Edexcel A Level Maths: Pure

Your notes

2.13 Further Modelling with Functions

Contents

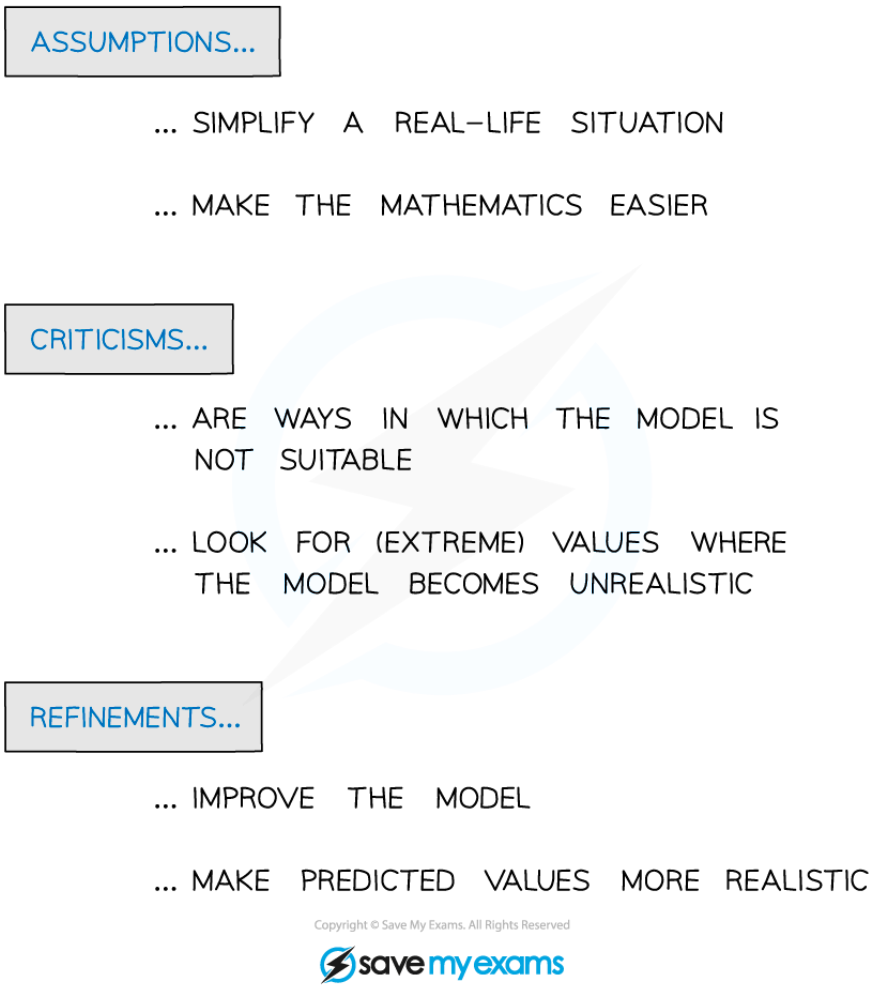
2.13.1 Further Modelling with Functions

Your notes

Further Modelling with Functions

Assumptions, criticisms and refinements in modelling

- Modelling is a large part of A level mathematics
- **Assumptions** and **refinements** to models often need to be considered
- Real-life general knowledge and common sense can really help in mathematical modelling



How do I solve modelling problems?

- There will be no one-size-fits-all step-by-step guide to solving modelling questions

- A combination of skills and **problem-solving** skills will be needed

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BOYLE'S IDEAL GAS LAW STATES THAT, FOR A CONSTANT TEMPERATURE, THE PRESSURE OF A GAS IS INVERSELY PROPORTIONAL TO THE VOLUME OF ITS CONTAINER.
A COMPANY IS TO USE THIS AS A MODEL TO FIND A SUITABLE CONTAINER FOR A PARTICULAR GAS THEY NEED TO STORE.

a) USING P FOR PRESSURE (MEASURED IN Pa, PASCALS) AND V FOR VOLUME (MEASURED IN m^3, CUBIC METRES) WRITE DOWN AN EQUATION FOR THE MODEL TO BE USED.

b) IT IS KNOWN THAT A CONTAINER WITH A VOLUME OF $5m^3$ CREATES A PRESSURE OF 350 000 Pa. FIND THE VALUE OF THE CONSTANT OF PROPORTIONALITY

c) FIND THE PRESSURE IF THE VOLUME OF THE CONTAINER IS 750 m^3

d) GIVEN THAT THE COMPANY MUST KEEP THE PRESSURE BELOW 120 000 Pa FIND, TO THE NEAREST CUBIC METRE, THE SMALLEST VOLUME OF CONTAINER THEY COULD USE

e) STATE ONE ASSUMPTION AND ONE PROBLEM IN USING THIS MODEL

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a) $P \propto \frac{1}{V}$ ← INVERSE PROPORTION

$\therefore P = \frac{k}{V}$ WHERE k IS A CONSTANT

b) SUBSTITUTE P AND V INTO MODEL EQUATION

$350\,000 = \frac{k}{5}$

$k = 1\,750\,000$ ← CAREFUL WITH ALL THOSE ZEROS

STORE IN CALCULATORS' MEMORY

c) $P = \frac{1\,750\,000}{V}$

$P = \frac{1\,750\,000}{750}$

$P = 2330$ Pa (3sf)

d) $P < 120\,000$

$\frac{1\,750\,000}{V} < 120\,000$

$V > \frac{1\,750\,000}{120\,000}$

$V > 14.583...$

$\therefore$ SMALLEST CONTAINER IS $15\,m^3$

e) ASSUMPTION:
THE AMOUNT OF GAS THE COMPANY NEEDS TO STORE DOES NOT CHANGE

PROBLEM:

PROBLEM:
KEEPING THE TEMPERATURE OF THE GAS / CONTAINER CONSTANT

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Examiner Tip

- Reciprocal graphs generally have two parts/curves
  - Only one – usually the positive – may be relevant to the model
  - Think about why **x/t/θ** can only take positive values?

Worked example

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The function

$$f(d) = \frac{L}{d+1}$$

where $d \geq 0$ and L is a constant

is used to model the number of leaves on a tree during the autumn season.

d is the number of days since the start of the autumn season.

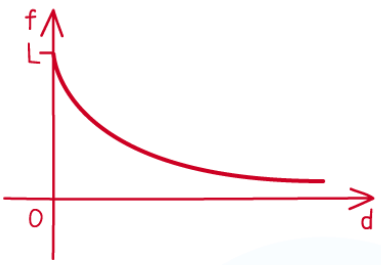
(a) What does the constant L represent?

(b) How long does it take for the number of leaves on the tree to halve?

(c) If the autumn season is 90 days and a tree started with 13500 leaves, how many leaves will be left on the tree at the end of the season

(d) Give a possible problem with the application of this model

A SKETCH OF THE GRAPH MAY HELP



RECIPROCAL GRAPH
d ⩾ 0

a) L REPRESENTS THE NUMBER OF LEAVES

ON THE TREE AT THE START OF AUTUMN (WHEN $d=0$)

b) WHEN HALVED $f=\frac{L}{2}$

$\therefore \frac{L}{2}=\frac{L}{d+1}$

$d+1=2$

$d=1$

IT TAKES ONE DAY FOR THE NUMBER OF LEAVES TO HALVE

c) $d=89 \quad L=13500$

$f=\frac{13500}{89+1}$

$f=150$

150 LEAVES ARE LEFT ON THE TREE AT THE END OF THE SEASON

d) IT IS HARD TO PINPOINT EXACTLY WHEN THE AUTUMN SEASON STARTS

THERE ARE MANY ANSWERS TO THESE TYPES OF QUESTIONS

LOOK FOR EXTREMES:
- ANYTHING STRANGE AT/NEAR $x=0$
- DO NEGATIVES GET INVOLVED EVEN THOUGH ALL VALUES SHOULD BE POSITIVE?
- WHAT HAPPENS FOR LARGE VALUES OF x

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