Edexcel A Level Maths: Pure

Your notes

7.5 Implicit Differentiation

Contents

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7.5.1 Implicit Differentiation

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Implicit Differentiation

What is implicit differentiation?

- An equation connecting x and y is not always easy to write **explicitly** in the form $y = f(x)$ or $x = f(y)$
- However you can still differentiate such an equation **implicitly** using the **chain rule:**

$$1.\quad \frac{d}{dx} f(y) = f'(y)\frac{dy}{dx}$$

- $\frac{d}{dx}$ MEANS "THE DERIVATIVE WITH RESPECT TO x (OF WHAT FOLLOWS)". SO $\frac{d}{dx} f(y)$ HERE IS DERIVATIVE OF f(y) WITH RESPECT TO x.
- $f'(y)$ IS THE DERIVATIVE OF f(y) WITH RESPECT TO y.
- IT IS IMPORTANT TO KEEP THIS STRAIGHT! (SEE THE EXAMPLE BELOW).

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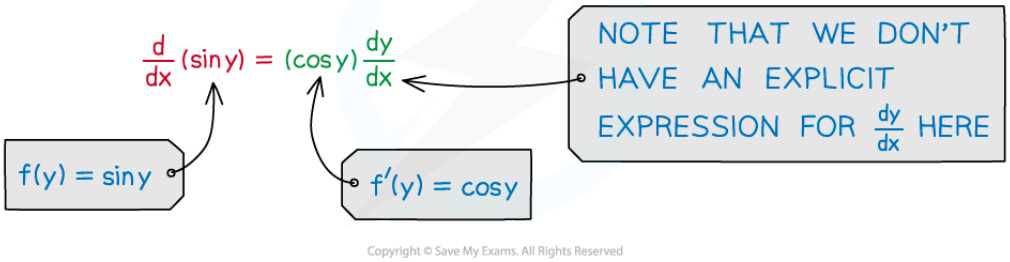
e.g. FIND THE DERIVATIVE OF siny WITH RESPECT TO x.



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- Combining this with the **product rule** gives us:

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2. $\frac{d}{dx}(f(x)g(y)) = f'(x)g(y) + f(x)g'(y)\frac{dy}{dx}$

- $f'(x)$ IS THE DERIVATIVE OF $f(x)$ WITH RESPECT TO x.
- $g'(y)$ IS THE DERIVATIVE OF $g(y)$ WITH RESPECT TO y.

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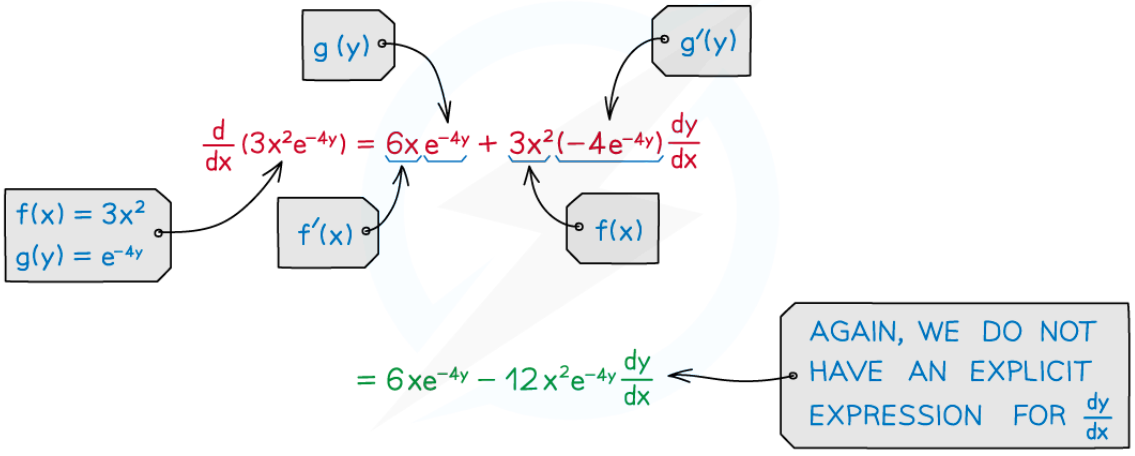
e.g. FIND THE DERIVATIVE OF $3x^2e^{-4y}$ WITH RESPECT TO x.



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- These two special cases are especially useful:

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3. $\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$

SPECIAL CASE OF 1. WITH $f(y) = y^n$

4. $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$

SPECIAL CASE OF 2. WITH $f(x) = x$ AND $g(y) = y$

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- When ***x*** and ***y*** are connected in an **equation** you can differentiate both sides with respect to ***x*** and rearrange to find a formula (usually in terms of ***x*** **and** ***y***) for **d*y*/d*x***
  - Note that **d*y*/d*x*** is a single algebraic object
  - When rearranging do not treat **d*y*/d*x*** as a fraction
  - Especially do not try to separate **d*y*** and **d*x*** and treat them as algebraic objects on their own!

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e.g. A CURVE IS DESCRIBED BY THE EQUATION

$$\sin y = 3x^2e^{-4y}$$

USE IMPLICIT DIFFERENTIATION TO FIND AN EXPRESSION FOR $\frac{dy}{dx}$.

$$\frac{d}{dx}(\sin y) = \frac{d}{dx}(3x^2e^{-4y})$$

DIFFERENTIATE BOTH SIDES WITH RESPECT TO x

$$\cos y\frac{dy}{dx} = 6xe^{-4y} - 12x^2e^{-4y}\frac{dy}{dx}$$

WE ALREADY WORKED OUT THESE DERIVATIVES ABOVE!

$$(12x^2e^{-4y} + \cos y)\frac{dy}{dx} = 6xe^{-4y}$$

REARRANGE TO MAKE $\frac{dy}{dx}$ THE SUBJECT

$$\frac{dy}{dx} = \frac{6xe^{-4y}}{12x^2e^{-4y} + \cos y}$$

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Examiner Tip

- When using implicit differentiation you will not always be able to write **dy/dx** simply as a function of **x**.
- However, this does not stop you from answering questions involving the derivative.

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Worked example

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A curve C has the equation

$$e^{3x} = y^2 - 4xy$$

a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

b) Hence find the gradient of C at the point $(0, -1)$.

a) $\frac{d}{dx}(e^{3x}) = \frac{d}{dx}(y^2 - 4xy)$

DIFFERENTIATE BOTH SIDES WITH RESPECT TO x

$3e^{3x} = 2y\frac{dy}{dx} - 4y - 4x\frac{dy}{dx}$

FORMULA 3

FORMULA 4

$3e^{3x} + 4y = (2y - 4x)\frac{dy}{dx}$

REARRANGE TO MAKE $\frac{dy}{dx}$ THE SUBJECT

$\frac{dy}{dx} = \frac{3e^{3x} + 4y}{2y - 4x}$

b) AT $(0, -1)$ $x = 0$ AND $y = -1$

SUBSTITUTE x- AND y-COORDINATES INTO THE EQUATION FOR $\frac{dy}{dx}$

SO $\frac{dy}{dx} = \frac{3e^{3(0)} + 4(-1)}{2(-1) - 4(0)} = \frac{1}{2}$

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