Edexcel A Level Maths: Pure

Your notes

5.1 Basic Trigonometry

Contents

5.1.1 Trigonometry - Definitions

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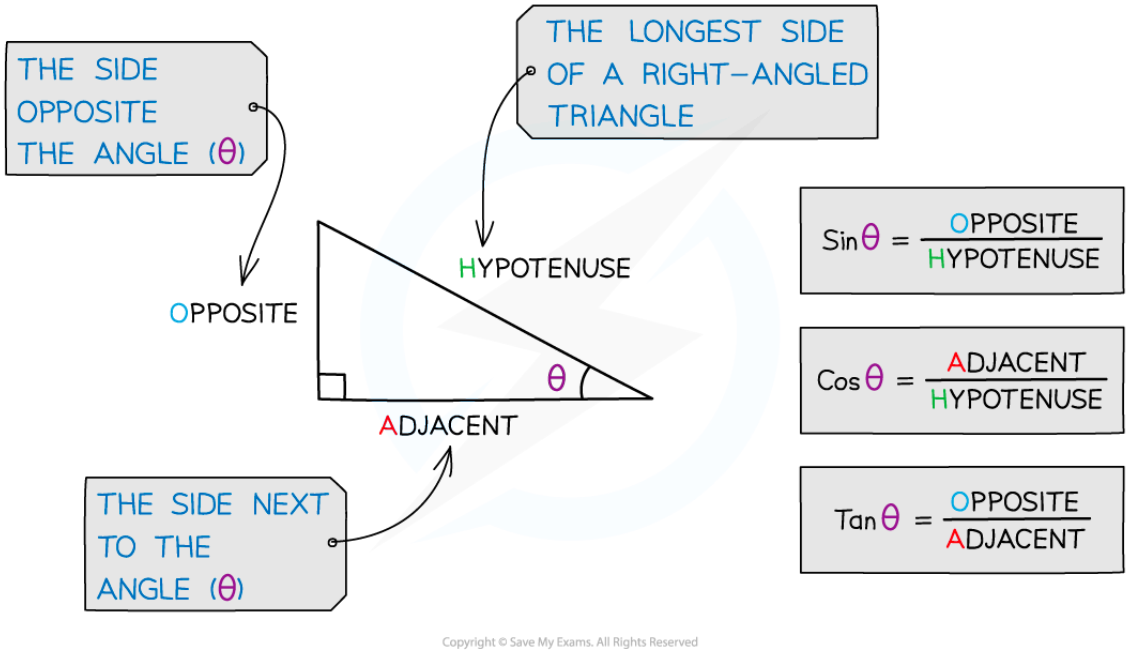
Trigonometry - Definitions

What is trigonometry?

- Trigonometry looks at the relationship between side lengths and angles of triangles
- It comes from the Greek words trigonon meaning 'triangle' and metron meaning 'measure'
- The three trigonometric functions Sine, Cosine and Tangent come from ratios of side lengths in right angled triangles



What is the unit circle and how does it help with trigonometry?

- The unit circle is a circle with radius 1 and centre (0, 0)
- It can be used to calculate trig values as a co-ordinate point (x, y) on the circle
- (x, y) = (cos θ, sin θ), where θ is the angle measured anticlockwise from the positive x-axis
- It allows us to calculate **sin**, **cos** and **tan** for angles greater than 90°

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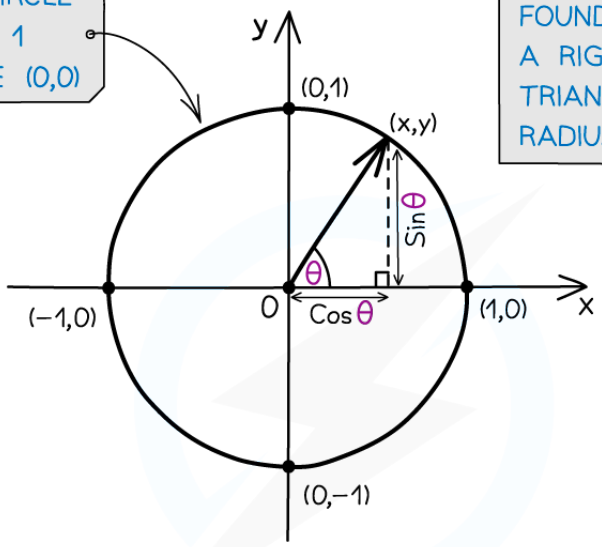
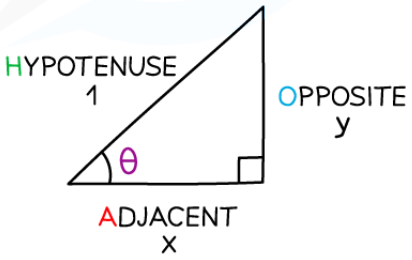
THE UNIT CIRCLE HAS RADIUS 1 AND CENTRE (0,0)

TRIG VALUES CAN BE FOUND BY MAKING A RIGHT ANGLED TRIANGLE WITH THE RADIUS AND x-AXIS



ANY POINT (x,y) ON THE UNIT CIRCLE CAN BE FOUND USING $(\cos\theta, \sin\theta)$



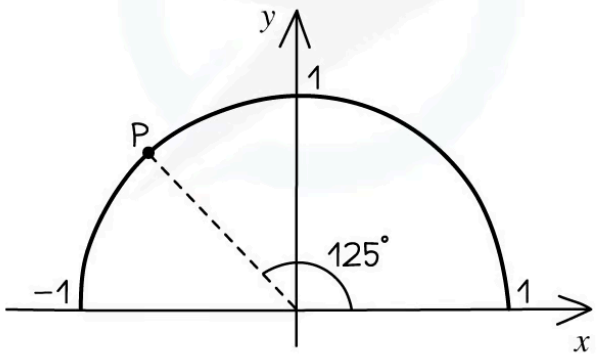
$$\cos\theta = \frac{A}{H} = \frac{x}{1} = x$$

$$\sin\theta = \frac{O}{H} = \frac{y}{1} = y$$

Worked example

a) The co-ordinates of a point on the unit circle, to 3 s.f., are (0.629, 0.777).
Find θ to the nearest degree.

b) Find the coordinates of point P on the unit circle.
Give your answer to 2 d.p.



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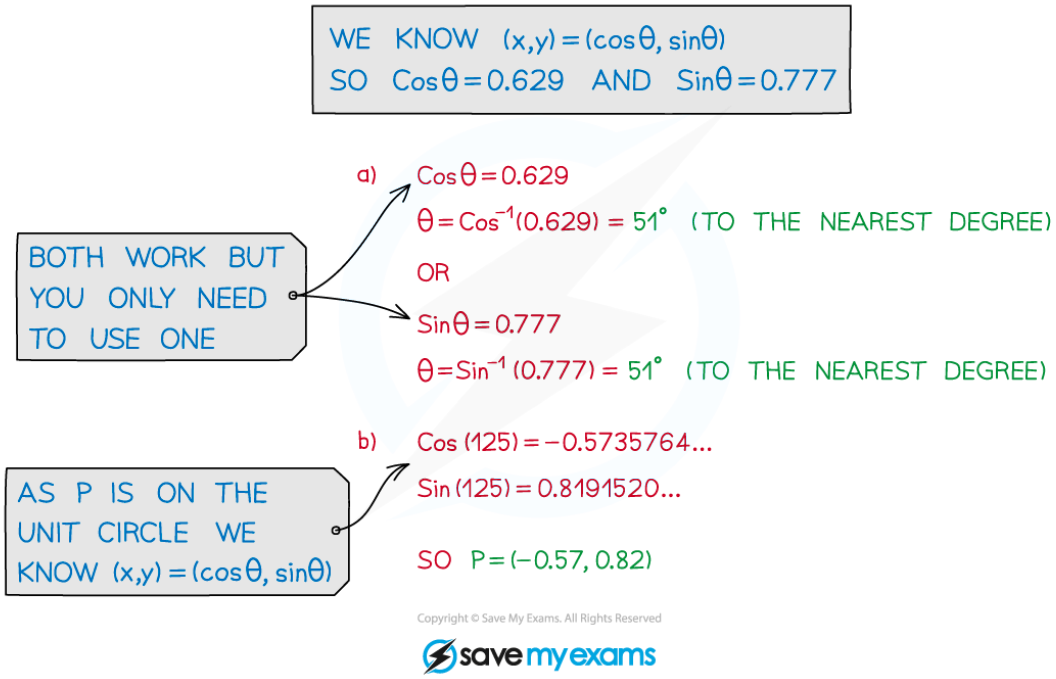

Your notes

5.1.2 Right-Angled Triangles

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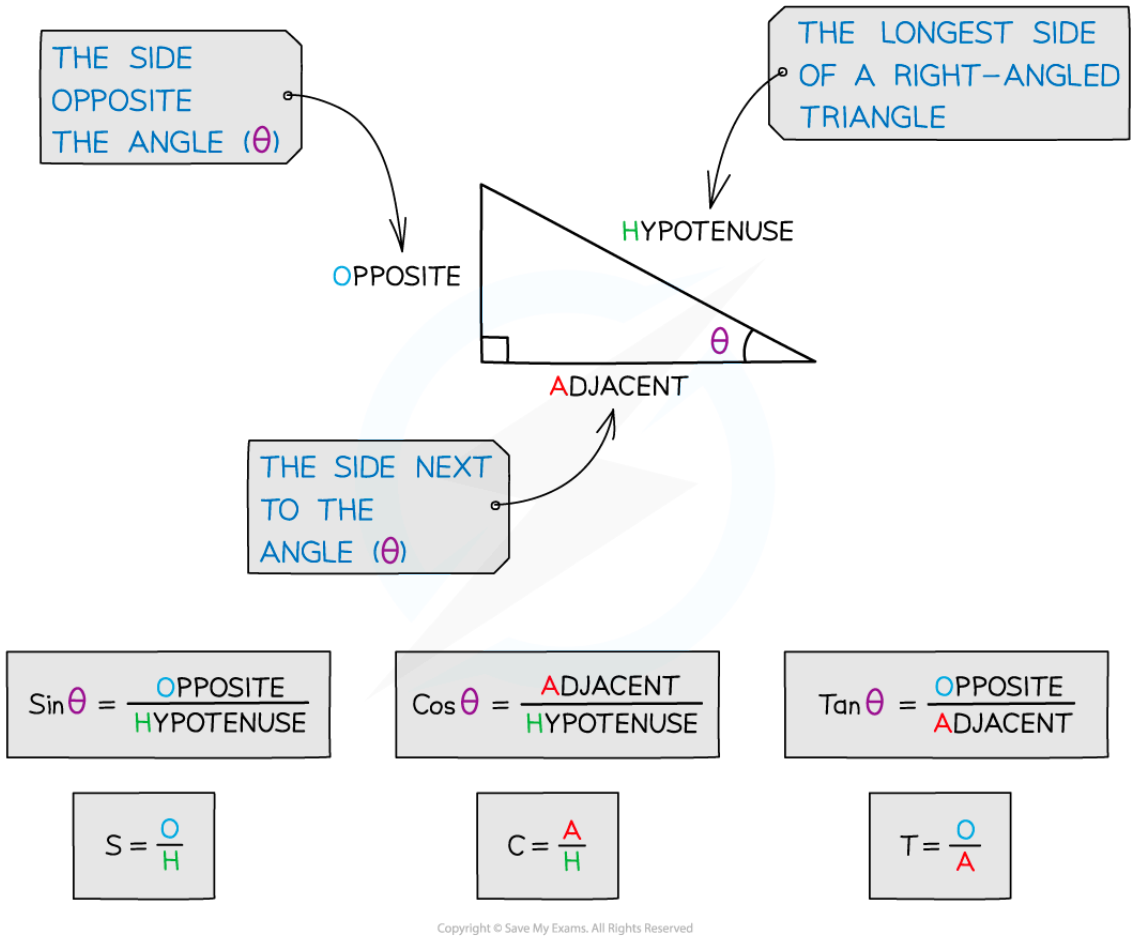
Right-Angled Triangles

What is SOHCAHTOA?

- SOH, CAH, and TOA are an easy way to remember the three basic trigonometry ratios
- They show the relationship between an angle and the different sides of a right-angled triangle which we can label **Hypotenuse**, **Adjacent** and **Opposite**
- We use them to find missing angles and sides for right-angled triangles
  - **Hypotenuse** is the longest side of a right-angled triangle
  - **Adjacent** is the side next to the angle you are working with (in between the angle and the right angle)
  - **Opposite** is the side opposite the angle you are working with



- We use the inverse trig functions to find missing angles

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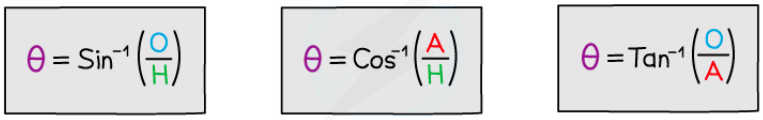
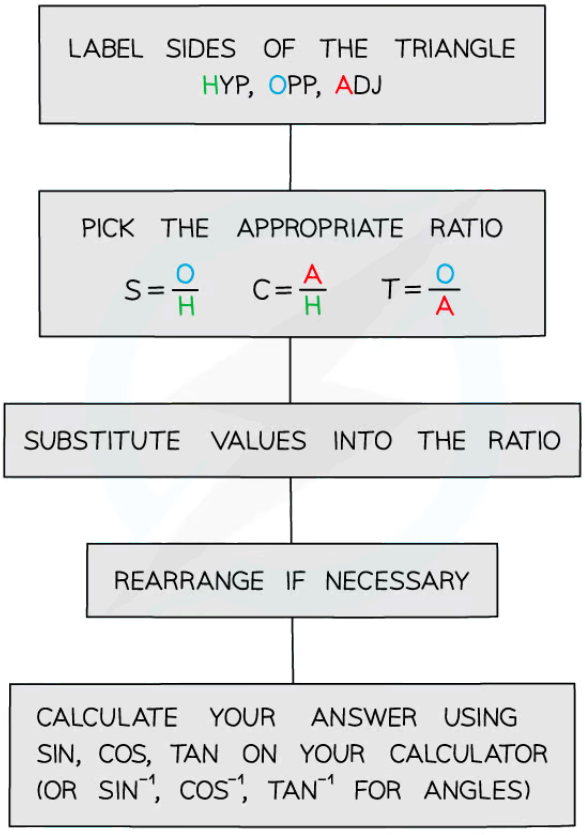


How do I use SOHCAHTOA?



Examiner Tip

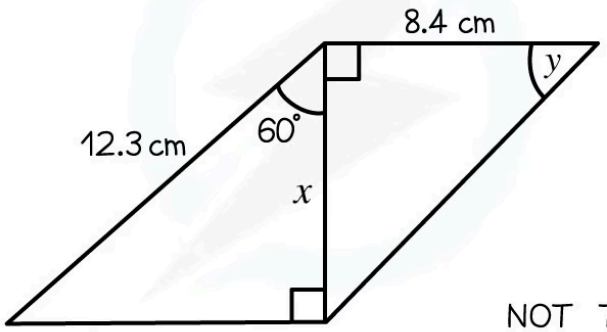
- Remember SOHCAHTOA are only for calculating missing sides and angles for right-angled triangles
  - For non-right-angled triangles you will need to use the Sine or Cosine rules.
- Make sure your calculator is in degree mode (D).
- Always check your answers are sensible...
  - Should my answer be longer or shorter than the sides I have already i.e. is it the hypotenuse?
  - Is my angle sensible...
  - Have I remembered to use $^{-1}$?
- Check if the question asks you to round your answer.

Your notes

Worked example

Find the values of x and y in the following diagram.

Give your answers to 3 significant figures.



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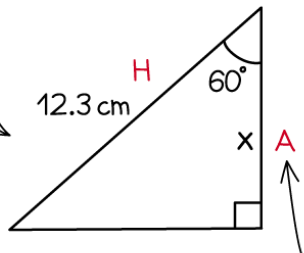
TO FIND x

SKETCH AND LABEL THE TRIANGLE



$\text{Cos}\,\theta = \frac{A}{H}$

WE KNOW H AND WE WANT TO FIND A SO MUST NEED CAH

$\text{Cos}\,60 = \frac{x}{12.3}$

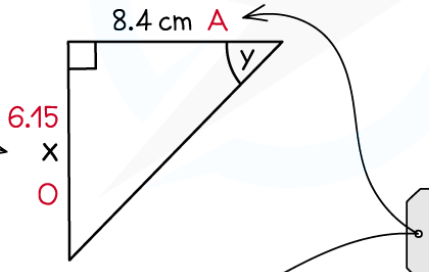
SUBSTITUTE VALUES AND REARRANGE

$x = 12.3 \times \text{Cos}\,60$

$x = 6.15\ \text{cm}$

TO FIND y

NOW WE'VE FOUND x=6.15, SKETCH AND LABEL THE TRIANGLE



WE HAVE A AND O SO MUST NEED TOA

$\text{Tan}\,\theta = \frac{O}{A}$

$\text{Tan}\,y = \frac{6.15}{8.4}$

USE Tan^{-1} TO CALCULATE THE ANGLE

$y = \text{Tan}^{-1}\left(\frac{6.15}{8.4}\right)$

$y = 36.209...$

$y = 36.2$


Your notes

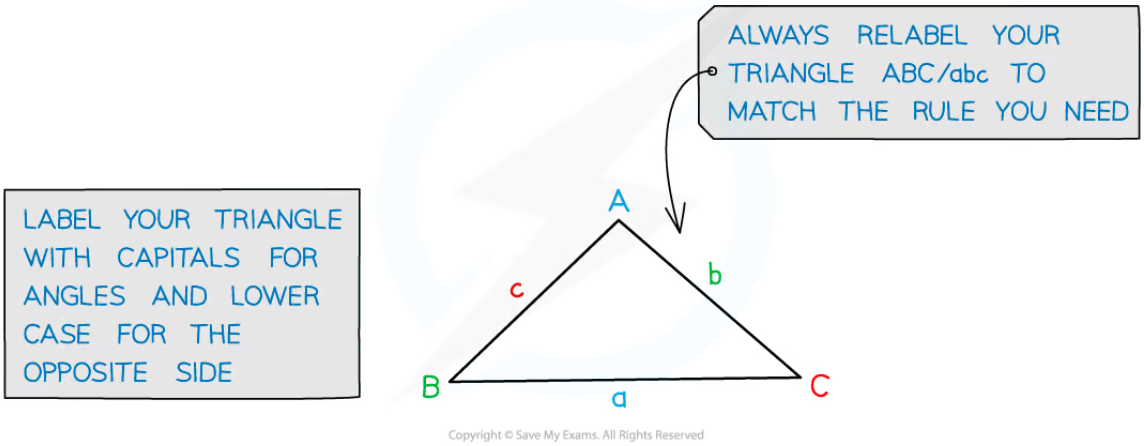
5.1.3 Non-Right-Angled Triangles

Your notes

Non-Right-Angled Triangles

What do I do if there isn't a right-angle?

- The Sine Rule, Cosine Rule and Area Formula can be used for ANY type of triangle
- They can help us calculate missing side lengths, angles and area



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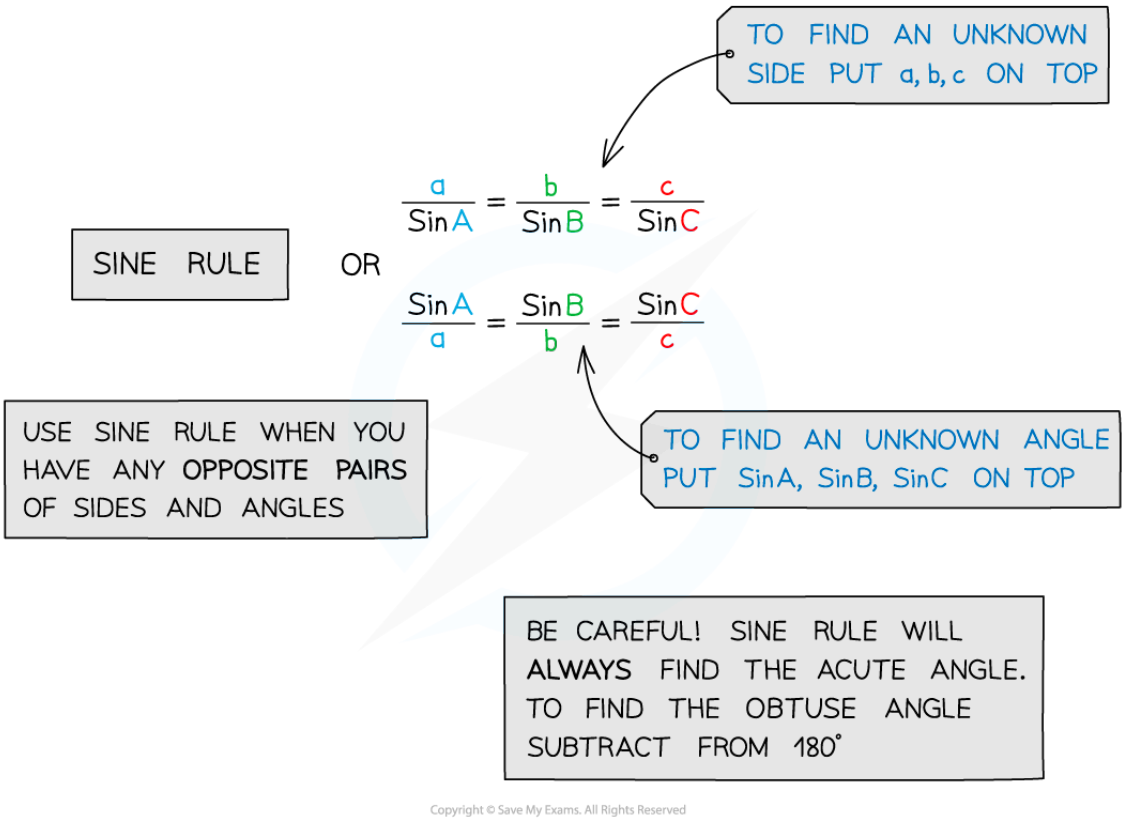

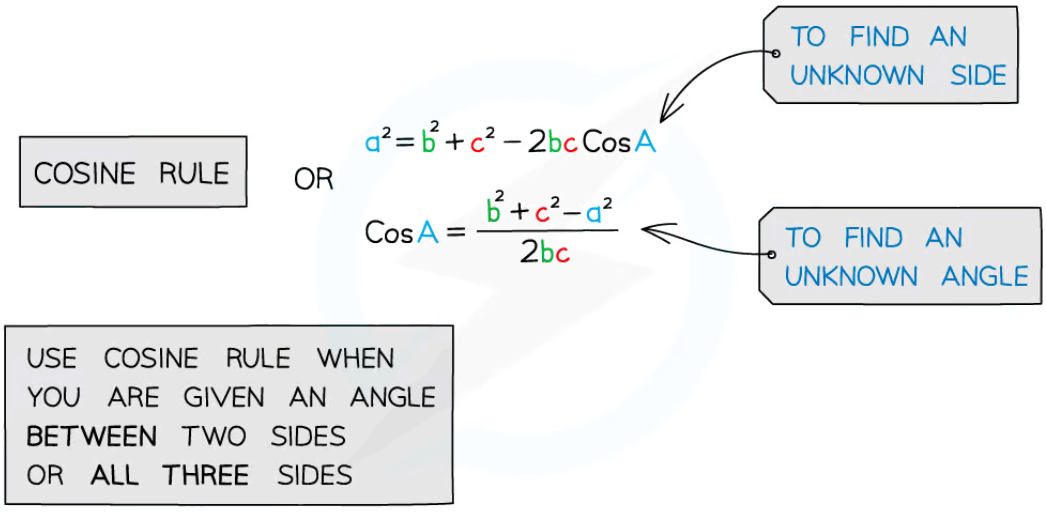

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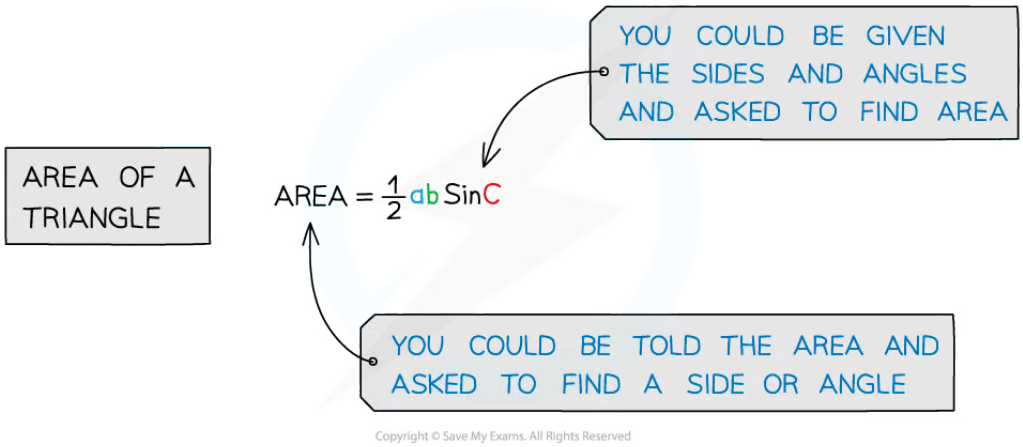

How do I know which rule to use?

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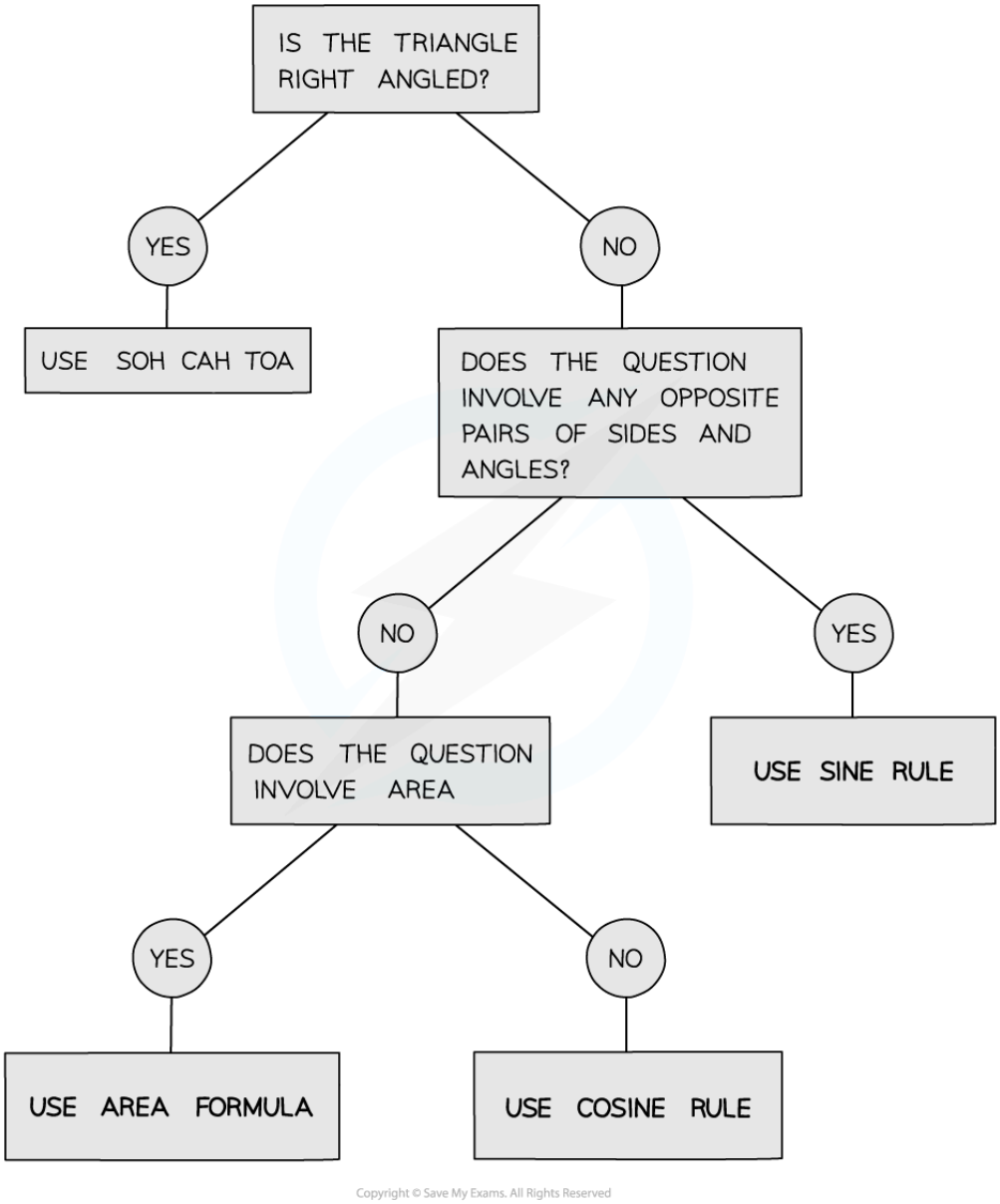


- Once you know which rule to use, simply label your triangle, substitute in the values and rearrange if necessary

Examiner Tip

- If you are given two angles in a triangle, finding the missing angle by subtracting from 180° might help.
- Make sure you are careful to pick the correct rule to use, you'll need to remember all the rules as they aren't in the formula booklet.
- Make sure your calculator is in degree mode (D).
- Always check your answers are sensible...
  - Have I remembered to square root at the end?
  - Have I remembered to use $^{-1}$?
- Check if the question asks you to round your answer.


Your notes

Worked example

Your notes

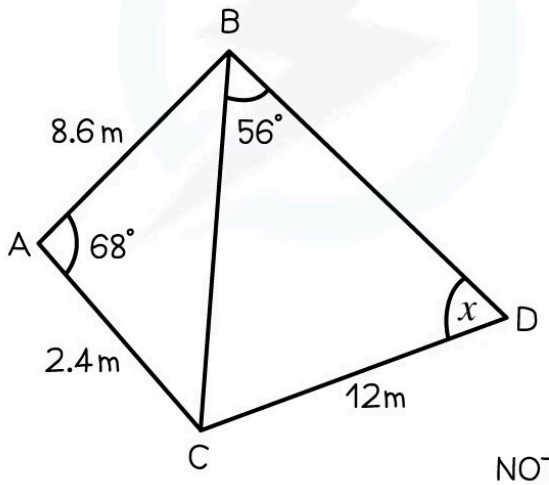
ABCD is a quadrilateral as shown in the diagram below.

a) Find the area of ABC.

Angle x is an acute angle.

b) Work out the size of angle x.

Give your answers to 3 significant figures.



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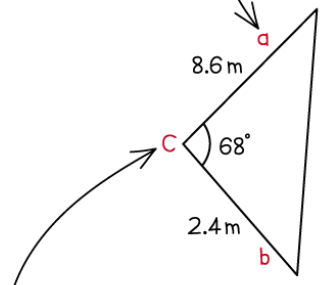
a)



IT DOESN'T MATTER WHICH WAY AROUND YOU LABEL a AND b

SUBSTITUTE AND CALCULATE

RELABEL DIAGRAM TO MAKE THE GIVEN ANGLE C

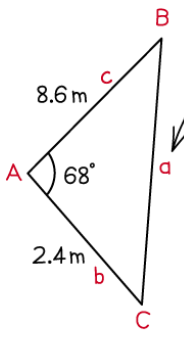
$$\text{AREA} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2} \times 8.6 \times 2.4 \times \sin 68$$

$$= 9.568...$$

$$= 9.57 m^2 \quad (3SF)$$

b)



WE HAVE AN ANGLE BETWEEN TWO SIDES SO WE CAN START BY FINDING THE LENGTH OF BC USING COSINE RULE

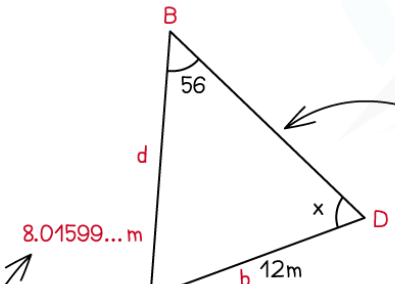
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 2.4^2 + 8.6^2 - 2 \times 2.4 \times 8.6\cos 68$$

$$a^2 = 64.256...$$

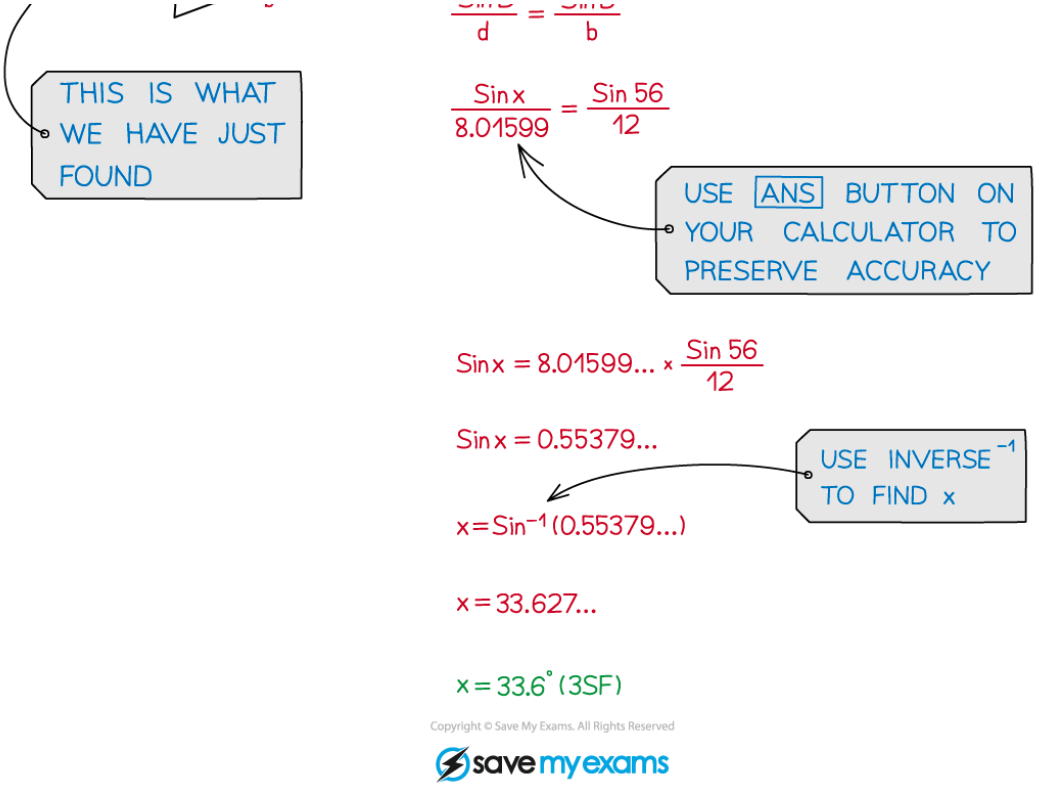
DON'T FORGET TO $\sqrt{\text{SQUARE ROOT}}$

$$a = \sqrt{64.256...} = 8.01599...$$



WE NOW HAVE OPPOSITE PAIRS OF SIDES AND ANGLES SO CAN USE SINE RULE TO FIND x

$$\frac{\sin D}{} \quad \frac{\sin B}{}$$



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