(c) How many permutations of the letters "ABCDEFGH" contain the string "ABC". 2

9. (a) How many different strings can be made from the letters in "MISSISSIPPI". 2

(b) What is the expansion of $(x+y)^5$. 2

(c) Draw the Hasse diagram for the POSET $(\{2,4,5,10,12,20,25\},|)$. 2

10 (a) Check whether $(\mathbb{R},\leq)$ is a POSET or not? 2

(b) Suppose that R is the relation on the set of strings of English letters such that aRb iff $l(a)=l(b)$, where $l(x)$ is the length of the string x. Is R an equivalence relation? 2

(c) Draw the directed graph of the relation R on the set $A=\{1,2,3,4\}$, where $R=\{(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,3),(4,1),(4,3)\}$ 2

End of Questions

END-SEMESTER EXAMINATION, February-2025
Discrete Mathematics (MA 3001)

Programme: MCA **Semester: 1st**
Full Marks: 60 **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Understand and apply rules of logic to distinguish between valid and invalid arguments and use them to prove mathematical statements.	L2, L3	1(a, b, c) 2(a, b, c) 3(a, b)	14
Comprehend sets, their various operations and use them to analyze functions and its various concepts as well as study sequences and summations	L3, L4	3(c), 4(a, b, c) 5(a)	12
Analyze the searching and sorting algorithms and use the growth of functions to study the time complexity of algorithms as well as apply some of the important concepts of number theory to divisibility and modular arithmetic, integer representation of algorithms, congruence and cryptography	L3, L4, L5	5(b, c), 6(a, b, c), 7(a, b)	14
Construct proofs by mathematical induction and formulate recursive definitions and develop structural induction	L4, L5	7(c), 8(a, b)	6
Apply different counting techniques to solve various problems.	L3, L4	8(c), 9(a, b)	6
Implement relations and their properties to analyze equivalence relations and partial orderings	L4, L5	9(c) 10(a, b, c)	8

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1. (a) Determine whether these statements are true or false. 2

(i) $1+1=2$ **if and only if** $2+3=4$

(ii) **If** $1+1=3$, **then** $2+2=4$.

(b) Construct a truth table for the compound proposition $(p \vee q) \rightarrow (p \oplus q)$. 2

(c) Evaluate the expressions $(01111 \wedge 10101) \vee 01000$. 2

2\. (a) Prove or disprove that the given conditional statement is a tautology. 2

$$\neg(p \rightarrow q) \rightarrow \neg q$$

(b) Determine the truth value of $\exists x(x^4 < x^2)$ if the domain consists of all real numbers. 2

(c) What rules of inferences is used in each of these arguments? 2
"If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today."

3\. (a) Give a proof by contradiction of the theorem "If $3n+2$ is odd, then n is odd." 2

(b) Solve the system 2

$$7x_1 - 8x_2 + 5x_3 = 5,$$
$$-4x_1 + 5x_2 - 3x_3 = -3,$$
$$x_1 - x_2 + x_3 = 0.$$

(c) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 2

Find $A^{[3]}$.

4\. (a) Find $\sum_{k=50}^{100} k^2$. 2

(b) Check whether the following sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if $a_n = 2(-4)^n + 3$. 2

(c) Determine whether the function $f: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ is one-one and onto if $f(m,n) = m+n$. Justify your answer. 2

5\. (a) Use bubble sort to sort 6, 9, 8, 11, 4, 5 showing the lists obtained at each step. 2

(b) Show that x^3 is $O(x^4)$ but that x^4 is not $O(x^3)$. 2

(c) What is the worst-case complexity of the binary search in terms of the number of comparisons made? 2

6\. (a) Evaluate $(99^2 \bmod 32)^3 \bmod 15$. 2

(b) Find the inverse of 13 *mod* 2436 2

(c) Use CRT to find the solution of the system 2

$$x \equiv 2 \bmod 3$$
$$x \equiv 1 \bmod 4$$
$$x \equiv 3 \bmod 5$$

7\. (a) Decrypt the message "$RKNZ\ \ BCRE$" for $key = 13$. 2

(b) Find $f(2), f(3), f(4), f(5)$, if f is defined recursively by $f(n+1) = f(n) + 2f(n)f(n+1) + 4f(n+1)$ where $f(0) = 1$ and $f(1) = 3$. 2

(c) Use mathematical induction to show that 2

$$1 \cdot 2 + 2 \cdot 3 + 3.4 + \cdots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}$$

8\. (a) Prove that every amount of postage stamp of 12 cents or more can be formed using just 4 cents and 5 cents stamps. 2

(b) Give the recursive definitions of the sequence $a_n = 1 + (-1)^n$, for $n \geq 1$. 2