ITER, SOA (Deemed to be) University, Bhubaneswar

MCA I^{st} Semester
Assignment 6, December 2025
Subject: Discrete Mathematics (MA 3001)
Sections: 25C2A1, 25C2A2, 25C2B1, & 25C2B2

Answer all questions

6.1 The Basics of Counting

1. An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?
2. A multiple choice test contains 10 questions. There are four possible answers for each question.
   (a) In how many ways can a student answer the questions on the test if the student answer every question?
   (b) In how many ways can a student answer the questions on the test if the student can leave answers blank?
3. How many different three letter initials can people have?
4. How many bit strings of length 10 both begin and end with a 1?
5. How many positive integers between 50 and 100
   (a) are divisible by 7? Which integers are these?
   (b) are divisible by 11? Which integers are these?
   (c) are divisible by 3 and by 4? Which integers are these?

6.2 The Pigeonhole Principle

6. Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.
7. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.
   (a) How many socks must he take out to be sure that he has at least two socks of the same color?
   (b) How many socks must he take out to be sure that he has at least two black socks?
8. A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

(a) How many balls must she select to be sure of having at least three balls of the same color?

(b) How many balls must she select to be sure of having at least three blue balls?

6.3 Permutations and Combinations

9. Suppose that there are eight runners in a race. The winner receives a gold medal, the second-place finisher receives a silver medal and the third-place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties?

10. How many permutations of the letters $ABCDEFGHIJ$ contain the string BA and FGH?

11. The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contains

    (a) exactly one vowel?

    (b) exactly two vowels?

    (c) at least one vowel?

    (d) at least two vowels?

12. How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [Hint: First position the men and then consider possible positions for the women.]

13. Thirteen people on a softball team show up for a game.

    (a) How many ways are there to choose 10 players to take the field?

    (b) How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?

    (c) Of the 13 people who show up, there are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a women?

6.4 Binomial Coefficients

14. Find the expansion of

    (a) $(x+y)^5$

    (b) $(x-y)^6$

    (c) $(3x-y^2)^4$

    (d) $(3x^4-2y^3)^5$

    (e) $(x+\frac{1}{x})^7$

15. Find the coefficient of x^5y^8 in $(x+y)^{13}$.

16. How many terms are there in the expansion of $(x+y)^{100}$ after like terms are collected?

17. What is the coefficient of x^7 in $(1+x)^{11}$?

18. What is the coefficient of x^8y^9 in the expansion of $(3x+2y)^{17}$?

19. What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x-3y)^{200}$?

20. Use the binomial theorem to find the coefficient of x^ay^b in the expansion of $(2x^3-4y^2)^7$ where
    (a) $a=9, b=8$
    (b) $a=12, b=6$
    (c) $a=0, b=14$

6.5 Generalized Permutations and Combinations

21. How many ways are there to choose eight coins from a piggy bank containing 100 identical pennies and 80 identical nickels?

22. Suppose that a large family has 14 children, including two sets of identical triplets, three sets of identical twins, and two individual children. How many ways are there to seat these children in a row of chairs if the identical triplets or twins cannot be distinguished from one another?

23. How many different strings can be made from the letters in MISSISSIPPI, using all the letters?

24. How many ways are there to deal hands of seven cards to each of five players from a standard deck of 52 cards?

25. Find n if
    (a) $P(n,4) = 12P(n,2)$.
    (b) $C(n,5) = C(n,2)$.
    (c) $C(n,3) = P(n,2)$.