MID-SEMESTER EXAMINATION, December-2023
Discrete Mathematics (MA 3001)

Programme: MCA **Semester:1st**
Full Marks: 30 **Time: 2 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
To get introduced to the basic concepts of Discrete Mathematics such as set theory, relations and functions, propositional logic and Decision problems of propositional logic. Ability to apply the knowledge for modelling and solving the problem in computing sciences.	L1, L2, L3, L4	1, 2, 3, 5	24
To get introduced to Induction and recursion	L1, L2, L5	4	6
To get introduced to Algorithms, The Growth of Functions, Complexity of Algorithms, Divisibility and Modular Arithmetic, Integer Representations and Algorithms, Primes and Greatest Common Divisors, Solving Congruences, Applications of Congruences, Cryptography and their applications	L2		
To understand the principle of counting techniques such as mutual inclusive exclusive principle, pigeonhole principle, permutations and combinations and summation. Ability to apply them to real life counting problems. Ability to write recurrence relations and generating functions.	L2, L3		

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

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Answer all questions. Each question carries equal mark.

1. (a) Show that p ↔ q ≡ (p → q) ∧ (q → p) (Using truth table). 2

(b) State the inverse and contrapositive statements of the given statement. 2
If I come to class then there is going to be a quiz.

(c) Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology 2

2. (a) Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people. 2
I. $\forall x(C(x) \rightarrow F(x))$
II. $\exists x(C(x) \wedge F(x))$

(b) Translate each of these statement into logical expressions using predicates, quantifiers, and logical connectives. 2
I. Something is not in the correct place
II. One of your tool is not in the correct place, but it is in excellent condition.

(c) I. Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent. Justify your answer. 2
II. Find the truth value of $\forall x \exists y(x + y = 0)$ *and* $\exists y \forall x(x + y = 0)$ where the domain for each variable is set of real numbers.

3. (a) Let $A_i = \{\dots, -2, -1, 0, 1, 2, \dots, i\}$ for $i = 1, 2, 3, \dots$. Find 2

I. $\bigcup_{i=1}^{n+1} A_i$

II. $\bigcap_{i=3}^{n} A_i$

(b) Given sets A and B in a universe U, draw the Venn diagrams of each of these sets. 2
I. $A \longrightarrow B = \{x \in U \mid x \in A \longrightarrow x \in B\}$
II. $A \leftrightarrow B = \{x \in U \mid x \in A \leftrightarrow x \in B\}$

(c) I. Let f(x) = ax + b and g(x) = cx + d, where a, b, c, and d are constant. Determine necessary and sufficient conditions on a, b, c, and d so that $f \circ g = g \circ f$ 2
II. Show that the function f(x) = ax + b from **R** to **R** where a and b are constant with a≠0 is invertible, and find the inverse of f.

4. (a) Use mathematical induction to prove the following generalization of one of De Morgan's laws: 2

$$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$$

whenever $A_1, A_2, \dots, A_n$ are subsets of a universal set U and $n \geq 2$.

(b) Conjecture a formula for the sum of the first n positive odd integers. Then prove your conjecture using mathematical induction. 2

(c) Give a recursive definition of the sequence $\{a_n\}, n = 1, 2, 3, \dots$ if 2

I. $a_n = (4n - 2)$

II. $a_n = 10^n$

III. $a_n = n(n + 1)$

IV. $a_n = n^2$

5. (a) State and proof of Chinese remainder theorem. 2

(b) Solve the following linear congruence $19x \equiv 4 \pmod{141}$ 2

(c) Find $11^{644} \bmod 645$ 2

End of Questions