# Edexcel A Level Maths: Pure

Your notes

## 9.1 Parametric Equations

### Contents





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# 9.1.1 Parametric Equations – Basics

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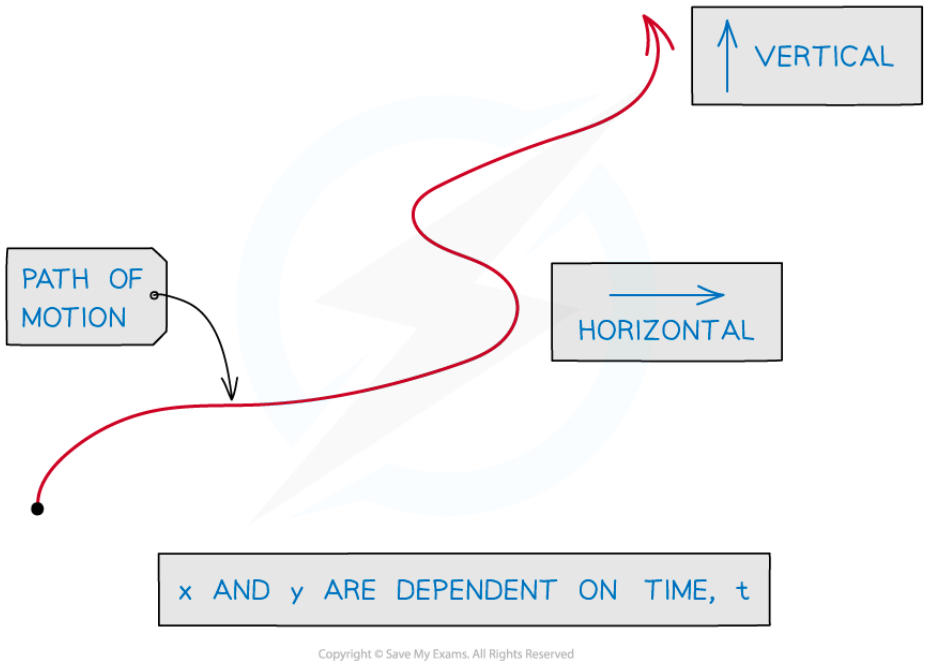
## Parametric Equations – Basics

### What are parametric equations?

- Graphs are usually described by a **Cartesian equation**
  - The equation involves **x** and **y** only
- Equations like this can sometimes be rearranged into the form, **y = f(x)**



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- In **parametric equations** both **x** and **y** are dependent on a third variable
  - This is called a **parameter**
  - **t** and **θ** are often used as parameters
- A common example …
  - **x** is the horizontal position of an object
  - **y** is the vertical position of an object
  - and the position of the object is dependent on time **t**
- **x** is a function of **t**, **y** is a function of **t**
  - **x = f(t)**
  - **y = g(t)**

### What do I do with parametric equations?



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- It is still possible to plot a graph of **y** against **x** from their parametric equations

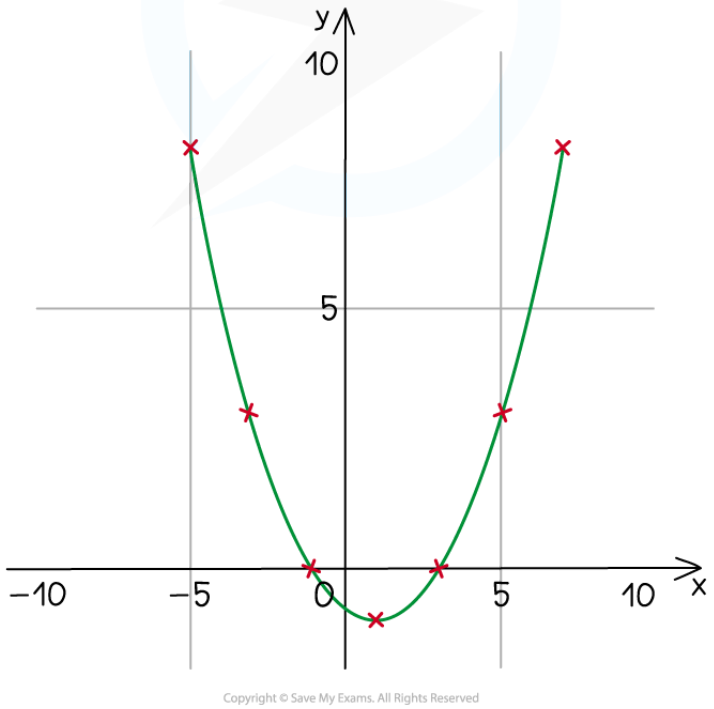
**PLOTTING A GRAPH FROM PARAMETRIC EQUATIONS**

e.g. PLOT THE GRAPH GIVEN BY THE PARAMETRIC EQUATIONS $x = 2t + 1$ AND $y = t^2 - 1$ FOR $-3 \leqslant t \leqslant 3$.

**CONSTRUCT A TABLE OF VALUES**

| t | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| x | −5 | −3 | −1 | 1 | 3 | 5 | 7 |
| y | 8 | 3 | 0 | −1 | 0 | 3 | 8 |

**PLOT THE GRAPH OF y AGAINST x**

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- Also see Parametric Equations – Sketching Graphs



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## What is the link between parametric equations and circles?

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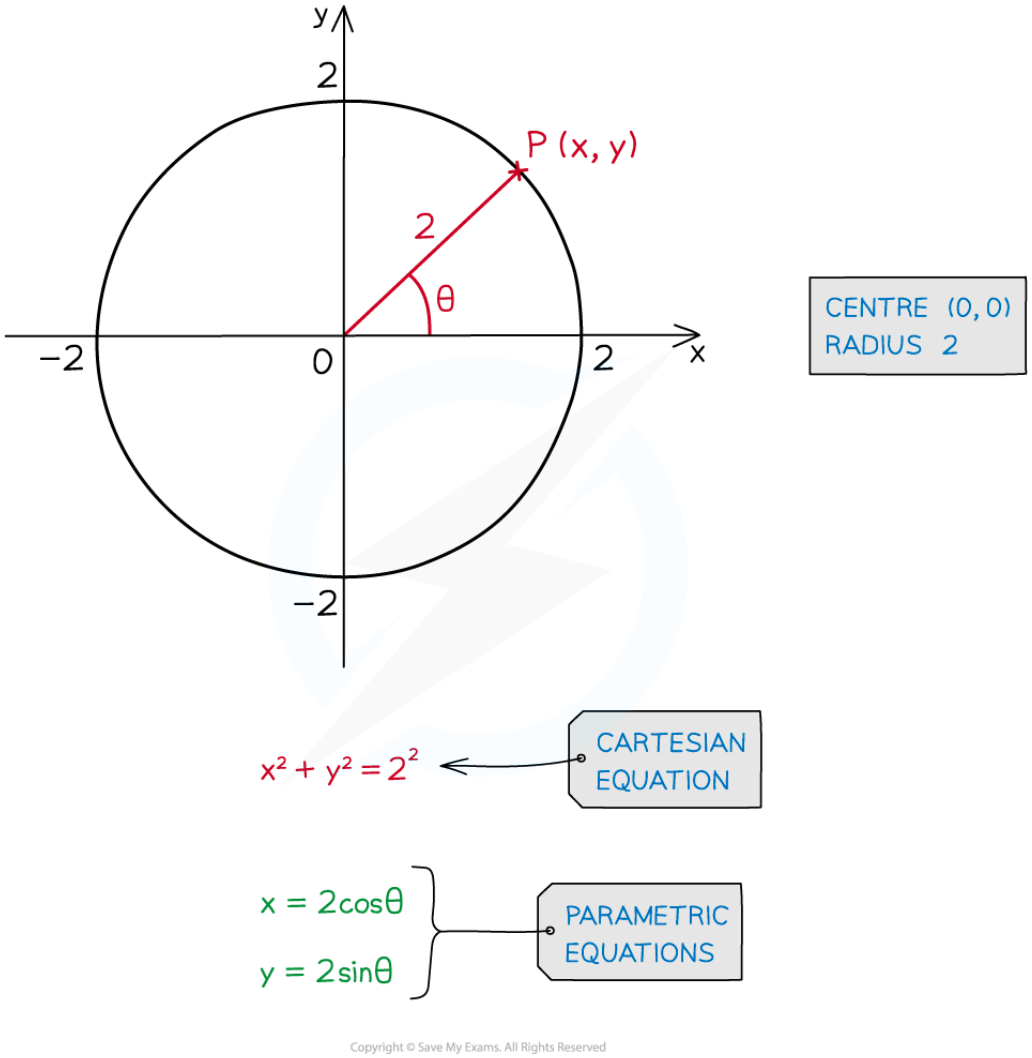


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- For a circle, centre (0, 0) and radius **r**
  - **x = rcos θ**
  - **y = rsin θ**
  - (Note that **r** is constant, this is not two parameters)
- For a circle, centre (**a**, **b**) and radius **r**
  - **x = rcos θ + a**
  - **y = rsin θ + b**



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## Worked example

(a) Write down the radius and the centre of the circle defined by the parametric equations

$$x = 3\cos\theta$$

and

$$y = 3\sin\theta - 4$$

(b) Hence write down the Cartesian equation of the circle

a) CENTRE: (0, −4)
RADIUS: 3

$x = r\cos\theta + a$
$y = r\sin\theta + b$

b) $x^2 + (y+4)^2 = 9$ ← $r^2 = 3^2 = 9$

CENTRE (0, −4)

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# 9.1.2 Parametric Equations - Eliminating the Parameter

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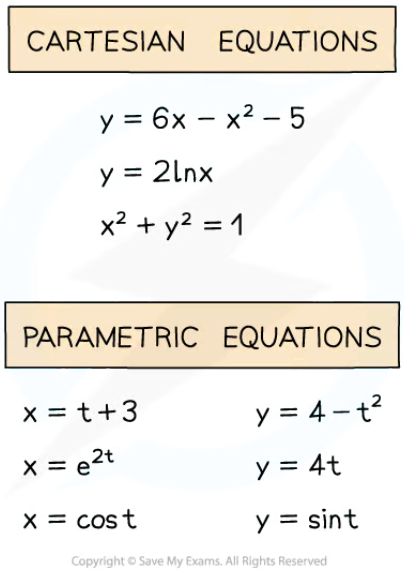
## Parametric Equations - Eliminating the Parameter

### What does eliminating the parameter mean?




- In parametric equations, **x = f(t)** and **y = g(t)**
- There is still a connection directly linking **x** and **y**
  - This will be the **Cartesian** equation of the graph

### How do I find the Cartesian equation from parametric equations?



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e.g. FIND A CARTESIAN EQUATION FOR THE CURVE GIVEN PARAMETRICALLY AS

$$x = \frac{t-3}{4} \qquad y = e^{2t}$$

STEP 1 REARRANGE ONE EQUATION TO MAKE t THE SUBJECT

$4x = t - 3$

$t = 4x + 3$

BOTH EQUATIONS EASY TO REARRANGE

STEP 2 SUBSTITUTE INTO OTHER EQUATION

$y = e^{2(4x+3)}$

STEP 3 REARRANGE INTO CARTESIAN FORM

$y = e^{8x+6}$

NOT MUCH TO DO HERE!

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- STEP 1: Rearrange one of the equations to make t the subject
  - Either **t = p(x)** or **t = q(y)**
- STEP 2: Substitute into the other equation
- STEP 3 Rearrange into the desired (Cartesian) form

**How do I eliminate t when trig is involved?**



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e.g. FIND A CARTESIAN EQUATION FOR THE CURVE GIVEN PARAMETRICALLY AS

$x = 3 + \cos t \qquad y = 2 - \sin t$

STEP 1 REARRANGE BOTH EQUATIONS INTO THE FORM $\cos t = \ldots$ AND $\sin t = \ldots$

$\cos t = x - 3$

$\sin t = 2 - y$

STEP 2 SQUARE BOTH SIDES OF BOTH EQUATIONS

$\cos^2 t = (x-3)^2$

$\sin^2 t = (2-y)^2$

STEP 3 ADD THE EQUATIONS TOGETHER

$\cos^2 t + \sin^2 t = (x-3)^2 + (2-y)^2$

STEP 4 USE THE TRIG IDENTITY "$\sin^2 x + \cos^2 x \equiv 1$" TO ELIMINATE $t$

$1 = (x-3)^2 + (2-y)^2$

STEP 5 REARRANGED INTO DESIRED FORM

$(x-3)^2 + (y-2)^2 = 1$

$(y-2)^2 = (2-y)^2$

EQUATION OF A CIRCLE CENTRE: (3, 2) RADIUS: 1

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- STEP 1 Rearrange both equations into the forms "cos t = ..." and "sin t = ..."
- STEP 2 Square BOTH sides of BOTH equations
- STEP 3 Add the equations together
- STEP 4 The trig identity "$\sin^2 x + \cos^2 x \equiv 1$" eliminates **t**
- STEP 5 Rearrange into desired (Cartesian) form
  - This technique is seen in Trigonometric Identities


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## Examiner Tip

When choosing which equation to rearrange, aim for "as simple as possible":

- Linear equations are simpler than quadratics
  - eg Rearrange $x = 2t + 3$
    or

    $y = 3t^2 + 3t - 4$ ?
- Single exponential terms are quite easy to deal with
  - eg $x = e^t \rightarrow t = \ln x$

Trig identities may be needed and remember **squared** terms are good!

- eg $\sin^2 x + \cos^2 x \equiv 1$



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## Worked example

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Find the Cartesian equation for the curve C defined by the parametric equations

$$x = 3\sin 2t$$

and

$$y = 2\cos 2t$$

- THIS IS A sin/cos BASED QUESTION
- AIM FOR "SQUARE AND ADD"

STEP 1 REARRANGE BOTH INTO FORM "sin t = ..." AND "cos t = ..."

$\frac{x}{3} = \sin 2t$ $\frac{y}{2} = \cos 2t$

STEP 2 & 3 "SQUARE AND ADD"

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = \sin^2 2t + \cos^2 2t$

STEP 4 ELIMINATE t

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

STEP 5 REARRANGE INTO DESIRED FORM

$\frac{x^2}{9} + \frac{y^2}{4} = 1$ ← EQUATION OF AN ELIPSE



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$4x^2 + 9y^2 = 36$

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# 9.1.3 Parametric Equations - Sketching Graphs


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## Parametric Equations - Sketching Graphs

### How do I sketch a graph from parametric equations?

- **Plotting** a graph is covered in Parametric Equations - Basics



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e.g. SKETCH THE GRAPH OF y AGAINST x FOR THE PARAMETRIC EQUATIONS

$x = 3t + 1 \quad y = t^2 - 4$

$3t + 1 = 0$

$t = -\frac{1}{3} \longrightarrow y = (-\frac{1}{3})^2 - 4$

$\left(0, -\frac{35}{9}\right) \quad y = -\frac{35}{9}$

y-AXIS INTERCEPT
$x = 0$

$t^2 - 4 = 0$

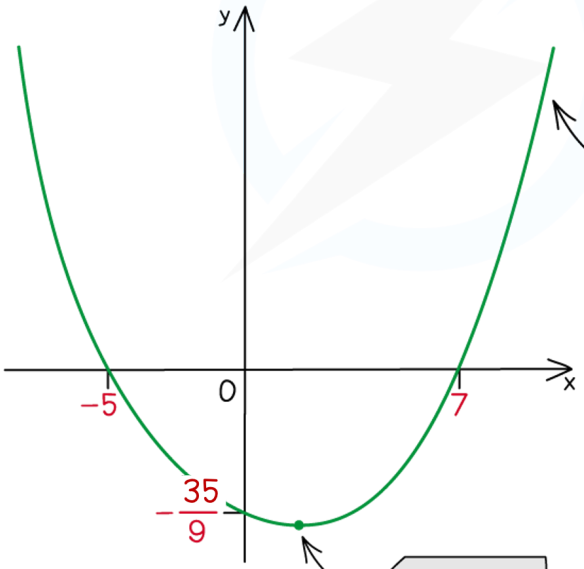
$t = \pm 2 \longrightarrow x = 3 \times 2 + 1 = 7$

$x = 3 \times -2 + 1 = -5$

$(-5, 0)$

$(7, 0)$

x-AXIS INTERCEPT(S)
$y = 0$




y WILL BE POSITIVE FOR LARGE x

TURNING POINT

IT IS NOT REQUIRED BUT THE CARTESIAN EQUATION IS $9y = x^2 - 2x - 35$ WHICH IS A QUADRATIC

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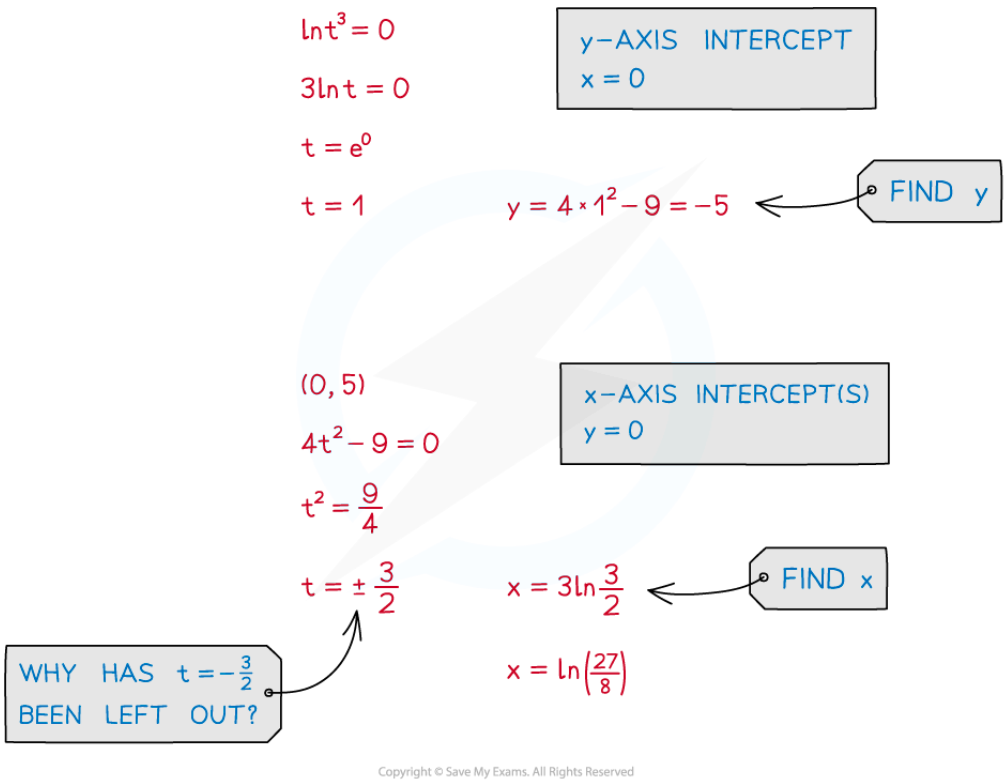
- Still find the key features of a graph …
  - … the **y**-axis intercept
  - … the **x**-axis intercept(s)
  - … asymptotes
  - … location of (and if required coordinates of) stationary points (see Parametric Differentiation)
- **Sketch** these points and join up accordingly

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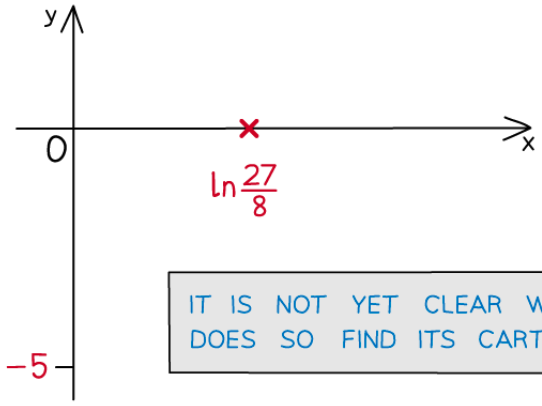



IT IS NOT YET CLEAR WHAT ELSE THE GRAPH DOES SO FIND ITS CARTESIAN EQUATION

$x = \ln t^3$

$x = 3\ln t$

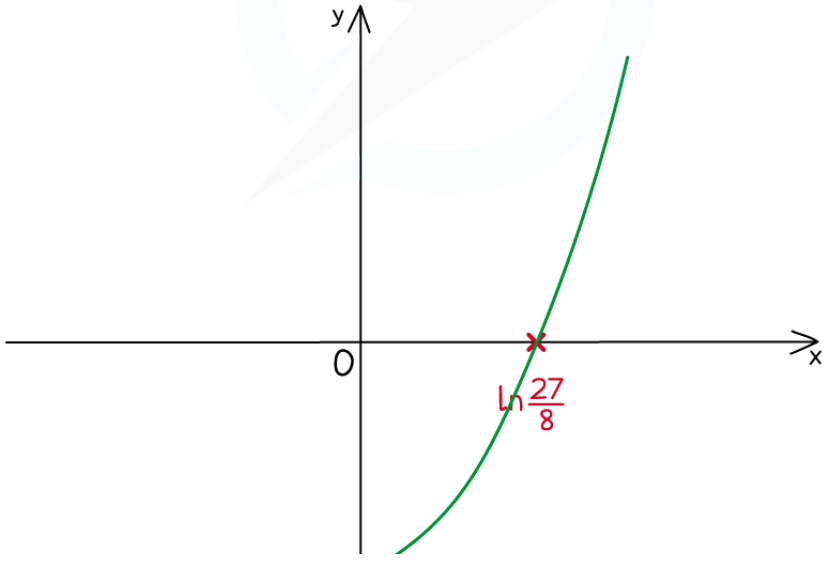
$\ln t = \frac{x}{3}$

$t = e^{\frac{x}{3}}$

$y = 4\left(e^{\frac{x}{3}}\right)^2 - 9$

$y = 4e^{\frac{2}{3}x} - 9$

A POSITIVE EXPONENTIAL GRAPH
∘ ASYMPTOTE AT $y = -9$



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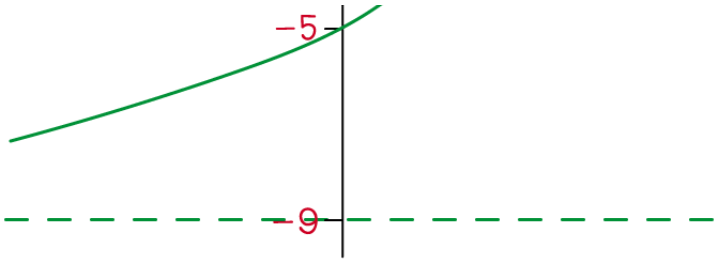


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## Examiner Tip

- Not all curves defined parametrically lead to familiar shaped graphs and it may be worth plotting a few extra points by calculating them.
- Your calculator may be able to produce a table of values quickly, however, remember you are sketching and not plotting.
- It may be easier to find the Cartesian equation first and draw the graph from that – this will depend on the question.
- It is only a **definite** strategy if you cannot make progress otherwise.
- If you are given the sketch of a graph it is usually only for reference and can be used to check answers for axes intercepts, etc.



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## Worked example

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Sketch the graph of the curve defined by the parametric equations

$$x = e^t \text{ and } y = t^2 - t$$

stating clearly the coordinates of any points where the curve intercepts the coordinate axes.

DEDUCE $x > 0$ SO y–AXIS IS AN ASYMPTOTE

$x = e^t = 0$
NO SOLUTIONS

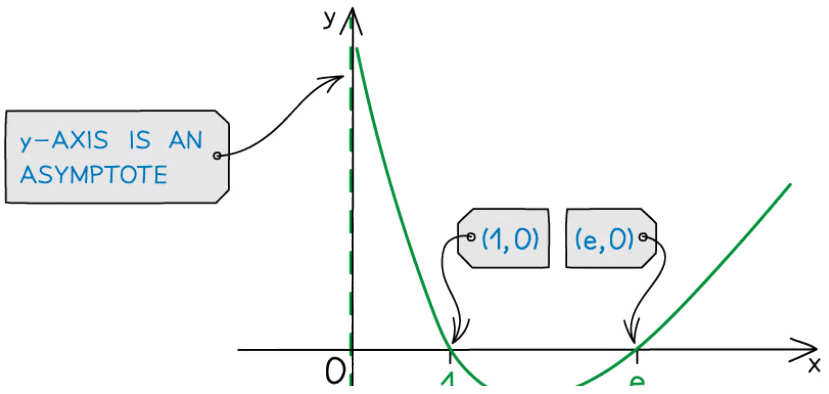
y–AXIS INTERCEPT
$x = 0$

$y = t^2 - t = 0$
$t(t-1) = 0$
$t = 0, \quad t = 1$

x–AXIS INTERCEPT(S)
$y = 0$

AT $t = 0$, $x = e^0 = 1$
AT $t = 1$, $x = e^1 = e$

THERE WILL BE A TURNING POINT BETWEEN $x = 1$ AND $x = e$



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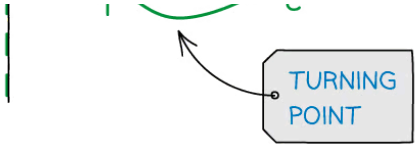


WHY IS THE GRAPH THIS WAY UP?

ANSWER: AS $t \to -\infty$, $x \to 0$, $y \to \infty$

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