Edexcel A Level Maths: Pure

Your notes

2.10 Combinations of Transformations

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Combinations of Transformations

What are combinations of graph transformations?

- When you alter a function in certain ways, the effects on the graph of the function can be described by geometrical transformations
- In additional to single transformations, you need to be able to interpret the effects of multiple transformations applied to the same function

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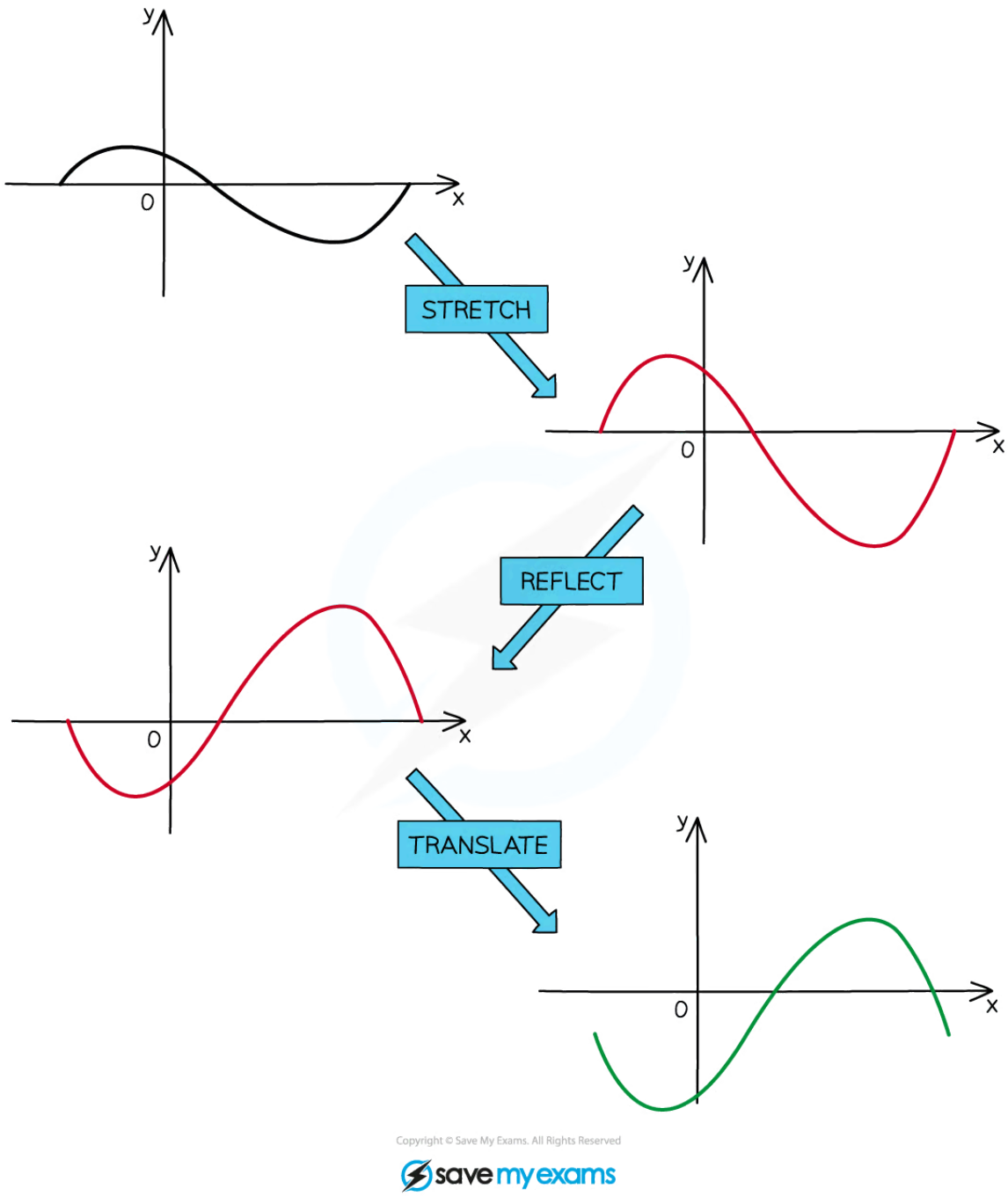

How do I combine two or more graph transformations?

- Make sure you understand the effects of individual **translations**, **stretches**, and **reflections** on the graph of a function (see the previous pages)
- When applying **combinations** of these transformations, apply them to the graph **one at a time** according to the following guidelines:
  - First apply any **horizontal** transformations **inside** the brackets (if you have more than one transformation inside the brackets, **translations** must be applied **before stretches and reflections**)
    - $y = kf(\mathbf{a}x + \mathbf{b}) + c$
  - Then apply any **vertical** transformations **outside** the brackets (if you have more than one transformation outside the brackets, **stretches and reflections** must be applied **before translations**)
    - $y = \mathbf{k}f(ax + b) + \mathbf{c}$


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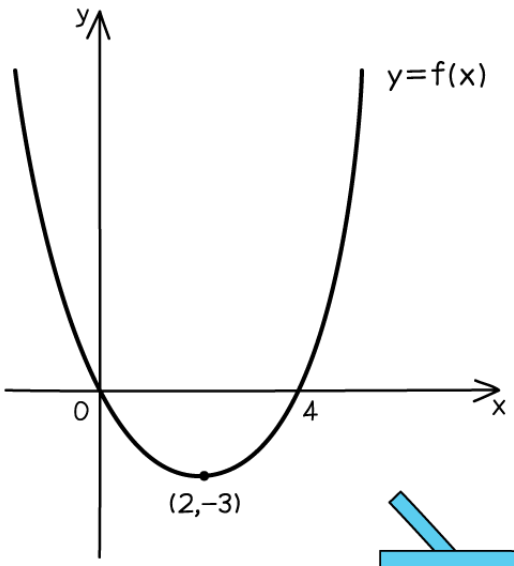
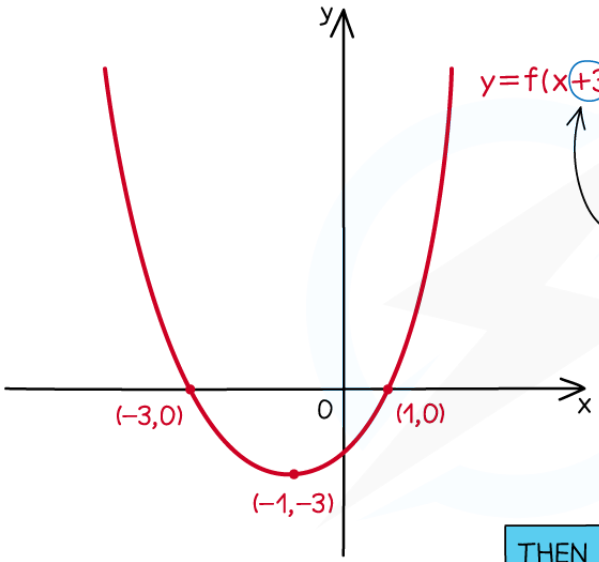


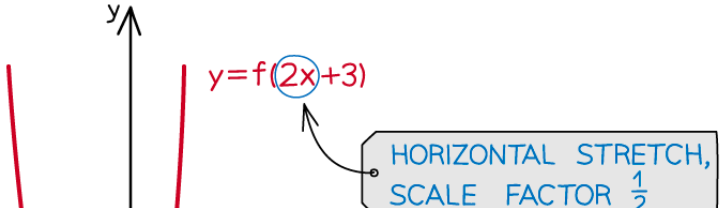
e.g. TO TRANSFORM THIS GRAPH OF $y=f(x)$ INTO THE GRAPH OF $y=-f(2x+3)-1$ START WITH THE TRANSFORMATIONS INSIDE THE BRACKETS...

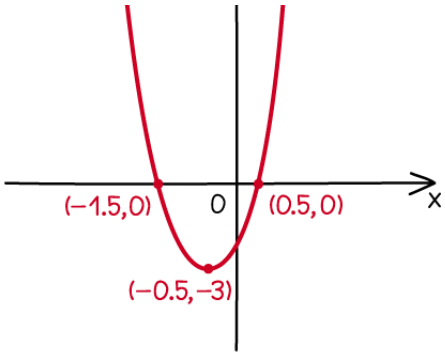
TRANSLATE



HORIZONTAL TRANSLATION BY THE VECTOR $\begin{pmatrix}-3\\0\end{pmatrix}$

THEN STRETCH





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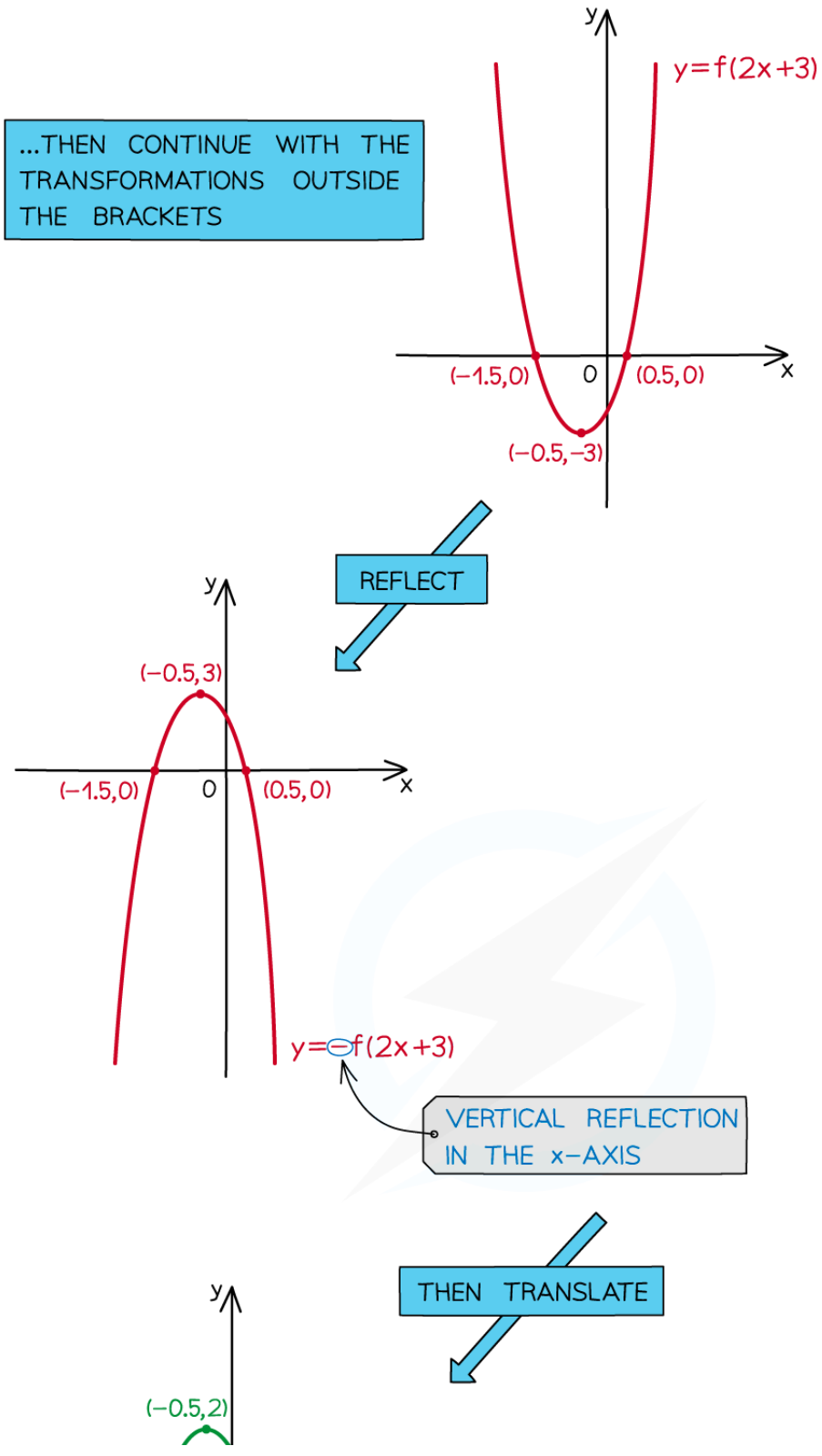

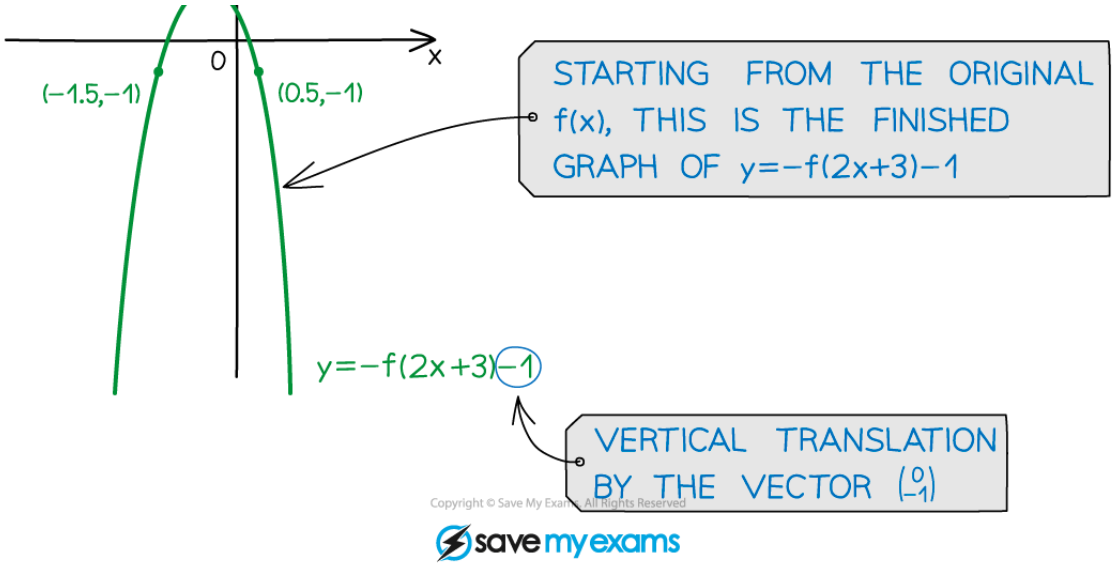

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- Any asymptotes of the function are also affected by the combined transformation (perform the transformations one at a time in the same order as above)

Examiner Tip

- Be sure to apply transformations in the correct order – applying them in the wrong order can produce an incorrect transformation.
- When you sketch a transformed graph, indicate the new coordinates of any points that are marked on the original graph.
- Try to indicate the coordinates of points where the transformed graph intersects the coordinate axes (although if you don't have the equation of the original function this may not be possible).
- If the graph has asymptotes, don't forget to sketch the asymptotes of the transformed graph as well.

Worked example

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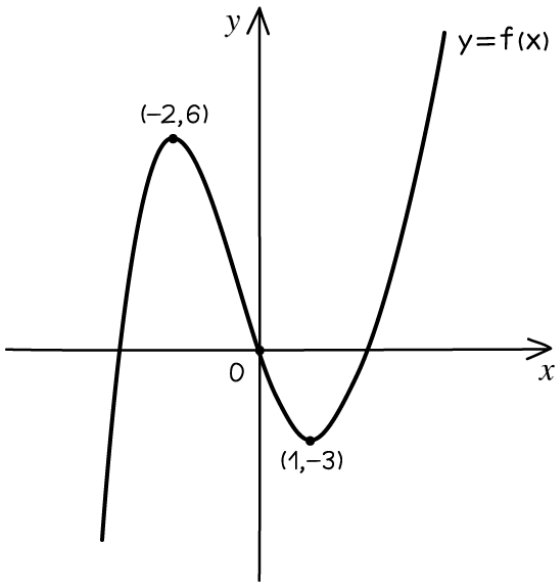
The diagram below shows the graph of $y = f(x)$.



Sketch the graph of $y = 3f(x) - 2$.

YOU DON'T NEED TO DRAW 2 GRAPHS HERE! JUST CONSIDER THE EFFECTS OF THE TRANSFORMATIONS ON THE MARKED POINTS AND SKETCH THE GRAPH AROUND THEM

3f(x)−2 IS A STRETCH BY SCALE FACTOR 3 AROUND THE x−AXIS, FOLLOWED BY A TRANSLATION BY THE VECTOR $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$.

THE EFFECTS ON THE POINTS ARE:

	STRETCH		TRANSLATION	
(−2,6)	→	(−2,18)	→	(−2,16)
(0,0)	→	(0,0)	→	(0,−2)

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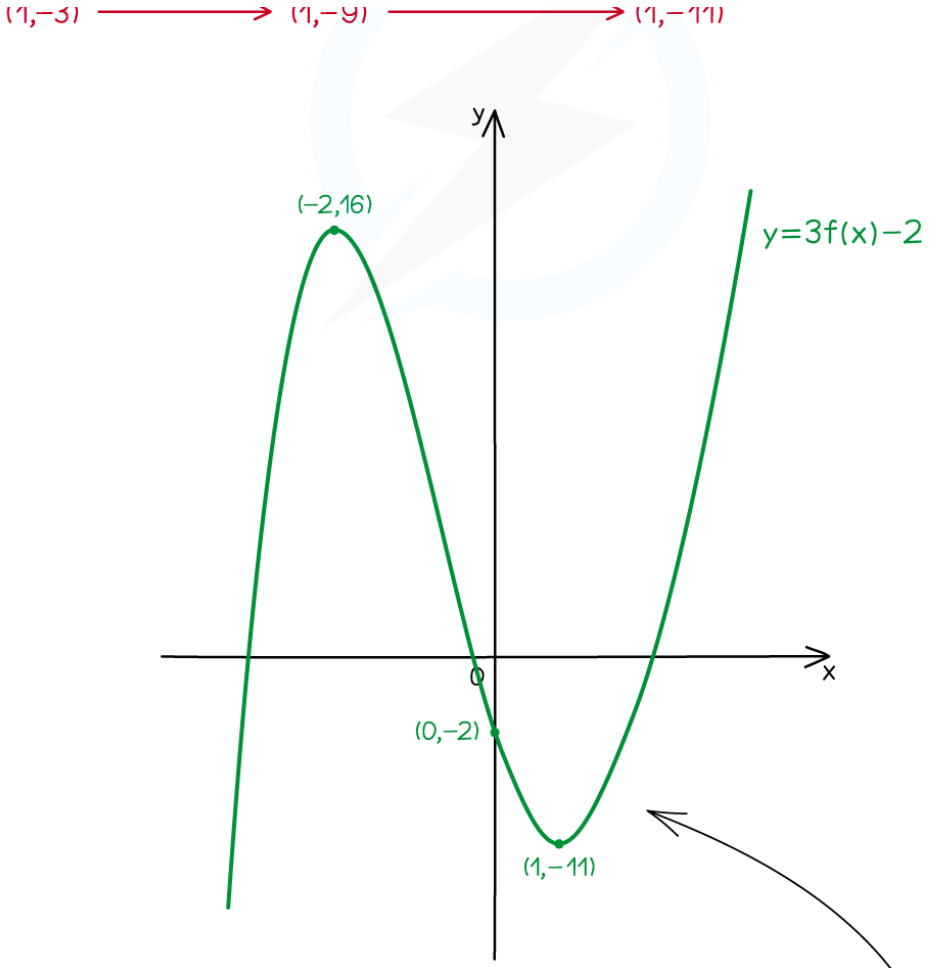

THIS IS A CASE WHERE THE ORDER MATTERS.
IF WE DID THE TRANSLATION BEFORE THE STRETCH, THEN (−2,6) WOULD GO TO (−2,12), (0,0) WOULD GO TO (0,−6), AND (1,−3) WOULD GO TO (1,−15). THAT WOULD GIVE US THE GRAPH OF

$$3(f(x)-2)=3f(x)-6$$