Edexcel A Level Maths: Pure

Your notes

5.7 Further Trigonometric Equations

Contents

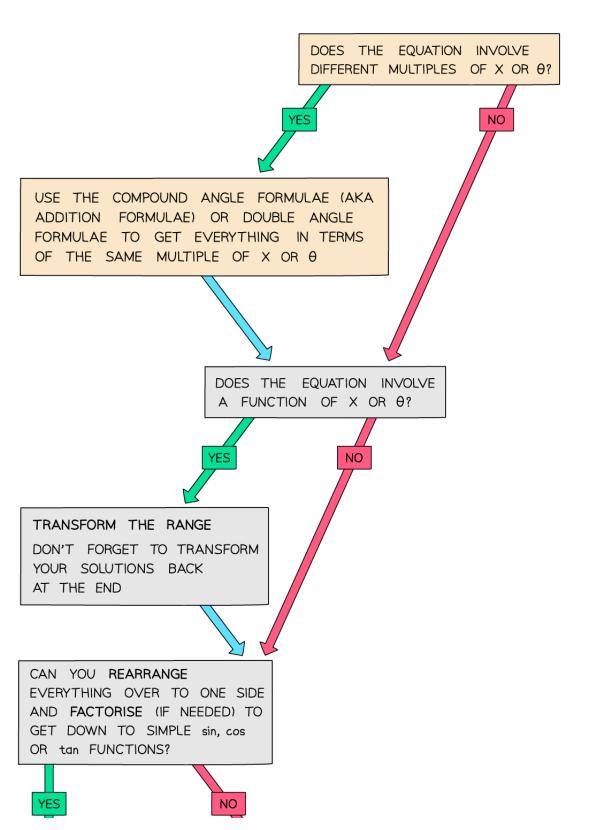# 5.7.1 Strategy for Further Trigonometric Equations

Your notes

Strategy for Further Trigonometric Equations

How to approach solving harder trig equations

- You can solve harder trig equations, such as those involving **reciprocal** and **inverse** functions in a variety of different ways
  - Using **further trigonometric identities**
  - Using **compound or double angle** formulas
  - Factorising **quadratic** trig equations
  - Then finding all solutions using **CAST** or **sketching graphs**
- The final rearranged equation you solve will involve **sin**, **cos** or **tan** – don't try to solve an equation with **cosec**, **sec**, or **cot** directly
- If you're having trouble solving a trig equation, this flowchart might help:

Your notes

DOES THE EQUATION INVOLVE DIFFERENT MULTIPLES OF X OR θ?

YES

NO

USE THE COMPOUND ANGLE FORMULAE (AKA ADDITION FORMULAE) OR DOUBLE ANGLE FORMULAE TO GET EVERYTHING IN TERMS OF THE SAME MULTIPLE OF X OR θ

DOES THE EQUATION INVOLVE A FUNCTION OF X OR θ?

YES

NO

TRANSFORM THE RANGE

DON'T FORGET TO TRANSFORM YOUR SOLUTIONS BACK AT THE END

CAN YOU **REARRANGE** EVERYTHING OVER TO ONE SIDE AND **FACTORISE** (IF NEEDED) TO GET DOWN TO SIMPLE sin, cos OR tan FUNCTIONS?

YES

NO

Your notes

DOES IT INVOLVE AN **IDENTITY**?

$\tan x \equiv \dfrac{\sin x}{\cos x}$ OR $\sin^2 x + \cos^2 x \equiv 1$

USE THESE TO CONVERT EVERYTHING INTO ONE TRIG FUNCTION

FURTHER IDENTITIES

$1 + \tan^2 x \equiv \sec^2 x$

$1 + \cot^2 x \equiv \operatorname{cosec}^2 x$

$\cot x \equiv \dfrac{1}{\tan x} \equiv \dfrac{\cos x}{\sin x}$

IS IT **LINEAR** OR **QUADRATIC**?

LINEAR

CAN I REARRANGE IT? IS THERE A COMMON FACTOR?

QUADRATIC

CAN I REARRANGE IT? DO I NEED TO FACTORISE? YOU SHOULD END UP WITH TWO SOLUTIONS

IF YOU HAVE SOLVED YOUR EQUATION FOR $\sec, \operatorname{cosec}$ OR $\cot$, USE THE IDENTITIES

$\sec x \equiv \dfrac{1}{\cos x}$ $\operatorname{cosec} x \equiv \dfrac{1}{\sin x}$ $\cot x \equiv \dfrac{1}{\tan x}$

TO CONVERT TO $\cos, \sin$ OR $\tan$ BEFORE TRYING TO SOLVE FOR X OR θ

DO ALL MY SOLUTIONS ACTUALLY EXIST?

REMEMBER SOLUTIONS TO $\sin x = k$ AND $\cos x = k$ ONLY EXIST WHERE $-1 \leqslant k \leqslant 1$ WHEREAS SOLUTIONS TO $\tan x = k$ EXIST FOR ALL VALUES OF k

FIND THE PRINCIPLE VALUE

Your notes

ARE SOLUTIONS WITHIN GIVEN RANGE?
DO I HAVE **ALL** SOLUTIONS?

YES

NO

USE **CAST**

$90°$

S   A

$180°$ —— $x°$ —— $0°$

T   C

$270°$

SKETCH THE GRAPH

USE YOUR KNOWLEDGE OF GRAPH SYMMETRIES AND PERIODICITY TO FIND ALL OTHER SOLUTIONS

$y = \cos x$

$y = \sin x$

$y = \tan x$

CHECK WHETHER YOU HAVE FOUND **ALL SOLUTIONS** AND THAT THEY ARE **WITHIN THE GIVEN RANGE**
(DON'T FORGET TO TRANSFORM YOUR SOLUTIONS BACK AT THE END IF THEY WERE FROM A TRANSFORMED RANGE)

💡 Examiner Tip

- Try to use identities and formulas to reduce the equation into its simplest terms.
- Don't forget to check the function range and ensure you have included all possible solutions.
- If the question involves a function of x or θ ensure you transform the range first (and ensure you transform your solutions back again at the end!).

✏️ Worked example

Your notes
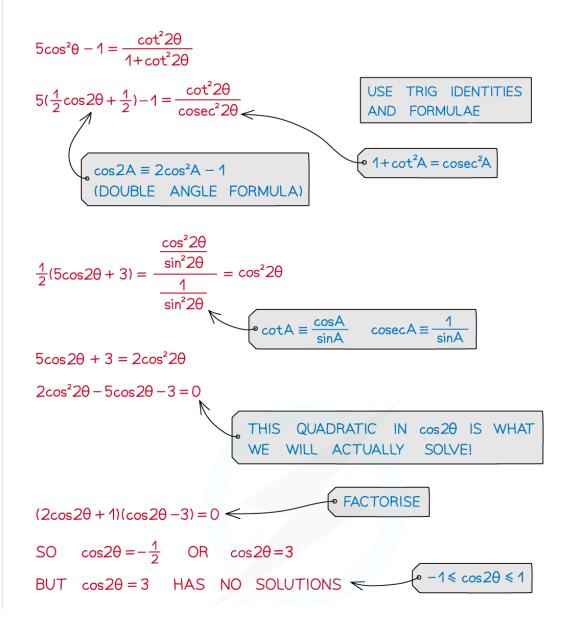
> ❓ Find the solutions for the equation
>
> $$\left(1 + \cot^2 2\theta\right)\left(5\cos^2\theta - 1\right) = \cot^2 2\theta$$
>
> in the interval $0 \le \theta < 360°$.

$$5\cos^2\theta - 1 = \frac{\cot^2 2\theta}{1 + \cot^2 2\theta}$$

$$5\left(\tfrac{1}{2}\cos 2\theta + \tfrac{1}{2}\right) - 1 = \frac{\cot^2 2\theta}{\text{cosec}^2 2\theta}$$

USE TRIG IDENTITIES AND FORMULAE

$\cos 2A \equiv 2\cos^2 A - 1$
(DOUBLE ANGLE FORMULA)

$1 + \cot^2 A = \text{cosec}^2 A$

$$\tfrac{1}{2}(5\cos 2\theta + 3) = \frac{\dfrac{\cos^2 2\theta}{\sin^2 2\theta}}{\dfrac{1}{\sin^2 2\theta}} = \cos^2 2\theta$$

$\cot A \equiv \dfrac{\cos A}{\sin A} \qquad \text{cosec}\, A \equiv \dfrac{1}{\sin A}$

$$5\cos 2\theta + 3 = 2\cos^2 2\theta$$

$$2\cos^2 2\theta - 5\cos 2\theta - 3 = 0$$

THIS QUADRATIC IN $\cos 2\theta$ IS WHAT WE WILL ACTUALLY SOLVE!

FACTORISE

$$(2\cos 2\theta + 1)(\cos 2\theta - 3) = 0$$

SO $\cos 2\theta = -\tfrac{1}{2}$ OR $\cos 2\theta = 3$

$-1 \le \cos 2\theta \le 1$

BUT $\cos 2\theta = 3$ HAS NO SOLUTIONS

SO WE WANT SOLUTIONS TO $\cos 2\theta = -\frac{1}{2}$

IN THE RANGE $0 \leq 2\theta < 720°$ ← **TRANSFORM THE RANGE**

$2\theta = \cos^{-1}(-\frac{1}{2}) = 120°$ ← **FIND PRINCIPAL VALUE**

OR $2\theta = 360 - 120 = 240°$ ← **USE CAST TO FIND OTHER VALUE IN RANGE $0 \leq 2\theta < 360°$**

SO $2\theta = 120°, 240°, 480°, 600°$ ← **ADD 360° TO FIND ADDITIONAL ANSWERS IN RANGE $360 \leq 2\theta < 720°$**

$\theta = 60°, 120°, 240°, 300°$ ← **CONVERT BACK TO θ**