Edexcel A Level Maths: Pure

Your notes

5.2 Trigonometric Functions

Contents

5.2.1 Graphs of Trigonometric Functions


Your notes

Graphs of Trigonometric Functions

Graphs of trigonometric functions

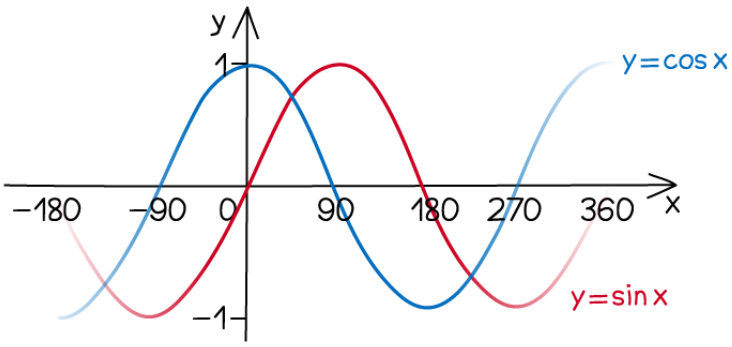
- The Trigonometric Functions Sin, Cos and Tan all have special periodic graphs that you need to be able to sketch and remember
- You'll need to know their properties and how to sketch them to solve equations and for transforming trig functions

Your notes

$y = \sin x$ AND $y = \cos x$

Sin x AND Cos x ARE ALWAYS IN THE RANGE −1 TO 1

Sin x PASSES THROUGH THE ORIGIN
Cos x PASSES THROUGH 1

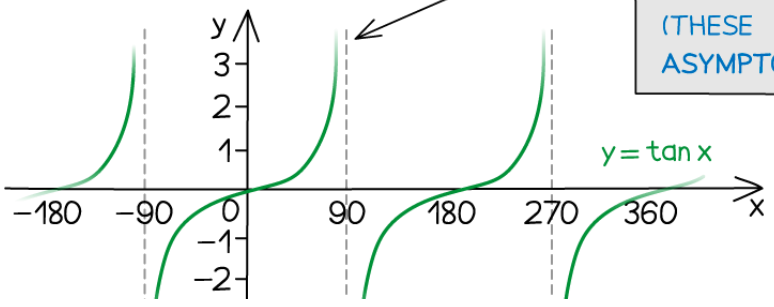


Sin x AND Cos x ARE PERIODIC REPEATING EVERY 360°

Sin x HAS ROTATIONAL SYMMETRY ABOUT THE ORIGIN SO $\sin(-x) = -\sin(x)$
Cos x IS SYMMETRICAL THROUGH THE y−AXIS SO $\cos(-x) = \cos(x)$

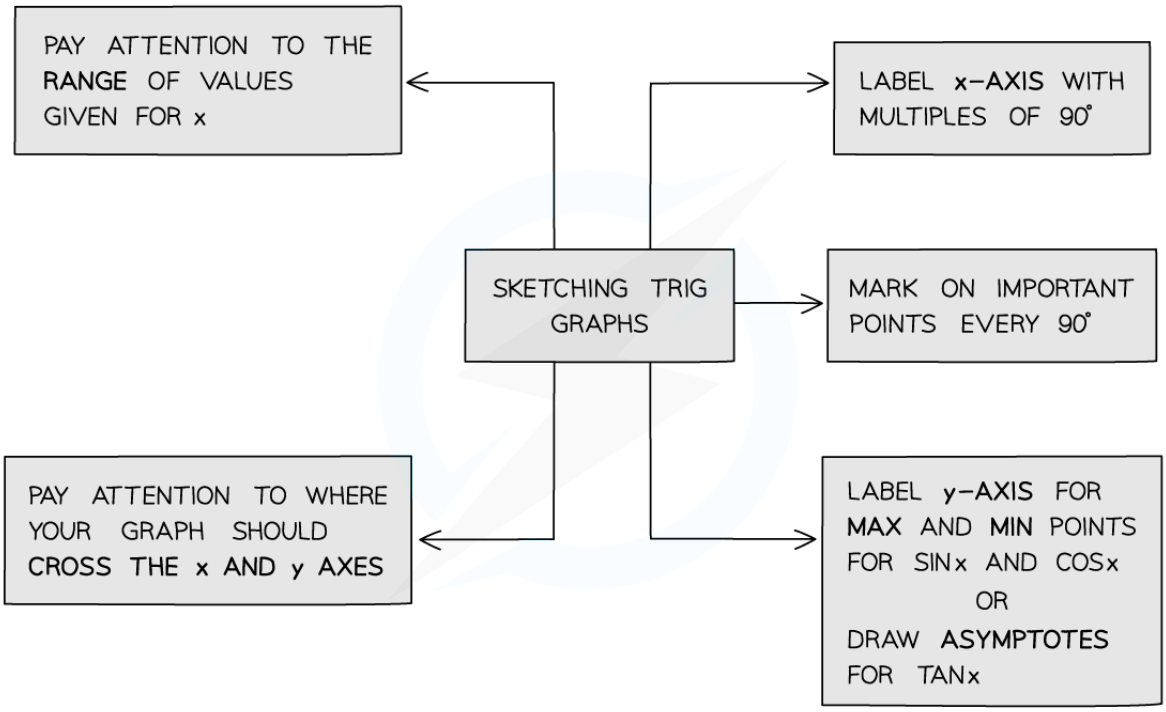
$y = \tan x$

Tan x IS UNDEFINED AT ±90°, ±270°, ±450°... MEANING IT RANGES FROM $-\infty$ TO $+\infty$ (THESE POINTS ARE CALLED ASYMPTOTES)



Your notes

Sketching trigonometric graphs



Using trigonometric graphs

- By sketching the graph you can read off all the solutions in a given range (or interval)
- Your calculator will only give you the principal value
- You should recognise any values/angles associated with **exact values**
- You should be able to spot the pattern of solutions using the **symmetry** and **periodicity** of the graph

Your notes

STEP 1: SKETCH THE GRAPH FOR THE GIVEN FUNCTION AND RANGE

STEP 2: DRAW A HORIZONTAL LINE TO SEE WHERE THE SOLUTIONS SHOULD BE, YOU MAY BE ABLE TO READ DIRECTLY OFF THE GRAPH TO FIND ALL OF THE SOLUTIONS

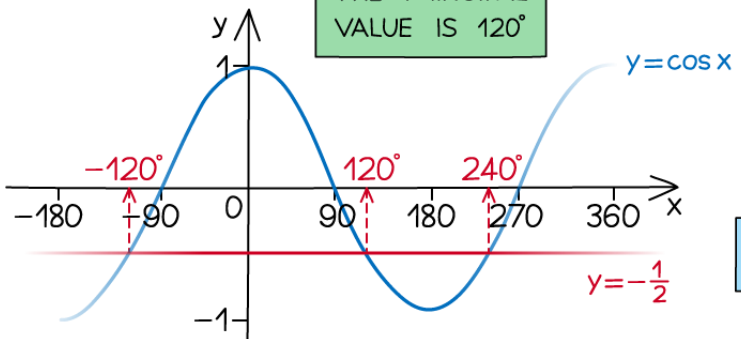
STEP 3: IF NOT, USE YOUR CALCULATOR TO FIND THE PRINCIPAL VALUE (YOU MAY ALREADY KNOW IT IF IT IS A TRIG EXACT VALUE) AND MARK THIS ON THE GRAPH

STEP 4: USE THE SYMMETRY OF THE GRAPH TO FIND ALL OTHER SOLUTIONS BY ADDING / SUBTRACTING FROM POINTS OF REPETITION WITHIN THE SPECIFIED RANGE

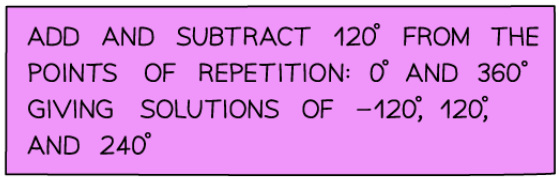
FOR THE SOLUTIONS TO THE EQUATION $\cos x = -\frac{1}{2}$ IN THE RANGE $-180^\circ \leqslant x \leqslant 360^\circ$

HERE THE FUNCTION IS $y = \cos x$ AND THE RANGE IS $-180^\circ \leqslant x \leqslant 360^\circ$

THE PRINCIPAL VALUE IS 120°



DRAW THE LINE $y = -\frac{1}{2}$



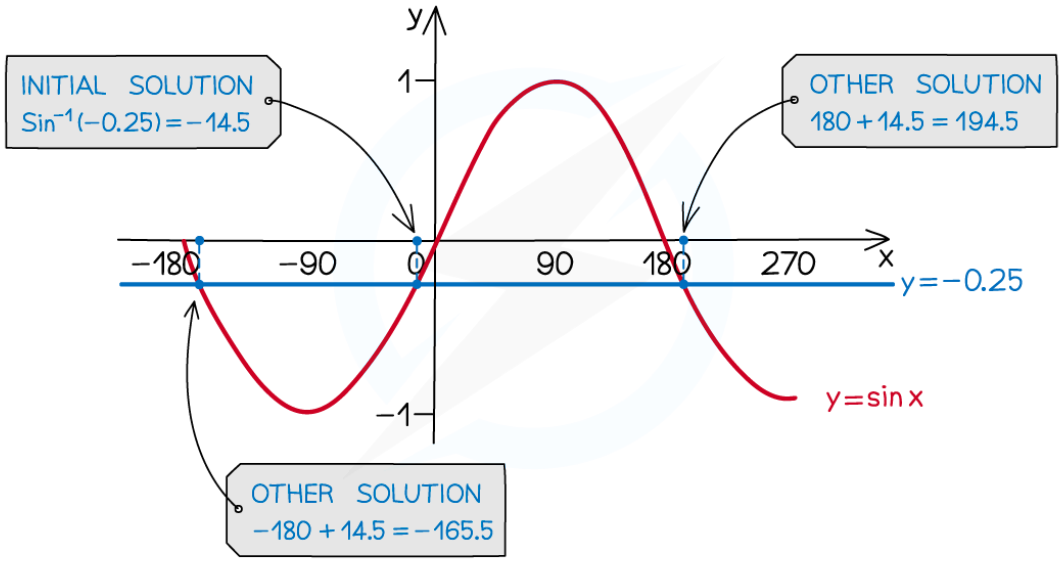
Your notes

Examiner Tip

- Always sketch with a pencil and draw a smooth curve and pay attention to the key features of each graph:
  - Where it crosses the x and y axes
  - How often it repeats
  - Whether it is symmetrical
- Remember, when answering exam questions that ask for solutions, a sketch will help ensure you give **all** the appropriate solutions for a given interval.

Worked example

By sketching a graph, find all the solutions to $\sin x = -0.25$ for $-180° \leq x \leq 270°$.



ALL SOLUTIONS ARE:
$x = -165.5$, $x = -14.5$ AND $x = 194.5$

Your notes

5.2.2 Transformations of Trigonometric Functions


Your notes

Transformations of Trigonometric Functions

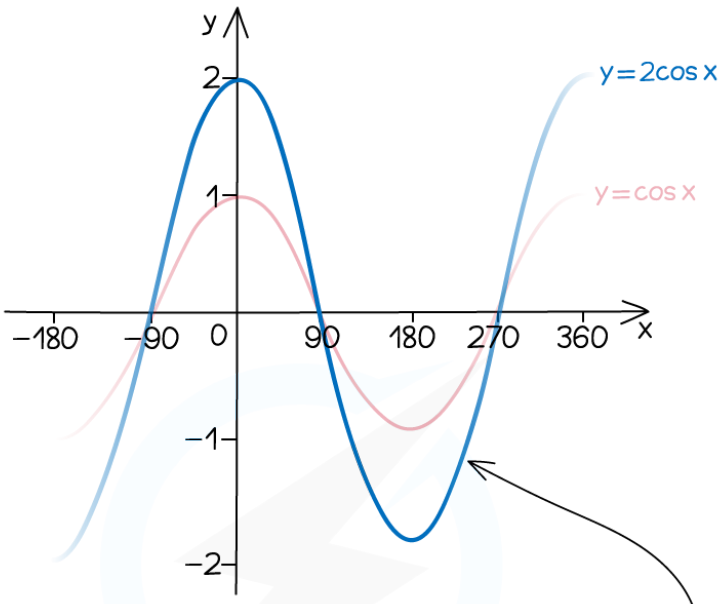
Transformations of trigonometric functions

- You should already have a good idea about **translating**, **stretching** and **reflecting** basic functions (see **Transformations of Functions**)
- The basic principles are exactly the same for transforming any trigonometric graph
  - $y = n \cos x$ gives a **vertical stretch or squash**
  - $y = \sin nx$ gives a **horizonal stretch or squash**
  - $y = \tan (x + c)$ gives a **horizonal translation**

Your notes

$y=n\cos x$ GIVES A VERTICAL STRETCH OR SQUASH

THE GRAPH OF $y=\cos x$ STRETCHED VERTICALLY BY A FACTOR OF 2 MAKING IT TWICE AS TALL



STEP 1: SKETCH THE ORIGINAL sin, cos OR tan FUNCTION
STEP 2: PICK KEY CO-ORDINATE POINTS TO PLOT THE NEW GRAPH
STEP 3: SKETCH IN THE NEW TRANSFORMED GRAPH

FOR A VERTICAL STRETCH (OR SQUASH)

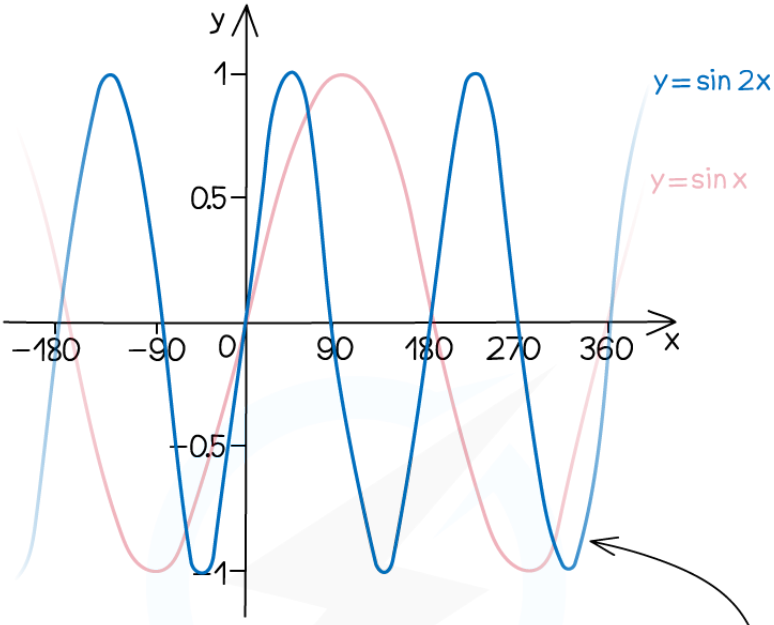
- IF $n>1$, THE GRAPH IS **STRETCHED VERTICALLY** BY A SCALE FACTOR OF n
- IF $0<n<1$, THE GRAPH IS **SQUASHED** BY THAT SCALE FACTOR
- IF $n<0$, THE GRAPH IS ALSO **REFLECTED** IN THE x-AXIS
- REMEMBER THE x COORDINATES STAY THE SAME; y COORDINATES ARE MULTIPLIED BY n

Your notes

Your notes

$y = \sin nx$ GIVES A HORIZONTAL STRETCH OR SQUASH

THE GRAPH OF $y = \sin x$ SQUASHED HORIZONTALLY BY A FACTOR OF 2 MEANING IT REPEATS ITSELF TWICE EVERY 360°



STEP 1: SKETCH THE ORIGINAL sin, cos OR tan FUNCTION
STEP 2: PICK KEY CO-ORDINATE POINTS TO PLOT THE NEW GRAPH
STEP 3: SKETCH IN THE NEW TRANSFORMED GRAPH

FOR A HORIZONTAL STRETCH (OR SQUASH)

- IF $n>1$, THE GRAPH IS **SQUASHED** HORIZONTALLY BY A SCALE FACTOR OF n
- IF $0<n<1$, THE GRAPH IS **STRETCHED** BY THAT SCALE FACTOR
- IF $n<0$, THE GRAPH IS ALSO **REFLECTED** IN THE y-AXIS
- REMEMBER THE y COORDINATES STAY THE SAME;

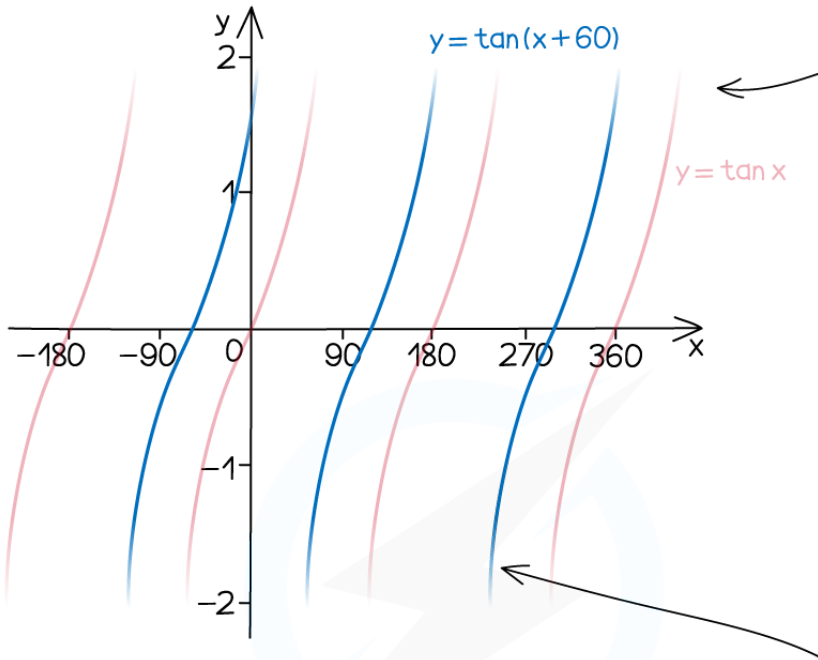
x COORDINATES ARE MULTIPLIED BY $\frac{1}{n}$

Your notes

Your notes

$y = \tan(x+c)$ GIVES A HORIZONTAL TRANSLATION

THE GRAPH OF $y = \tan x$ TRANSLATED HORIZONTALLY TO THE LEFT BY 60



STEP 1: SKETCH THE ORIGINAL sin, cos OR tan FUNCTION
STEP 2: PICK KEY CO-ORDINATE POINTS TO PLOT THE NEW GRAPH
STEP 3: SKETCH IN THE NEW TRANSFORMED GRAPH

FOR A HORIZONTAL TRANSLATION

- IF $c>0$, THE GRAPH IS **TRANSLATED LEFT** BY c
  e.g. $\sin(x+40)$ IS $\sin x$ MOVED 40 LEFT
- IF $c<0$, THE GRAPH IS **TRANSLATED RIGHT** BY c
  e.g. $\cos(x-20)$ IS $\cos x$ MOVED 20 RIGHT
- REMEMBER THE y COORDINATES STAY THE SAME; x COORDINATES WILL BE $\pm c$

Your notes

Examiner Tip

- Always sketch with a pencil and draw a smooth curve.
- When you sketch the transformation of a graph, be sure to indicate the new coordinates of any points that are marked on the original graph.
- Try to indicate the coordinates of points where the new graph intersects the axes.

Worked example

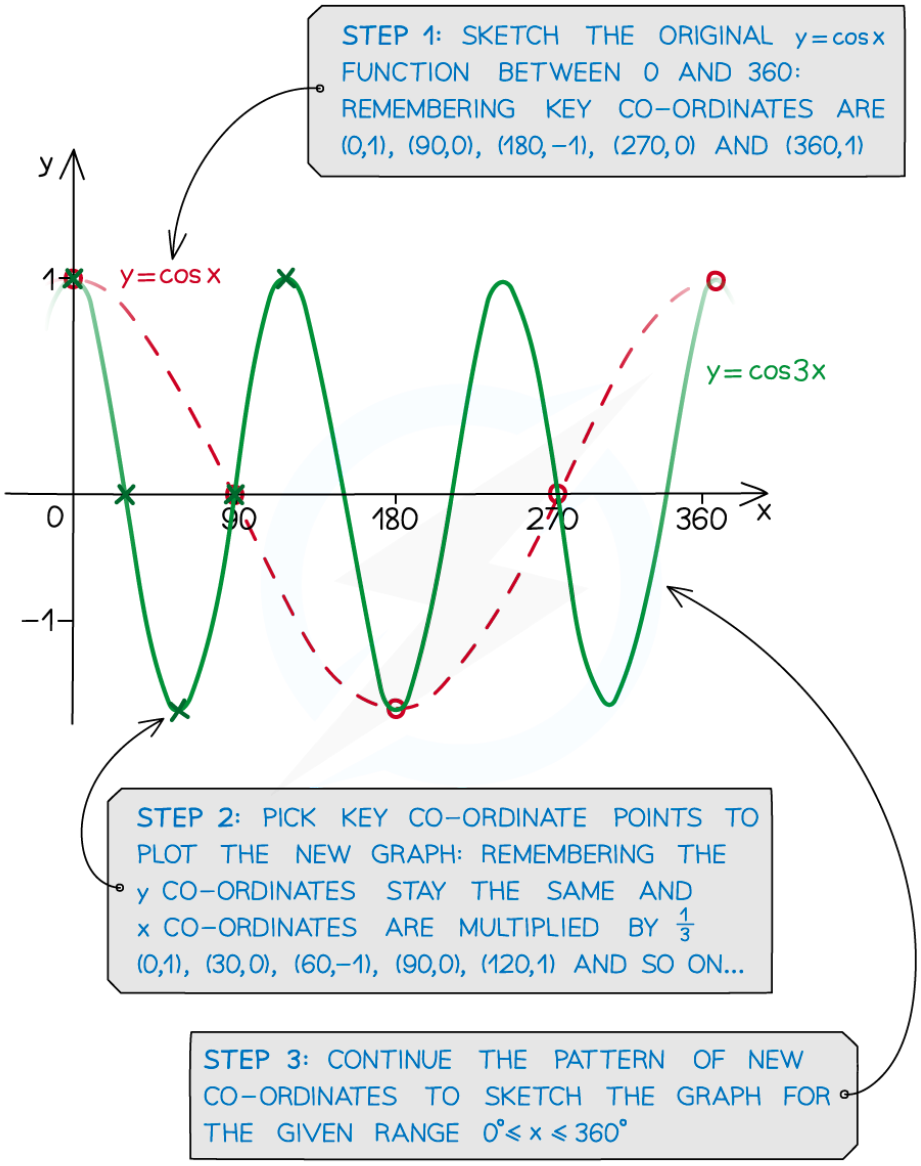
Your notes

Your notes

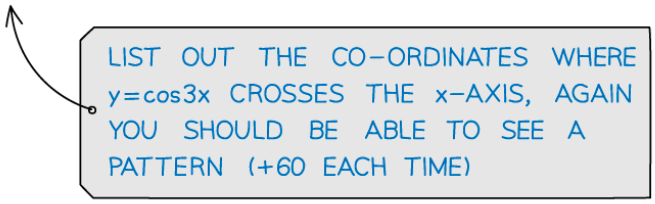
a) Sketch the graph of $y = \cos 3x$, for $0° \leq x \leq 360°$.

b) Write down all the values of x where $\cos 3x = 0$, for $0° \leq x \leq 360°$.

a)



b) (30,0), (90,0), (150,0), (210,0), (270,0), (330,0)



Your notes