



# Edexcel A Level Maths: Pure



Your notes

## 4.1 Binomial Expansion

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#### \* 4.1.1 Binomial Expansion



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## 4.1.1 Binomial Expansion

### Binomial Expansion

What is the binomial expansion?

$$\begin{aligned}
 (a+b)^n &= {}^nC_0(a)^n(b)^0 \\
 &+ {}^nC_1(a)^{n-1}(b)^1 \\
 &+ {}^nC_2(a)^{n-2}(b)^2 \\
 &+ {}^nC_3(a)^{n-3}(b)^3 \\
 &+ \dots \\
 &+ {}^nC_r(a)^{n-r}(b)^r \\
 &+ \dots \\
 &+ {}^nC_{n-1}(a)^1(b)^{n-1} \\
 &+ {}^nC_n(a)^0(b)^n
 \end{aligned}$$

$${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \quad n \in \mathbb{N}$$

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- Look at the pattern
  - Start at  ${}^nC_0$ , then  ${}^nC_1$ ,  ${}^nC_2$ , etc
  - Powers of **a** start at **n** and decrease by 1
  - Powers of **b** start at 0 and increase by 1
- There are shortcuts but these hide the pattern
  - ${}^nC_0 = {}^nC_n = 1$
  - ${}^nC_1 = {}^nC_{n-1} = n$
  - ${}^nC_r = {}^nC_{n-r}$
  - $(b)^0 = (a)^0 = 1$

- Use the shortcuts once familiar with the pattern
- ! means factorial

$$(a+b)^n = a^n + na^{n-1}b + {}^nC_2a^{n-2}b^2 + \dots + b^n$$

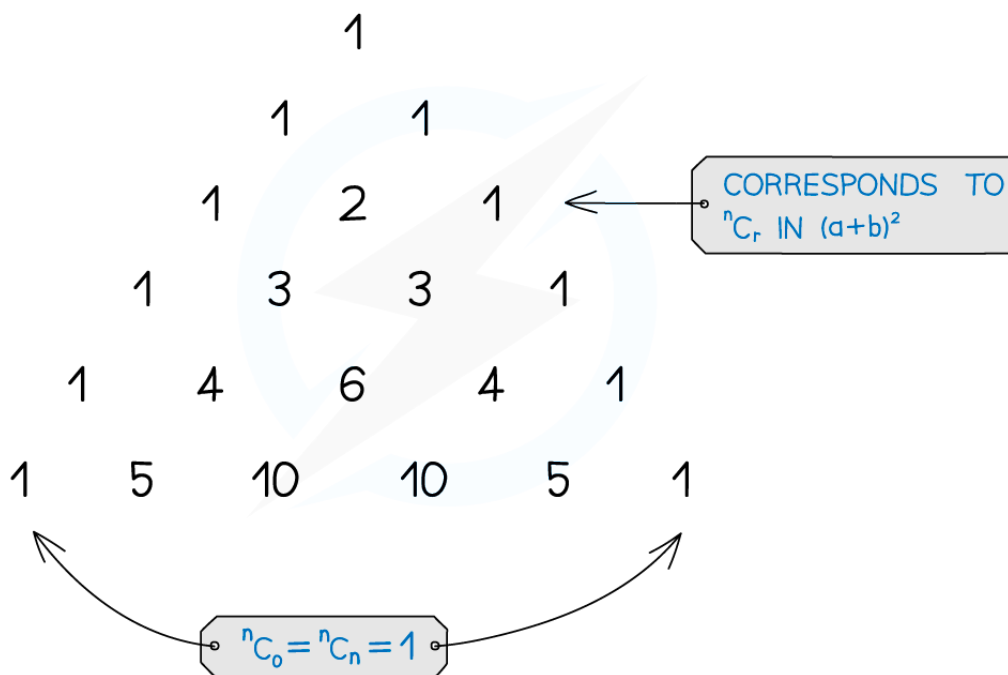
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- This is given in the formula booklet

### What about Pascal's triangle?

#### PASCAL'S TRIANGLE



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- Pascal's triangle is an alternative to  ${}^nC_r$
- It is useful for lower values of  $n$



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- For larger  $n$  it is slow and prone to arithmetic errors

## How do I expand brackets with binomial expansion?

$a=3, b=2x$

e.g.  $(3+2x)^4 = 3^4$

$+ 4(3^3)(2x)$

$+ {}^4C_2(3)^2(2x)^2$

$+ {}^4C_3(3)(2x)^3$

$+ (2x)^4$

$= 81 + 216x + 216x^2 + 96x^3 + 16x^4$

USE A LINE FOR EACH TERM

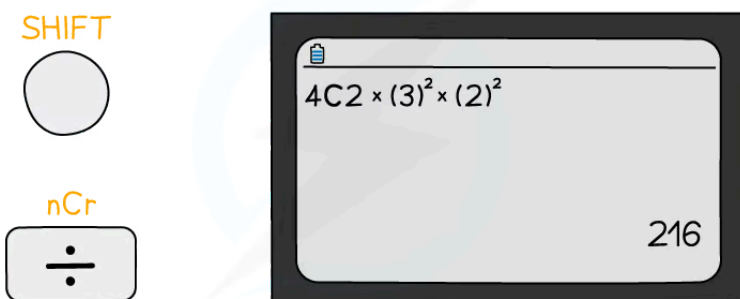
USE BRACKETS AND CALCULATOR FOR ACCURACY

THIS 2 WILL GET CUBED

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- Use a line for each term to make things easier to read and follow
- Use brackets, particularly helpful when negatives involved
- Use a calculator for  ${}^nC_r$



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### Worked example



Find the first four terms, in ascending powers of  $x$ , in the expansion of  $(2 - 3x)^6$

USE SEPARATE LINES AND BRACKETS TO KEEP WORKING CLEAR

$$(2-3x)^6 \approx (2)^6$$

$$+ 6(2)^5(-3x)$$

$$+ {}^6C_2(2)^4(-3x)^2$$

$$+ {}^6C_3(2)^3(-3x)^3$$

TYPE

$${}^6C_3 \times (2)^3 \times (-3)^3$$

INTO CALCULATOR

$$= 64 - 576x + 2160x^2 - 4320x^3$$

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## Applications of Binomial Expansion

### What are binomial expansions used for?

- Expanding brackets, it is usual to be asked for either ...
  - ... the first few terms only

e.g. FIND, IN ASCENDING POWERS OF  $x$ ,  
THE FIRST THREE TERMS OF  $(5+2x)^9$

$$(5+2x)^9 \approx (5)^9 + 9(5)^8(2x) + {}^9C_2(5)^7(2x)^2$$

$$= 1\,953\,125 + 7\,031\,250x + 11\,250\,000x^2$$

APPROXIMATELY EQUAL  
TO AS 3 TERMS ONLY

ASCENDING POWERS  
OF  $x$  SO START  
WITH CONSTANT  
TERM ( $x^0$ )

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- or (the coefficient of) a particular term

e.g. FIND THE COEFFICIENT OF  $x^6$  IN THE  
EXPANSION OF  $(3+2x)^8$

START HERE, FOR " $x^6$ "  
" $(b)^6$ " IS NEEDED

$${}^8C_6 (3)^2 (2x)^6$$

$$= 16\,128x^6$$

MATCHES  
POWER  
OF " $b$ "

TOTAL OF POWERS IS " $n$ "  
WHICH HERE IS 8

SO THE COEFFICIENT OF  $x^6$  IS 16 128

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## ■ Solving problems in unknowns



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e.g. THE COEFFICIENT OF THE  $x^3$  TERM IN THE EXPANSION OF  $(p+2x)^5$  IS 2000.  
FIND THE VALUE OF  $p$ , GIVEN  $p > 0$

ONLY NEED THE  
TERM IN  $x^3$

$${}^5C_3(p)^2(2x)^3 = 80p^2x^3$$

$$80p^2 = 2000$$

$$p^2 = 25$$

$$~~p = -5~~ \quad p = 5$$

SAY WHY THIS  
SOLUTION IS  
REJECTED

REJECT SINCE  $p > 0$

$$p = 5$$

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