Edexcel A Level Maths: Pure

Your notes

2.5 Polynomials

Contents

Your notes

2.5.1 Expanding Brackets

Expanding Brackets

What do I need to know about expanding brackets?

- To expand brackets, the rule is that each term in one set of brackets must be multiplied by each term in the other set of brackets

$$(a+b)(x+y+z) = a(x+y+z) + b(x+y+z)$$
$$= ax + ay + az + bx + by + bz$$

$$(a+b+c)(x+y+z) = a(x+y+z) + b(x+y+z) + c(x+y+z)$$
$$= ax + ay + az + bx + by + bz + cx + cy + cz$$

- 'FOIL' is a special case of this, when each set of brackets contains two terms you can mulitply in order
  - e.g. to multiply out or expand these brackets (2x + 3)(x - 5)
    - First (**2x** + 3)(**x** - 5) gives $2x^2$
    - Outside (**2x** + 3)(x **- 5**) gives -10x
    - Inside (2x + **3**)(**x** - 5) gives +3x
    - Last (2x **+ 3**)(x **- 5)** gives -15
  - $(2x + 3)(x - 5) = 2x^2 - 10x + 3x - 15 = 2x^2 - 7x - 15$
- If you spot something like $(a + b)^2$ or $(a + b)^3$ just write the brackets out twice or three times respectively
  - e.g. $(3x + 2)^2 = (3x + 2)(3x + 2) = 9x^2 + 6x + 6x + 4 = 9x^2 + 12x + 4$
- If you are trying to expand something like $(\boldsymbol{a} + \boldsymbol{b})^{\boldsymbol{n}}$ for powers of ***n*** greater than 2 or 3, use the **binomial expansion**
- If you have to expand more than two sets of brackets, just expand them two at a time

Your notes

Worked example

Expand and simplify $4x(x-3y)(3x+2y-5)$.

EXPAND $4x(x-3y)$ FIRST

$4x(x-3y) = 4x^2 - 12xy$

MULTIPLY EACH TERM IN THE FIRST BRACKET BY EACH TERM IN THE SECOND BRACKET

$(4x^2 - 12xy)(3x+2y-5)$

$= 12x^3 + 8x^2y - 20x^2 - 36x^2y - 24xy^2 + 60xy$

$= 12x^3 - 28x^2y - 20x^2 - 24xy^2 + 60xy$

COLLECT LIKE TERMS

Worked example

Your notes

Expand $(x-4)(x+3)\left(x^2+x-1\right)$

FIRST EXPAND $(x-4)(x+3)=x^2-x-12$

THEN CONTINUE: $(x^2-x-12)(x^2+x-1)$

$=x^2(x^2+x-1)-x(x^2+x-1)-12(x^2+x-1)$

$=x^4+x^3-x^2-x^3-x^2+x-12x^2-12x+12$

$=x^4-14x^2-11x+12$

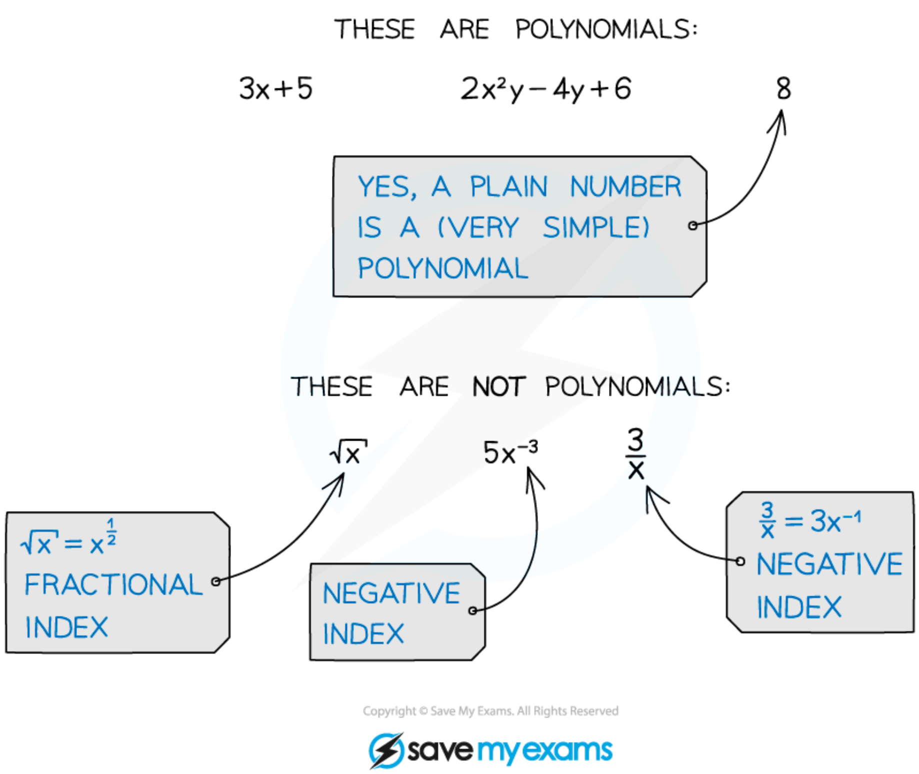
2.5.2 Polynomial Division

Your notes

Polynomial Division

What is a polynomial?

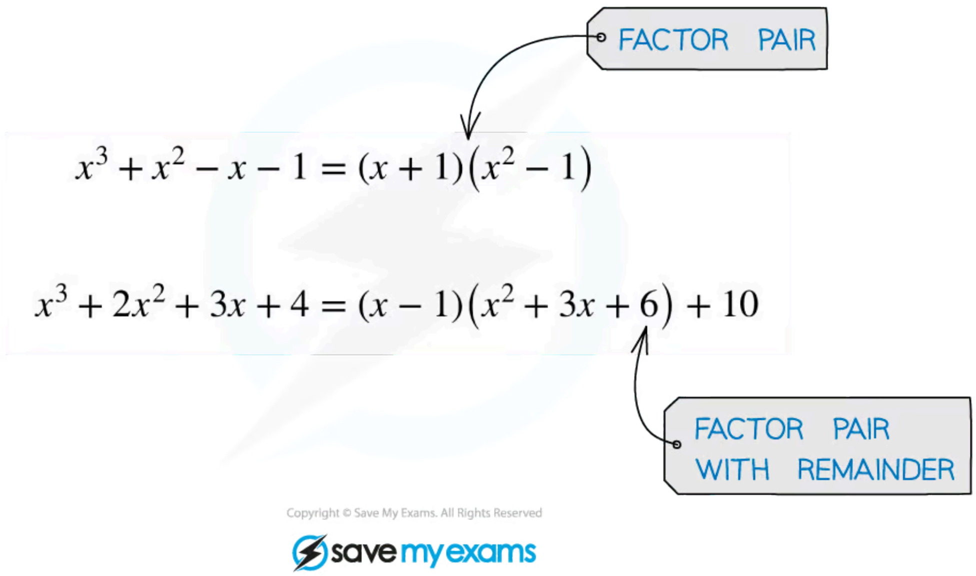
- A **polynomial** is an algebraic expression consisting of a finite number of terms, with non-negative integer indices only



What is polynomial division?

- Polynomial division is a method for splitting polynomials into factor pairs (with or without an accompanying remainder term)

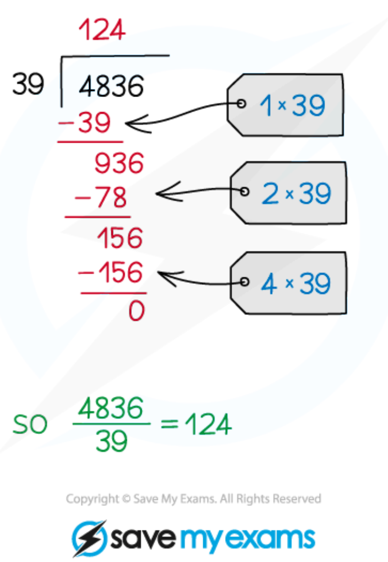
Your notes



- At A level you will most frequently use it to factorise polynomials, or when dealing with improper (ie 'top-heavy') algebraic fractions

How do I divide polynomials?

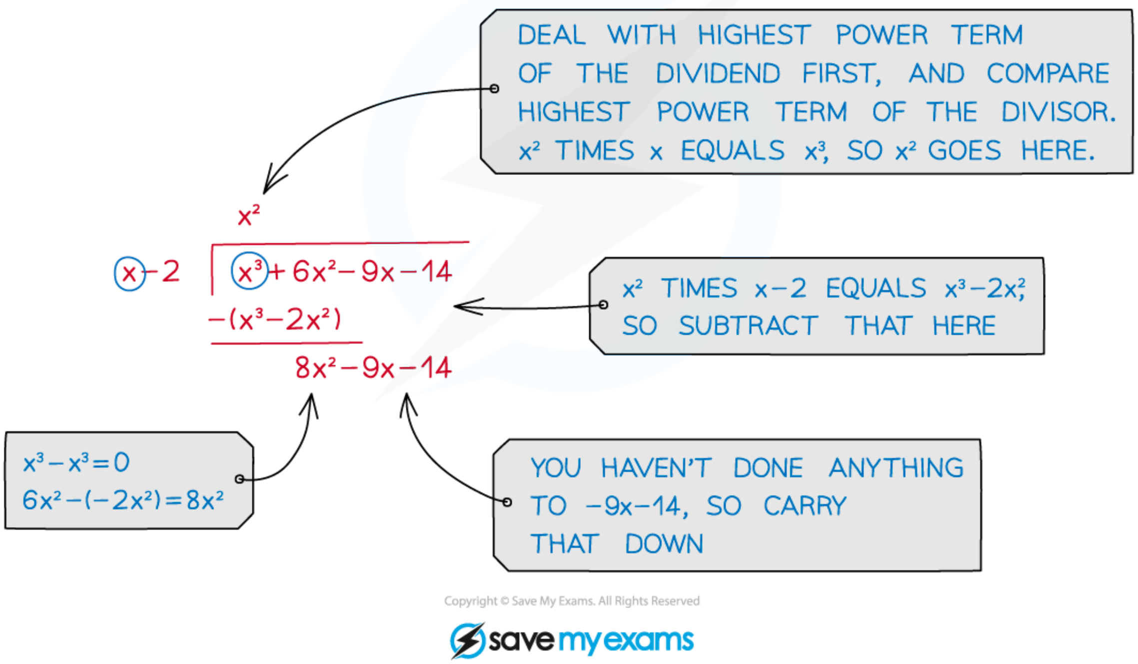
- The method used for polynomial division is just like the long division method (sometimes called 'bus stop division') used to divide regular numbers:



- At A level you will normally be dividing a polynomial dividend of degree 3 or 4 by a divisor in the form ***(x* ± *p*)**

- The answer to a polynomial division question is built up term by term, working downwards in powers of the variable (usually x)
- Start by dividing by the highest power term
- Write out this multiplied by the divisor and subtract

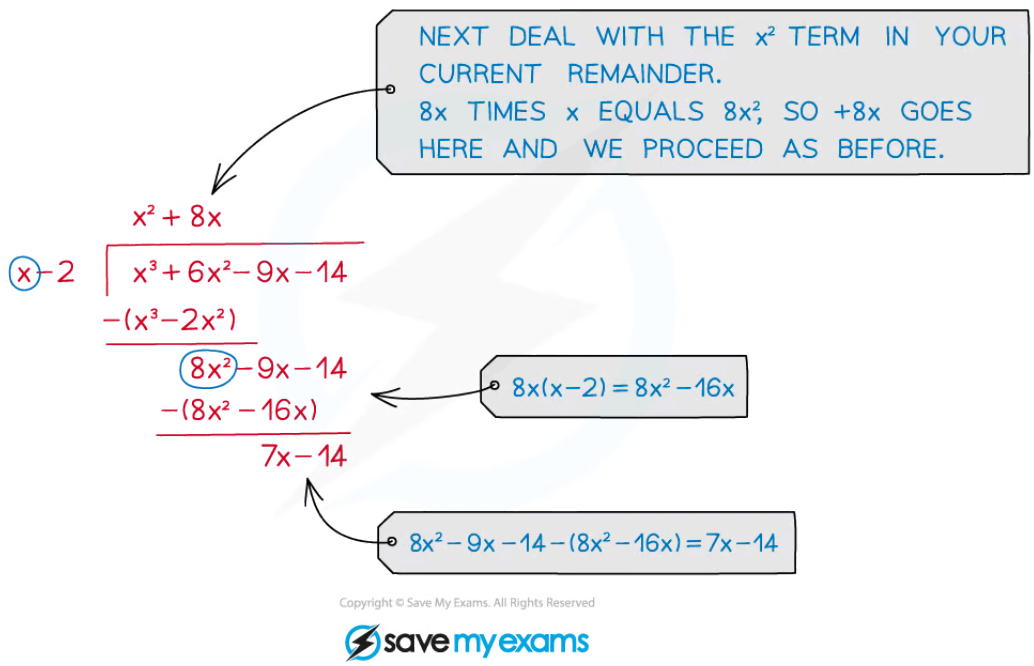
Divide the polynomial $f(x) = x^3 + 6x^2 - 9x - 14$ by $(x - 2)$.



- Continue to divide by each reducing power term and subtracting your answer each time

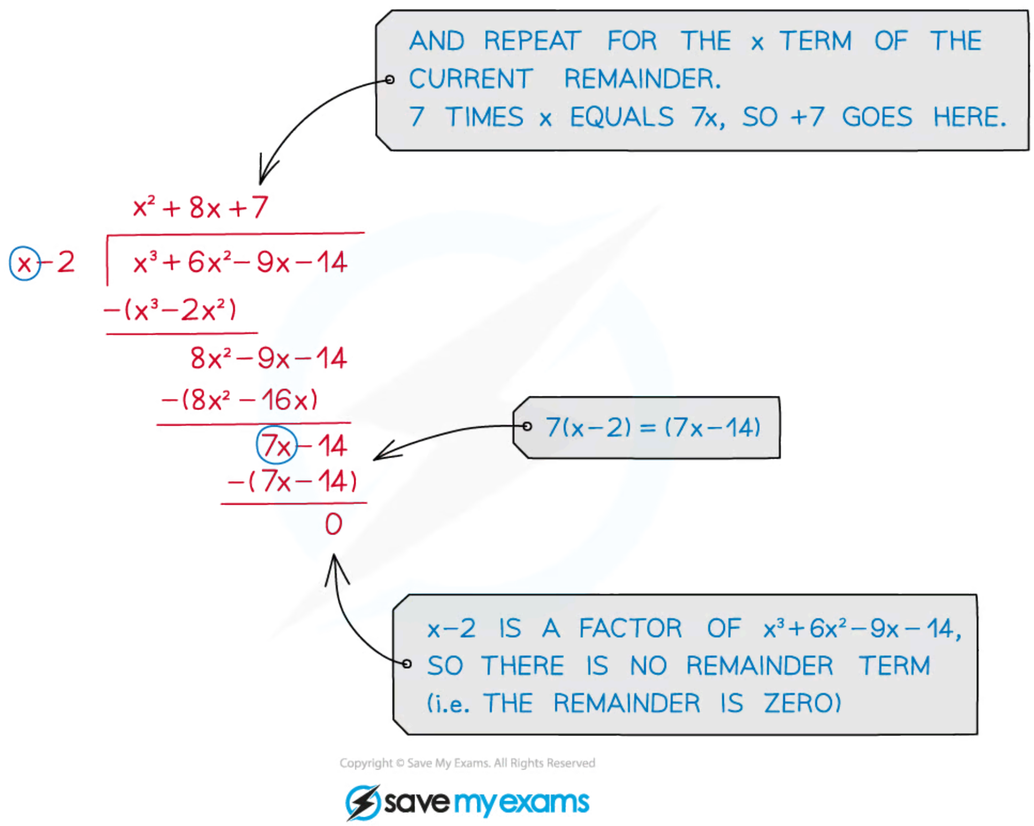
Your notes

Your notes



- Continue until you are left with zero

Your notes



- If the divisor is not a factor of the polynomial then there will be a remainder term left at the end of the division

Worked example

Your notes

Your notes

❓ For the polynomial $f(x) = x^3 + 6x^2 - 9x - 14$:

a) Divide $f(x)$ by $(x + 7)$.

b) Find the remainder when $f(x)$ is divided by $(x + 3)$.

a)

$$
\begin{array}{r|l}
 & x^2 - x - 2 \\
\hline
x+7 & x^3 + 6x^2 - 9x - 14 \\
 & -(x^3 + 7x^2) \\
\hline
 & -x^2 - 9x - 14 \\
 & -(-x^2 - 7x) \\
\hline
 & -2x - 14 \\
 & -(-2x - 14) \\
\hline
 & 0
\end{array}
$$

THE x^2 TERM OF THE REMAINDER HERE IS NEGATIVE, SO $-x$ GOES ON TOP

THE x TERM OF THE REMAINDER HERE IS NEGATIVE, SO -2 GOES ON TOP

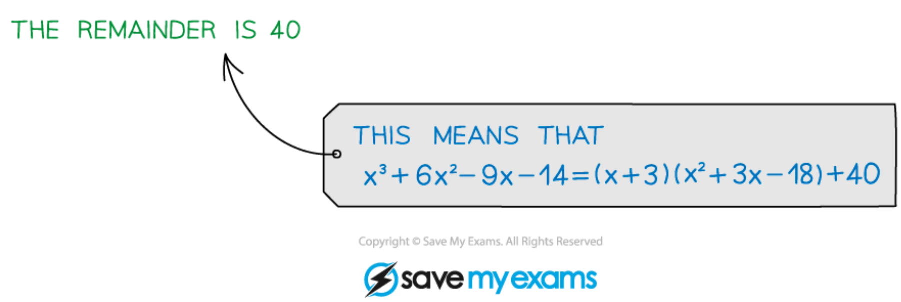
$$\frac{x^3 + 6x^2 - 9x - 14}{x + 7} = x^2 - x - 2$$

THIS MEANS THAT $x^3 + 6x^2 - 9x - 14 = (x+7)(x^2 - x - 2)$

b)

$$
\begin{array}{r|l}
 & x^2 + 3x - 18 \\
\hline
x+3 & x^3 + 6x^2 - 9x - 14 \\
 & -(x^3 + 3x^2) \\
\hline
 & 3x^2 - 9x - 14 \\
 & -(3x^2 + 9x) \\
\hline
 & -18x - 14 \\
 & -(-18x - 54) \\
\hline
 & 40
\end{array}
$$

THE x TERM OF THE REMAINDER HERE IS NEGATIVE, SO -18 GOES ON TOP


THE REMAINDER IS 40
THIS MEANS THAT
$x^3+6x^2-9x-14=(x+3)(x^2+3x-18)+40$
Copyright © Save My Exams. All Rights Reserved
save my exams

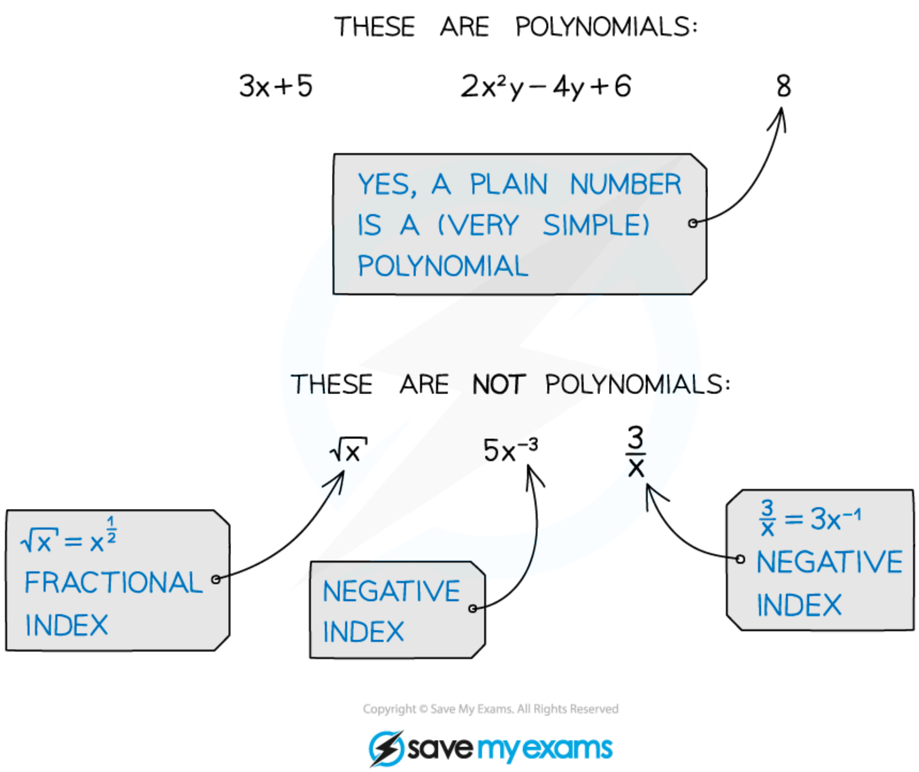
Your notes

2.5.3 Factor Theorem

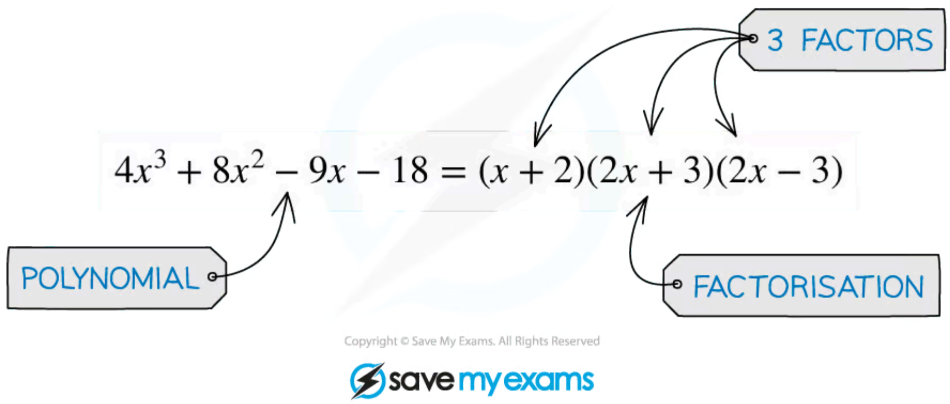
Factor Theorem

What is the factor theorem?

- The factor theorem is a very useful result about polynomials
- A **polynomial** is an algebraic expression consisting of a finite number of terms, with non-negative integer indices only



- At A level you will most frequently use the factor theorem as a way to simplify the process of factorising polynomials

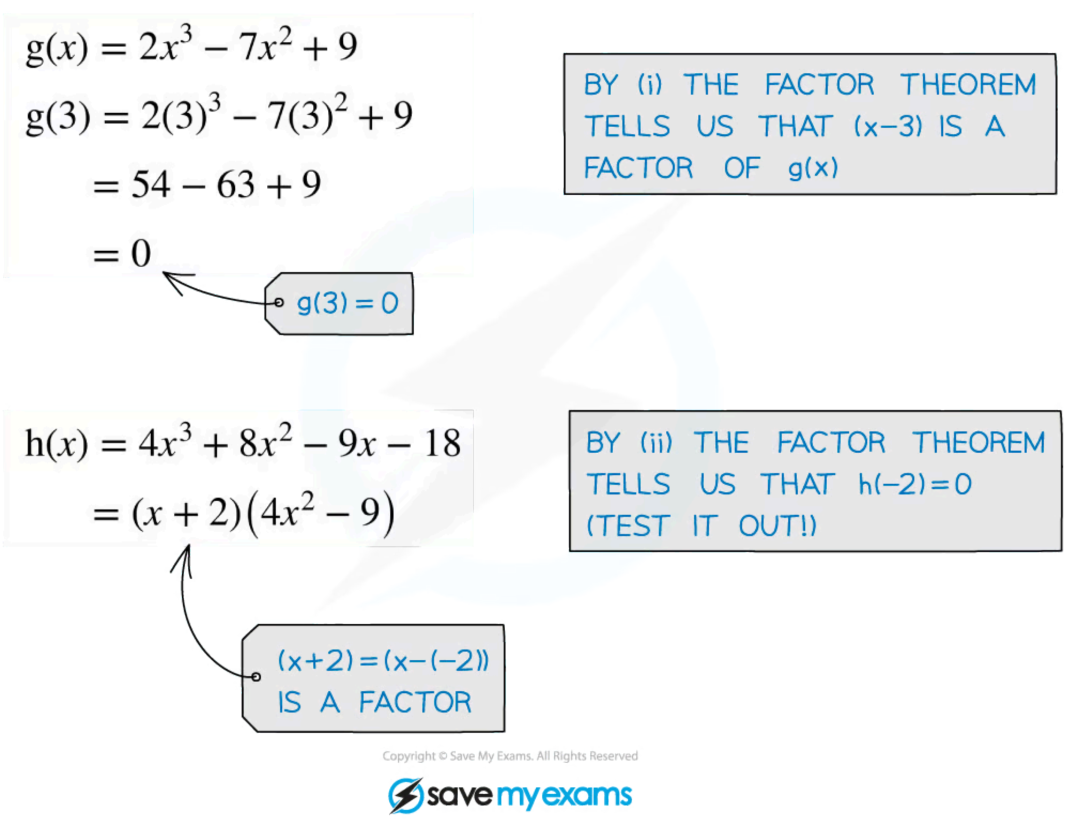


What do I need to know about the factor theorem?

- For a polynomial **f(*x*)** the factor theorem states that:
  - If **f(*p*) = 0**, then **(*x* – *p*)** is a factor of **f(*x*)**

  AND

  - If **(*x* – *p*)** is a factor of **f(*x*)**, then **f(*p*) = 0**



Your notes

Examiner Tip

- In an exam, the values of ***p*** you need to find that make **f(*p*) = 0** are going to be integers close to zero.
  - Try ***p*** = 1 and -1 first, then 2 and -2, then 3 and -3
  - It is very unlikely that you'll have to go beyond that.

Worked example

For the polynomial $f(x) = x^3 + 6x^2 - 9x - 14$:

a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

b) Use the factor theorem to find another factor of $f(x)$.

a) $f(2) = (2)^3 + 6(2)^2 - 9(2) - 14$

$= 8 + 24 - 18 - 14 = 0$

FOR $(x-2)$ TO BE A FACTOR, $f(2)$ MUST BE EQUAL TO ZERO, NOT $f(-2)$

SO $(x-2)$ IS A FACTOR OF $f(x)$

b) $f(1) = (1)^3 + 6(1)^2 - 9(1) - 14$

$= 1 + 6 - 9 - 14 = -16$

TRY $x=1$ FIRST

$f(-1) = (-1)^3 + 6(-1)^2 - 9(-1) - 14$

$= -1 + 6 + 9 - 14 = 0$

$f(1) \neq 0$, SO TRY $x=-1$ NEXT

SO $(x+1)$ IS ANOTHER FACTOR OF $f(x)$

$f(-1) = 0$, SO BY THE FACTOR THEOREM $(x-(-1)) = (x+1)$ IS A FACTOR OF $f(x)$

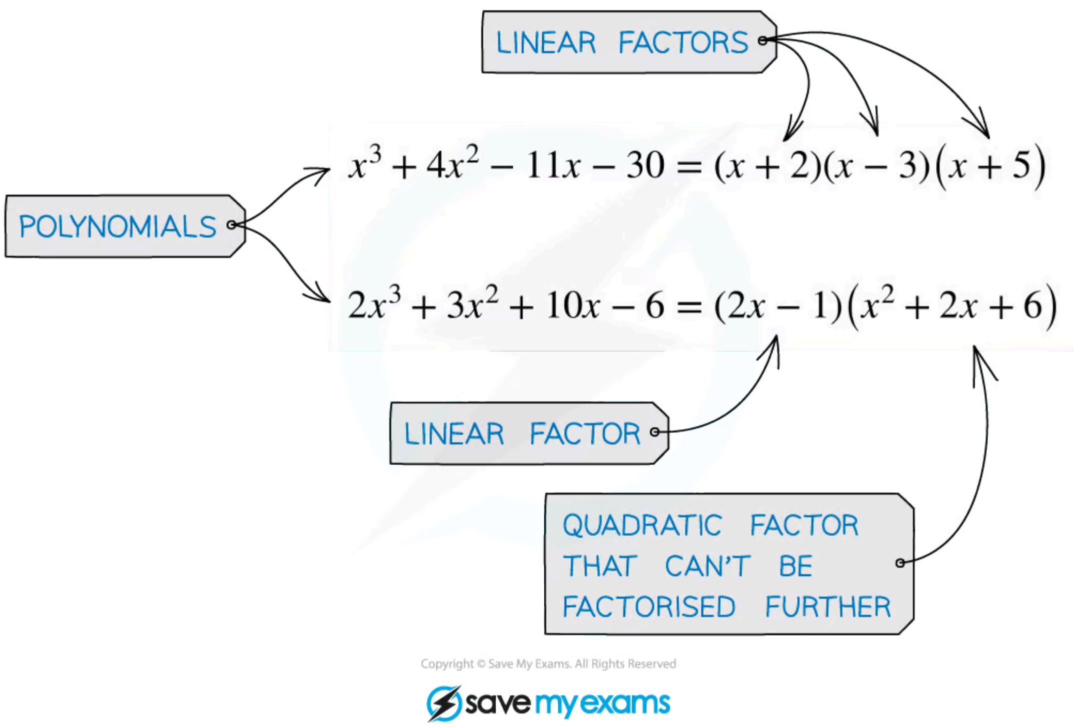
2.5.4 Factorisation

Your notes

Polynomial Factorisation

What is polynomial factorisation?

- Factorising a polynomial combines the **factor theorem** with the method of **polynomial division**
- The goal is to break down a polynomial as far as possible into a product of linear factors



How do I factorise a polynomial?

- At A level you will usually be asked to factorise a **cubic** – i.e. a polynomial where the highest power of x is 3

Your notes

THESE ARE ALL EXAMPLES OF CUBICS:

$$x^3 + 2x^2 - 5x + 7$$

$$3 - x^2 - 4x^3$$

$$x^3 + 2x$$

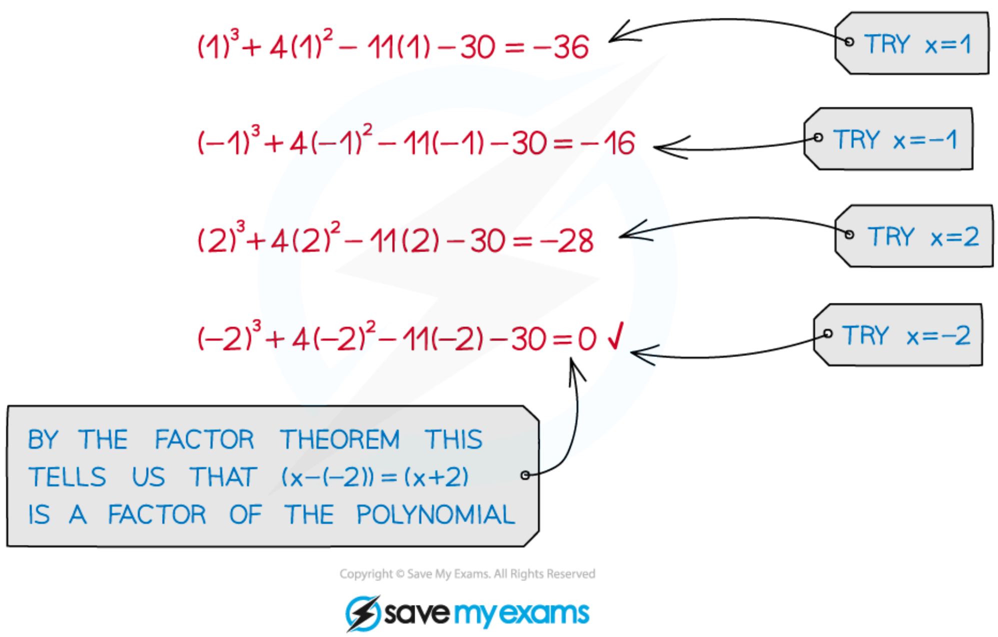
$$2x^3$$

- To factorise a cubic polynomial **f(x)** follow the following steps:

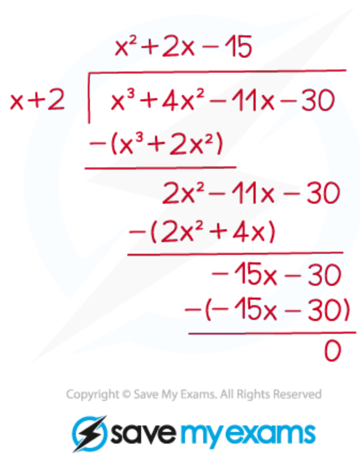
Fully factorise $x^3 + 4x^2 - 11x - 30$.

- Step 1. Find a value ***p*** that makes **f(*p*)** = 0

Your notes



- Step 2. Use polynomial division to divide **f(*x*)** by **(*x* – *p*)**



- Step 3. Use the result of your division to write

$$f(x) = (x - p)(ax^2 + bx + c)$$

Your notes

$$\text{SO} \quad x^3+4x^2-11x-30 = (x+2)(x^2+2x-15)$$

- Step 4. If the quadratic **($ax^2 + bx + c$)** is factorisable, factorise it and write **f(x)** as a product of three linear factors (if the quadratic is not factorisable, then your result from Step 3 is the final factorisation)

$$x^2+2x-15 = (x+5)(x-3)$$

$$\text{SO} \quad x^3+4x^2-11x-30 = (x+2)(x+5)(x-3)$$

Examiner Tip

- The method outlined above can be logically extended to factorise a polynomial of any degree.

Worked example

Your notes

Your notes

Fully factorise $x^3 + 6x^2 - 9x - 14$.

STEP 1:

$f(1) = (1)^3 + 6(1)^2 - 9(1) - 14$

$= 1 + 6 - 9 - 14 = -16$

TRY $x=1$ FIRST

$f(-1) = (-1)^3 + 6(-1)^2 - 9(-1) - 14$

$= -1 + 6 + 9 - 14 = 0$

$f(1) \neq 0$, SO TRY $x=-1$ NEXT

SO $(x+1)$ IS A FACTOR OF $f(x)$

$f(-1) = 0$, SO BY THE FACTOR THEOREM $(x-(-1)) = (x+1)$ IS A FACTOR OF $f(x)$

STEP 2:

$$\begin{array}{r|l} & x^2 + 5x - 14 \\ x+1 & x^3 + 6x^2 - 9x - 14 \\ & -(x^3 + x^2) \\ & \quad 5x^2 - 9x - 14 \\ & \quad -(5x^2 + 5x) \\ & \qquad -14x - 14 \\ & \qquad -(-14x - 14) \\ & \qquad\qquad 0 \end{array}$$

STEP 3:

SO $x^3 + 6x^2 - 9x - 14 = (x+1)(x^2 + 5x - 14)$

STEP 4:

$(x^2 + 5x - 14) = (x+7)(x-2)$

SO $x^3 + 6x^2 - 9x - 14 = (x+1)(x+7)(x-2)$

Your notes