Edexcel A Level Maths: Pure
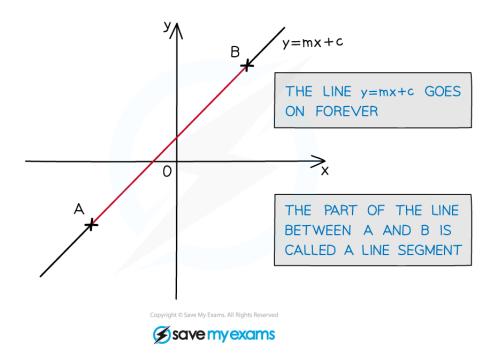
3.1 Equation of a Straight Line

Contents

3.1.1 Basic Coordinate Geometry

Basic Coordinate Geometry

What is basic coordinate geometry?

- Basic coordinate geometry refers to working with points, lines and shapes on the coordinate axis
- **Cartesian** coordinates are simply the **x -y** coordinate system
- Using coordinates you can
    - Calculate the distance between two points (length of a line)
    - Divide lines in m:n ratio,
    - Find the mid-point of a line
    - Calculate the area of a triangle (or other shapes)
    - Find missing coordinates using any of the above
- Most of these involve working with straight line graphs
- **Straight line graphs** are of the form **y = mx + c**
    - **m** is the gradient
    - **c** is the **y**-axis intercept
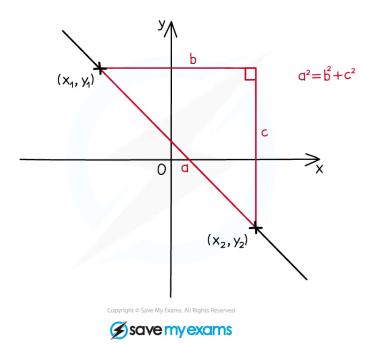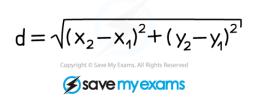   - Part of a straight line is called a **line segment**

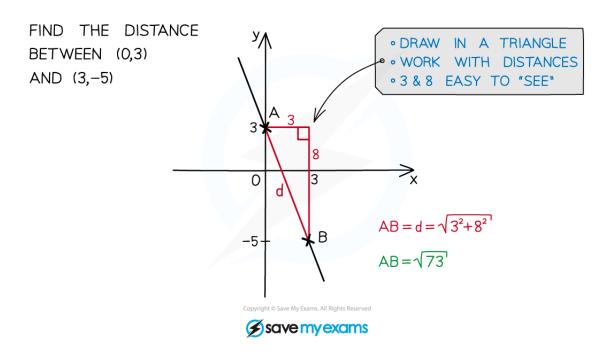## How do I find the length of a line (segment)?

- By finding the distance, **d**, between two points (x_1, y_1) (x_2, y_2)
- **Pythagoras' Theorem** ($a^2 = b^2 + c^2$) is used to find the length of a line

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Your notes

FIND THE DISTANCE BETWEEN (0,3) AND (3,−5)

- DRAW IN A TRIANGLE
- WORK WITH DISTANCES
- 3 & 8 EASY TO "SEE"

$AB = d = \sqrt{3^2 + 8^2}$

$AB = \sqrt{73}$
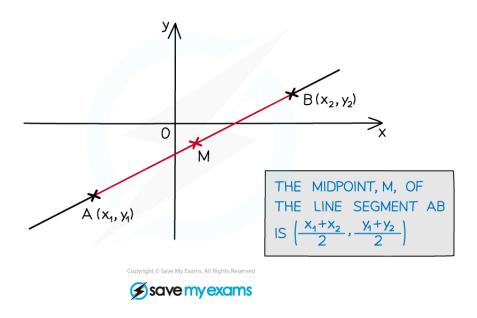
How do I find the midpoint of a line (segment)?

THE MIDPOINT, M, OF THE LINE SEGMENT AB IS $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

- M is often used as the midpoint between two points (x_1, y_1) (x_2, y_2)
- It is the average of both the **x** and **y** coordinates

Your notes

> 💡 **Examiner Tip**
>
> - Work with the **square** of a distance for as long as possible as this avoids early rounding errors and surds.
> - Only square root when forced to or for a final answer, and use the ANS button (and other memory features) on your calculator.
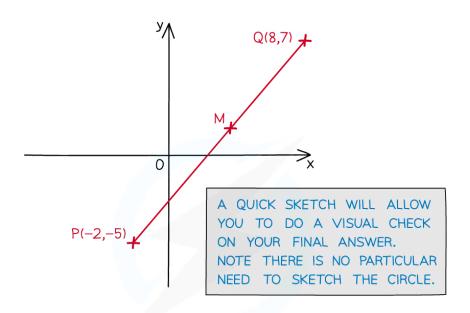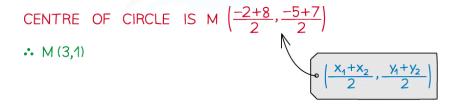
🖊 **Worked example**

Your notes

Your notes

? The line segment PQ is the diameter of a circle with coordinates P(-2, -5) and Q(8, 7).

Find the coordinates of the centre of the circle and the length of the diameter, giving your answer in the form $a\sqrt{b}$, where a and b are integers to be found.

A QUICK SKETCH WILL ALLOW YOU TO DO A VISUAL CHECK ON YOUR FINAL ANSWER. NOTE THERE IS NO PARTICULAR NEED TO SKETCH THE CIRCLE.

CENTRE OF CIRCLE IS M $\left(\dfrac{-2+8}{2}, \dfrac{-5+7}{2}\right)$

$\therefore$ M (3,1)

$\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

LENGTH OF DIAMETER IS d, $d = \sqrt{(8-(-2))^2 + (7-(-5))^2}$

AFTER SOME PRACTICE AIM TO GO STRAIGHT TO THIS LINE:
"x HAS INCREASED BY 10"
"y HAS INCREASED BY 12"

$d = \sqrt{10^2 + 12^2}$

$d = 2\sqrt{61}$

✏️
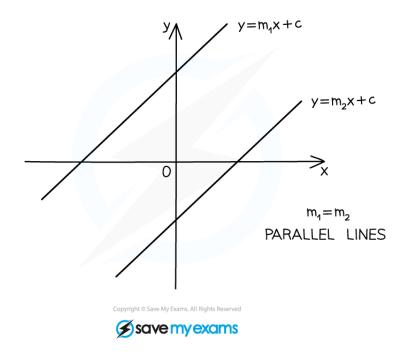Your notes

Your notes

3.1.2 Parallel & Perpendicular Gradients

Parallel & Perpendicular Gradients

What are parallel lines?

- **Parallel** lines are equidistant meaning they never meet
- **Parallel** lines have **equal gradients**
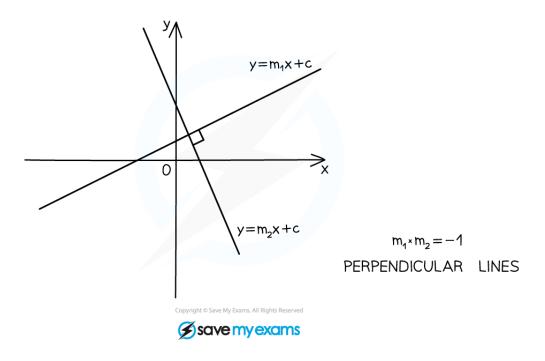
What are perpendicular lines?

- **Perpendicular** lines meet at **right angles**
- The **product** of their **gradients** is -1

What are collinear lines?

- **Collinear** lines are two line segments that are part of the **same** line
- They have **equal gradients** and **pass** through, or **meet** at, a **point**

Your notes
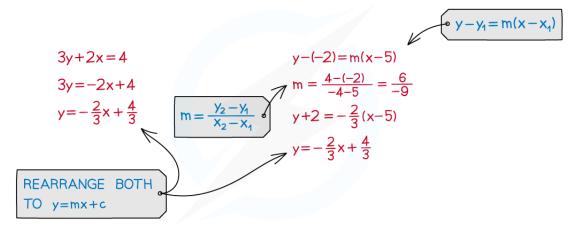
- **Rearrange** their equations into the same form and they will be identical

e.g. DETERMINE IF THE LINES $3y+2x=4$ AND THE LINE PASSING THROUGH $(5,-2)$ AND $(-4,4)$ ARE COLLINEAR OR NOT

$$y-y_1=m(x-x_1)$$

$3y+2x=4$

$3y=-2x+4$

$y=-\frac{2}{3}x+\frac{4}{3}$

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$y-(-2)=m(x-5)$

$m=\frac{4-(-2)}{-4-5}=\frac{6}{-9}$

$y+2=-\frac{2}{3}(x-5)$

$y=-\frac{2}{3}x+\frac{4}{3}$

REARRANGE BOTH TO $y=mx+c$

EQUATIONS ARE THE SAME WHEN REARRANGED SO THE TWO LINES ARE COLLINEAR

How do I tell if lines are parallel or perpendicular?

- Rearrange equations into the form **y = mx + c**
- **m** is the **gradient**

1. $2x - 4y + 3 = 0$ 2. $3x - 2 = 6y$ 3. $4x + 2y = 1$

   $4y = 2x + 3$ $y = \frac{3}{6}x - \frac{2}{6}$ $2y = -4x + 1$

   $y = \frac{1}{2}x + \frac{3}{4}$ $y = \frac{1}{2}x - \frac{1}{3}$ $y = -2x + \frac{1}{2}$

EASY TO COMPARE IN $y = mx + c$ FORM

GRADIENTS OF EQUATIONS 1 AND 2 ARE EQUAL SO LINES ARE PARALLEL.
$\frac{1}{2} \times -2 = -1$ SO EQUATION 3 IS PERPENDICULAR TO THE OTHER TWO

💡 Examiner Tip

- Exam questions are good at "hiding" parallel and perpendicular lines.
- For example a **tangent** and a **radius** are **perpendicular.**
- **Parallel lines** could be implied by phrases like "… at the same rate …"

Worked example

Your notes

> Determine whether the following pairs of straight-line graphs are parallel, perpendicular or neither:
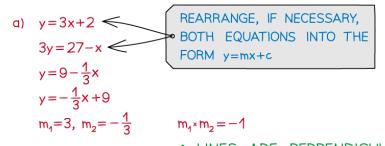>
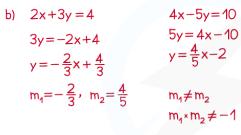> (a) $y = 3x + 2$ and $3y = 27 - x$
>
> (b) $2x + 3y = 4$ and $4x - 5y = 10$
>
> (c) $x + y = 2$ and $3x + 3y = 6$

a) $y = 3x + 2$

REARRANGE, IF NECESSARY, BOTH EQUATIONS INTO THE FORM $y = mx + c$

$3y = 27 - x$

$y = 9 - \frac{1}{3}x$

$y = -\frac{1}{3}x + 9$

$m_1 = 3, \quad m_2 = -\frac{1}{3}$ $m_1 \times m_2 = -1$

$\therefore$ LINES ARE PERPENDICULAR

b) $2x + 3y = 4$ $4x - 5y = 10$

$3y = -2x + 4$ $5y = 4x - 10$

$y = -\frac{2}{3}x + \frac{4}{3}$ $y = \frac{4}{5}x - 2$

$m_1 = -\frac{2}{3}, \quad m_2 = \frac{4}{5}$ $m_1 \neq m_2$

$m_1 \times m_2 \neq -1$
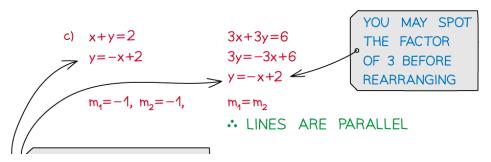
$\therefore$ LINES ARE NEITHER PARALLEL NOR PERPENDICULAR

THIS WILL MEAN THEY INTERSECT

c) $x + y = 2$ $3x + 3y = 6$

$y = -x + 2$ $3y = -3x + 6$

$y = -x + 2$

YOU MAY SPOT THE FACTOR OF 3 BEFORE REARRANGING

$m_1 = -1, \quad m_2 = -1,$ $m_1 = m_2$

$\therefore$ LINES ARE PARALLEL

THESE "LINES" ARE
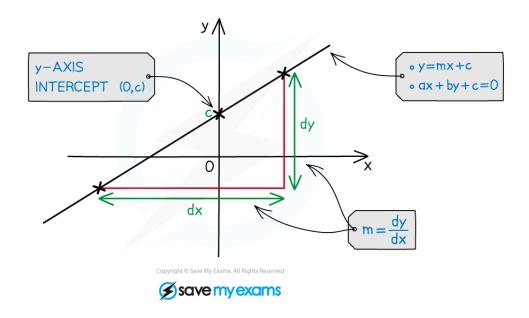COLLINEAR — THE SAME
STRAIGHT LINE

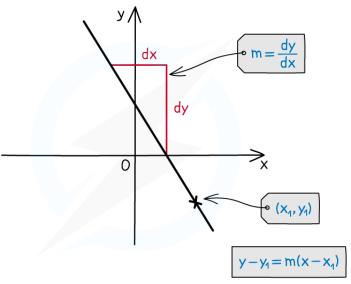## 3.1.3 Equation of a Straight Line

Equation of a Straight Line

What is the equation of a straight line?

- **y = mx + c** is the equation for any straight line
- **m** is **gradient** given by "difference in **y**" ÷ "difference in **x**" or **dy/dx**
- **c** is the **y**-axis **intercept**
- Alternative form is **ax + by + c = 0** where **a**, **b** and **c** are integers

How do I find the equation of a straight line?

Copyright © Save My Exams. All Rights Reserved

- **Two** features of a straight line are needed
  - gradient, **m**
  - a point the line passes through, (x_1, y_1)
- The equation can then be found using **y – y_1 = m(x – x_1)**
- This can be arranged into either **y = mx + c** or **ax +by + c = 0**

e.g. $m = -\frac{2}{3}$ $(x_1, y_1) = (-2, 5)$

$$y - 5 = -\frac{2}{3}(x - (-2))$$

$$y - 5 = -\frac{2}{3}x - \frac{4}{3}$$

"y = mx + c"　　　　"ax + by + c = 0"

$y = -\frac{2}{3}x - \frac{4}{3} + 5$ 　　　$\frac{2}{3}x + y - 5 + \frac{4}{3} = 0$

$y = -\frac{2}{3}x + \frac{11}{3}$ 　　　$\frac{2}{3}x + y - \frac{11}{3} = 0$

　　　　　　　　　　$2x + 3y - 11 = 0$

How do I find the gradient of a straight line?
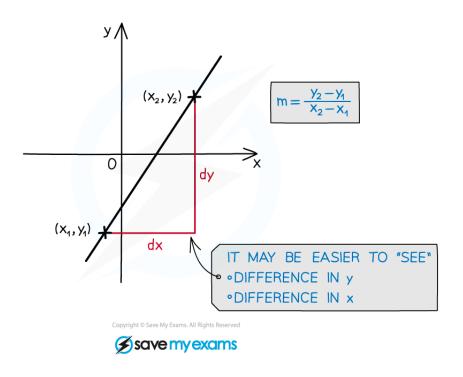
- There are lots of ways to find the gradient of a line

- Using two points on a line to find the change in y divided by change in x

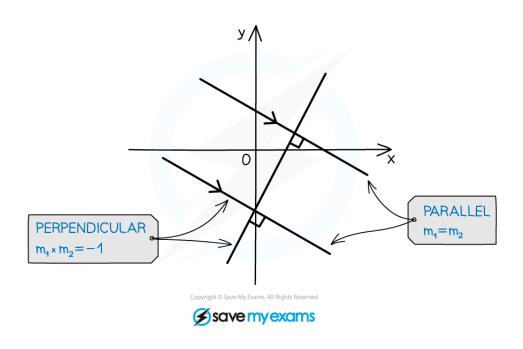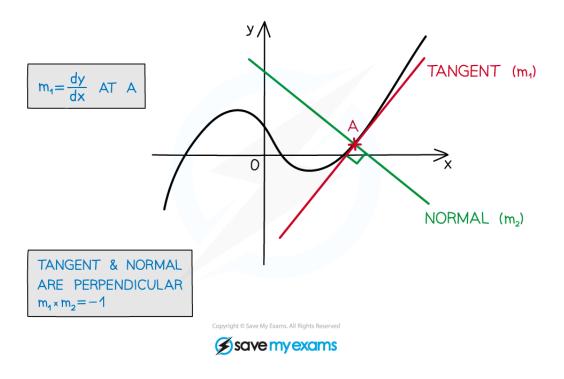$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Using the fact that lines are **parallel** or **perpendicular** to another line (see Parallel and Perpendicular Gradients)

- Using **Tangents** and **Normals** – **Differentiation** (see Gradients, Tangents & Normals)

$$m_1 = \frac{dy}{dx} \text{ AT A}$$

TANGENT (m_1)

A

NORMAL (m_2)

TANGENT & NORMAL ARE PERPENDICULAR
$$m_1 \times m_2 = -1$$

- Other ways
  - **Collinear** lines are the same straight line so gradients are equal
  - Angle facts and **circle theorems**
    eg. a **radius** and **tangent** are **perpendicular**

💡 **Examiner Tip**

- Working with straight lines can involve lots of algebra, but **sketching** a diagram will always help.
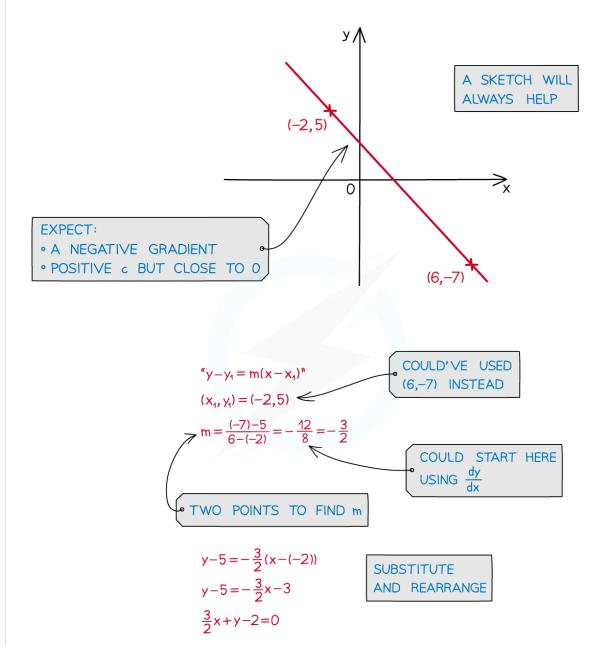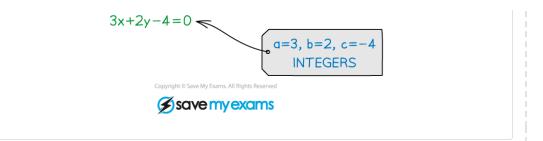- Use a **sketch** to check if answers seem about right.

✏️ **Worked example**

✏️
Your notes

Your notes

> ? The line l_1 passes through the points $(-2, 5)$ and $(6, -7)$.
>
> Find the equation l_1, giving your answer in the form
>
> $ax + by + c = 0$ where a, b and c are integers to be found.

A SKETCH WILL ALWAYS HELP

$(-2, 5)$

$(6, -7)$

0

EXPECT:
- A NEGATIVE GRADIENT
- POSITIVE c BUT CLOSE TO 0

"$y - y_1 = m(x - x_1)$"

$(x_1, y_1) = (-2, 5)$

COULD'VE USED $(6, -7)$ INSTEAD

$m = \dfrac{(-7) - 5}{6 - (-2)} = -\dfrac{12}{8} = -\dfrac{3}{2}$

COULD START HERE USING $\dfrac{dy}{dx}$

TWO POINTS TO FIND m

$y - 5 = -\dfrac{3}{2}(x - (-2))$

$y - 5 = -\dfrac{3}{2}x - 3$

$\dfrac{3}{2}x + y - 2 = 0$

SUBSTITUTE AND REARRANGE

$$3x + 2y - 4 = 0$$

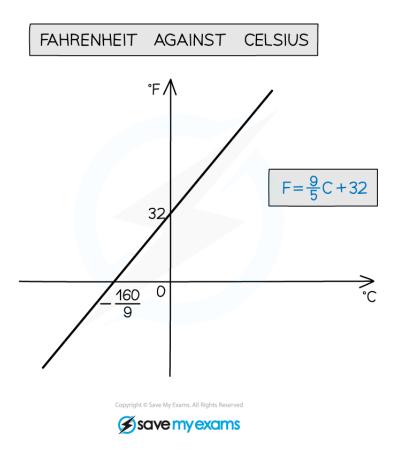a=3, b=2, c=−4
INTEGERS

save my exams

3.1.4 Modelling with Straight Lines

Modelling with Straight Lines

What is modelling with straight lines?

- **Modelling** means applying the mathematics to a **real-life** situation
- The model is used to **find** or **predict** future values
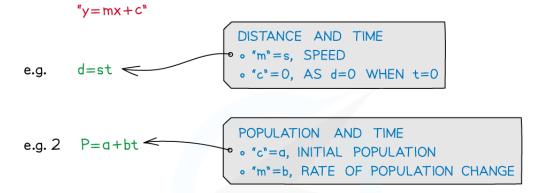- Models will often need **refining** to improve their accuracy

FAHRENHEIT AGAINST CELSIUS

$$F = \frac{9}{5}C + 32$$

How do I work with straight line models?

- Turn the words from the question into mathematical equations
- In **y = mx + c** constants "**m**" and "**c**" will have a meaning
  - gradient "**m**" will be the **rate of change**
  - eg extension in a spring per 1 kg of mass added
  - **y**-axis intercept "**c**" will be the **initial value**
  - eg resting length of a spring

MODELLING WITH STRAIGHT LINES

"y = mx + c"

e.g.   d = st ⟵   DISTANCE AND TIME
                     ◦ "m" = s, SPEED
                     ◦ "c" = 0, AS d = 0 WHEN t = 0

e.g. 2   P = a + bt ⟵   POPULATION AND TIME
                           ◦ "c" = a, INITIAL POPULATION
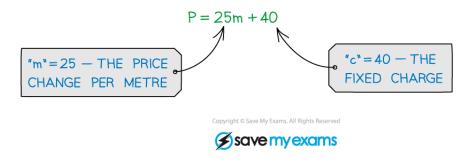   ◦ "m" = b, RATE OF POPULATION CHANGE

e.g. 3   A FENCE BUILDER CHARGES £25
          PER METRE OF FENCE AND A FIXED
          CHARGE OF £40.
          USING m FOR THE METRES REQUIRED
          WRITE DOWN AN EQUATION FOR THE
          PRICE, £P, THE FENCE BUILDER CHARGES.

$$P = 25m + 40$$

"m" = 25 — THE PRICE CHANGE PER METRE   "c" = 40 — THE FIXED CHARGE

- The skills needed are covered in the other pages of the section Equation of a Straight Line

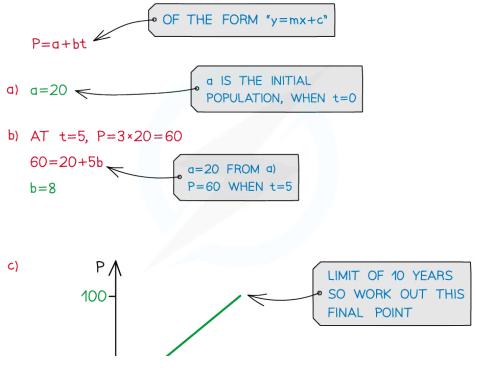### Worked example

Your notes

An endangered animal is introduced to a new area with an initial population of 20. Scientists use the linear model $P = a + bt$ to predict the future population of the animal where P is the population and t is the time in years since the animal was first introduced to the area.
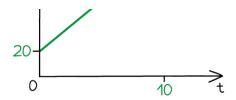
(a) Write down the value of the constant a.

(b) After 5 years scientists observe the population has tripled since the animal was first introduced to the area. Work out the value of b.

(c) Sketch the graph of population for the first 10 years

(d) Suggest one criticism of this model

$P = a + bt$

OF THE FORM "y=mx+c"

a) $a = 20$

a IS THE INITIAL POPULATION, WHEN t=0

b) AT t=5, P=3×20=60

$60 = 20 + 5b$

$b = 8$

a=20 FROM a)
P=60 WHEN t=5

c)

LIMIT OF 10 YEARS SO WORK OUT THIS FINAL POINT

d) POPULATION INCREASES INDEFINITELY

> POPULATION GROWTH IS USUALLY MODELLED EXPONENTIALLY — SLOW AT FIRST THEN MUCH QUICKER