ITER, SOA(Deemed to be) University, Bhubaneswar

MCA I^{st} Semester
Assignment 2, October 2025
Subject: Discrete Mathematics (MA 3001)
Sections: 25C2A1, 25C2A2, 25C2B1, & 25C2B2

Answer all questions

2.1 Sets

1. Suppose that $A = \{2, 4, 6\}$, $B = \{2, 6\}$, $C = \{4, 6\}$, and $D = \{4, 6, 8\}$. Determine which of these sets are subsets of which other of these sets.

2. Determine whether each of these statements is true or false.
   (a) $\{0\} \in \{0\}$
   (b) $\phi \subset \{0\}$
   (c) $\{\phi\} \subset \{\phi, \{\phi\}\}$
   (d) $\{\{\phi\}\} \subset \{\{\phi\}, \{\phi\}\}$
   (e) $\{x\} \in \{x\}$
   (f) $\{x\} \subset \{\{x\}\}$

3. Use a Venn diagram to illustrate the relationship $A \subseteq B$ and $B \subseteq C$

4. Find the power set of $\{\phi, \{\phi\}\}$.

5. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$.

6. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find
   (a) $A \times B$
   (b) $B \times A$

7. Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find
   (a) $A \times B \times C$
   (b) $C \times B \times A$
   (c) $C \times A \times B$
   (d) $B \times B \times B$

8. Find A^2 and A^3 if $A = \{0, 1, 3\}$.

9. Use a Venn diagram to illustrate the set of all months of the year whose names do not contain the letter R in the set of all months of the year.

10. Translate each of these quantifications into english and determine its truth value.

(a) $\forall x \in \mathbb{R}(x^2 \neq -1)$

(b) $\exists x \in \mathbb{Z}(x^2 = 2)$

(c) $\forall x \in \mathbb{Z}(x^2 \in \mathbb{Z})$

(d) $\exists x \in \mathbb{R}(x^3 = -1)$

2.2 Set Operations

11. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find
    (a) $A \bigcap B$
    (b) $A \bigcup B$
    (c) $A - B$
    (d) $B - A$
    (e) $A \bigoplus B$

12. Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$ and $A \bigcap B - \{3, 6, 9\}$.

13. Prove the first and second De Morgan law using set builder form.

14. Show that if A and B are sets, then
    (a) $A - B = A \bigcap \overline{B}$.
    (b) $(A \bigcap B) \bigcup (A \bigcap \overline{B}) = A$.

15. Let $A_i = \{1, 2, 3, \cdots i\}$ for $i = 1, 2, 3, \cdots$. Find
    (a) $\bigcup_{i=1}^{n} A_i$
    (b) $\bigcap_{i=1}^{n} A_i$

2.3 Functions

16. Find these values.
    (a) $\lfloor -0.1 \rfloor$
    (b) $\lceil -2.99 \rceil$
    (c) $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$
    (d) $\lceil \lfloor \frac{1}{2} \rfloor + \lceil \frac{1}{2} \rceil + \frac{1}{2} \rceil$
    (e) $\lfloor \frac{1}{2} \cdot \lfloor \frac{5}{2} \rfloor \rfloor$

17. Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if $f(m, n) = m + n$ and $f(m, n) = m^2 + n^2$. Justify your answer.

18. Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$, if $f(x) = 2x+1$ and $f(x) = (x^2 + 1)/(x^2 + 2)$. Justify.

19. Let $f(x) = \lfloor \frac{x^2}{3} \rfloor$. Find $f(S)$ if $S = \{1, 5, 7, 11\}$.

20. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from $\mathbb{R}$ to $\mathbb{R}$.

21. Show that if x is a real number, then $x-1 < \lfloor x \rfloor \leqslant x \leqslant \lceil x \rceil < x+1$.

22. Draw graphs of the function $f(x) = \lfloor x + \frac{1}{2} \rfloor$.

2.5 Sequence amd Summation

23. Find a_6 of the sequence $\{a_n\} = 2 \cdot (-3)^n + 5^n$.

24. Find the first five terms of the sequence $a_n = na_{n-1} + n^2 a_{n-2}, a_0 = 1, a_1 = 1$.

25. Check whether the following sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if
    (a) $a_n = 0$
    (b) $a_n = 2(-4)^n + 3$

26. Compute
    (a) $\sum_{i=1}^{10} 3$
    (b) $\sum_{j \in S} 1$, where $S = \{1, 3, 5, 7\}$
    (c) $\sum_{j=2}^{8} (-3)^j$
    (d) $\sum_{j=0}^{8} (2 \cdot 3^j + 3 \cdot 2^j)$
    (e) $\sum_{i=1}^{2} \sum_{j=1}^{3} (i+j)$
    (f) $\sum_{i=0}^{3} \sum_{j=0}^{2} (3i+2j)$

27. Find $\sum_{k=99}^{200} k^2(k-3)$.

2.7 Matrices

28. Find $A+B$, where $A = \begin{bmatrix} -1 & 0 & 5 & 6 \\ -4 & -3 & 5 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 9 & -3 & 4 \\ 0 & -2 & -1 & 2 \end{bmatrix}$

29. Find AB, if $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$.

30. Find a matrix A such that

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 1 \\ 4 & 0 & 3 \end{bmatrix} A = \begin{bmatrix} 7 & 1 & 3 \\ 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}.$$

[Hint: Finding A requires that you solve systems of linear equations.]

31. Show that $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix}$.

32. Let $A = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$. Find

(a) Find A^{-1}.

(b) Find A^3.

(c) Find $(A^{-1})^3$.

33. Solve the system

$$\begin{aligned} 7x_1 - 8x_2 + 5x_3 &= 5 \\ -4x_1 + 5x_2 - 3x_3 &= -3 \\ x_1 - x_2 + x_3 &= 0 \end{aligned}$$

34. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find

(a) $A \vee B$

(b) $A \wedge B$

(c) $A \odot B$

(d) $A^{[3]}$