Your notes

Edexcel A Level Maths: Pure

4.3 Arithmetic Sequences & Series

Contents

4.3.1 Arithmetic Sequences

Arithmetic Sequences

What do I need to know about arithmetic sequences?

- In an **arithmetic sequence**, the difference between consecutive terms in the sequence is constant

- That constant difference is known as the **common difference** of the sequence
- You need to know the nth **term formula** for an arithmetic sequence

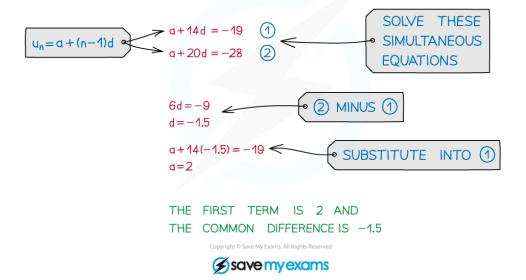$$u_n = a + (n-1)d$$

- *a* is the first term
- *d* is the common difference

Your notes

💡 Examiner Tip

If you know two terms in an arithmetic sequence you can find **a** and **d** using simultaneous equations.

e.g. THE 15th TERM OF AN ARITHMETIC SEQUENCE IS -19.
THE 21st TERM IS -28.
FIND THE FIRST TERM AND THE COMMON DIFFERENCE.

$$u_n = a + (n-1)d$$

$$a + 14d = -19 \quad ①$$
$$a + 20d = -28 \quad ②$$

SOLVE THESE SIMULTANEOUS EQUATIONS

$$6d = -9$$
$$d = -1.5$$

② MINUS ①

$$a + 14(-1.5) = -19$$
$$a = 2$$

SUBSTITUTE INTO ①

THE FIRST TERM IS 2 AND THE COMMON DIFFERENCE IS -1.5

Worked example

? The first three terms in an arithmetic sequence are $12, k^2, 5k, \ldots$

Find two possible values of k.

THIS IS AN ARITHMETIC SEQUENCE SO THERE'S A SINGLE COMMON DIFFERENCE

$d = k^2 - 12$ ← 2ⁿᵈ TERM MINUS 1ˢᵗ TERM

AND

$d = 5k - k^2$ ← 3ʳᵈ TERM MINUS 2ⁿᵈ TERM

SO $\quad k^2 - 12 = 5k - k^2$

$2k^2 - 5k - 12 = 0$

$(2k + 3)(k - 4) = 0$ ← SOLVE THE QUADRATIC

$k = -\dfrac{3}{2} \quad$ OR $\quad k = 4$

Your notes

4.3.2 Arithmetic Series

Arithmetic Series

How do I find the sum of an arithmetic series?

- An **arithmetic series** is the sum of the terms of an **arithmetic sequence**

1, 3, 5, 7, 9, ...   IS   AN   ARITHMETIC   SEQUENCE

1 + 3 + 5 + 7 + 9 + ...   IS   AN   ARITHMETIC   SERIES

- The following formulae will let you find the sum of the first **n** terms of an arithmetic series:

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right) \text{ or } S_n = \frac{n}{2}\left(a + l\right)$$

  - *a* is the first term
  - *d* is the common difference
  - *l* is the last term

- You can use whichever formula is more convenient for a given question
- The **a** and the **d** in those formulae are exactly the same as the ones used with **arithmetic sequences**

How do I derive the arithmetic series formula?

- Learn this proof of the arithmetic series formula – you can be asked to give it on the exam:
  - Write the terms out once in order
  - Write the terms out again in reverse order
  - Add the two sums together
    - The terms will pair up to give the same sum $2a + (n-1)d$
    - There will be *n* of these terms
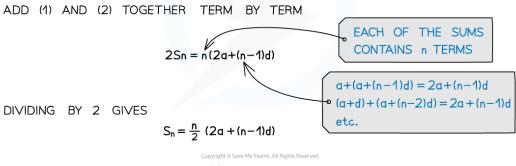 - Divide by two as two of the sums have been added together

THE SUM OF THE FIRST n TERMS IS

$$S_n = a + (a+d) + (a+2d) + \ldots + (a+(n-2)d) + (a+(n-1)d) \qquad (1)$$

NOW WRITE THE SAME SUM THE OTHER WAY ROUND

$$S_n = (a+(n-1)d) + (a+(n-2)d) + \ldots + (a+2d) + (a+d) + a \qquad (2)$$

ADD (1) AND (2) TOGETHER TERM BY TERM

$$2S_n = n(2a+(n-1)d)$$

EACH OF THE SUMS CONTAINS n TERMS

$$a+(a+(n-1)d) = 2a+(n-1)d$$
$$(a+d)+(a+(n-2)d) = 2a+(n-1)d$$
etc.

DIVIDING BY 2 GIVES

$$S_n = \frac{n}{2}(2a+(n-1)d)$$

> 💡 **Examiner Tip**
>
> The arithmetic series formulae are in the formulae booklet – you don't need to memorise them.
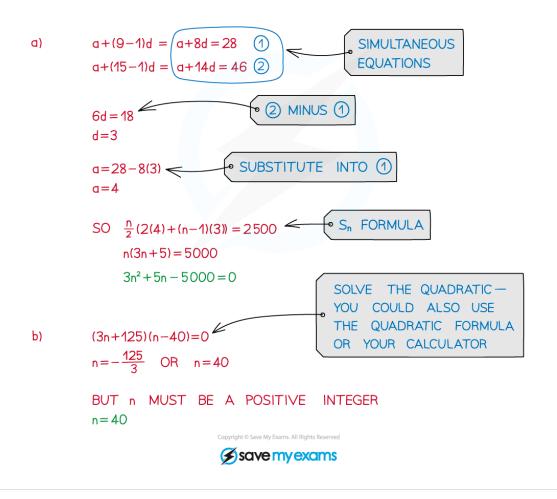
✏️ Worked example

> **❓** The ninth term of an arithmetic series is 28.
> The fifteenth term is 46. The sum of the first
> n terms is 2500.
>
> a) Show that $3n^2 + 5n - 5000 = 0$.
>
> b) Hence find the value of n.

a) $a + (9-1)d = \boxed{a + 8d = 28} \;①$
    $a + (15-1)d = \boxed{a + 14d = 46} \;②$

    SIMULTANEOUS EQUATIONS

    $6d = 18$ ② MINUS ①
    $d = 3$

    $a = 28 - 8(3)$ SUBSTITUTE INTO ①
    $a = 4$

    SO $\quad \dfrac{n}{2}(2(4) + (n-1)(3)) = 2500$ S_n FORMULA

    $\quad n(3n + 5) = 5000$

    $\quad 3n^2 + 5n - 5000 = 0$

    SOLVE THE QUADRATIC — YOU COULD ALSO USE THE QUADRATIC FORMULA OR YOUR CALCULATOR

b) $(3n + 125)(n - 40) = 0$

    $n = -\dfrac{125}{3} \quad$ OR $\quad n = 40$

    BUT n MUST BE A POSITIVE INTEGER
    $n = 40$