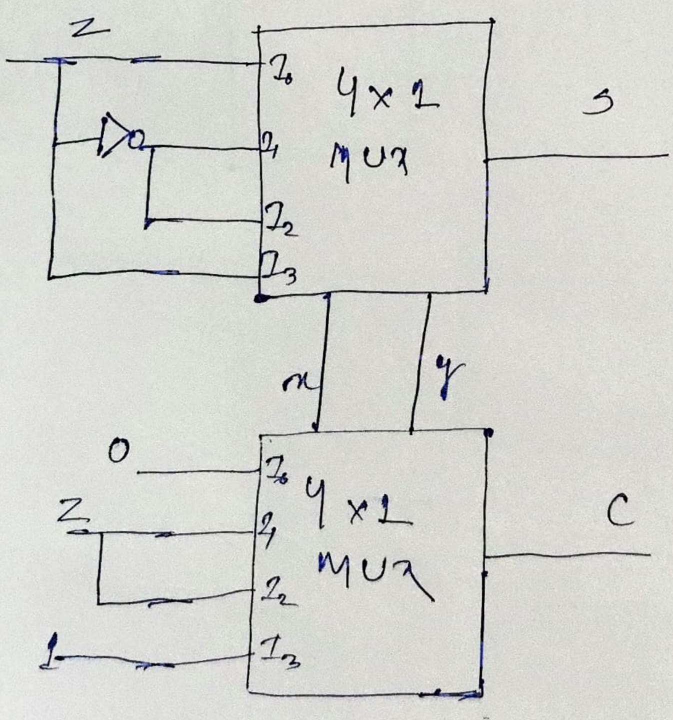
x	y	z	s	c
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

sum

carry



Full Adder using 8x1 MUX



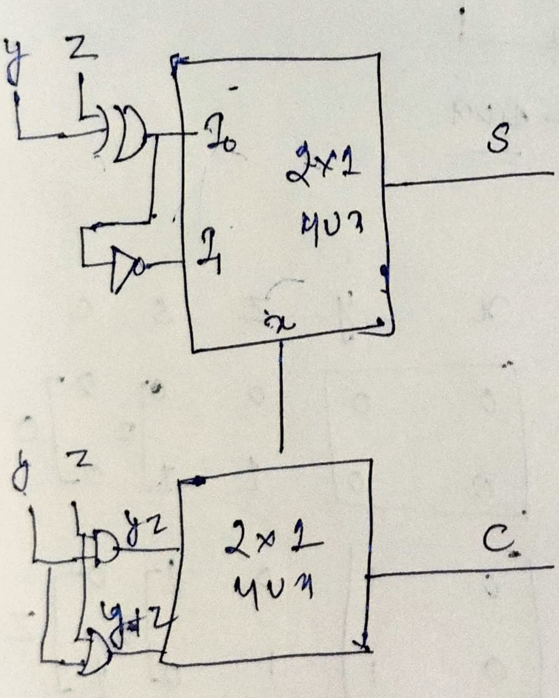
x	y	z	s	c	s	c
0	0	0	0	0	z	0
0	0	1	1	0		
0	1	0	1	0	$\bar{z}$	z
0	1	1	0	1		
1	0	0	1	0	$\bar{z}$	z
1	0	1	0	1		
1	1	0	0	1	z	1
1	1	1	1	1		

Full Adder using 4x1

x	y	z	s	c
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

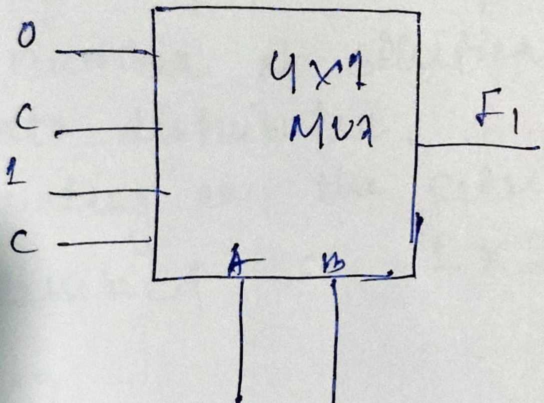
yz $y \oplus z$ $y \oplus z$ $y+z$

$s = \bar{y}z + y\bar{z} = y \oplus z$



Design the following function using 4x1 MUX.

$F_1 = ABC + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$.

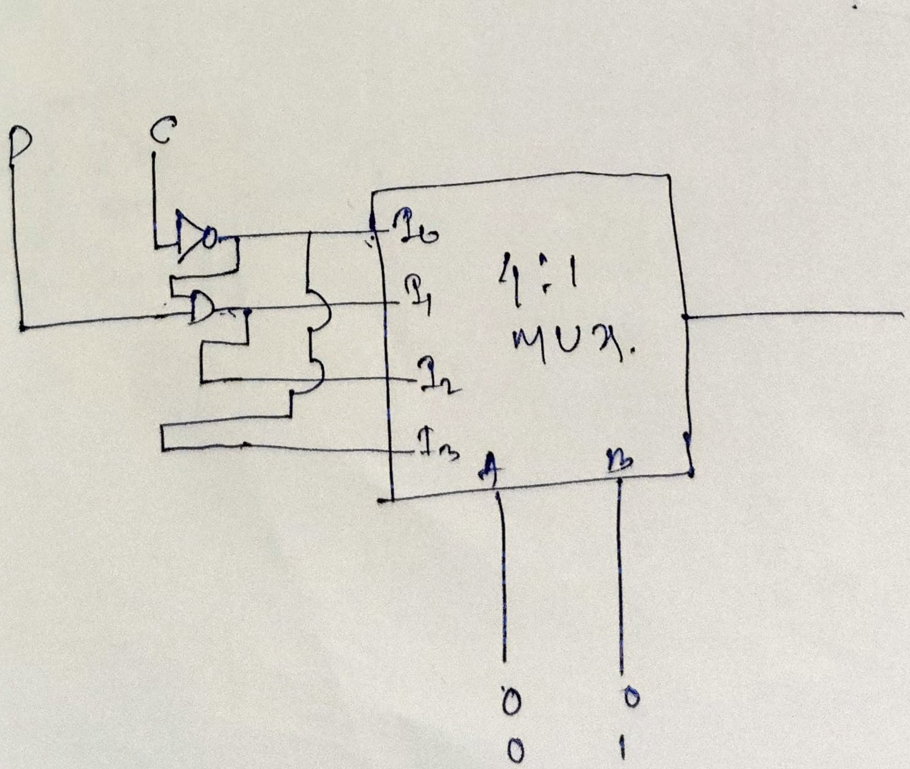


A	B	C	F_1	
0	0	0	0	0
0	0	1	0	
0	1	0	0	C
0	1	1	1	
1	0	0	1	1
1	0	1	1	
1	1	0	0	C
1	1	1	1	

dt- 25-11-25.

$F_2 = \sum m(0, 1, 5, 9, 12, 13) \rightarrow$ 4:1 mux.

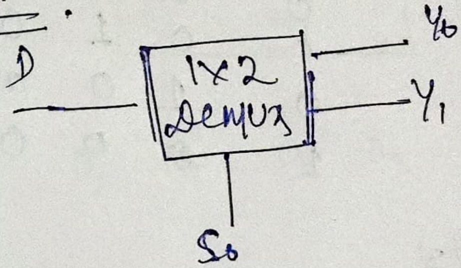
A	B	C	D		F_2
0	0	0	0	$\longrightarrow$	1
0	0	0	1	$\longrightarrow$	1
0	0	1	0	$\longrightarrow$	0
0	0	1	1	$\longrightarrow$	0
0	1	0	0	$\longrightarrow$	0
0	1	0	1	$\longrightarrow$	1
0	1	1	0	$\longrightarrow$	0
0	1	1	1	$\longrightarrow$	0
1	0	0	0	$\longrightarrow$	0
1	0	0	1	$\longrightarrow$	1
1	0	1	0	$\longrightarrow$	0
1	0	1	1	$\longrightarrow$	0
1	1	0	0	$\longrightarrow$	1
1	1	0	1	$\longrightarrow$	1
1	1	1	0	$\longrightarrow$	0
1	1	1	1	$\longrightarrow$	0



Demultiplexer
or
demux.

- → Single input 2^n output
- n → number of selection line.
- → Data distributer.
- → depending on the selection line demux are classified as 1×2, 1×4, 1×8, 1×16 - - -

1×2



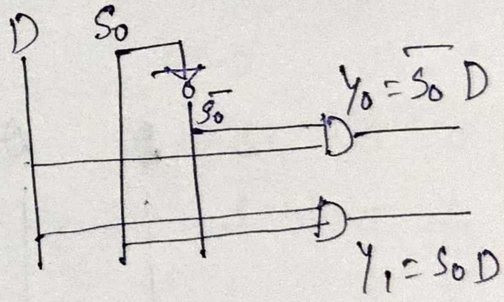
S_0	Y_1	Y_0
0	0	D
1	D	0

Table

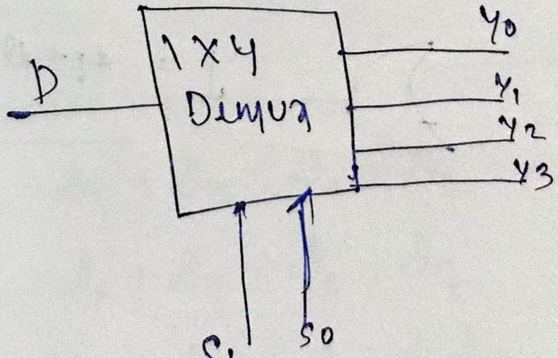
Expression:

$Y_0 = \overline{S_0} D$

$Y_1 = S_0 D$.



1×4



S_1	S_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	D
0	1	0	0	D	0
1	0	0	D	0	0
1	1	D	0	0	0

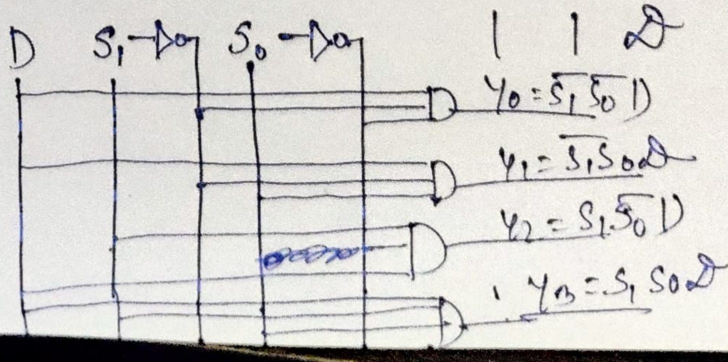
Expression:

$Y_0 = \overline{S_1}\,\overline{S_0} D$

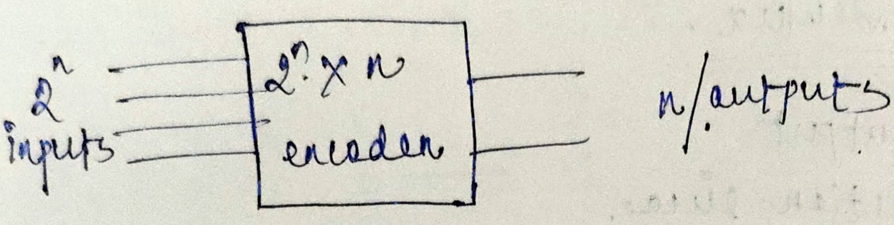
$Y_1 = \overline{S_1} S_0 D$

$Y_2 = S_1 \overline{S_0} D$

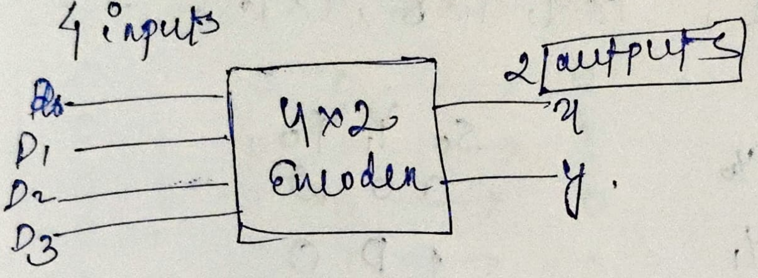
$Y_3 = S_1 S_0 D$.



Encoder.

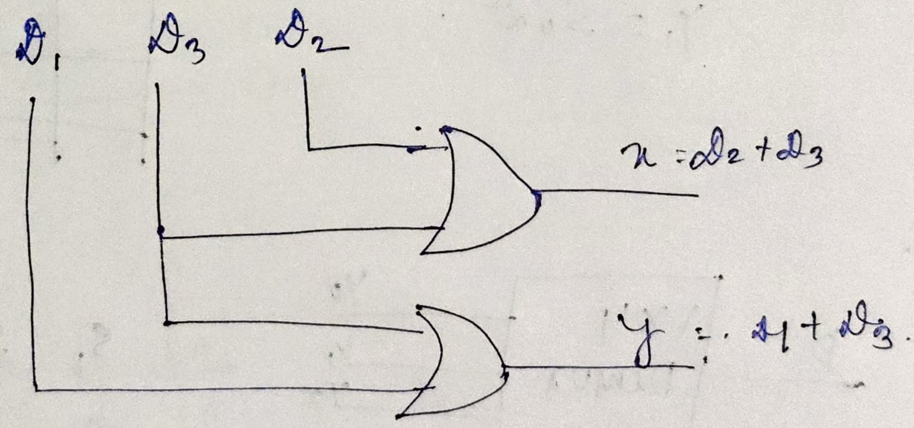


→ 2^n inputs and n outputs.

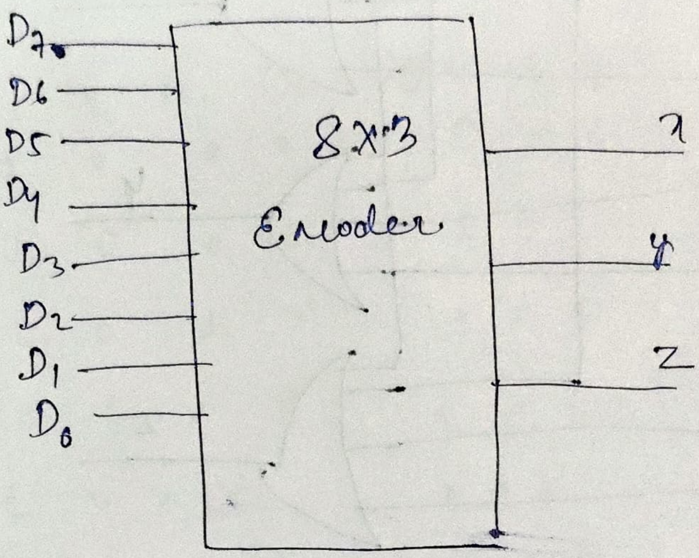


D_3	D_2	D_1	D_0	x	y
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1

$$x = D_2 + D_3$$
$$y = D_1 + D_3$$



8X3



D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0	x	y	z
							1	0	0	0
						1		0	0	1
					1			0	1	0
				1				0	1	1
			1					1	0	0
		1						1	0	1
	1							1	1	0
1								1	1	1

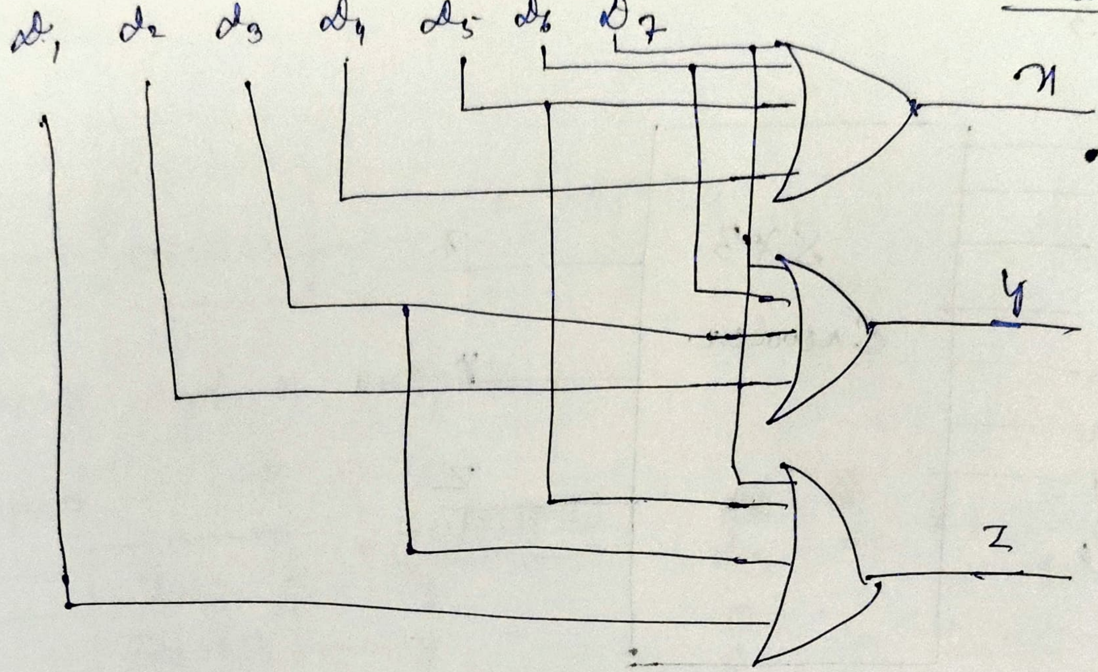
Expression :

$x = D_4 + D_5 + D_6 + D_7$

$y = D_2 + D_3 + D_6 + D_7$

$z = D_1 + D_3 + D_5 + D_7$.

dt - 26 - 11 - 25



Priority Encoder

→ In normal encoder no bit at the output are present to represent all '0' combinations for the input.

→ If more than one input bits are '1' simultaneously the o/p can not be predicted.

for example: In 8×3 encoder if D_6 and D_1 are '1' simultaneously then the output is neither 110 nor 001 rather the output is 111.

For this reason we will assign priorites to the Databits. and design priority encoder.

Design a 4×2 priority encoder

D_3 having priority
D_0 having lowest priority

→

Design a 4x2 priority encoder.

D_3 having high
D_0 lowest priority.

D_3	D_2	D_1	D_0	V	x	y
0	0	0	0	0	x	x
0	0	0	1	1	0	0
0	0	1	x	1	0	1
0	1	x	x	1	1	0
1	x	x	x	1	1	1

K map for V

D_3D_2 \ D_1D_0	00	01	11	10
00		1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$V = D_3 + D_2 + D_1 + D_0$

Kmap x.

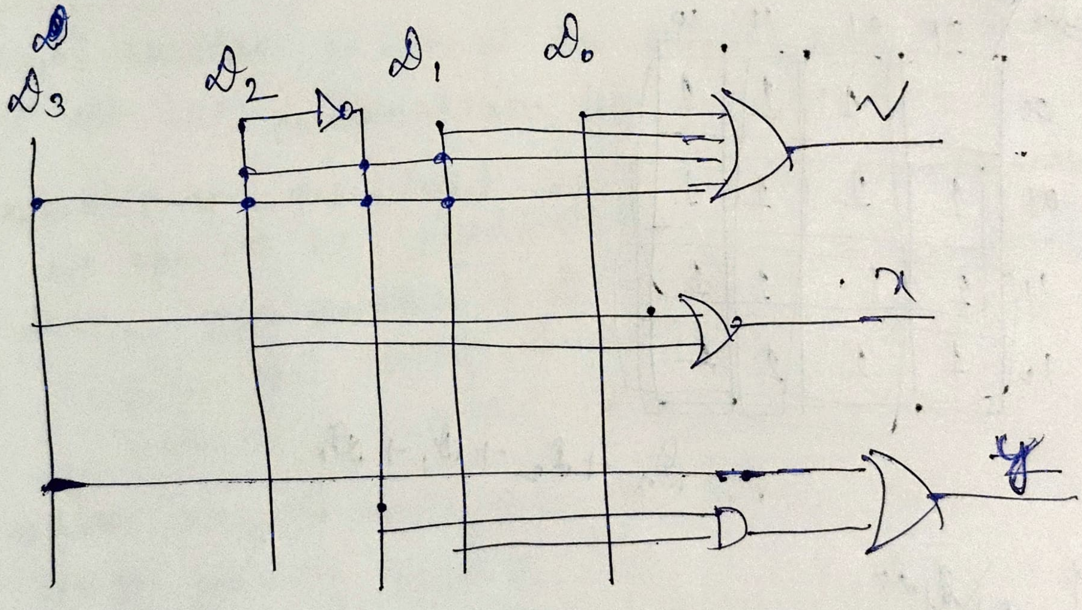
D_3D_2 \ D_1D_0	00	01	11	10
00	X			
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$x = D_3 + D_2$.

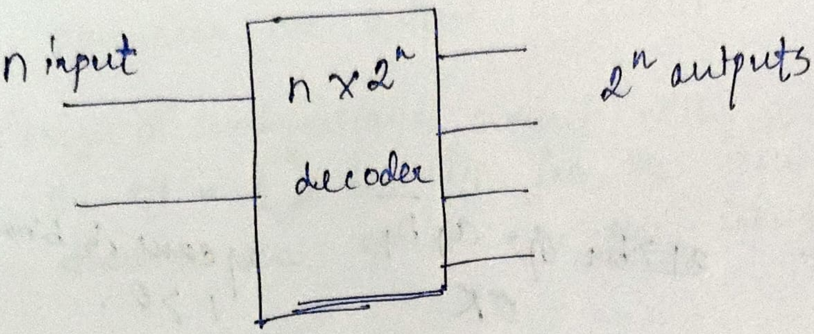
Kmap-y

$D_3D_2 \backslash D_1D_0$	00	01	11	10
00	X		1	1
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

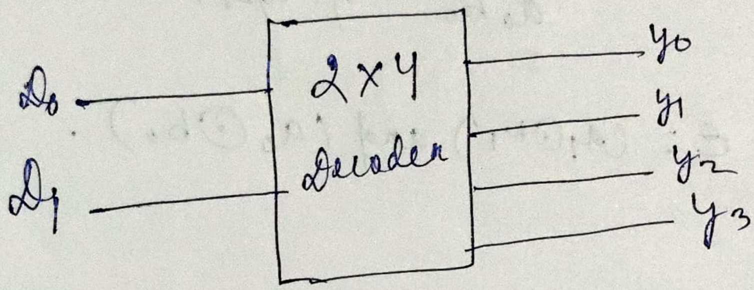
$$y = D_3 + \bar{D}_2 D_1 .$$



Decoder



Design a 2x4 decoder.



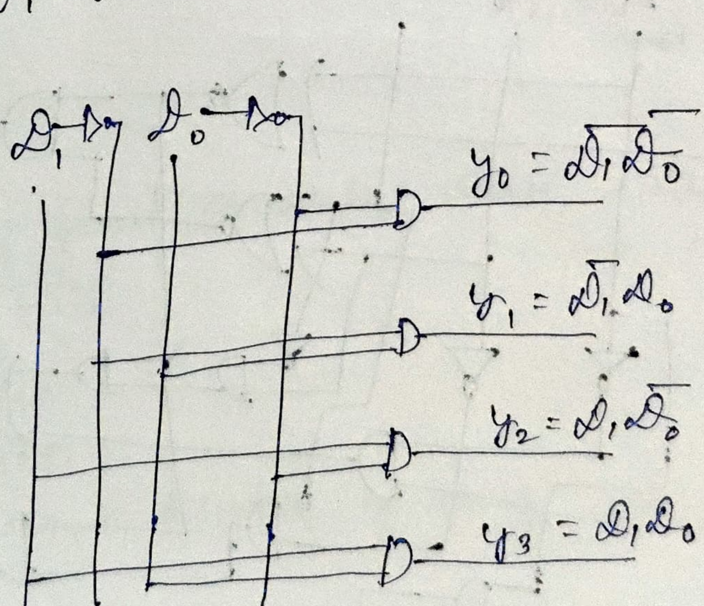
D_1	D_0	y_0	y_1	y_2	y_3
0	0	1			
0	1		1		
1	0			1	
1	1				1

$y_0 = \bar{D}_1 \bar{D}_0$

$y_1 = \bar{D}_1 D_0$

$y_2 = D_1 \bar{D}_0$

$y_3 = D_1 D_0$



Comparator

Let

$A = a_1 a_0$
$B = b_1 b_0$

① (i) $A > B$ $\quad a_1 > b_1$ $\quad G = a_1 \overline{b_1}$ means in binary $1 > 0$.

OR

(ii) $a_1 = b_1 \quad a_0 > b_0$ $\quad a_1 \odot b_1$ and $a_0 \overline{b_0}$ $\quad G = a_1\overline{b_1} + (a_1 \odot b_1)(a_0\overline{b_0})$

② Equality $A = B$

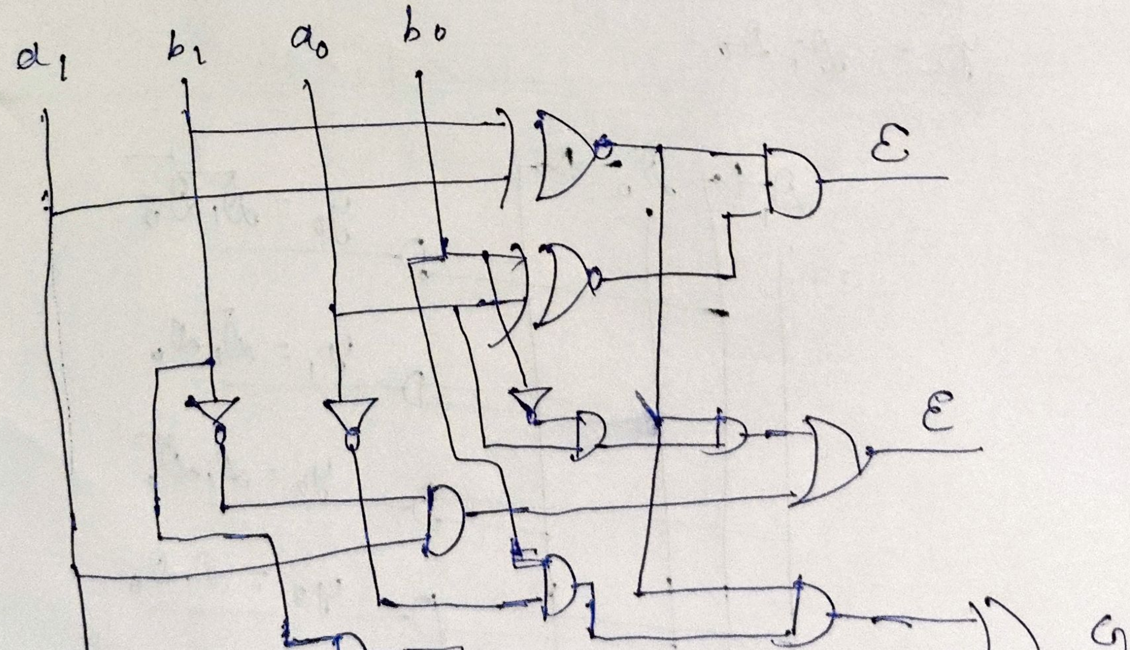
$a_1 = b_1 \quad a_0 = b_0$ $\quad E = (a_1 \odot b_1)$ and $(a_0 \odot b_0)$.

③ $A < B$

$a_1 < b_1$ $\quad L = \bar{a}_1 b_1 + (a_1 \odot b_1)(\bar{a}_0 b_0)$.

(ii) $a_1 = b_1 \quad a_0 < b_0$

E :



Q Design a combinational circuit that has 3 inputs and 1 output the output is 1 when the value of the input is < 3 otherwise the output is ZERO (0).

Q Design a combinatiayal circuit with 3 i/p and 3 o/p the o/p is 1 > the i/p for the inputs 0, 1, 2, 3, 4, 5. otherwise the o/p is 1 < the inputs.

01 - 31/11/25

Difference between combinatiayal circuit and sequential circuit.

Combinatiayal	Sequential
→ The o/p depends upon present values of the inputs only.	→ The o/p depends upon present value of the inputs as well as past values of the o/ps.
→ There is no feed back path or memory.	→ There must be feedback path or memory.
→ Ex: Adder, subtractor, MUX, encoder.	→ Ex: Flipflops, counter, shift register.

Flipflops:

→ Basic memory element. It has two stable states.

→ S.R Flipflop

→ D flipflop

→ JK flipflop

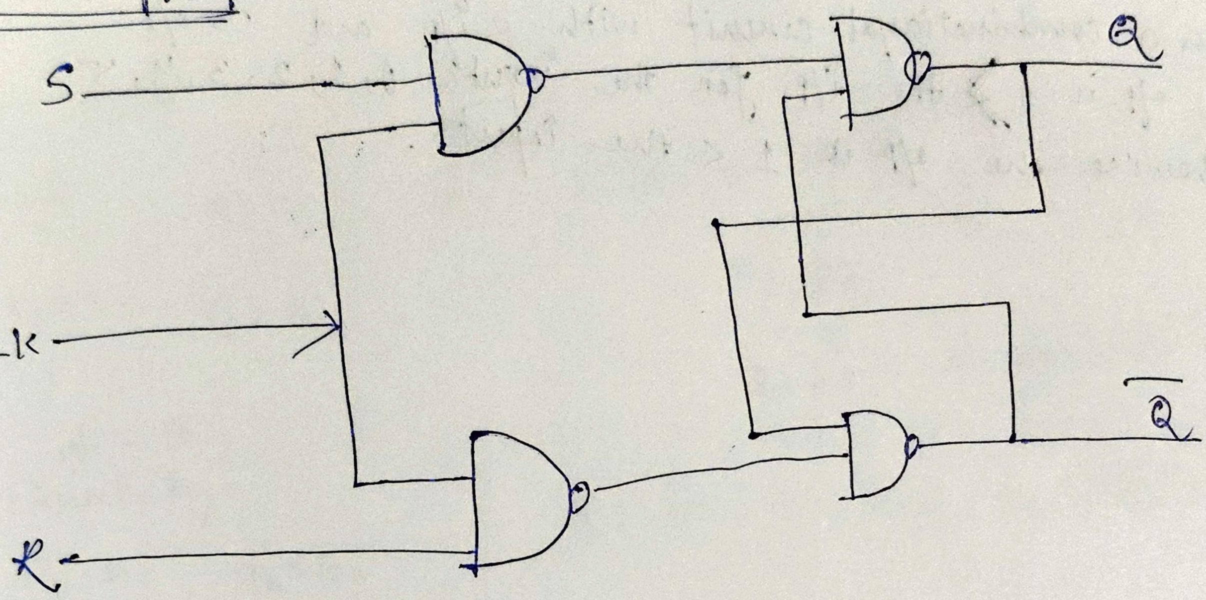
→ T flipflop.

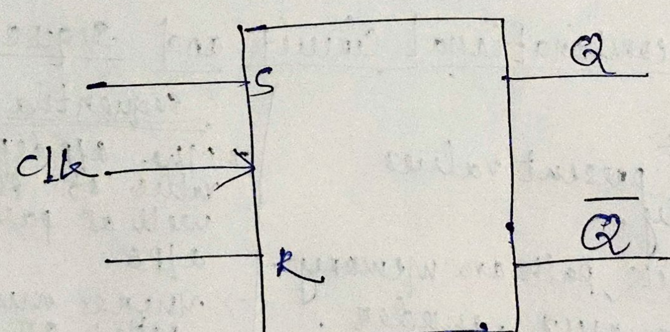
→ flipflops can be designed by using NAND gates or NOR gates.

→ Always there is a clock input in the flipflop.

dt-1-12-25

→ S-R Flipflop





Block diagram

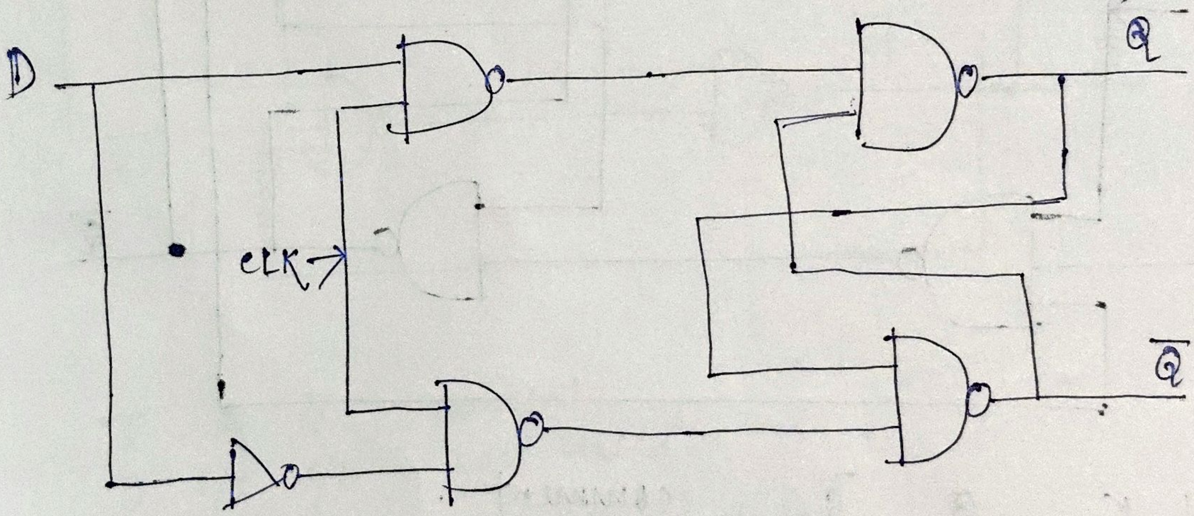
CLK	S	R	Q	$\overline{Q}$	comment
1	0	0	0/1	1/0	→ Last state
1	0	1	0	1	→ Reset
1	1	0	1	0	→ Set
1	1	1	1	1	→ Invalid

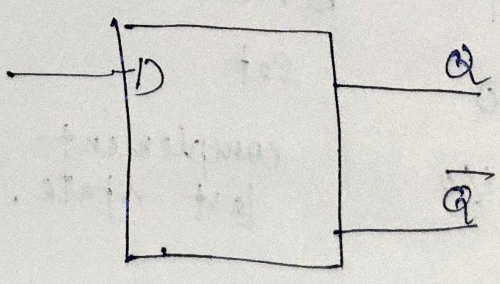
Reset and Set: Two stable states of flipflop.

∴ That's why S-R flip flop - Set-Reset flipflop -

Dt-02-12-25

D-FLIPFLOP (Delay) F.F.

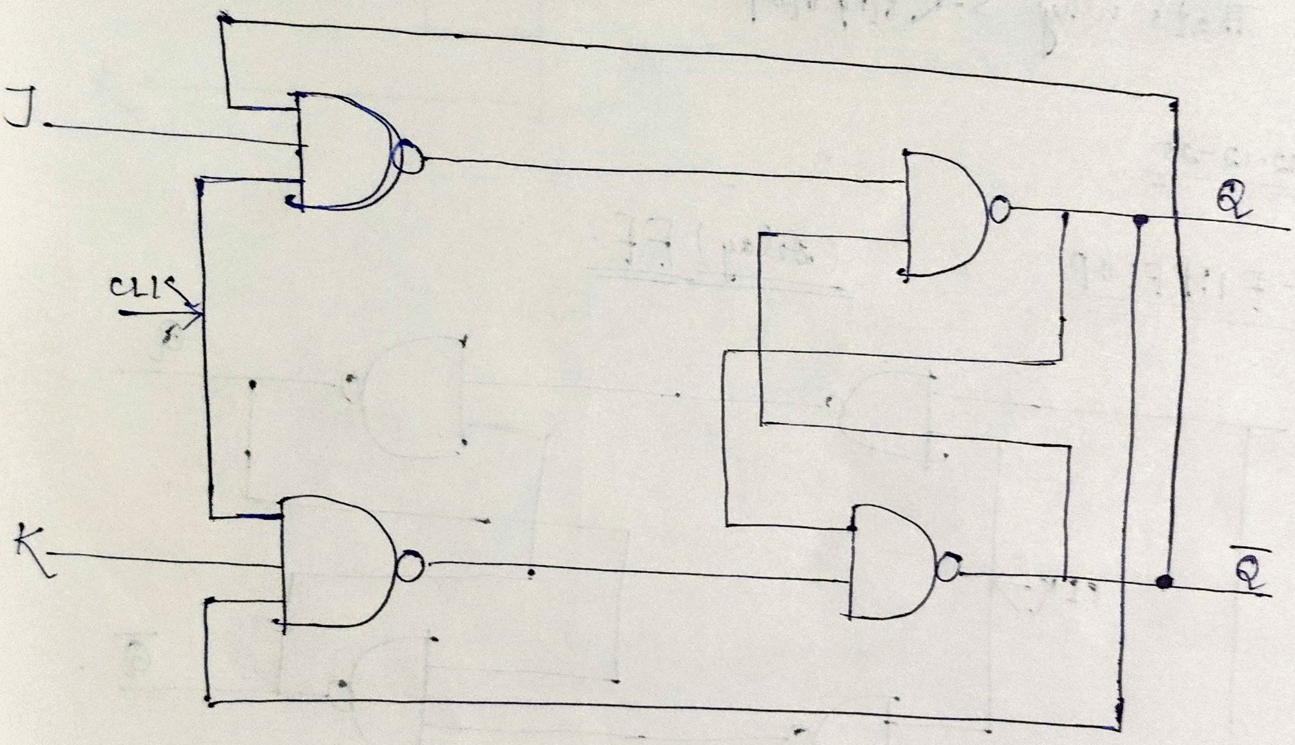




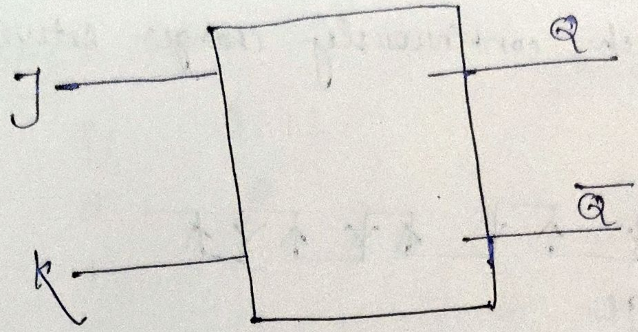
Block Diagram

D	Q	$\overline{Q}$	comment
0	0	1	→ Reset
1	1	0	→ Set.

JK FLIPFLOP :

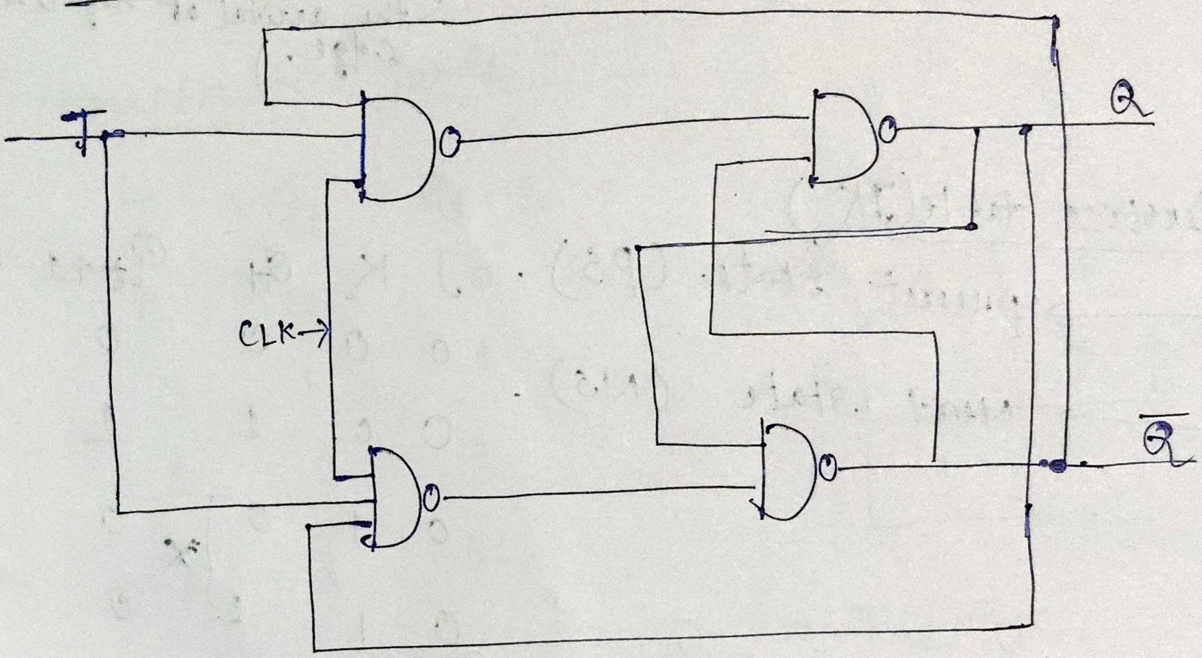


J	K	Q	$\overline{Q}$	comment
0	0	0/1	1/0	Last state
0	1	0	1	Reset
1	0	1	0	Set
1	1	0/1	1/0	complement of last state.



Block diagram.

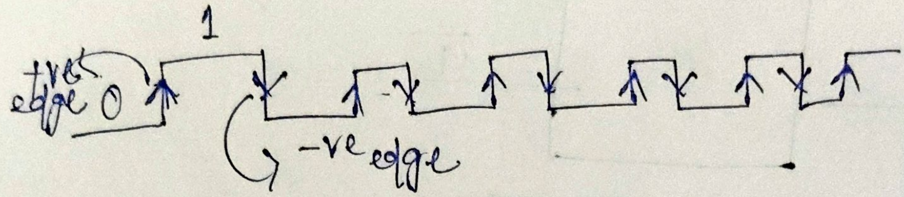
T- FLIPFLOP.



T	Q	$\overline{Q}$	comment
0	0/1	1/0	Last state
1	1/0	0/1	complement of last state or toggle.

CLOCK PULSE

→ It is a signal which continuously changes between zero and one.



→ FlipFlops are always edge triggered.

[-ve edge triggered].

(→It changes state at the arrival of negative edge.)

Characteristics table (JK)

Q_t ⟶ present state (PS).

Q_{t+1} ⟶ Next state (NS).

J	K	Q_t	Q_{t+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1		0 → 1
1	1		1 → 0

J \ K Q_t	00	01	11	10
0		1		
1	1	1		1

$$\boxed{Q_{t+1} = \overline{K}Q_t + J\overline{Q_t}}$$

characteristic equation for JK FF.

dt - 03-12-25

characteristics table - D.

D	Q_t	Q_{t+1}
0	0	0
0	1	0
1	0	1
1	1	1

D \ Q_t	0	1
0		
1	1	1

$$\boxed{Q_{t+1} = D}$$

characteristics table - T.

T	Q_t	Q_{t+1}
0	0	0
0	1	1
1	0	1
1	1	0

T \ Q_t	0	1
0		1
1	1	

$$Q_{t+1} = T \oplus Q_t .$$

characteristic table SR

S	R	Q_t	Q_{t+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	X
1	1	1	X

Excitation table

→ In this table we will find the input to the flip flop by taking present state (PS) and next state (NS).

JK :

Q_t	Q_{t+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Excitation table - D :

Q_t	Q_{t+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

Excitation Table - T

Q_t	Q_{t+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Excitation Table - SR

Q_t	Q_{t+1}	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

Convert SR FF to D FF

S R ⟶ D

Excitation SR

D	Q_t	Q_{t+1}	S	R
0	0	0	0	X
0	1	0	0	1
1	0	1	1	0
1	1	1	X	0

K map for S

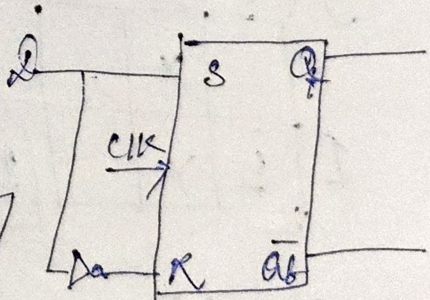
D \ Q_t	0	1
0		
1	1	X

$\boxed{S = D}$

K map for R

D \ Q_t	0	1
0	X	1
1		

$\boxed{x = \bar{D}}$



Block diagram for SR to D

convert SR to JK

SR ⟶ JK .

J	K	Q_t	Q_{t+1}	S	R
0	0	0	0	0	X
0	0	1	1	X	0
0	1	0	0	0	X
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	1	X	0
1	1	0	1	1	0
1	1	1	0	0	1

K map of S

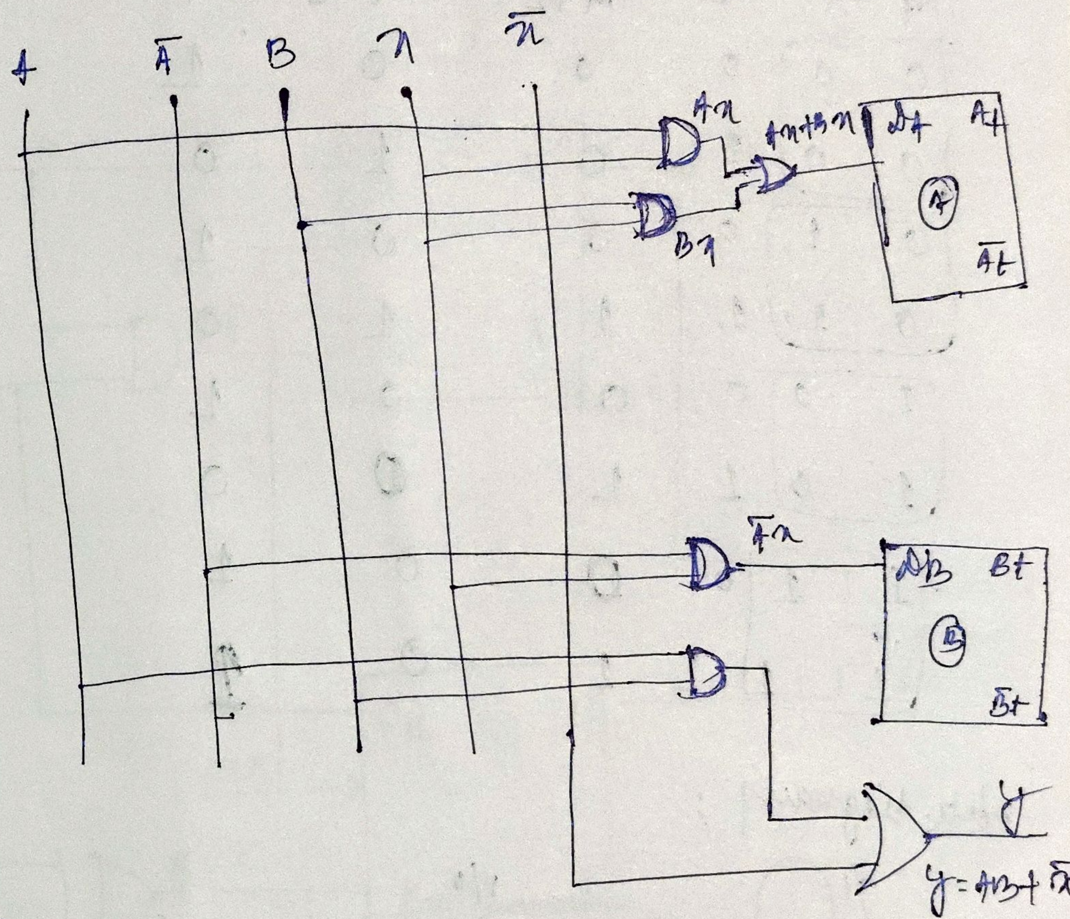
J \ KQ_t	00	01	11	10
0		X		
1	1	X		1

$S = J\bar{Q}_t$

dt - 08-12-25

Analysis of clocked sequential circuit (cont)

→ characteristics equation.
→ state table.
→ State diagram.



$A_t \to$ PS D_A
$B_t \to$ PS D_B
$x \to$ input
$y \to$ output

→ Characteristic equation

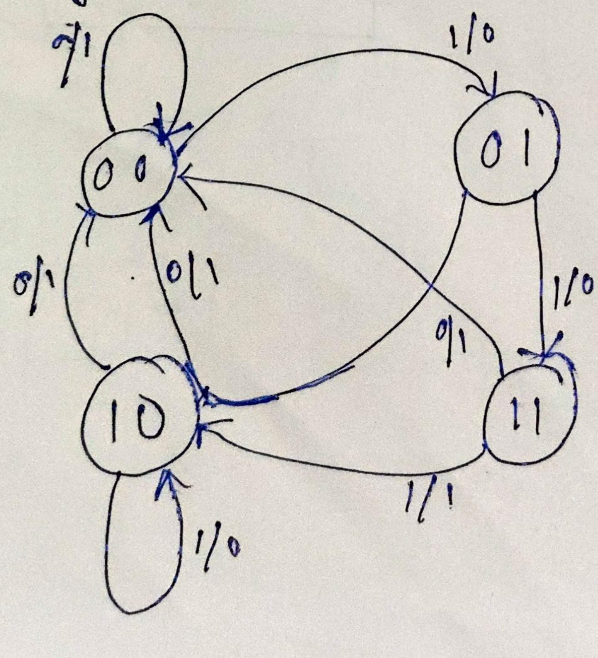
→ $A_{t+1} = A_t \cdot y + B_t \cdot x$

→ $B_{t+1} = \overline{A_t}\, x$

→ State table

A_t	B_t	x	A_{t+1}	B_{t+1}	y
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	1

State diagram :



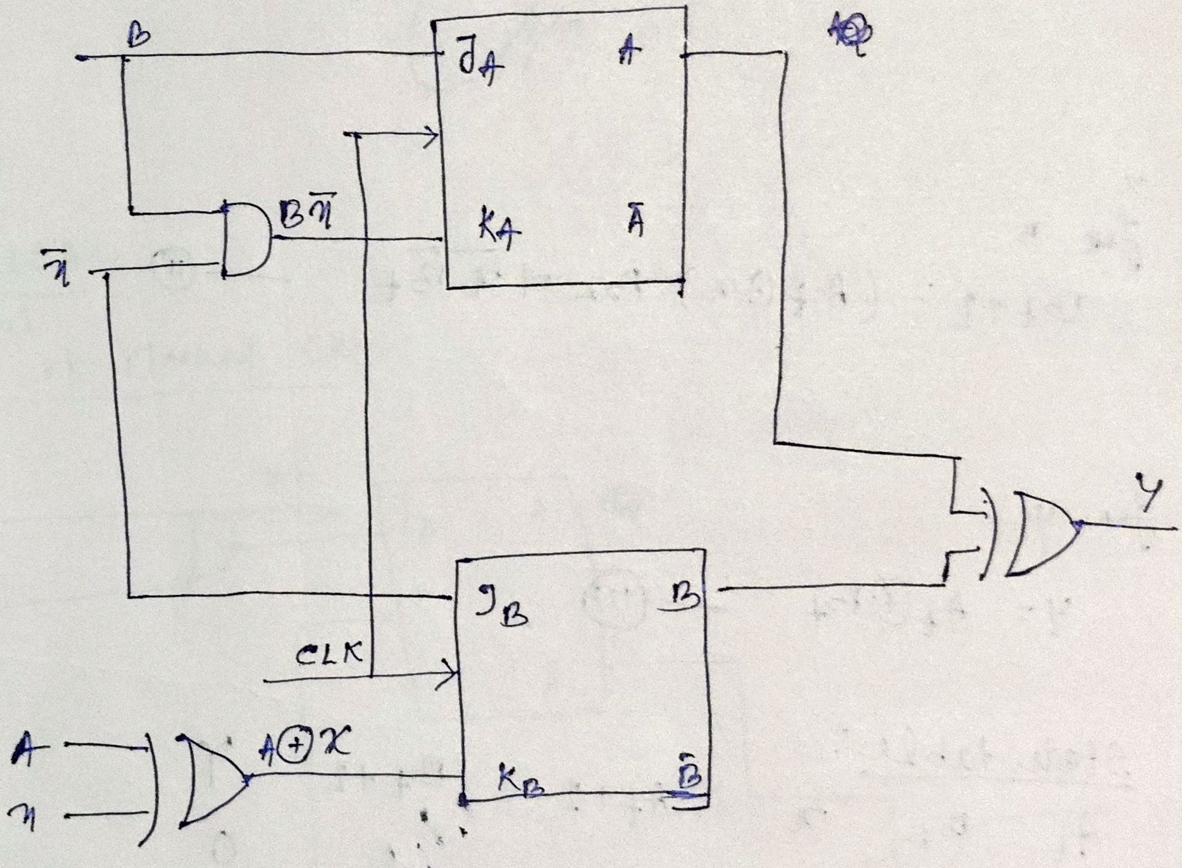
dt- 09-12-25:

Eg. Analyze the given circuit

(i) Write characteristic equation

(ii) Write state table

(iii) Draw state diagram.



$A_t, B_t \longrightarrow PS$

$x \longrightarrow$ input

$y \longrightarrow$ output.

Characteristics equation.

$$\boxed{Q_{t+1} = \bar{K}Q_t + J\bar{Q}_A}$$

$J_A = B_t$, $K_A = B_t\bar{x}$

$J_B = \bar{x}$, $K_B = A_t \oplus x$

$y = A_t \oplus B_t$

Characteristics equation:

$$A_{t+1} = \overline{B_t \bar{x}} \cdot A_t + B_t \bar{A}_t$$

$$= (\bar{B}_t + x) A_t + B_t \bar{A}_t$$

$$= A_t \bar{B}_t + A_t x + \bar{A}_t B_t$$

$$A_{t+1} = A_t \oplus B_t + A_t x. \quad \text{—— (I)}$$

x

for B

$$B_{t+1} = (A_t \odot x) B_t + \bar{x} \bar{B}_t \quad \text{—— (II)}$$

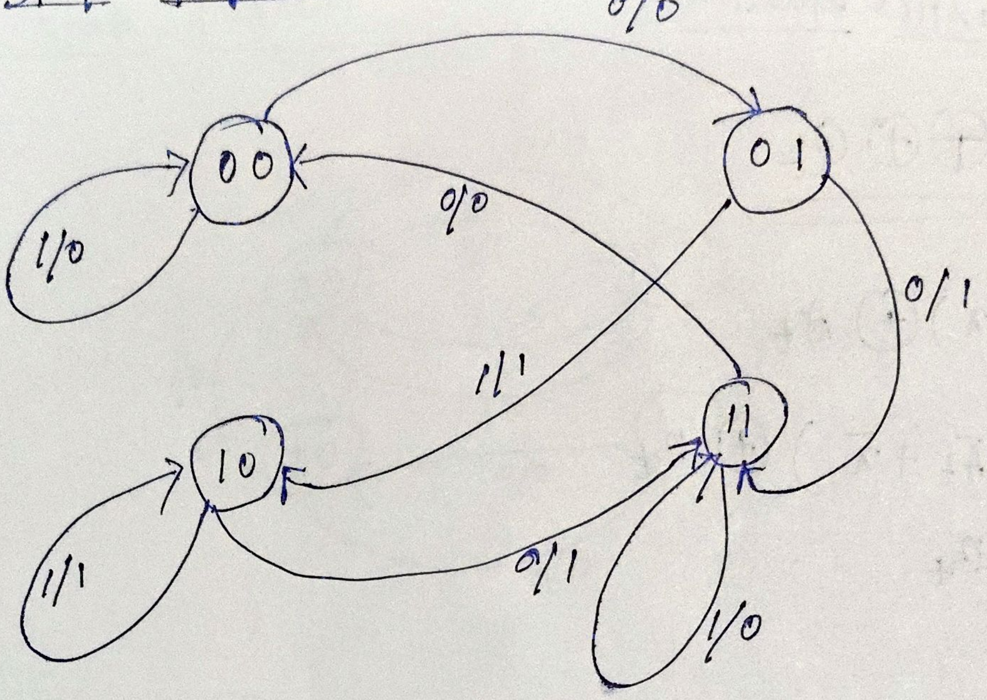
for y

$$y = A_t \oplus B_t \quad \text{—— (III)}$$

State table:

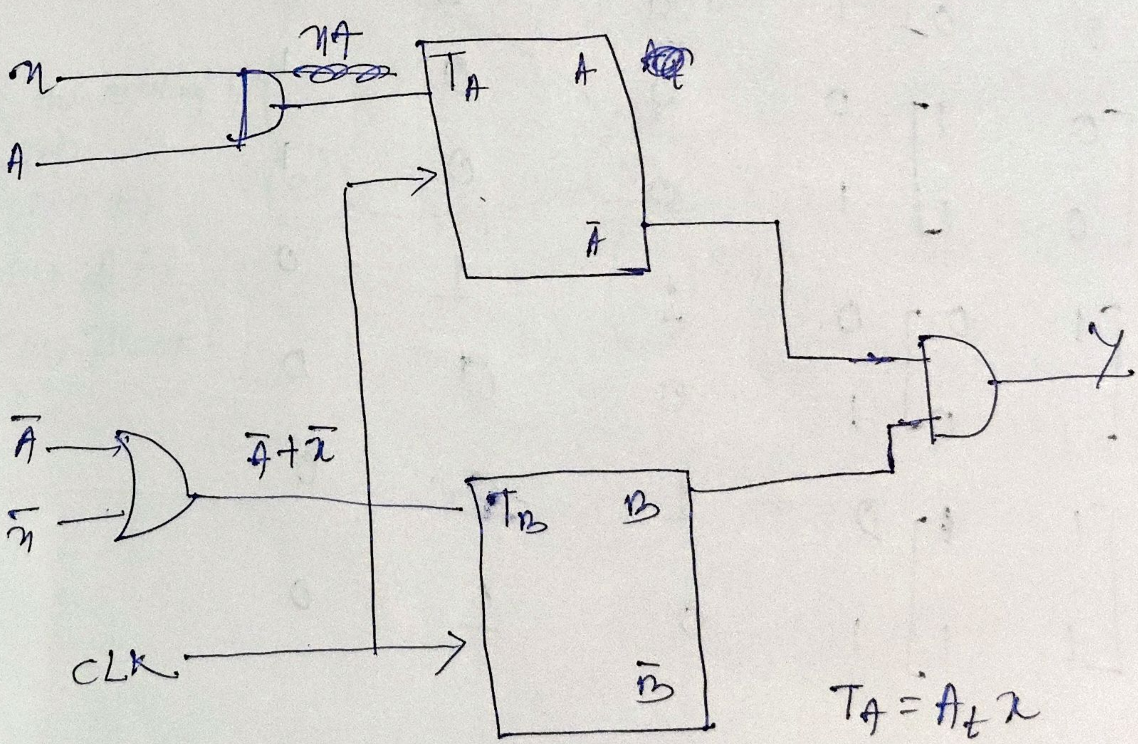
A_t	B_t	x	A_{t+1}	B_{t+1}	y
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	1	0	1
1	0	0	1	1	1
1	0	1	1	0	1
1	1	0	0	0	0
1	1	1	1	1	0

State diagram:



T- FLIP FLOP

sequential analysis of clocked ckt.



$A_t, B_t \rightarrow PS$

$x \rightarrow i/p$

$y \rightarrow o/p$

$T_A = A_t x$

$T_B = \bar{A_t} + \bar{x}$

$Y = \bar{A_t} B_t$

characteristics equation

$$Q_{t+1} = T \oplus Q_t$$

$$A_{t+1} = (A_t x) \oplus A_t$$

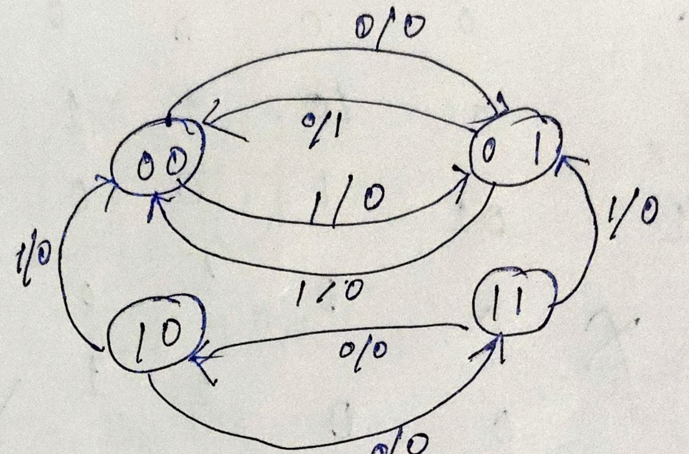
$$B_{t+1} = (\bar{A}_t + \bar{x}) \oplus B_t$$

$$Y = \bar{A}_t B_t$$

State Table

A_t	B_t	x	A_{t+1}	B_{t+1}	Y
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	0	0	0
1	1	0	1	0	0
1	1	1	0	1	0

State diagram



dt - 10-12-25

Design of clocked sequential circuit :

(Q) A sequential circuit has 1 i/p and 1 o/p.

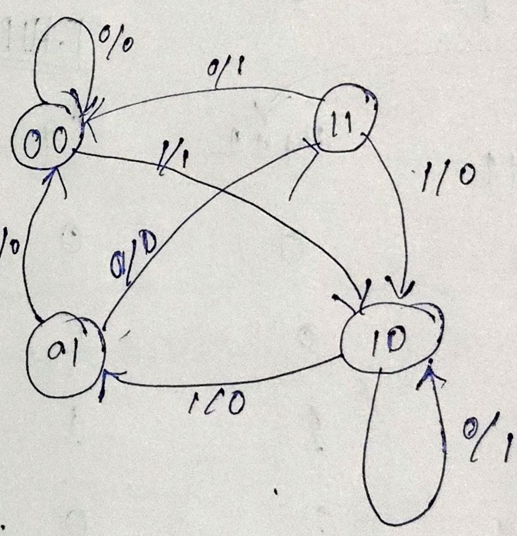
the state diagram is as shown in the figure.

→ Design the sequential circuit with

(i) T FF

(ii) D FF

(iii) JK FF.



PTO →

T.FLIPFLOP

A_t	B_t	x	A_{t+1}	B_{t+1}	T_A	T_B	Y
0	0	0	0	0	0	0	0
0	0	1	1	0	1	0	1
0	1	0	1	1	1	0	0
0	1	1	0	0	0	1	0
1	0	0	1	0	0	0	1
1	0	1	0	1	1	1	0
1	1	0	0	0	1	1	1
1	1	1	1	0	0	1	0

Kmap for T_A.

$A_t \backslash B_t x$	00	01	11	10
0		1		1
1		1		1

$$T_A = \bar{B}_t x + B_t \bar{x}$$
$$= B_t \oplus x$$

Kmap for T_B.

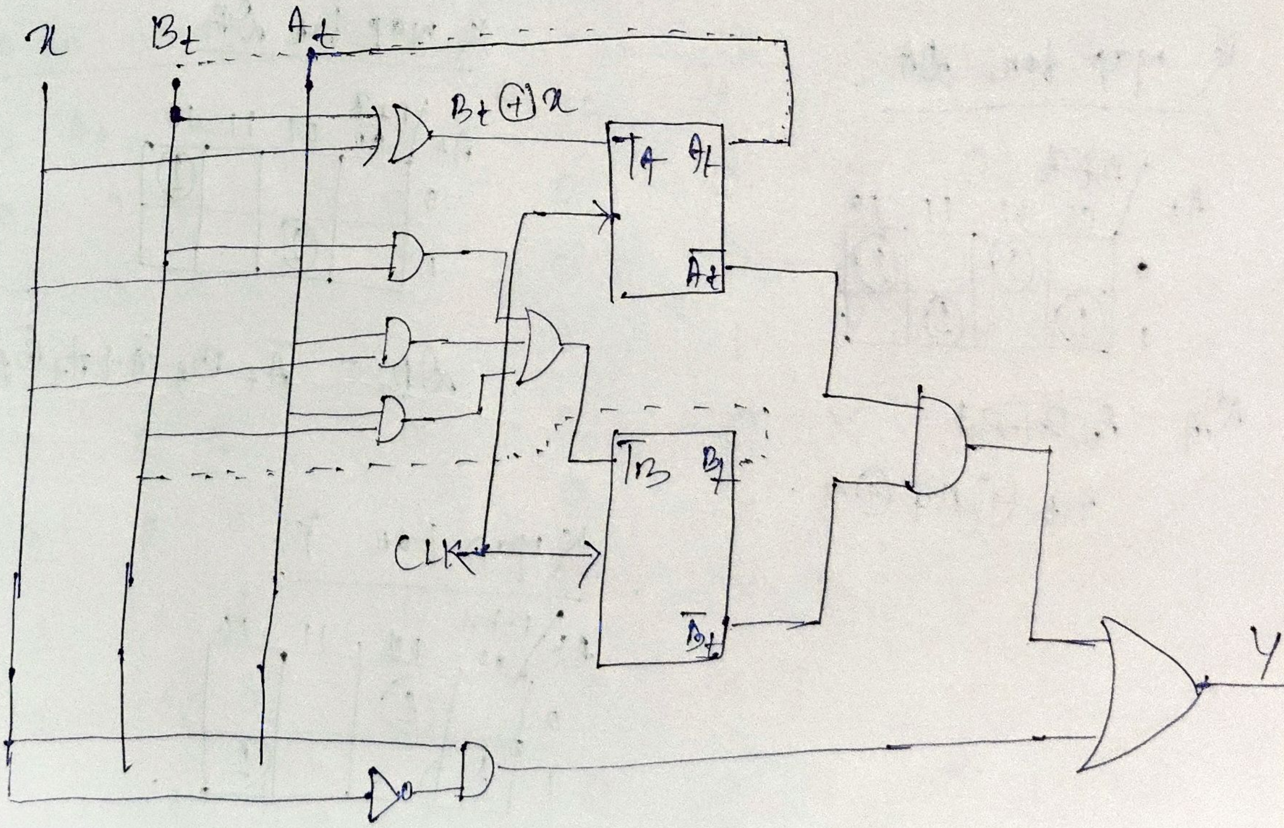
$A_t \backslash B_t x$	00	01	11	10
0			1	
1		1	1	1

$$T_B = B_t x + A_t x + A_t B_t$$

Kmap for Y.

$A_t \backslash B_t x$	00	01	11	10
0		1		
1	1			1

$$Y = A_t \bar{x} + \bar{A}_t \bar{B}_t x$$



D-FF

A_t	B_t	x	A_{t+1}	B_{t+1}	D_A	D_B	Y
0	0	0	0	0	0	0	0
0	0	1	1	0	1	0	1
0	1	0	1	1	1	1	0
0	1	1	0	0	0	0	0
1	0	0	1	0	1	0	1
1	0	1	0	1	0	1	0
1	1	0	0	0	0	0	1
1	1	1	1	0	1	0	0

K map for D_A

A_t \ $B_t x$	00	01	11	10
0		1		1
1	1		1	

$D_A = \cancel{A_t B_t x}$

$A_t \oplus B_t \oplus x$

K map for D_B

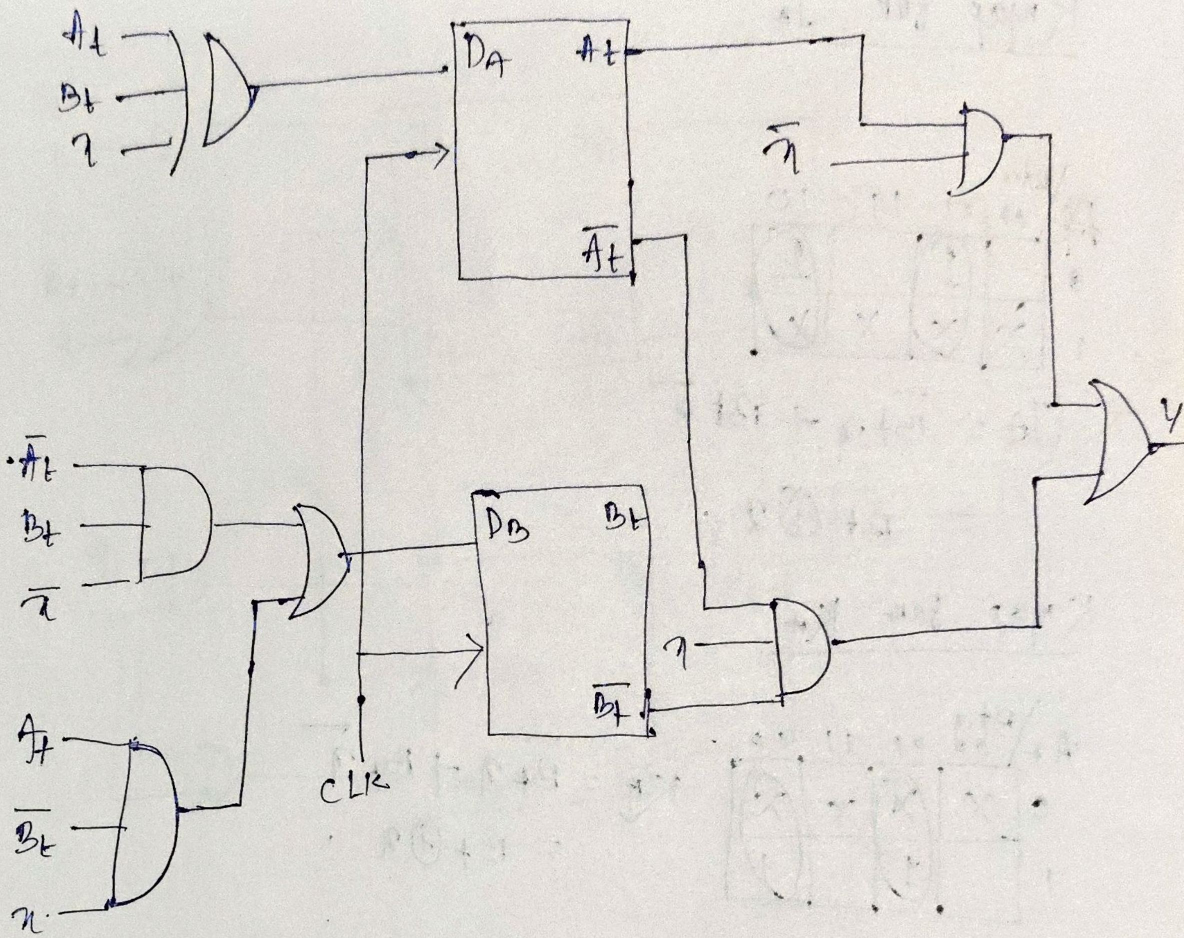
A_t \ $B_t x$	00	01	11	10
0				1
1		1		

$D_B = \bar{A}_t B_t \bar{x} + A_t \bar{B}_t x$

K map for Y

A_t \ $B_t x$	00	01	11	10
0		1		
1	1			1

$Y = A_t \bar{x} + \bar{A}_t \bar{B}_t x$



dt - 12 - 12 - 25.

Disign using JK FF :

A_t	B_t	x	A_{t+1}	B_{t+1}	J_A	K_A	J_B	K_B	Y
0	0	0	0	0	0	X	0	X	0
0	0	1	1	0	1	X	0	X	1
0	1	0	1	1	1	X	X	0	0
0	1	1	0	0	0	X	X	1	0
1	0	0	1	0	X	0	0	X	1
1	0	1	0	1	X	1	1	X	0
1	1	0	0	0	X	1	X	1	1
1	1	1	1	0	X	0	X	1	0

Kmap for J_A

$A_t \backslash B_t x$	00	01	11	10
0		1		1
1	X	X	X	X

$$J_A = \bar{B_t} x + B_t \bar{x}$$

$$= B_t \oplus x.$$

Kmap for K_A

$A_t \backslash B_t x$	00	01	11	10
0	X	X	X	X
1		1		1

$$K_A = \bar{B_t} x + B_t \bar{x}$$

$$= B_t \oplus x.$$

Kmap for J_B

$A_t \backslash B_t x$	00	01	11	10
0			X	X
1		1	X	X

$$J_B = A_t x$$

Kmap for K_B

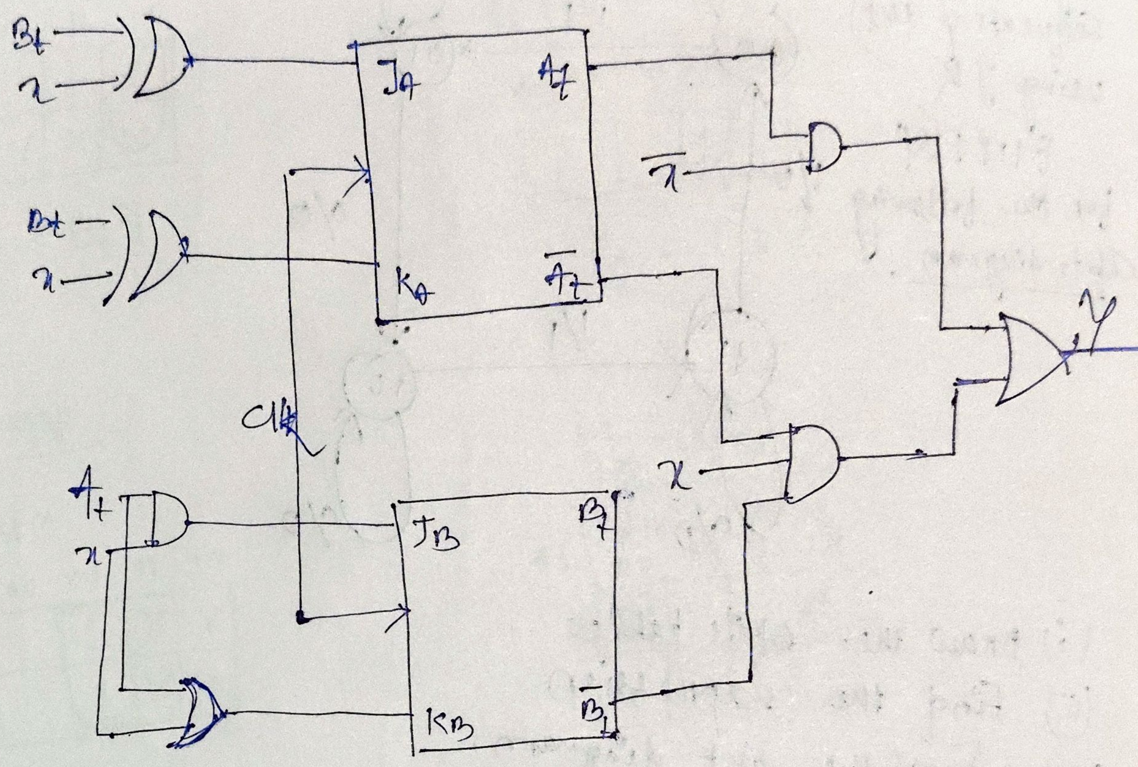
$A_t \backslash B_t x$	00	01	11	10
0	X	X	1	
1	X	X	1	1

$$K_B = A_t + x.$$

Kmap for Y

$A_t \backslash B_t x$	00	01	11	10
0		1		
1	1			1

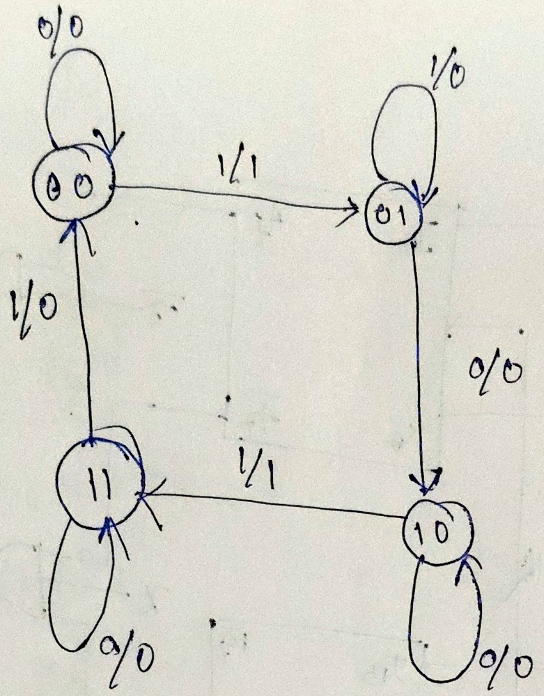
$$Y = A_t \bar{x} + \bar{A_t} \bar{B_t} x$$



Bt
x
Bt
x
JA
KA
At
Āt
x̄
x
Y
Clk
At
x
JB
KB
Bt
B̄t

(Q).

Design the clocked sequential ckt using JK Flip Flop for the following state diagram.



(i) Draw the state table.

(ii) Find the expression

(iii) Draw the ckt diagram.

State table

A_t	B_t	x	A_{t+1}	B_{t+1}	J_A	K_A	J_B	K_B	Y
0	0	0	0	0	0	X	0	X	0
0	0	1	0	1	0	X	1	X	1
0	1	0	1	0	1	X	X	1	0
0	1	1	0	1	0	X	X	0	0
1	0	0	1	0	X	0	0	X	0
1	0	1	1	1	X	0	1	X	1
1	1	0	1	1	X	0	X	0	0
1	1	1	0	0	X	1	X	1	0

Kmap for J_A.

A_t \ $B_t x$	00	01	11	10
0				1
1	X	X	X	X

$J_A = B_t \bar{x}$.

K_A Kmap.

A_t \ $B_t x$	00	01	11	10
0	X	X	X	X
1			1	

$K_A = B_t x$.

J_B Kmap.

A_t \ $B_t x$	00	01	11	10
0		1	X	X
1		1	X	X

$J_B = x$.

K_B Kmap.

A_t \ $B_t x$	00	01	11	10
0	X	X		1
1	X	X	1	

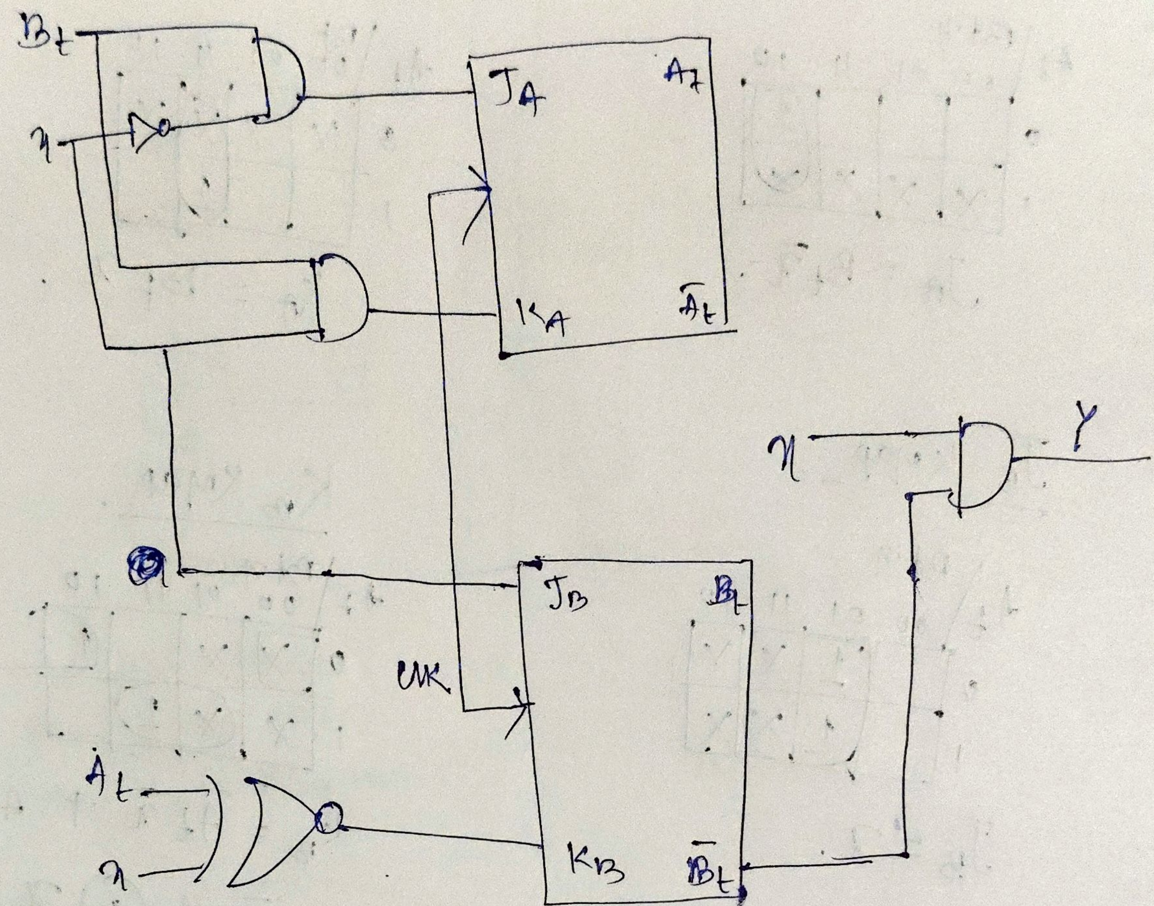
$K_B = \bar{A_t}\bar{x} + A_t x$.

$= A_t \odot x$.

Kmap for Y.

A_t \ B_t	00	01	11	10
0		1		
1		1		

$Y = \bar{B_t} x$



B_t
x
J_A
A_t
K_A
$\bar{A}_t$
x
Y
CLK
J_B
B_t
A_t
x
K_B
$\bar{B}_t$

dt - 15-12-25 :

Important.

Analysis.

-> ckt given

-> state equation

-> state table

-> state diagram.

Design

-> state diagram given

-> State table

-> Excitation table

-> Kmap to find inputs to the ff

-> Circuit diagram.

COUNTER :-

-> It will count number of states.

-> (number of clock pulses).

-> 'n' bit counter can count 2^n number of states

i.e. 0 to 2^n-1

-> To design 'n' bit counter 'n' number of FlipFlops are needed.

-> counters are two types.

* Asynchronous counter

** Synchronous Counter.

Asynchronous Counter :

-> When different clockpulses are given to difference flipflops then the counter is asynchronous counter.

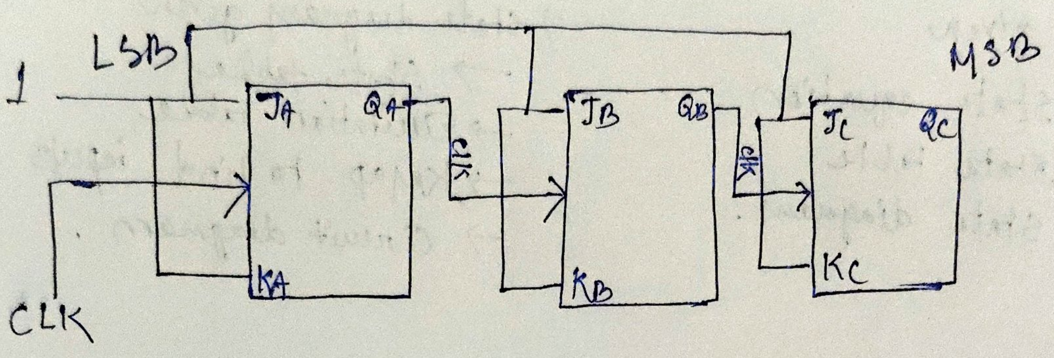
-> The output of 1st flipflop is given as a clockpulse to second flipflop and so on.

-> An Asynchronous counter can be designed by JK ff or T ff with $J=K=1$ or $T=1$

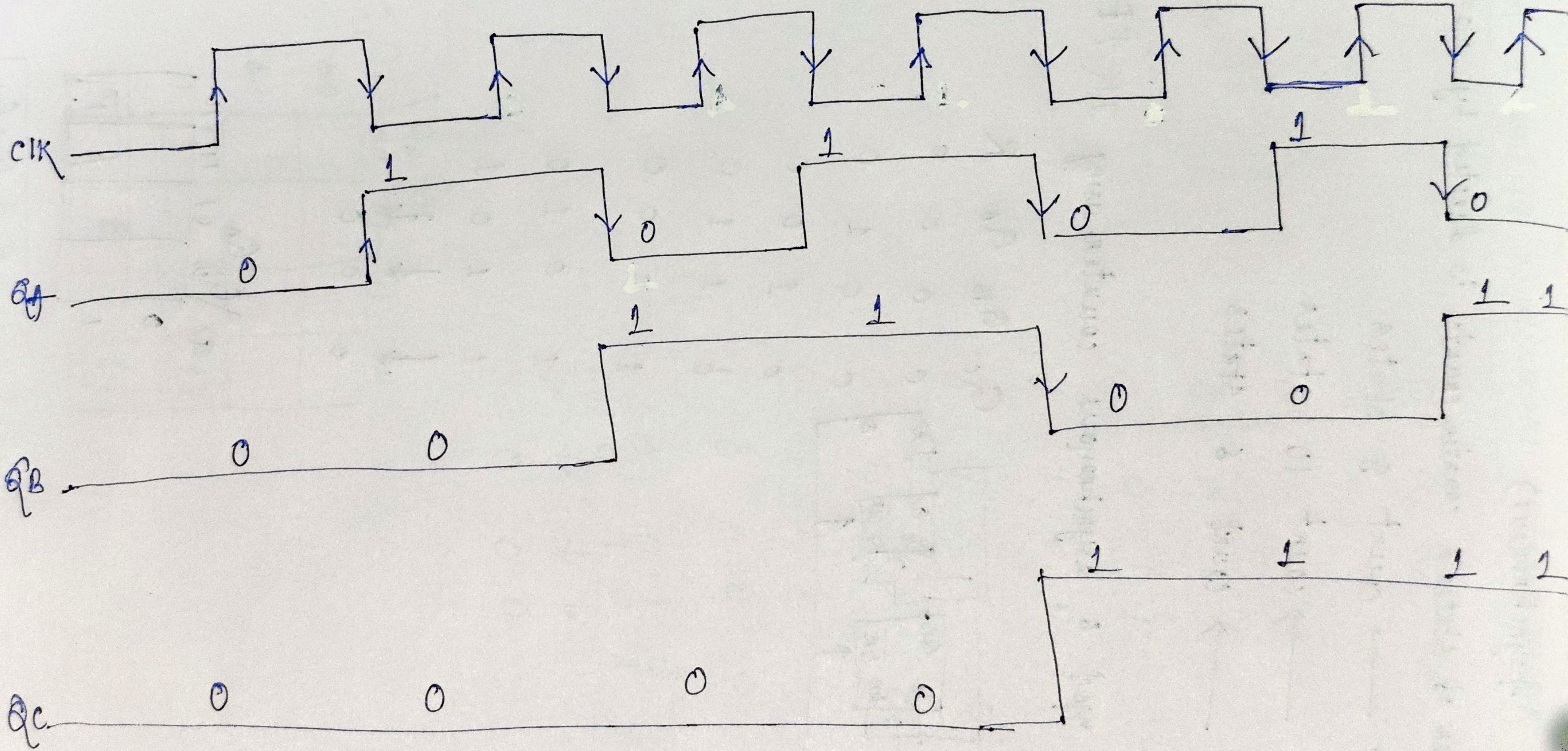
-> For n bit counter design we need 'n' number of ffs.

-> Flipflops are negative edge triggered i.e it will change its state on the arrival of the -ve edge of the clock pulses.

Design a 3bit asynchronous counter using JK FLIPFLOP.



CLK	Q_C	Q_B	Q_A
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
6	1	0	1
7	1	1	0
8	1	1	1



Timing diagram

Mod counter (Asynchronous)

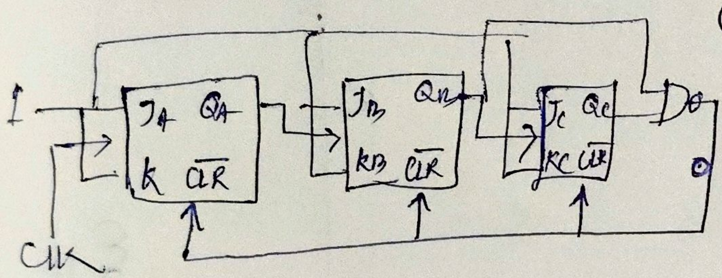
→ The number of states a counter counts is denoted by modulos (mod).

mod-8 ⟶ count 8 states

mod-10 ⟶ count 10 states

mod-6 ⟶ count 6 states

→ Design a mod 6 asynchronous counter using JK FF



Q_C	Q_B	Q_A	R
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1

0 0 0

Q_C \ $Q_B Q_A$	00	01	11	10
0				
1			1	1

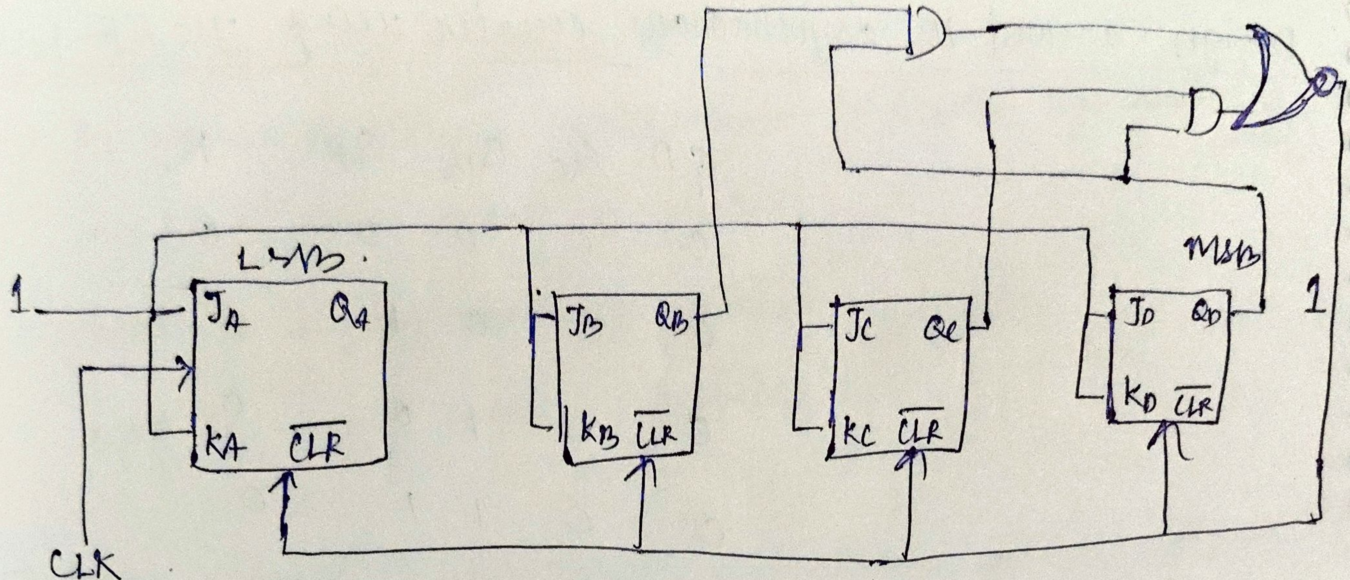
$$\boxed{R = Q_C Q_B}$$

Design a mod-10 asynchronous counter using JK FF

Q_D	Q_C	Q_B	Q_A	R
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
↓	↓	↓	↓	
0	0	0	0	

Q_DQ_C \ Q_BQ_A	00	01	11	10
00				
01				
11	1	1	1	1
10		0	1	1

$$R = Q_C Q_D + Q_D Q_B.$$



BCD

Decimal.

Synchronous Counter

* The same clock pulse is given to all the flipflops.

→ JK, D, T

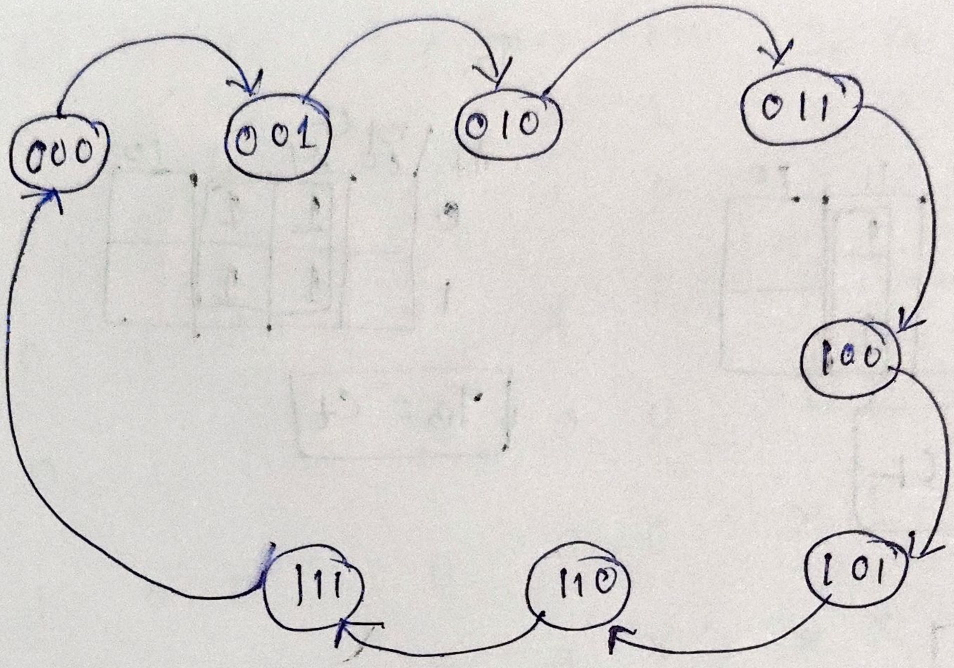
→ PS and NS and system excitation table.

→ state diagram.

→ K map to find input to the ff.

→ circuit diagram.

→ Design a 3 bit Synchronous counter using T ff.



A_t	B_t	C_t	A_{t+1}	B_{t+1}	C_{t+1}	T_A	T_B	T_C
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

Kmap T_A =

$A_t \backslash B_tC_t$	00	01	11	10
0			1	
1			1	

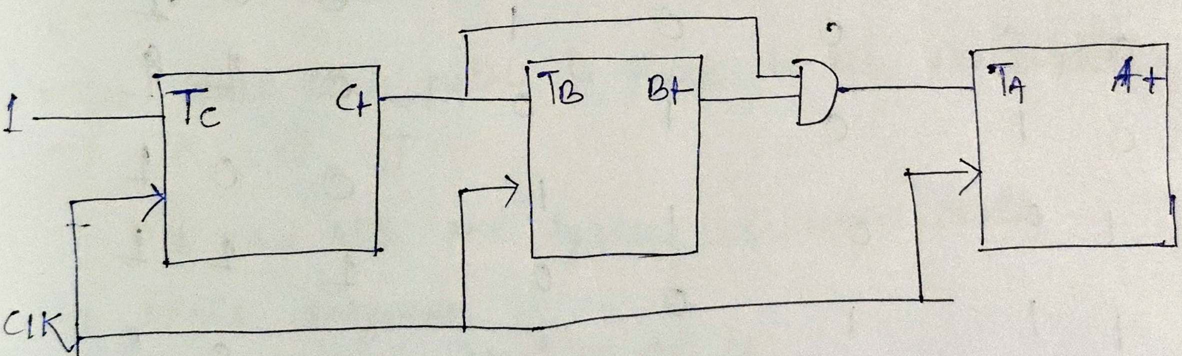
$$\boxed{T_A = B_tC_t}$$

T_B =

$A_t \backslash B_tC_t$	00	01	11	10
0		1	1	
1		1	1	

$$\boxed{T_B = C_t}$$

$$\boxed{T_C = 1}$$



Design a Mod-5 Synchronous Counter using JK FF

A_t	B_t	C_t	A_{t+1}	B_{t+1}	C_{t+1}	J_A	K_A	$J_B K_B$	$J_C K_C$
0	0	0	0	0	1	0	X	0 X	1 X
0	0	1	0	1	0	0	X	1 X	X 1
0	1	0	0	1	1	0	X	X 0	1 X
0	1	1	1	0	0	1	X	X 1	X 1
1	0	0	0	0	0	X	1	0 X	0 X
1	0	1	X	X	X	X	X	X X	X X
1	1	0	X	X	X	X	X	X X	X X
1	1	1	X	X	X	X	X	X X	X X

K map - J_A

$A_t \backslash B_tC_t$	00	01	11	10
0			1	
1	X	X	X	X

$J_A = B_tC_t$

K_A

$A_t \backslash B_tC_t$	00	01	11	10
0	X	X	X	X
1	1	X	X	X

$K_A = \cancel{A_tB_tC_t}\ 1$

J_B

$A_t \backslash B_tC_t$	00	01	11	10
0		1	X	X
1		X	X	X

$J_B = C_t$

K_B

$A_t \backslash B_tC_t$	00	01	11	10
0	X	X	1	
1	X	X	X	X

$K_B = C_t$

J_C

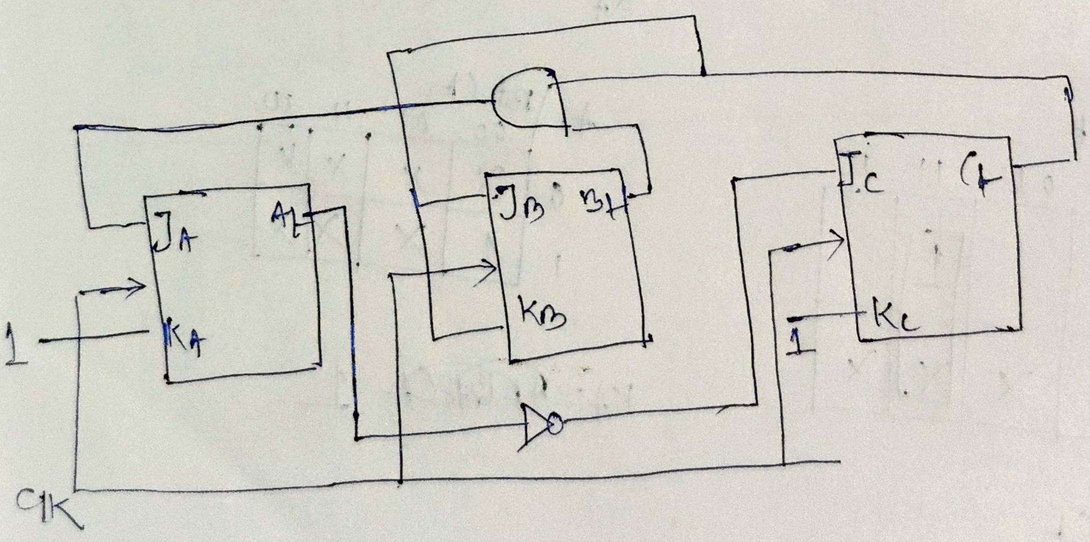
$A_t \backslash B_tC_t$	00	01	11	10
0	1	X	X	1
1		X	X	X

$J_C = \overline{A_t}$

K_C

$A_t \backslash B_tC_t$	00	01	11	10
0	X	1	1	X
1	X	X	X	X

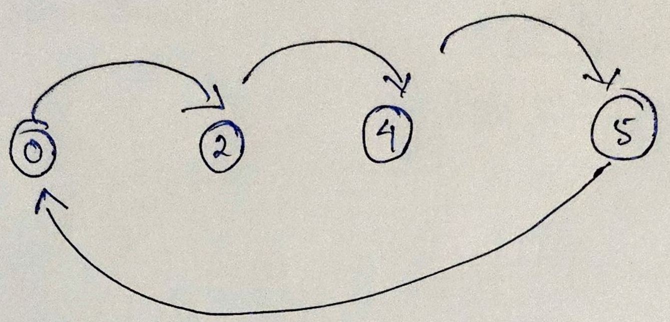
$K = 1$



Skipping state

Design the counter having following state diagram.

using TFF



A_t	B_t	C_t	A_{t+1}	B_{t+1}	C_{t+1}	T_A	T_B	T_C
0	0	0	0	1	0	0	1	0
0	0	1	x	x	x	x	x	x
0	1	0	1	0	0	1	1	0
0	1	1	x	x	x	x	x	x
1	0	0	1	0	1	0	0	1
1	0	1	0	0	0	1	0	1
1	1	0	x	x	x	x	x	x
1	1	1	x	x	x	x	x	x

$T_A =$

A_t \ B_tC_t	00	01	11	10
0		X	X	1
1		1	X	X

$= C_t + B_t$

$T_B =$

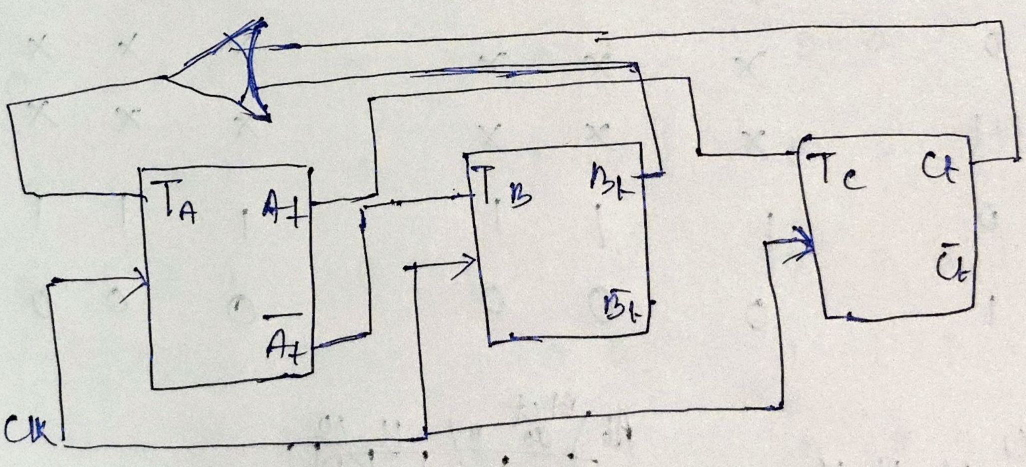
A_t \ B_tC_t	00	01	11	10
0	1	X	X	1
1			X	X

$= \overline{A_t}$

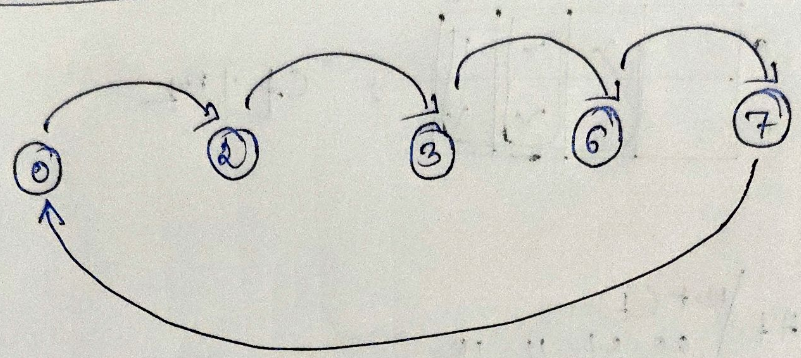
$T_C =$

A_t \ B_tC_t	00	01	11	10
0		X	X	
1	1	1	X	X

$= A_t$



Using D-flip flop



State table

A_t	B_t	C_t	A_{t+1}	B_{t+1}	C_{t+1}	D_A	D_B	D_C
0	0	0	0	1	0	0	1	0
0	0	1	X	X	X	X	X	X
0	1	0	0	1	1	0	1	1
0	1	1	1	1	0	1	1	0
1	0	0	X	X	X	X	X	X
1	0	1	X	X	X	X	X	X
1	1	0	1	1	1	1	1	1
1	1	1	0	0	0	0	0	0

D_A =

$A_t \backslash B_tC_t$	00	01	11	10
0		X	1	
1	X	X		1

$= A_t \oplus C_t$

D_B =

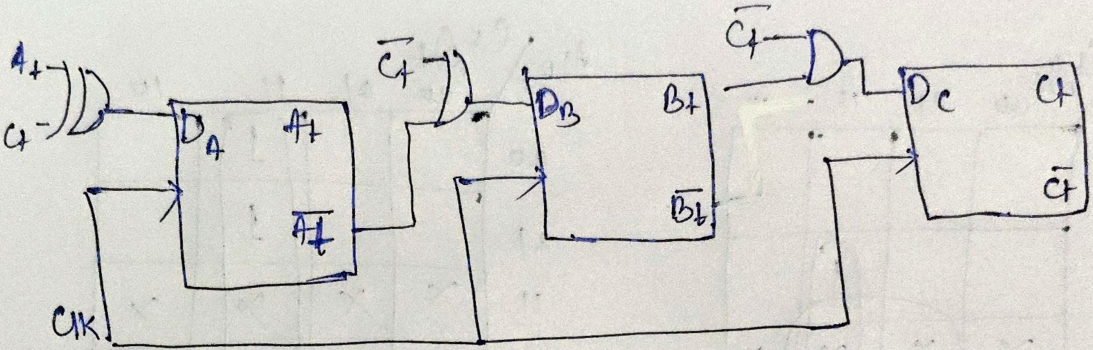
$A_t \backslash B_tC_t$	00	01	11	10
0	1	X	1	1
1	X	X		1

$= \bar{A}_t + \bar{C}_t$

D_C =

$A_t \backslash B_tC_t$	00	01	11	10
0		X		1
1	X	X		1

$= B_t\bar{C}_t$



Q. Design a mod 10 synchronous counter using T-Flipflop.

A_t	B_t	C_t	D_t	A_{t+1}	B_{t+1}	C_{t+1}	D_{t+1}	T_A	T_B	T_C	T_D
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	1	0	0	0	1	1
0	0	1	0	0	0	1	1	0	0	0	1
0	0	1	1	0	1	0	0	0	1	1	1
0	1	0	0	0	1	0	1	0	0	0	1
0	1	0	1	0	1	1	0	0	0	1	1
0	1	1	0	0	1	1	1	0	0	0	1
0	1	1	1	1	0	0	0	1	1	1	1
1	0	0	0	1	0	0	1	0	0	0	1
1	0	0	1	0	0	0	0	1	0	0	1
1	0	1	0	x	x	x	x	x	x	x	x
1	0	1	1	x	x	x	x	x	x	x	x
1	1	0	0	x	x	x	x	x	x	x	x
1	1	0	1	x	x	x	x	x	x	x	x
1	1	1	0	x	x	x	x	x	x	x	x
1	1	1	1	x	x	x	x	x	x	x	x

Kmap

T_A =

A_tB_t \ C_tD_t	00	01	11	10
00				
01			1	
11	X	X	X	X
10	X	1	X	X

$T_A = A_tD_t + B_tC_tD_t$

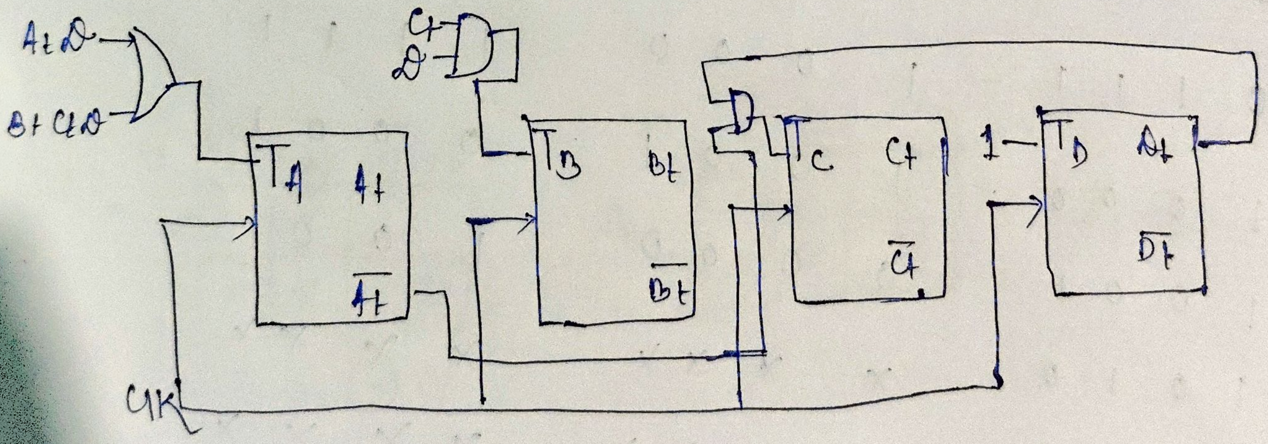
A_tB_t \ C_tD_t	00	01	11	10
00			1	
01			1	
11	X	X	X	X
10			X	X

$T_B = C_tD_t$

A_tB_t \ C_tD_t	00	01	11	10
00		1	1	
01		1	1	
11	X	X	X	X
10				

$T_D = 1$.

$T_C = \overline{A_t}D_t$



dt- 22-12-25.

Design an up down 3 bit Synchronous counter using T ff.

	Y	A_t	B_t	C_t	A_{t+1}	B_{t+1}	C_{t+1}	T_A	T_B	T_C
	0	0	0	0	0	0	1	0	0	1
	0	0	0	1	0	1	0	0	1	1
	0	0	1	0	0	1	1	0	0	1
up	0	0	1	1	1	0	0	1	1	1
	0	1	0	0	1	0	1	0	0	1
	0	1	0	1	1	1	0	0	1	1
	0	1	1	0	1	1	1	0	0	1
	0	1	1	1	0	0	0	1	1	1
	1	0	0	0	1	1	1	1	1	1
	1	0	0	1	0	0	0	0	0	1
	1	0	1	0	0	0	1	0	1	1
	1	0	1	1	0	1	0	0	0	1
down	1	1	0	0	0	1	1	1	1	1
	1	1	0	1	1	0	0	0	0	1
	1	1	1	0	1	0	1	0	1	1
	1	1	1	1	1	1	0	0	0	1

$T_C = 1$.

$T_A =$

$MA_t \backslash B_tC_t$	00	01	11	10
00			1	
01			1	
11	1			
10	1			

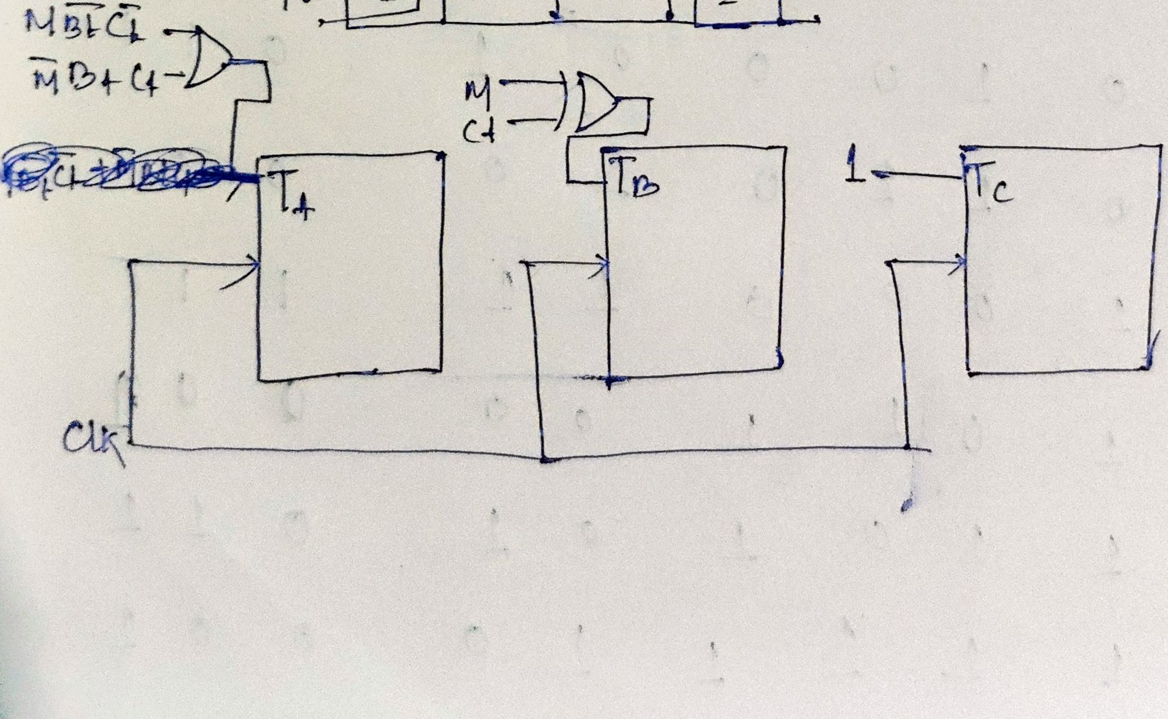
$= M\overline{B_t}\,\overline{C_t} + \overline{M}B_tC_t$

$T_B =$

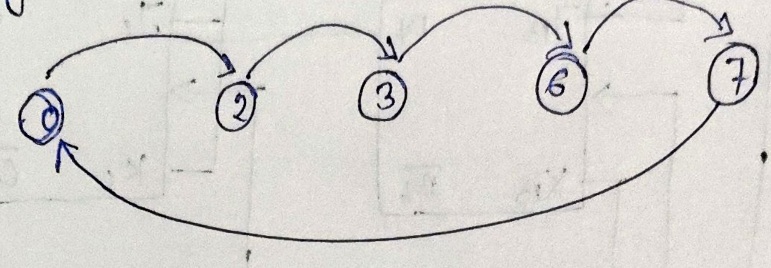
$MA_t \backslash B_tC_t$	00	01	11	10
00		1	1	
01		1	1	
11	1			1
10	1			1

$\overline{M}C_t + M\overline{C_t}$

$= M \oplus C_t$



Design a Synchronous counter using JK FF for the following State diagrams.



A_t	B_t	C_t	A_{t+1}	B_{t+1}	C_{t+1}	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	1	0	0	X	1	X	0	X
0	0	1	X	X	X	X	X	X	X	X	X
0	1	0	0	1	1	0	X	X	0	1	X
0	1	1	1	1	0	1	X	X	0	X	1
1	0	0	X	X	X	X	X	X	X	X	X
1	0	1	X	X	X	X	X	X	X	X	X
1	1	0	1	1	1	X	0	X	0	1	X
1	1	1	0	0	0	X	1	X	1	X	1

$J_B = 1$, $K_C = 1$.

$J_A =$

$A_t \backslash B_tC_t$	00	01	11	10
0		X	1	
1	X	X	X	X

$= C_t$

$K_A =$

$A_t \backslash B_tC_t$	00	01	11	10
0	X	X	X	X
1	X	X	1	

$= C_t$

$K_B =$

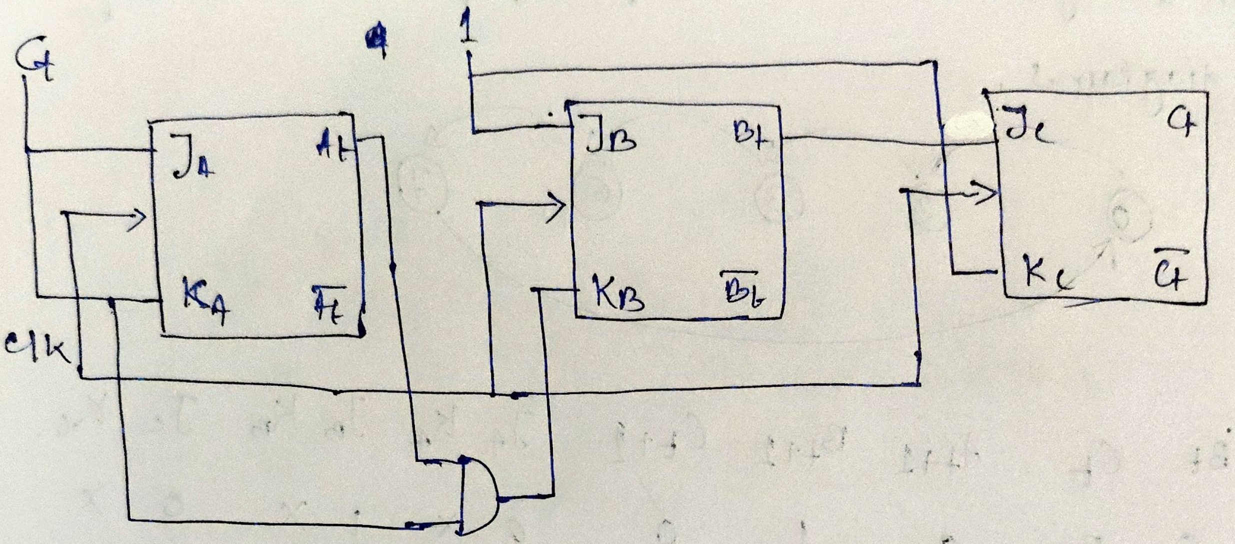
$A_t \backslash B_tC_t$	00	01	11	10
0	X	X		
1	X	X	1	1

$= A_t \overline{C_t}$

$J_C =$

$A_t \backslash B_tC_t$	00	01	11	10
0		X	X	1
1	X	X	X	1

$= B_t$



dt - 23 - 12 - 25.

design combinational ckt w/ 4 i/p and 4 o/p. Input represents Decimal digit in BCD and output is the 9's complement of the input.

BCD → Binary Coded Decimal.

→ Here each decimal digit will be represented by its 4 bit binary numbers.

g. 26 ⟶ 00100110.

Design a combinational ckt which converts binary code to Gray code.

Binary code				Gray code			
B_3	B_2	B_1	B_0	G_3	G_2	G_1	G_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	0	1	1	1	1
1	0	1	1	1	1	1	0
1	1	0	0	1	0	1	0
1	1	0	1	1	0	1	1
1	1	1	0	1	0	0	1
1	1	1	1	1	0	0	0