Edexcel A Level Maths: Pure

Your notes

6.2 Laws of Logarithms

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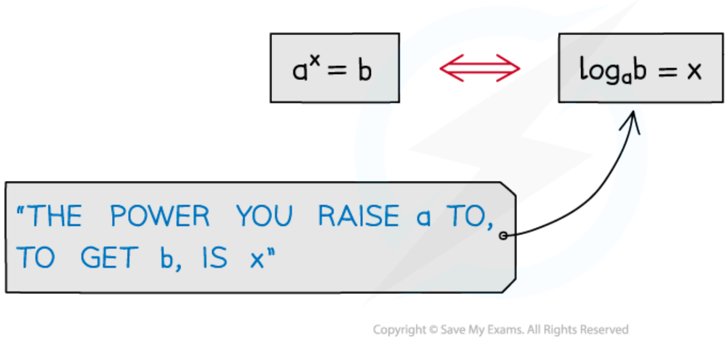
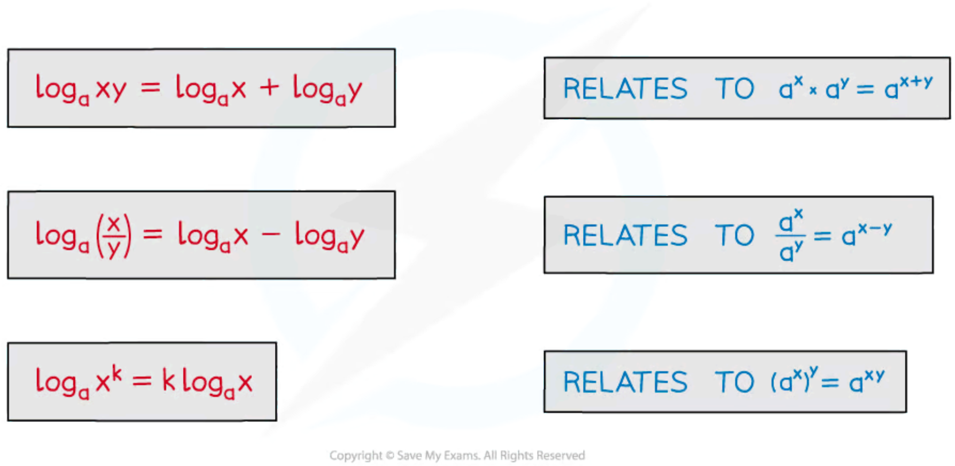
6.2.1 Laws of Logarithms

Laws of Logarithms

What are the laws of logarithms?



- There are many laws or rules of indices, for example
  - $a^m \times a^n = a^{m+n}$
  - $(a^m)^n = a^{mn}$
- There are equivalent laws of logarithms (for a > 0)
  - $\log_a xy = \log_a x + \log_a y$
  - $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
  - $\log_a x^k = k \log_a x$



- There are also some particular results these lead to

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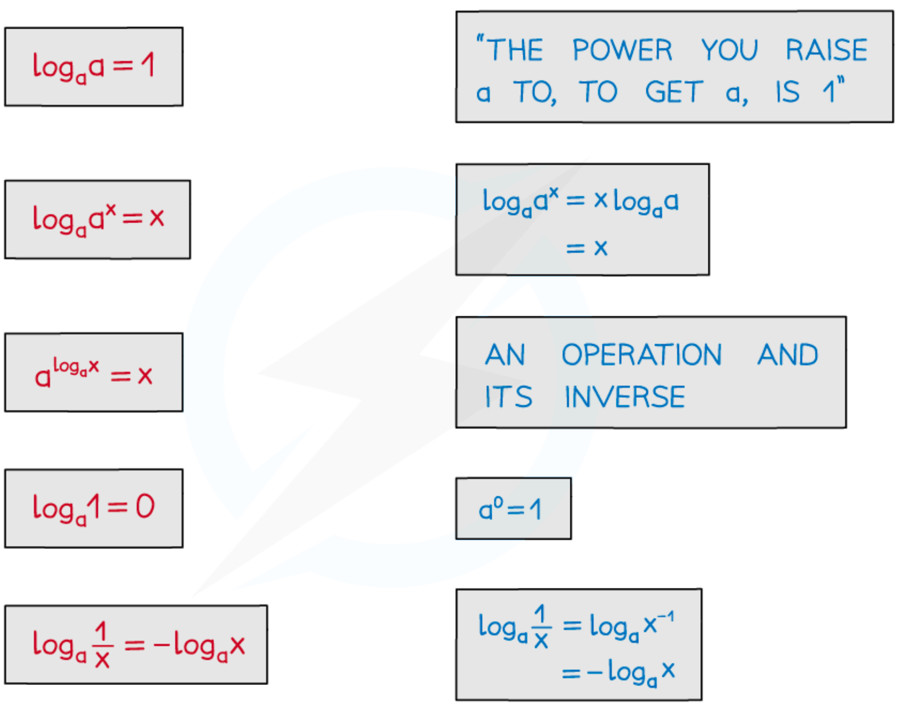
- $\log_a a = 1$
- $\log_a a^x = x$
- $a^{\log_a x} = x$
- $\log_a 1 = 0$
- $\log_a\left(\frac{1}{x}\right) = -\log_a x$



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- Two of these were seen in the notes Logarithmic Functions
- Beware ...
  - ... **log (x + y) ≠ log x + log y**
- Results apply to **ln** too
  - $\ln x \equiv \log_e x$
  - In particular $e^{\ln x} = x$ and $\ln(e^x) = x$

How do I use the laws of logarithms?

- Laws of logarithms can be used to ...
  - ... simplify expressions

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- ... solve logarithmic equations
- ... solve exponential equations

e.g. WRITE $3\log_2(2x+3) + \log_2 5 - 2\log_2(x+1)$ AS A SINGLE LOGARITHM

$$3\log_2(2x+3) + \log_2 5 - 2\log_2(x+1) = \log_2(2x+3)^3 + \log_2 5 - \log_2(x+1)^2$$

$$\boxed{k\log_a x = \log_a x^k}$$

$$= \log_2 5(2x+3)^3 - \log_2(x+1)^2$$

$$\boxed{\log_a x + \log_a y = \log_a(xy)}$$

$$= \log_2 \frac{5(2x+3)^3}{(x+1)^2}$$

$$\boxed{\log_a x - \log_a y = \log_a \frac{x}{y}}$$

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Examiner Tip

- Remember to check whether your solutions are valid
  - log (x+k) is only defined if x > -k
  - You will lose marks if you forget to reject invalid solutions

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Worked example

Solve the equation $\log_2(x+12) = 3\log_2 x - 2$

$\log_2(x+12) = 3\log_2 x - 2$

$\log_2(x+12) = \log_2 x^3 - 2$ ← $k\log_a x = \log_a x^k$

$\log_2 x^3 - \log_2(x+12) = 2$ ← LOGARITHMS ON SAME SIDE

$\log_2\left(\frac{x^3}{x+12}\right) = 2$ ← SINGLE LOGARITHM

$\frac{x^3}{x+12} = 2^2$ ← REWRITE

$x^3 = 4(x+12)$

$x^3 - 4x - 48 = 0$

REARRANGE AND SOLVE USING CALCULATOR

$x = 4$

NOTE: OTHER TWO SOLUTIONS CALCULATOR GIVES ARE IMAGINARY

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"ln"

Your notes

What is ln?

- **ln** is a function that stands for **natural logarithm**
- It is a logarithm where the **base** is the constant "**e**"
  - $\ln x \equiv \log_e x$
  - It is important to remember that **ln** is a function and **not a number**

What are the properties of ln?

- Using the **definition** of a **logarithm** you can see
  - $\ln 1 = 0$
  - $\ln \mathrm{e} = 1$
  - $\ln \mathrm{e}^x = x$
  - $\ln x$ is only defined for positive *x*
- As ln is a logarithm you can use the **laws of logarithms**
  - $\ln a + \ln b = \ln(ab)$
  - $\ln a - \ln b = \ln\left(\frac{a}{b}\right)$
  - $n \ln a = \ln(a^n)$

How can I solve equations involving e & ln?

- The functions e^x and $\ln x$ are **inverses** of each other
  - If $\mathrm{e}^{\mathrm{f}(x)} = \mathrm{g}(x)$ then $\mathrm{f}(x) = \ln \mathrm{g}(x)$
  - If $\ln \mathrm{f}(x) = \mathrm{g}(x)$ then $\mathrm{f}(x) = \mathrm{e}^{\mathrm{g}(x)}$
- If your equation involves "e" then try to get all the "e" terms on one side
  - If "e" terms are multiplied, you can add the powers
    - $\mathrm{e}^x \times \mathrm{e}^y = \mathrm{e}^{x+y}$
    - You can then apply ln to both sides of the equation
  - If "e" terms are added, try transforming the equation with a substitution
    - For example: If $y = \mathrm{e}^x$ then $\mathrm{e}^{4x} = y^4$
    - You can then solve the resulting equation (usually a quadratic)
    - Once you solve for *y* then solve for *x* using the substitution formula
- If your equation involves "ln", try to combine all "ln" terms together
  - Use the laws of logarithms to combine terms into a single term
  - If you have $\ln \mathrm{f}(x) = \ln \mathrm{g}(x)$ then solve $\mathrm{f}(x) = \mathrm{g}(x)$
  - If you have $\ln \mathrm{f}(x) = k$ then solve $\mathrm{f}(x) = \mathrm{e}^k$

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Your notes

Worked example

?

Solve the following equations. Give your answers in exact form.

(a) $e^{3x+2} = 5e^{x-3}$.

(b) $\ln(8x) - \ln(x+4) = 2$.

a) COLLECT THE "e" TERMS ON ONE SIDE: $\frac{e^{3x+2}}{e^{x-3}} = 5$

SIMPLIFY USING INDEX LAWS: $e^{2x+5} = 5$

$(3x+2) - (x-3)$

APPLY NATURAL log TO BOTH SIDES: $\cancel{\ln(e}^{2x+5}) = \ln 5$

USE $\ln(e^{f(x)}) = f(x)$ $\qquad 2x + 5 = \ln 5$

REARRANGE FOR x $\qquad x = \frac{1}{2}(-5 + \ln 5)$

WRITE $\frac{\ln a}{k}$ AS $\frac{1}{k}\ln a$

WRITE $\ln a + k$ as $k + \ln a$

b) COMBINE "ln" USING LAWS OF LOGARITHMS: $\ln \frac{8x}{x+4} = 2$

THE INVERSE OF $\ln(x)$ IS e^x: $\frac{8x}{x+4} = e^2$

REARRANGE FOR x: $8x = e^2x + 4e^2$

COLLECT x TERMS ON ONE SIDE

$8x - e^2x = 4e^2$

FACTORISE OUT THE x

$(8 - e^2)x = 4e^2$

$x = \frac{4e^2}{8 - e^2}$

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Examiner Tip

- Always simplify your answer if you can
  - for example, $\frac{1}{2}\ln 25 = \ln\sqrt{25} = \ln 5$
  - you wouldn't leave your final answer as $\sqrt{25}$ so don't leave your final answer as $\frac{1}{2}\ln 25$

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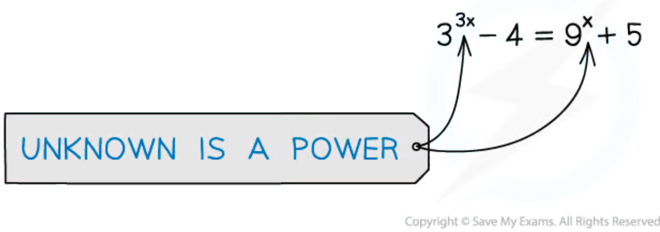
6.2.2 Exponential Equations

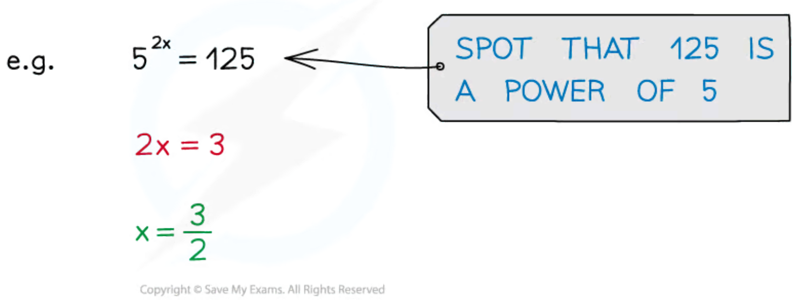
Your notes

Exponential Equations

What are exponential equations?



- An equation where the unknown is a power
- In simple cases the solutions can be "spotted"



How do I solve exponential equations?

- Laws of Indices are often needed to rewrite equations
- Laws of Logarithms are used to solve exponential equations
- **ln** (**log$_e$**) is often used
- Answers are often written in terms of **ln**

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Your notes

e.g. SOLVE $3^{2x-1} = 2^{6-x}$ GIVING YOUR ANSWER IN THE FORM $x = \frac{\ln p}{\ln q}$ WHERE p AND q ARE INTEGERS TO BE FOUND

STEP 1: TAKE LOGARITHMS OF BOTH SIDES

$$\ln 3^{2x-1} = \ln 2^{6-x}$$

STEP 2: USE LAWS OF LOGARITHMS TO REMOVE POWERS

$$(2x-1)\ln 3 = (6-x)\ln 2$$

STEP 3: REARRANGE TO ISOLATE x

$$2x\ln 3 - \ln 3 = 6\ln 2 - x\ln 2$$

$$2x\ln 3 + x\ln 2 = 6\ln 2 + \ln 3$$

$$x(2\ln 3 + \ln 2) = 6\ln 2 + \ln 3$$

$$x = \frac{6\ln 2 + \ln 3}{2\ln 3 + \ln 2}$$

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STEP 4: USE LAWS OF LOGARITHMS TO REWRITE IN CORRECT FORM

$$x = \frac{\ln 2^6 + \ln 3}{\ln 3^2 + \ln 2}$$

$$x = \frac{\ln 64 + \ln 3}{\ln 9 + \ln 2}$$

$$x = \frac{\ln 192}{\ln 18} \leftarrow \boxed{\begin{array}{l} p = 192 \\ q = 18 \end{array}}$$

$$\boxed{\log_a x + \log_a y = \log_a xy}$$

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e.g. SOLVE $6^{3x+2} = 42$, GIVING YOUR ANSWER CORRECT TO 3 SIGNIFICANT FIGURES

$$6^{3x+2} = 42$$

$$\ln(6^{3x+2}) = \ln 42$$

TAKE LOGARITHMS OF BOTH SIDES

$$(3x+2)\ln 6 = \ln 42$$

$$3x+2 = \frac{\ln 42}{\ln 6}$$

$\neq \ln 7$

$$3x = \frac{\ln 42}{\ln 6} - 2$$

$$x = \frac{\frac{\ln 42}{\ln 6} - 2}{3}$$

$$x = 0.0286777\ldots$$

AVOID USING CALCULATOR UNTIL NECESSARY

$$x = 0.0287 \quad (3 \text{ sf})$$

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What about hidden quadratics?

- Look out for "hidden" squared terms, these are hidden quadratics which will need to be solved
  - $4^x = (2^2)^x = 2^{2x} = (2^x)^2$
  - $e^{2x} = (e^2)^x = (e^x)^2$

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e.g. SOLVE $21e^x - 4 = 5e^{2x}$ GIVING YOUR SOLUTIONS IN THE FORM $x = a\ln b$, WHERE a AND b ARE INTEGERS

$$21e^x - 4 = 5e^{2x}$$

"HIDDEN" SQUARED TERM

$$5(e^x)^2 - 21(e^x) + 4 = 0$$

REARRANGE TO QUADRATIC FORM

$$(5e^x - 1)(e^x - 4) = 0$$

USE $y = e^x$ IF PREFERRED

$$e^x = \frac{1}{5} \qquad e^x = 4$$

$$x = \ln\frac{1}{5} \qquad x = \ln 4$$

$$x = \ln 5^{-1}$$

REWRITE IN CORRECT FORM

$$x = -\ln 5 \qquad x = \ln 4$$

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Worked example

Find the exact solution to $6 \times 3^{2x-3} - 2^{x+1} = 0$

$6 \times 3^{2x-3} = 2^{x+1}$

$\ln(6 \times 3^{2x-3}) = \ln 2^{x+1}$

STEP 1: LOGS OF BOTH SIDES

$\ln 6 + \ln 3^{2x-3} = (x+1)\ln 2$

$\ln 6 + (2x-3)\ln 3 = (x+1)\ln 2$

STEP 2: REMOVE POWERS

$\ln 6 + 2x\ln 3 - 3\ln 3 = x\ln 2 + \ln 2$

$2x\ln 3 - x\ln 2 = \ln 2 - \ln 6 + 3\ln 3$

$x(2\ln 3 - \ln 2) = \ln 2 - \ln 6 + 3\ln 3$

$$x = \frac{\ln \frac{2}{6} + \ln 3^3}{\ln 3^2 - \ln 2}$$

STEP 3: ISOLATE x

$$x = \frac{\ln \frac{1}{3} + \ln 27}{\ln 9 - \ln 2}$$

STEP 4: REWRITE USING LAWS OF LOGARITHMS

$$x = \frac{\ln 9}{\ln \frac{9}{2}}$$

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