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MID-SEMESTER EXAMINATION, December-2024
Discrete Mathematics (MA 3001)

Program: MCA **Semester:1st**
Full Marks: 30 **Time: 2 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Understand and apply rules of logic to distinguish between valid and invalid arguments and use them to prove mathematical statements.	L2, L3	1(a), 1(b), 1(c), 2(a), 2(b), 2(c)	12
Comprehend sets, their various operations and use them to analyze functions and its various concepts as well as study sequences and summations.	L3, L4	3(a), 3(b), 3(c), 4(a), 4(b), 4(c)	12
Analyze the searching and sorting algorithms and use the growth of functions to study the time complexity of algorithms as well as apply some of the important concepts of number theory to divisibility and modular arithmetic, integer representation of algorithms, congruence and cryptography.	L3, L4. L5	5(a), 5(b) 5(c)	6
Construct proofs by mathematical induction and formulate recursive definitions and develop structural induction.	L3, L4. L5	-	-
Apply different counting techniques to solve various problems.	L4, L5	-	-
Implement relations and their properties to analyze equivalence relations and partial orderings.	L3, L4. L5	-	-

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1. (a) Write the negation of the following statement. 2

"No student in your class has taken a course in logic programming."

(b) Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent. 2

(c) Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd. 2

2. (a) Use the rules of inference to prove the following argument. 2

"There is someone in this class who has been to France." "Everyone who goes to France visits Paris." Therefore, *"someone in this class has visited Paris."*

(b) State the converse, contrapositive and inverse of the given conditional statement. 2

"The home team wins whenever it is raining."

(c) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\sim Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \sim S(x))$, and $\exists x(\sim P(x))$ are true, then $\exists x(\sim R(x))$ is true. 2

3. (a) Show that for two sets A and B, $A - B = A \cap \overline{B}$ 2

(b) Evaluate 2

$$\sum_{i=0}^{3}\sum_{j=0}^{2}(3i+2j)$$

(c) Let 2

$$A=\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \text{ and } B=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 0 & 1\end{bmatrix}.$$

Find Boolean product, $A \odot B$.

4. (a) Determine whether the function $f:\mathbb{Z}\times\mathbb{Z}\rightarrow\mathbb{Z}$ is one-one and onto if $f(m,n)=m^2+n^2$. Justify your answer. 2

(b) Let $fog = gof$, where $f(x)=ax+b$ and $g(x)=cx+d$, where $a,b,c\,\&\,d$ are constants. Determine necessary and sufficient conditions on the constants so that the above condition satisfies. 2

(c) Let $f(x) = \left\lceil \frac{x}{5} \right\rceil$ Find $f(S)$ if $S = \{-1,0,2,4,7\}$. 2

5. (a) Use bubble sort to sort *d, f, k, m, a, b,* (as arrange in English alphabets) showing the lists obtained at each step. 2

(b) Find the least integer n such that $f(x)$ is $O(x^n)$ for $f(x)=(x+1)log\,(x^2+1)+3x^2$. 2

(c) Describe the time complexity of the Binary search algorithm. 2

End of Questions