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END-SEMESTER EXAMINATION, February-2024
Discrete Mathematics (MA 3001)

Programme: M.C.A. **Semester:1st**
Full Marks: 60 **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
To get introduced to the basic concepts of Discrete Mathematics such as set theory, relations and functions, propositional logic and Decision problems of propositional logic. Ability to apply the knowledge for modelling and solving the problem in computing sciences.	L1, L2, L3, L4	1, 2, 7, 8	24
To get introduced to Induction and recursion	L1, L2, L5	4	6
To get introduced to Algorithms, The Growth of Functions, Complexity of Algorithms, Divisibility and Modular Arithmetic, Integer Representations and Algorithms, Primes and Greatest Common Divisors, Solving Congruences, Applications of Congruences, Cryptography and their applications	L2	3, 9, 10	18
To understand the principle of counting techniques such as mutual inclusive exclusive principle, pigeonhole principle, permutations and combinations and summation. Ability to apply them to real life counting problems. Ability to write recurrence relations and generating functions.	L2, L3	5, 6	12

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1. (a) Show that if n is an integer and $n^3 + 5$ is odd, then n is even using 2
   I. a proof by contraposition
   II. a proof by contradiction

   (b) Translate each of these statements into logical expressions using predicates, quantifiers and logical connectives. 2

   I. No one is perfect.

II. All your friends are perfect.

(c) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. 2

2. (a) Determine whether f is function from Z to R if 2
   I. $f(n) = \pm n$.
   II. $f(n) = \sqrt{n^2+1}$.
   III. $f(n) = 1/(n^2-4)$.

(b) Determine whether each of these functions from Z to Z is one-to-one and onto 2
   I. $f(n) = n^2 + 1$.
   II. $f(n) = n^3$.

(c) Find $\sum_{k=50}^{100} k^2$ 2

3. (a) State and prove the Chinese remainder theorem. 2

(b) I. Solve $144x \equiv 4 (mod\ 233)$ 2
   II. Show that 937 is an inverse of 13 modulo 2436

(c) Encrypt the message **ATTACK** using the RSA system with n=53*61 and e=17. 2

4. (a) Conjecture a formula for the sum of the first n positive odd integers. Then prove your conjecture using mathematical induction. 2

(b) Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps. 2

(c) Give a recursive definition of the sequence $\{a_n\}, n = 1,2,3, \ldots$if 2
   I. $a_n = 6n$.
   II. $a_n = 10^n$.

5. (a) How many positive integers between 1000 and 9999 inclusive 2
   I. have distinct digits?
   II. are not divisible by either 5 or 7?

(b) I. How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are selected? 2
   II. How many must be selected from a standard deck of 52 cards to guarantee that at least three hearts are selected?

(c) A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes 2
   I. are there in total?
   II. contain exactly three heads?
   III. contain at least three heads?

6. (a) Find the coefficient of $x^{12}y^{13}$ in the following expressions: 2
   I. $(x+y)^{25}$
   II. $(2x-3y)^{25}$

(b) Find the sum value for the following: 2
   I. $\sum_{k=0}^{n} C(n,k)$
   II. $\sum_{k=0}^{n} (-1)^k C(n,k)$

(c) How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? 2

7. (a) Prove that the relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, \ldots$. 2

(b) Determine whether the relation R on a set of all people is reflexive, symmetric, antisymmetric, and transitive, where $(a,b) \in R$ if and only if 2
   I. a is taller that b.
   II. a has the same first name as b.

(c) Find the matrix represent each of the following relations on {1,2,3}. 2
   I. $\{(1,1),(1,2),(1,3)\}$
   II. $\{(1,2),(2,1),(2,2),(2,3)\}$
   III. $\{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$
   IV. $\{(1,3),(3,1)\}$

8. (a) How many non-zero entries does the matrix representing the relation R on $A = \{1,2,3,\ldots,1000\}$ have if R is 2
   I. $\{(a,b) | a > b\}$
   II. $\{(a,b) | a = b+2\}$

(b) Let $M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ be matrix representation of relation R_1 and R_2, respectively. Find $R_1 \cap R_2$ and $R_1 \cup R_2$. 2

(c) Draw the Hess diagram for the partial ordering $\{(A,B) | A \subseteq B\}$ on the power set $\mathcal{P}(S)$, where $S = \{a,b,c\}$. 2

9. (a) Show the step-by-step process to search for 19 in the list $1,2,3,5,6,7,8,10,12,13,15,16,18,19,20,22$ by using Binary Search Algorithm. 2

(b) Use the bubble sort to put 6, 2, 3, 1, 5, 4 into increasing order. 2

(c) Use the insertion sort to put the elements of the list 3, 2, 4, 1, 5 in increasing order. 2

10 (a) Show that 2
   I. $x^2 + 2x + 1$ is $O(x^2)$
   II. $7x^2$ is $O(x^3)$

(b) Show that 2
   I. $8x^3 + 5x^2 + 7$ is $\Omega(x^3)$
   II. $3x^2 + 8xlog(x)$ is $\Theta(x^2)$

(c) Describe the time complexity of the linear search algorithm. 2

End of Questions