

Edexcel A Level Maths: Pure



5.6 Compound & Double Angle Formulae

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5.6.1 Compound Angle Formulae

Your notes

Compound Angle Formulae

What are the compound angle formulae?

- There are six compound angle formulae (also known as addition formulae), two each for sin, cos and tan:
- For sin the +/- sign on the left-hand side matches the one on the right-hand side

$$\sin(A+B) \equiv \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) \equiv \sin A \cos B - \cos A \sin B$$

• For cos the +/- sign on the left-hand side is opposite to the one on the right-hand side

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

$$cos(A - B) \equiv cosAcosB + sinAsinB$$

■ For tan the +/- sign on the left-hand side matches the one in the numerator on the right-hand side, and is opposite to the one in the denominator

$$\tan(A+B) \equiv \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) \equiv \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

- You can **derive** the tan identity by:
 - Writing $tan(A+B) \equiv \frac{\sin(A+B)}{\cos(A+B)}$
 - Dividing the numerator and denominator by $\cos\!A\cos\!B$

Examiner Tip

All these formulae are in the formulae booklet - you don't have to memorise them.



Worked example	







- a) Express $tan(225^{\circ} 30^{\circ})$ in terms of $tan225^{\circ}$ and $tan30^{\circ}$.
- b) Hence show that $\tan 195^\circ = 2 \sqrt{3}$.

a)
$$tan(A-B) \equiv \frac{tanA - tanB}{1 + tanA tanB}$$
 FORMULA 6
ABOVE
$$tan(225^{\circ}-30^{\circ}) \equiv \frac{tan225^{\circ} - tan30^{\circ}}{1 + tan225^{\circ} tan30^{\circ}}$$

b)
$$\tan 225^{\circ} = 1$$
 $\tan 30^{\circ} = \frac{\sqrt{3}}{3}$ YOU SHOULD KNOW THESE!
$$\tan (195^{\circ}) = \tan (225^{\circ} - 30^{\circ})$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + (1)(\frac{\sqrt{3}}{3})} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$$

$$\begin{bmatrix} \times \frac{3}{3} \end{bmatrix} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$$

$$= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \times \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$

$$= \frac{12 - 6\sqrt{3}}{6}$$

$$= 2 - \sqrt{3}$$







5.6.2 Double Angle Formulae

Your notes

Double Angle Formulae

What are the double angle formulae?

- The double angle formulae are derived from the 'A+B' versions of the compound angle formulae:
 - $\sin(2A) \equiv 2\sin A \cos A$
 - $\cos(2A) \equiv \cos^2 A \sin^2 A$
 - $\cos(2A) \equiv 2\cos^2 A 1$
 - $\cos(2A) \equiv 1 2\sin^2 A$
 - $\tan(2A) \equiv \frac{2\tan A}{1 \tan^2 A}$
 - 1. $\sin 2A \equiv 2 \sin A \cos A$

$$sin(A+B) \equiv sinAcosB + cosAsinB$$

LET B=A. THEN
$$sin(2A) = sin(A+A)$$

$$= sinAcosA + cosAsinA$$

$$= 2sinAcosA$$

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2. $\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$



$$cos(A+B) \equiv cosAcosB - sinAsinB$$

LET B=A. THEN
$$cos(2A) \equiv cos(A+A)$$

$$\equiv cosAcosA - sinAsinA$$

$$\equiv cos^2A - sin^2A$$

THE OTHER TWO FORMS COME FROM REARRANGING
$$sin^2A + cos^2A \equiv 1$$
AND SUBSTITUTING

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3.
$$tan2A \equiv \frac{2tanA}{1-tan^2A}$$

$$tan(A+B) \equiv \frac{tanA + tanB}{1 - tanAtanB}$$

$$LET B=A. THEN$$

$$tan(2A) \equiv tan(A+A)$$

$$\equiv \frac{tanA + tanA}{1 - tanAtanA}$$

$$\equiv \frac{2tanA}{1 - tan^2A}$$

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Examiner Tip

- The double angle formulae are **not** included in the formulae booklet you have to know them.
- If you forget them, you can always derive them from the compound angle formulae (which are in the formula booklet) as shown above.



Worked example	







a) Show that $7\sin 2\theta - 3\sin \theta = 0$ can be written in the form

$$a\sin\theta(b\cos\theta + c) = 0$$

stating the values of a, b and c.

b) Hence solve, for $0 \le \theta < 360^\circ$, the equation $7\sin 2\theta - 3\sin \theta = 0$. Give your answers to 1 decimal place.

USE DOUBLE ANGLE

a) $7\sin 2\theta = 7(2\sin\theta\cos\theta) = 14\sin\theta\cos\theta$

WE CAN REWRITE THE EQUATION AS $14\sin\theta\cos\theta - 3\sin\theta = 0$ $\sin\theta (14\cos\theta - 3) = 0$ WITH a=1 b=14 c=-3

b) $\sin\theta (14\cos\theta - 3) = 0$

SO
$$\sin\theta = 0$$
 OR $\cos\theta = \frac{3}{14}$

IF
$$\cos\theta = \frac{3}{14}$$

$$\theta$$
 = 77.63 OR θ = 360 - 77.63 = 282.37



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USE CALCULATOR
TO FIND PRINCIPAL
VALUE

THEN FIND OTHER
VALUES IN THE
REQUIRED INTERVAL



THE SOLUTIONS ARE

 $\theta = 0^{\circ}$, 77.6°, 180°, 282.4° (1 d.p.)

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5.6.3 R addition formulae Rcos Rsin etc

Your notes

R addition formulae Rcos Rsin etc

How can I simplify expressions like $a \sin x \pm b \cos x$ or $a \cos x \pm b \sin x$?

- If **a** and **b** are positive constants, then
 - $a \sin x + b\cos x$ can be rewritten as $R \sin (x + \alpha)$
 - $a \sin x b\cos x$ can be rewritten as $R \sin (x \alpha)$
 - $a \cos x + b \sin x$ can be rewritten as $R \cos (x \alpha)$
 - $a \cos x b \sin x \cos \theta$ can be rewritten as $R \cos (x + \alpha)$
- In all four cases R > 0 and $0 < \alpha < 90^\circ$, with:

$$a = R \cos \alpha$$

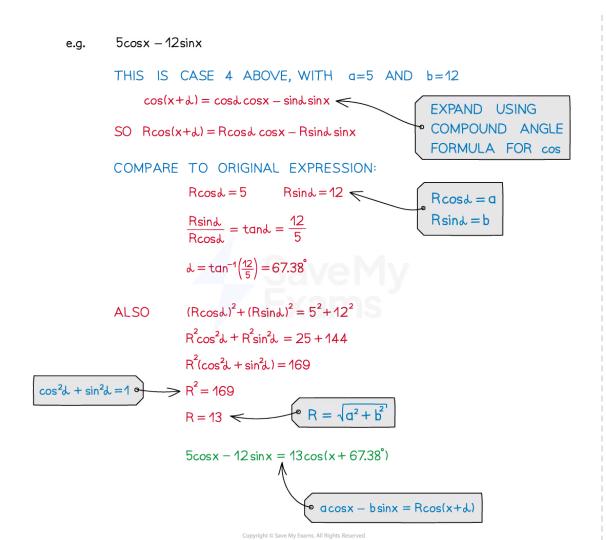
$$b = R \sin \alpha$$

$$R = \sqrt{a^2 + b^2}$$
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■ To rewrite the expression use the appropriate **compound angle formula** and equate coefficients



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Examiner Tip

• There is more than one way to rewrite a given expression, and the question in the exam will tell you which form to use.



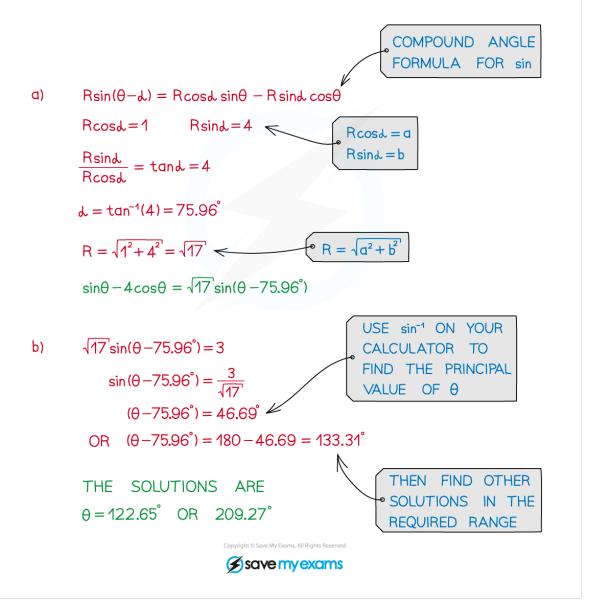
Worked example	







- a) Express $\sin\theta 4\cos\theta$ in the form $R\sin(\theta \alpha)$, where R > 0 and $0 < \alpha < 90^{\circ}$.
- b) Hence solve $\sin \theta 4\cos \theta = 3$ for $0 \le \theta < 360^{\circ}$.



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