Edexcel A Level Maths: Pure

Your notes

4.1 Binomial Expansion

Contents

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Your notes

Binomial Expansion

What is the binomial expansion?

$$(a+b)^n = {}^nC_0(a)^n(b)^0 + {}^nC_1(a)^{n-1}(b)^1 + {}^nC_2(a)^{n-2}(b)^2 + {}^nC_3(a)^{n-3}(b)^3 + \ldots + {}^nC_r(a)^{n-r}(b)^r + \ldots + {}^nC_{n-1}(a)^1(b)^{n-1} + {}^nC_n(a)^0(b)^n$$

$${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \qquad n \in \mathbb{N}$$

- Look at the pattern
  - Start at ${}^nC_0$, then ${}^nC_1$, ${}^nC_2$, etc
  - Powers of **a** start at **n** and decrease by 1
  - Powers of **b** start at 0 and increase by 1
- There are shortcuts but these hide the pattern
  - ${}^nC_0 = {}^nC_n = 1$
  - ${}^nC_1 = {}^nC_{n-1} = n$
  - ${}^nCr = {}^nC_{n-r}$
  - $(b)^0 = (a)^0 = 1$

- Use the shortcuts once familiar with the pattern
- ! means factorial

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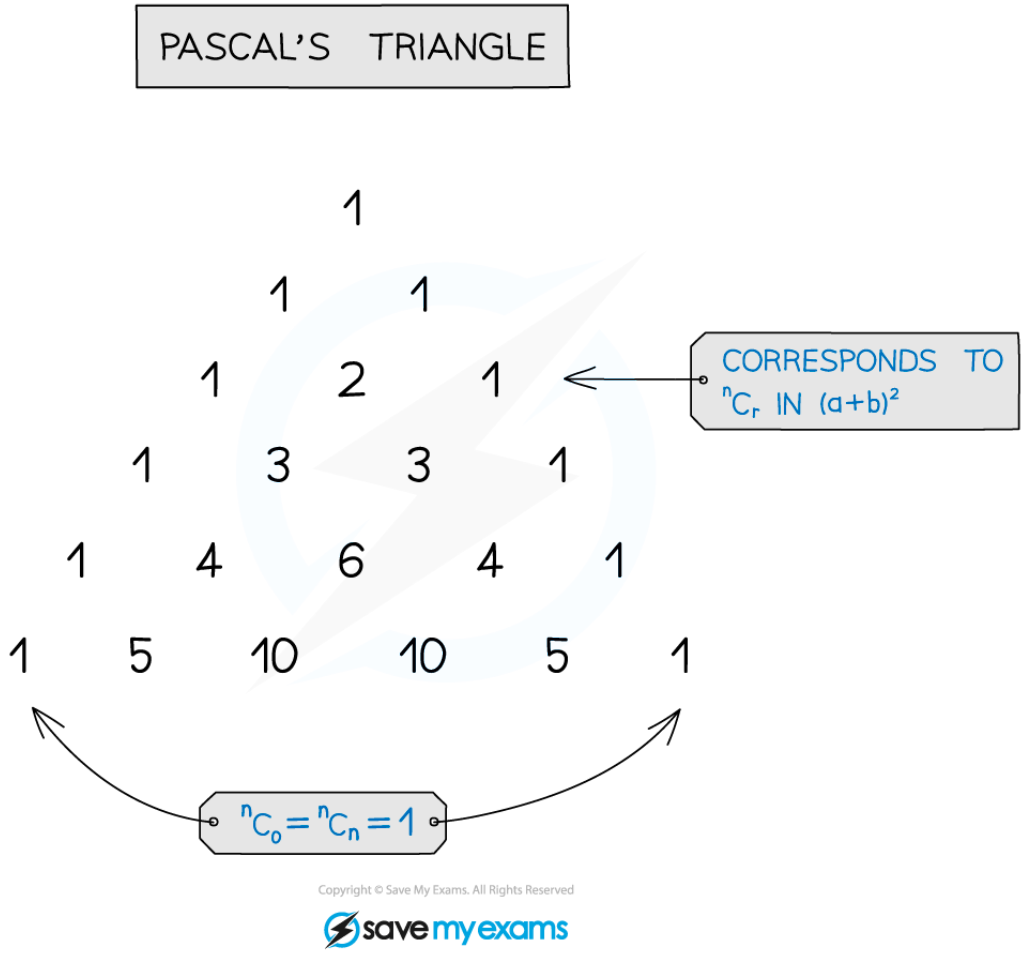
$$(a+b)^n = a^n + na^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + b^n$$

- This is given in the formula booklet

What about Pascal's triangle?



- Pascal's triangle is an alternative to ^nC_r
- It is useful for lower values of **n**

- For larger **n** it is slow and prone to arithmetic errors

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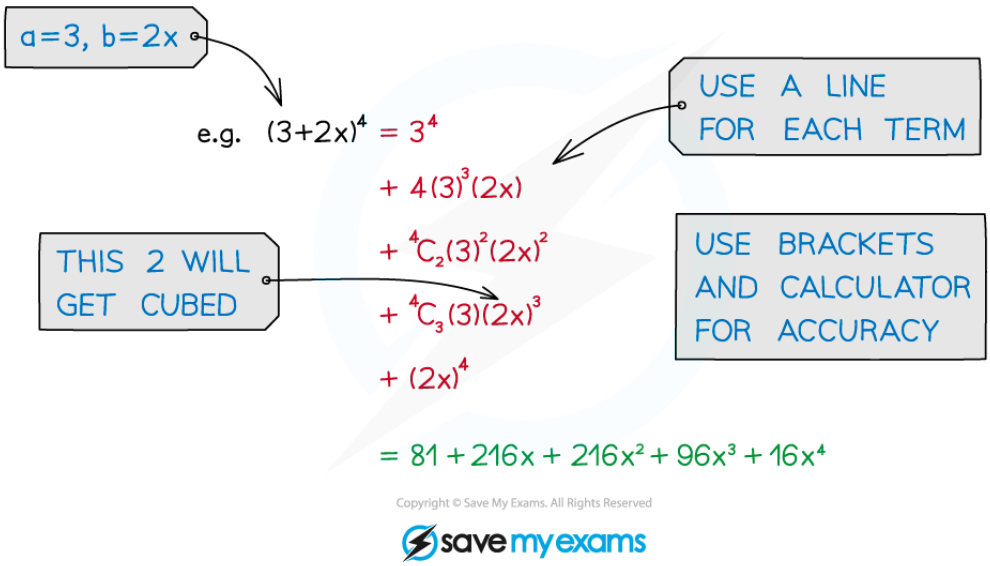
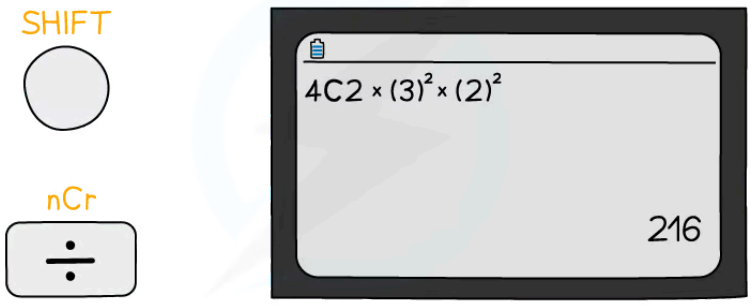
How do I expand brackets with binomial expansion?



- Use a line for each term to make things easier to read and follow
- Use brackets, particularly helpful when negatives involved
- Use a calculator for ${}^nC_r$



Worked example

Find the first four terms, in ascending powers of x, in the expansion of $(2-3x)^6$

USE SEPARATE LINES AND BRACKETS TO KEEP WORKING CLEAR

$$(2-3x)^6 \approx (2)^6$$
$$+ 6(2)^5(-3x)$$
$$+ {}^6C_2(2)^4(-3x)^2$$
$$+ {}^6C_3(2)^3(-3x)^3$$

TYPE ${}^6C_3 \times (2)^3 \times (-3)^3$ INTO CALCULATOR

$$= 64 - 576x + 2160x^2 - 4320x^3$$

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Applications of Binomial Expansion

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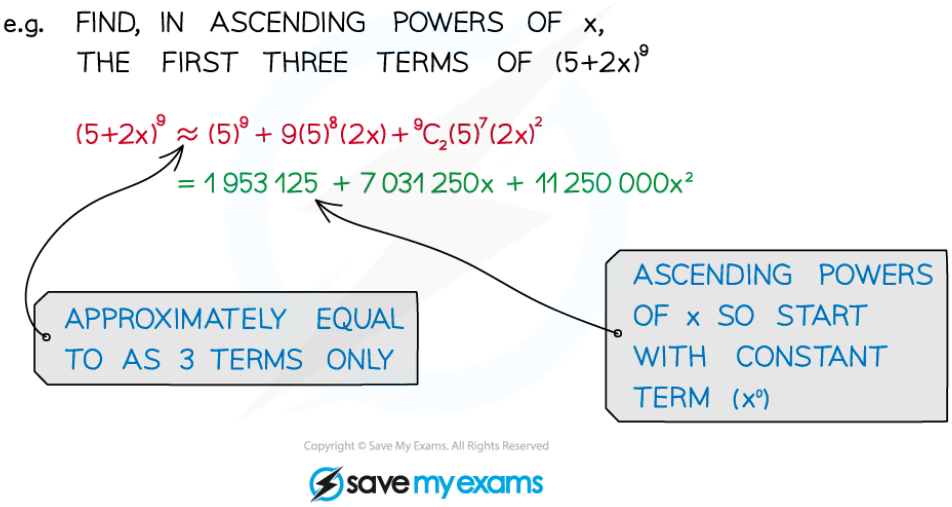
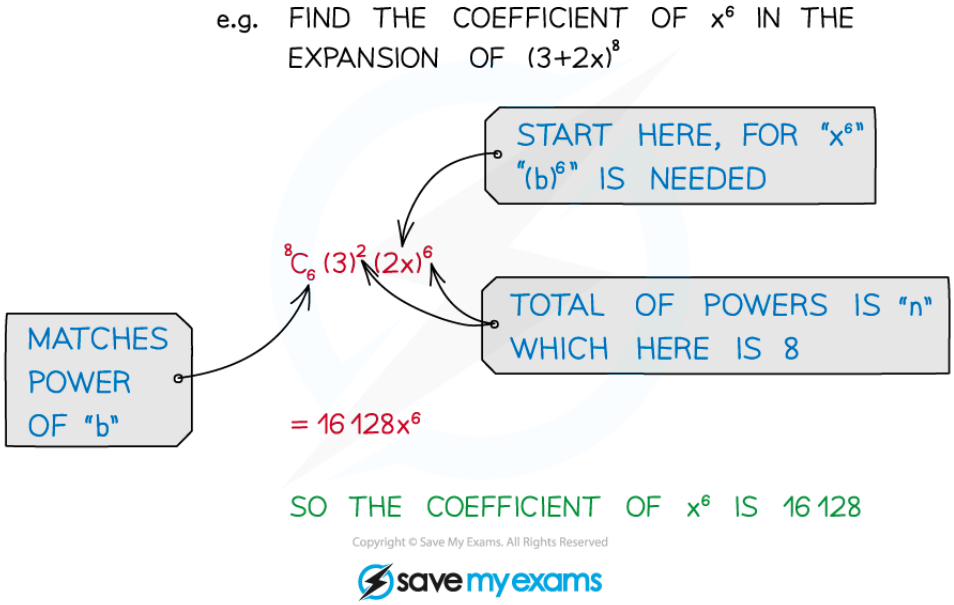
What are binomial expansions used for?

- Expanding brackets, it is usual to be asked for either ...
  - ... the first few terms only



- or (the coefficient of) a particular term



- **Solving problems** in unknowns

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