Edexcel A Level Maths: Pure

Your notes

2.12 Modelling with Functions

Contents

2.12.1 Modelling with Functions


Your notes

Modelling with Functions

What is a mathematical model?

- A **mathematical model** simplifies a real-world situation so it can be described using mathematics which can then be used to make predictions
  - The path a stone will take if thrown from the top of a cliff
  - The number of toys a factory can produce in a day
- **Assumptions** about the situation are made in order to simplify the mathematics
  - Air resistance on the stone can be ignored
  - The machines/people at the factory produce toys at a constant rate
- Models can be **refined** (improved) if further information is available or the model is compared to real-world data
  - The mass of the stone needs to be considered
  - 30-minutes downtime per day is allowed for machine repairs/maintenance

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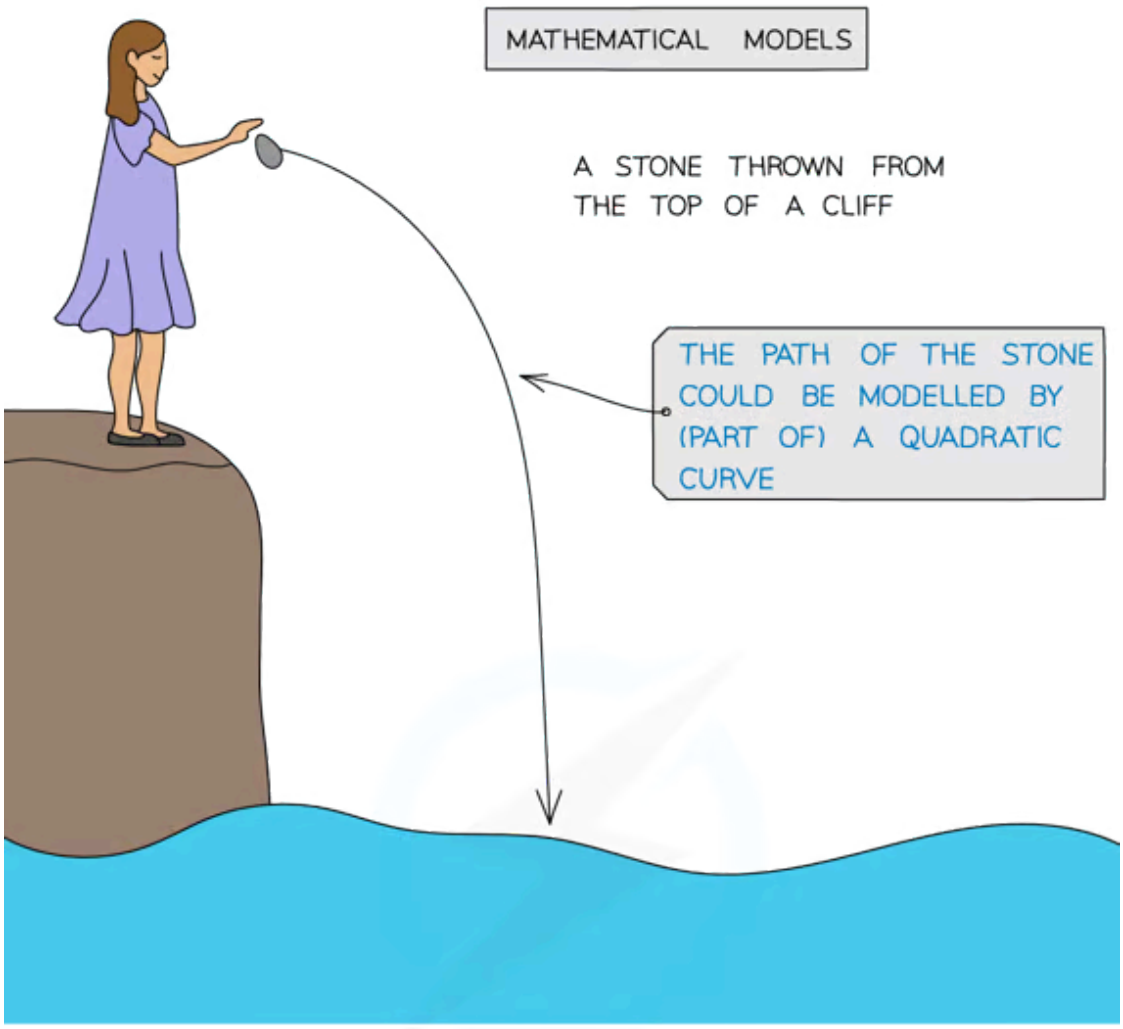

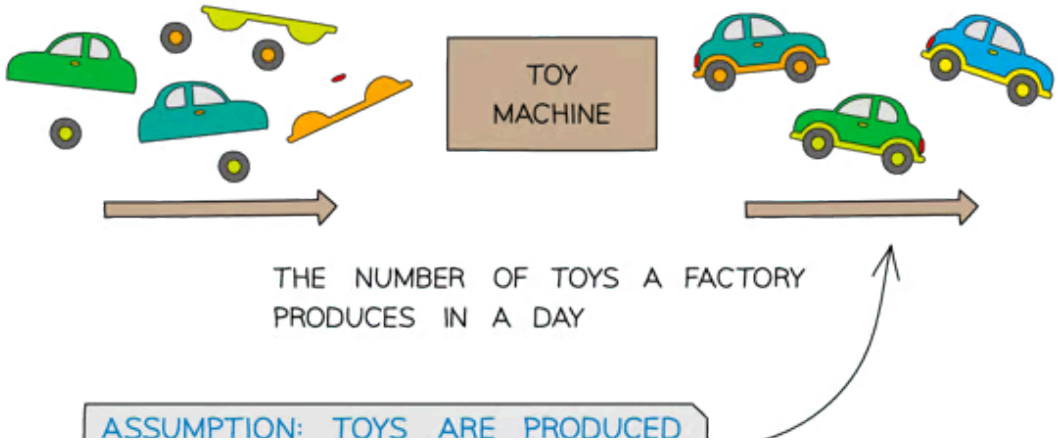

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How do I solve modelling problems?

- There will be no one-size-fits-all step-by-step guide to solving modelling questions
- A combination of skills and **problem-solving** skills will be needed

FOLLOWING A STEADY RISE IN SALES OF A PARTICULAR PRODUCT OVER THE PREVIOUS YEAR A SHOP MANAGER SUGGESTS A LINEAR MODEL OF THE FORM $N=64+4m$ SHOULD BE USED TO PREDICT FUTURE SALES OF THE PRODUCT WHERE N IS THE NUMBER OF PRODUCTS SOLD AND t IS THE NUMBER OF MONTHS AFTER 1 JANUARY 2020.

a) HOW MANY PRODUCTS ARE PREDICTED TO BE SOLD **DURING** JANUARY?

b) WHAT IS THE MEANING OF THE VALUE 4 IN THE MODEL?

c) HOW MANY PRODUCTS WOULD BE SOLD IN **TOTAL** DURING THE FIRST THREE MONTHS OF THE YEAR?

d) SUGGEST ONE CRITICISM OF THIS MODEL

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a) DURING JANUARY — SO AT END OF MONTH, $m=1$

$N=64+4\times1$

$N=68$

b) 4 IS THE GRADIENT, "m" IN "y=mx+c"

4 IS THE MONTHLY INCREASE IN THE NUMBER OF PRODUCTS SOLD

c) FIRST 3 MONTHS IN TOTAL, SO $m=1,2,3$

$m=1,\ N=68$
$m=2,\ N=64+4\times2=72$
$m=3,\ N=64+4\times3=76$

YOU COULD WRITE THESE STRAIGHT DOWN

$\therefore$ TOTAL SALES $= 68+72+76$
$= 216$

d) THE MODEL SUGGESTS THE NUMBER OF PRODUCTS WOULD CONTINUE TO RISE FOREVER. THIS IS UNLIKELY.

Examiner Tip

- Read – and re-read – the question carefully.Try to "get involved" in the context of the question – however boring it may be!
  - Imagine what happens to a stone as you throw it from a cliff
  - What would it be like to manage a toy factory?
- Draw a diagram – **sketch** a graph of the function being used as the model.
- If you are completely stuck try "doing something" with the maths. For example, if there's a quadratic function – sketch it, factorise it, solve it. Or if there's a polynomial – use f**actor theorem** and **algebraic division** to factorise it.


Your notes

Worked example

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A stone is thrown upwards from the top of a cliff.

The height, h metres, of the stone above the sea after t seconds is modelled by the quadratic function

$$h = 32 + 4.8t - 0.8t^2 \text{ where } t \geq 0$$

(a) Find the height of the stone after 4 seconds

(b) How long does the stone take to reach the sea?

(c) Write h in the form $h = -a(t-b)^2 + c$, where a, b and c are constants to be found

(d) Hence find the maximum height above the sea the stone reaches

(e) What is the significance of the number 32 in the model?

a) SUBSTITUTE t=4 INTO h

AT t=4, $h = 32 + 4.8 \times 4 - 0.8 \times 4^2$

h = 38.4m

b) AT SEA LEVEL, h=0

$32 + 4.8t - 0.8t^2 = 0$

USE YOUR CALCULATOR TO SOLVE

t=10 ~~t=-4~~

$t \geqslant 0$

IT TAKES 10 SECONDS FOR THE STONE TO REACH THE SEA

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c) RECOGNISE THE FORM AS COMPLETING THE SQUARE

$-0.8t^2 + 4.8t + 32 = -0.8(t^2-6t-40)$

$= -0.8[(t-3)^2-9-40]$

$= -0.8(t-3)^2+39.2$

d) MAX. HEIGHT WILL BE WHEN $-0.8(t-3)^2$ IS AT ITS MAXIMUM — WHICH IS ZERO

MAX. HEIGHT IS REACHED WHEN $t=3$

AT $t=3$, $h = 32 + 4.8\times3 - 0.8\times3^2$

$h = 39.2$

CAN BE WRITTEN STRAIGHT DOWN USING ANSWER TO c)

e) $h=32$ WHEN $t=0$ — i.e. AT THE START

32 (METRES) IS THE HEIGHT OF THE CLIFF