Edexcel A Level Maths: Pure

Your notes

5.8 Trigonometric Proof

Contents

5.8.1 Trigonometric Proof

Your notes

Trigonometric Proof

Proving trigonometric identities

- You can use trigonometric identities you already know to prove new identities
- Make sure you know the **simple trigonometric identities** and **further trigonometric identities**
- To prove an identity start on one side and proceed step by step until you get to the other side

e.g. SHOW THAT $\tan\theta + \cot\theta = 2\operatorname{cosec} 2\theta$

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

WRITE tan AND cot IN TERMS OF sin AND cos

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$\sin^2\theta + \cos^2\theta \equiv 1$

$$= \frac{1}{\frac{1}{2}\sin 2\theta}$$

$\sin 2\theta \equiv 2\sin\theta\cos\theta$

$$= \frac{2}{\sin 2\theta} = 2\operatorname{cosec} 2\theta$$

- Clever substitution into the **compound angle formulae** can be a useful tool for proving identities

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e.g. SHOW THAT $\cos\frac{\theta}{2}(1-\cos\theta) = \sin\frac{\theta}{2}\sin\theta$

$\cos\frac{\theta}{2}(1-\cos\theta) = \cos\frac{\theta}{2} - \cos\frac{\theta}{2}\cos\theta$

$= \cos(\theta - \frac{\theta}{2}) - \cos\frac{\theta}{2}\cos\theta$

$= (\cos\theta\cos\frac{\theta}{2} + \sin\theta\sin\frac{\theta}{2}) - \cos\frac{\theta}{2}\cos\theta$

$= \sin\frac{\theta}{2}\sin\theta$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$ WITH $A=\theta$ AND $B=\frac{\theta}{2}$

- Make sure you are confident handling fractions and fractions-within-fractions

e.g. SHOW THAT $\frac{1+\tan\theta}{1+\cot\theta} \equiv \tan\theta$

$\frac{1+\tan\theta}{1+\cot\theta} \equiv \frac{1+\tan\theta}{1+\frac{1}{\tan\theta}}$ $\cot\theta \equiv \frac{\cos\theta}{\sin\theta} \equiv \frac{1}{\tan\theta}$

$\equiv \frac{1+\tan\theta}{\left(\frac{\tan\theta+1}{\tan\theta}\right)}$

$\equiv (1+\tan\theta) \times \frac{\tan\theta}{\tan\theta+1}$

$\equiv \tan\theta$

- Always keep an eye on the 'target' expression – this can help suggest what identities to use

Examiner Tip

- Don't forget that you can start a proof from either end – sometimes it might be easier to start from the right-hand side (see the Worked Example)
- A number of trigonometric identities are given to you in the formulae booklet – make sure you know which ones are (and aren't) in there

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Worked example

Show that $8\cos^4\theta - 8\cos^2\theta + 1 \equiv \cos 4\theta$.

HERE IT WILL BE EASIER TO START ON THE RIGHT HAND SIDE

$\cos 4\theta \equiv 2\cos^2 2\theta - 1$ ← $\cos 2A = 2\cos^2 A - 1$ WITH $A = 2\theta$

$\equiv 2(2\cos^2\theta - 1)^2 - 1$ ← AND AGAIN, WITH $A = \theta$

$\equiv 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1$

$\equiv 8\cos^4\theta - 8\cos^2\theta + 1$

THE FORM OF "TARGET" HERE SUGGESTED THAT $\cos 2A = 2\cos^2 A - 1$ WOULD BE MORE USEFUL THAN $\cos 2A = 1 - 2\sin^2 A$ OR $\cos 2A = \cos^2 A - \sin^2 A$