

## **Edexcel A Level Maths: Pure**



## 2.7 Graphs of Functions

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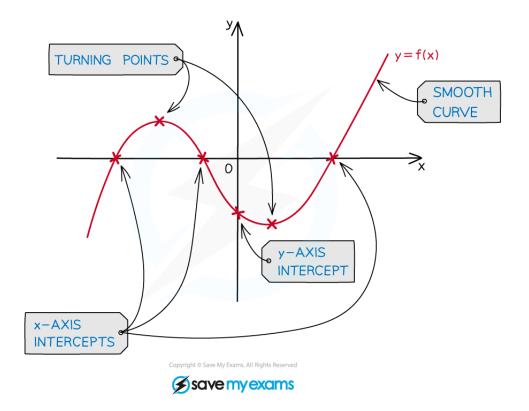
## 2.7.1 Sketching Polynomials

## Your notes

### **Sketching Polynomials**

#### Sketching the graph of a polynomial

• Remember a polynomial is any finite function with non-negative indices, that could mean a quadratic, cubic, quartic or higher power



- When asked to sketch a polynomial you'll need to think about the following
  - **y**-axis intercept
  - x-axis intercepts (roots)
  - turning points (maximum and/or minimum)
  - a smooth **curve** (this takes practice!)

#### How do I sketch a graph of a polynomial?

STEP 1 Find the y-axis intercept by setting x = 0

STEP 2 Find the x-axis intercepts (roots) by setting y = 0



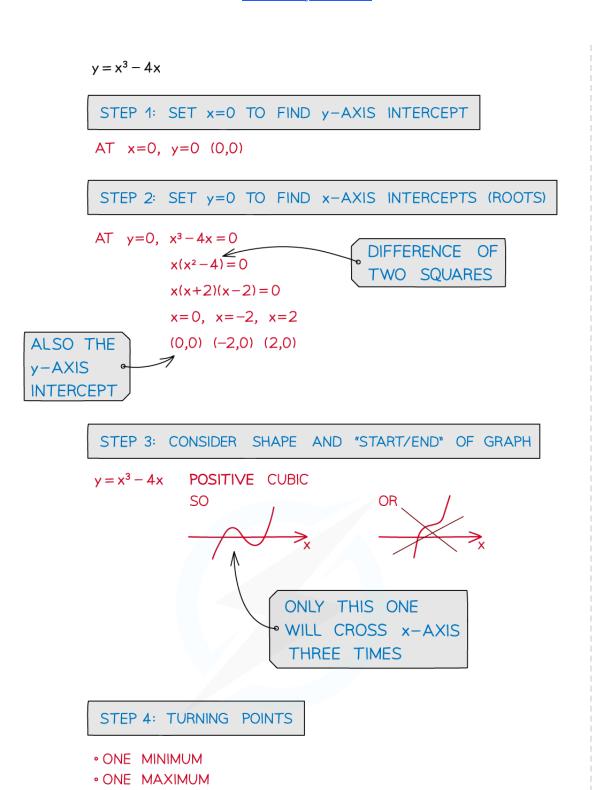
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STEP 3	Consider the snape and "start"/ end "of the graph"		
	eg. a <b>positive cubic</b> graph starts in third quadrant ("bottom left") and "ends" in first quadrant ("top right")		
STEP 4	Consider where any turning points should go		
STEP 5	Draw with a smooth curve		



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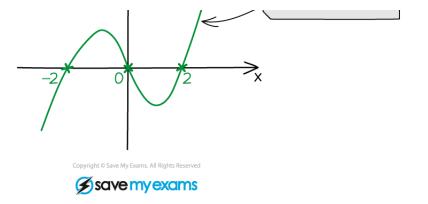
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SMOOTH CURVE







QUADRATICS ARE POLYNOMIALS (DEGREE 2)

ODD-DEGREE POLYNOMIALS MUST HAVE AT LEAST ONE x-AXIS INTERCEPT

EVEN-DEGREE POLYNOMIALS DO NOT NECESSARILY INTERCEPT THE x-AXIS

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- Coordinates of turning points can be found using differentiation
- Except with a point of inflection, repeated roots indicate the graph touches the x-axis



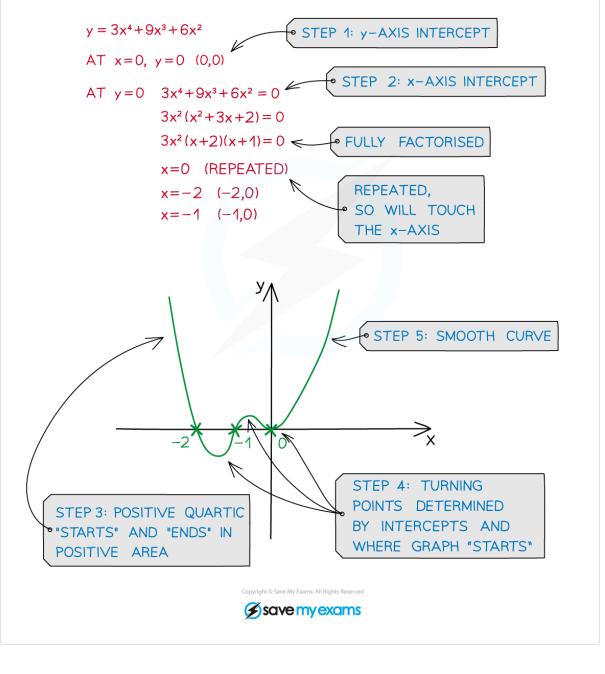
Worked example	
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Sketch the graph of  $y = 3x^4 + 9x^3 + 6x^2$ 



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## 2.7.2 Reciprocal Graphs - Sketching

# Your notes

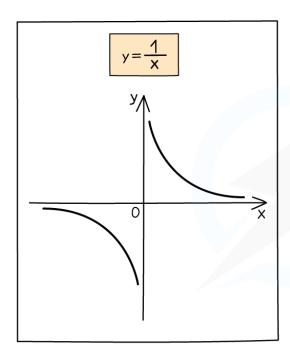
## Reciprocal Graphs - Sketching

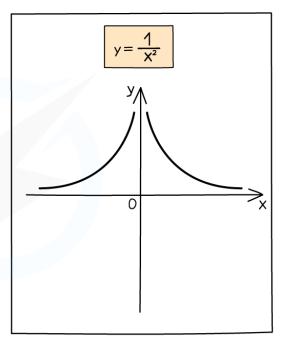
What are reciprocal graphs?

- Reciprocal graphs involve equations with an X term on the denominator e.g.  $\frac{1}{X}$
- There are two basic reciprocal graphs to know for A level

$$y = \frac{1}{x} \text{ and } y = \frac{1}{x^2}$$

• The second one of these is always positive





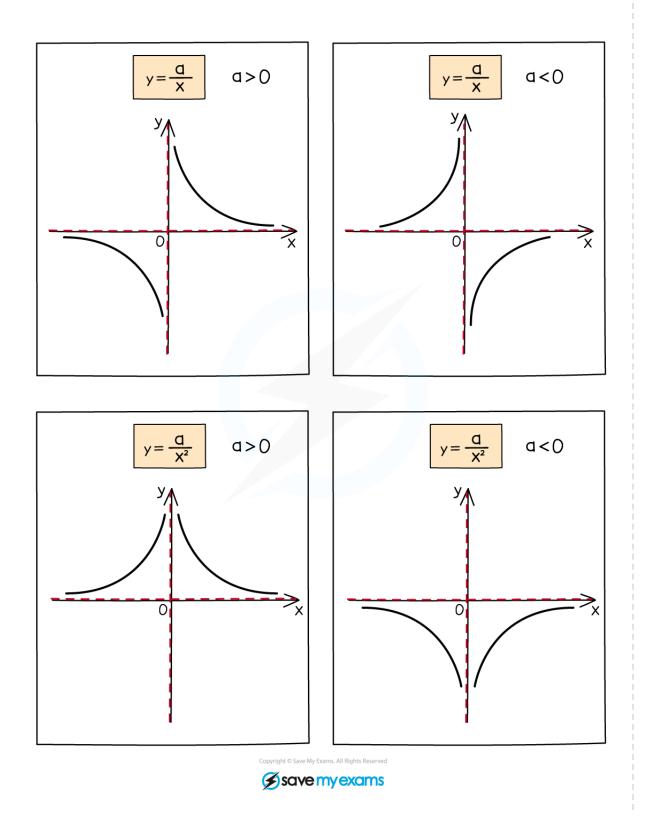
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#### More reciprocal graphs

• You also need to recognise graphs where the numerator is not one

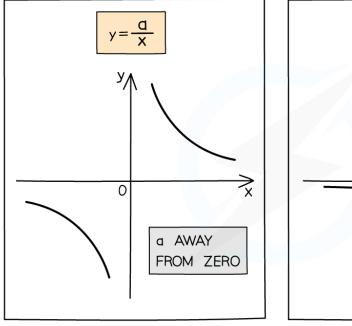


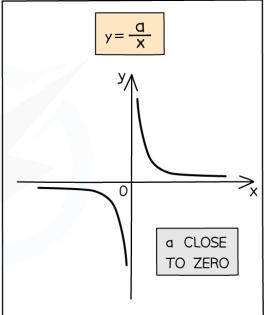




- The **sign** of **a** shows which part of the graph the curves are located
- The **size** of **a** shows how steep the curves are
  - The closer **a** is to 0 the more **L-shaped** the curves are







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- horizontal, y = 0 (x-axis)
  - vertical, **x** = **0** (**y**-axis)All have two asymptotes

#### How do I sketch a reciprocal graph?

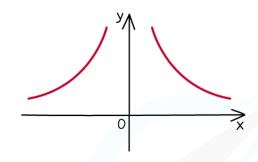




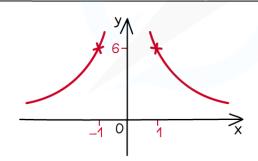
STEP 1: USE SIGN OF "a" TO LOCATE CURVES
USE SIZE OF "a" TO GAUGE STEEPNESS

a>0, CURVES IN POSITIVE AREAS
a=6, LARGE COMPARED TO 1, LESS L-SHAPED

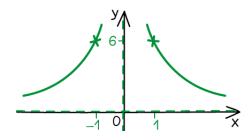
### STEP 2: SKETCH THE GRAPH



### STEP 3: LABEL x=1 & x=-1 AS A GUIDE



### STEP 4: DRAW ASYMPTOTES WITH A DOTTED LINE



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- STEP 1 Use the sign of "a" to locate the curves

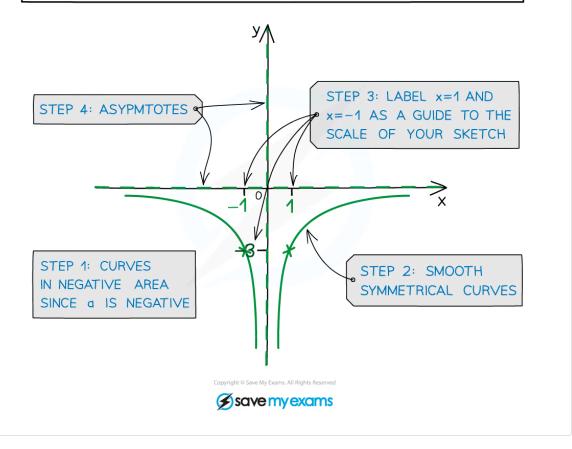
  and use the size of "a" to gauge the steepness of the curve
- STEP 2 Sketch the graph
- STEP 3 Label the points x = 1 and x = -1 as a guide to the scale of your graph
- STEP 4 Draw asymptotes with a dotted line Draw **asymptotes** with **dotted** lines
- These graphs do not intercept either axis
- **Graph transformations** of them could cross the axes (see Translations)







Sketch the graph of  $y = \frac{-3}{x^2}$ 

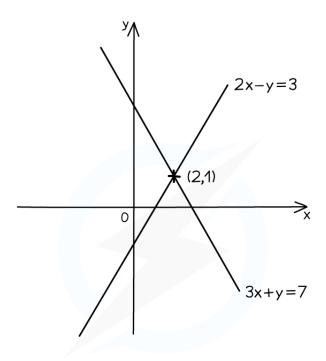


## 2.7.3 Solving Equations Graphically

# Your notes

## **Solving Equations Graphically**

Solving (simultaneous) equations graphically



- LINES INTERSECT AT (2,1)
- SOLVING 2x-y=3 AND 3x+y=7 SIMULTANEOUSLY IS x=2, y=1

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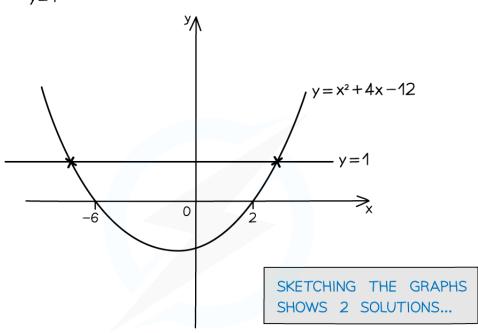
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- This is a way to solve **simultaneous equations** (see Simultaneous Equations)
- Coordinates of the intersections are the solutions

How do I solve simultaneous equations using a graph?

e.g. 
$$y=x^2+4x-12$$
  
 $y=1$ 





$$x^2 + 4x - 12 = 1$$

$$x^2 + 4x - 13 = 0$$

$$x=-2+\sqrt{17}$$
 OR  $x=-2-\sqrt{17}$   
 $y=1$   $y=1$ 

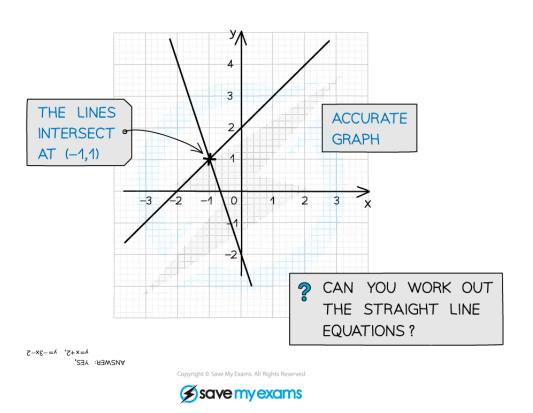
... AND ALGEBRA
FINDS THEM EXACTLY

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- Use graphs and algebra together
- **Sketch** a graph if it has not been given
- Read/interpret a graph if it has been given





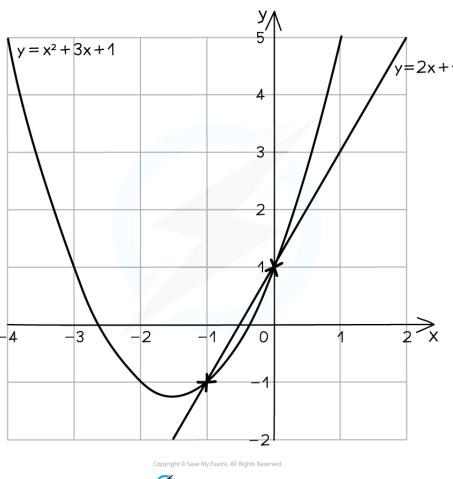
Your notes

• It can be difficult to tell from a sketch if graphs intersect once, more, or not at all



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## Examiner Tip

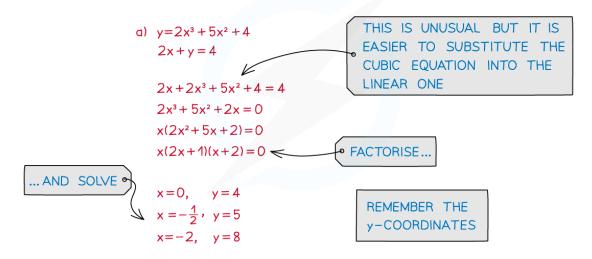
When writing out your solutions to simultaneous equations, always pair the correct x solution with the correct y solution

## Worked example





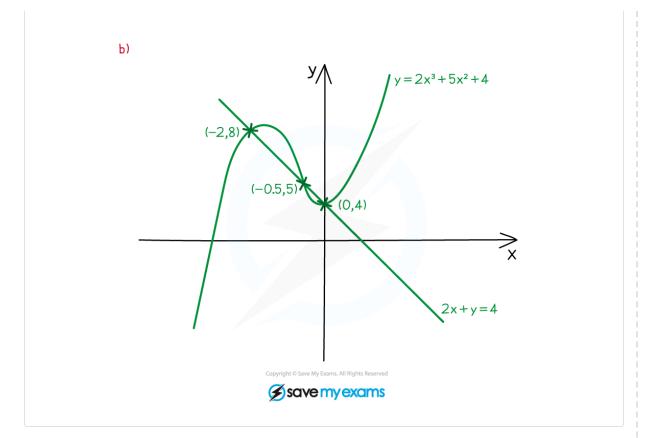
- (a) Find all points of intersection between the curve with equation  $y = 2x^3 + 5x^2 + 4$  and the straight line 2x + y = 4
- (b) Sketch the graphs on the same diagram clearly showing the points of intersection.



: INTERSECTIONS ARE (0,4),  $(-\frac{1}{2},5)$  AND (-2,8)











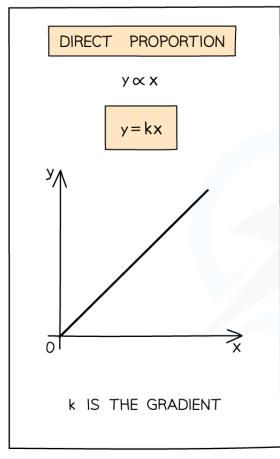
### 2.7.4 Proportional Relationships

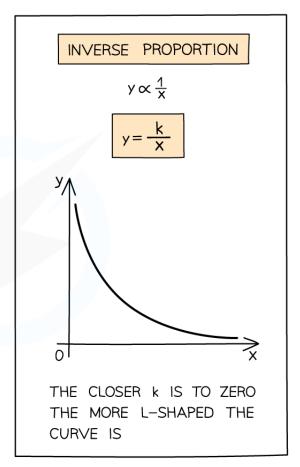
# Your notes

## **Proportional Relationships**

#### **Proportional relationships**

- Proportional relationships describe a proportional connection between two variables
- This can happen in two ways
  - Direct proportion y = kx
    - one variable increases or decreases the other does the same
  - Inverse proportion  $y = \frac{k}{x}$ 
    - one variable increases the other decreases and vice versa
- ullet Proportional relationships use the symbol ullet which means is proportional to





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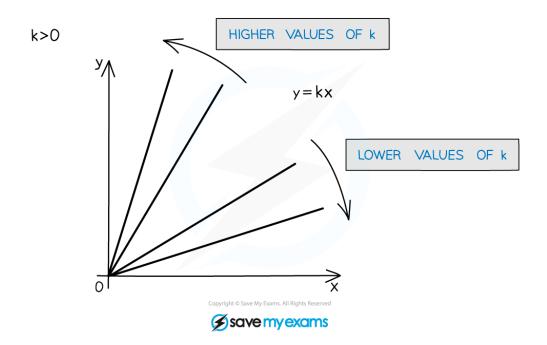
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- Both direct and inverse proportion can be represented graphically
  - Direct proportion creates a linear graph where k is the gradient
  - Inverse proportion creates a reciprocal graph

## Your notes

#### **Direct proportion**

- $Y \propto X$  means y is proportional to x
- y increases as x does, k determines the rate (gradient)
- by changing this to the equation y = kx we can substitute in given values and solve to find k
  - Note that this means the ratio of x and y is constant k = y / x

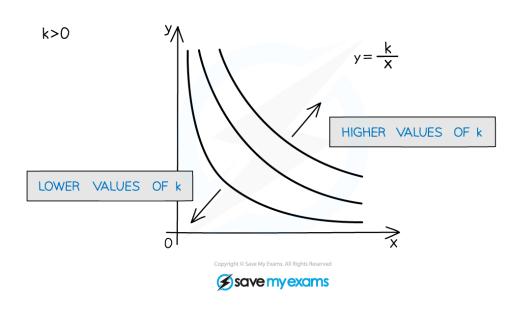


#### Inverse proportion

- $y \propto \frac{1}{x}$  means y is proportional to  $\frac{1}{x}$  or y is inversely proportional to x
- y decreases as x increases and vice versa, k determines the rate
- by changing this to the equation  $y = \frac{k}{x}$  we can substitute in given values and solve to find k
  - Note that this means the **product** of x and y is constant **k = xy**







### How do I work with proportional relationships?

STATEMENT	PROPORTION	EQUATION
"y IS DIRECTLY PROPORTIONAL TO x PLUS TWO"	y ∝ x+2	y=k(x+2)
"y IS INVERSELY PROPORTIONAL TO THE SQUARE OF x"	$y \propto \frac{1}{x^2}$	$y = \frac{k}{X^2}$
"y IS DIRECTLY PROPORTIONAL TO Inx"	y∝lnx	y=klnx



- Set up your proportional relationship using  $\propto$  then change to = k
- Be clear about what **y** is proportional to ...
  - "... the square of x" ( $x^2$ )
  - "... x plus four" (x + 4)



- Calculate or deduce the value of k from the information given or a graph
- Once you've found k sub it back in to your original proportion equation
- You can now find any values using this proportional relationship
- y = mx + c rearranges to y c = mx so (y c) is **directly** proportional to x
- **Proportional relationships** are often used in modelling

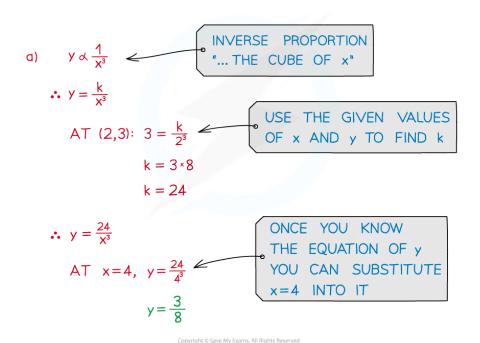


## Worked example





- (a) y is inversely proportional to the cube of x. Given that y = 3 when x = 2 find the value of y when x = 4.
- (b) Sketch the graph of y against x



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