Edexcel A Level Maths: Pure

Your notes

2.9 Transformations of Functions

Contents

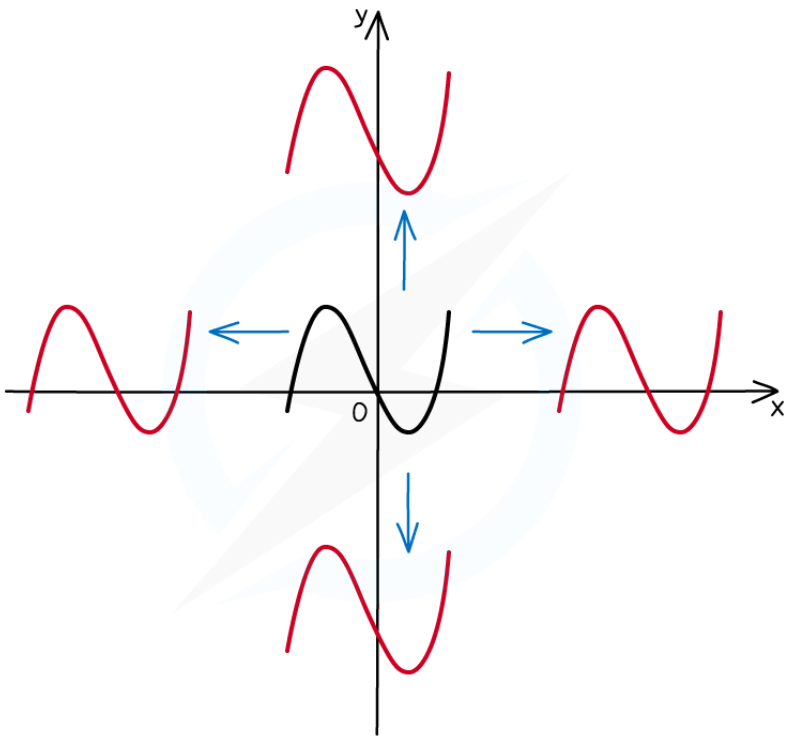
2.9.1 Translations

Your notes

Translations

What are graph transformations?

- When you alter a function in certain ways, the effects on the graph of the function can be described by geometrical transformations
- With a **translation** the shape, size, and orientation of the graph remain unchanged – the graph is merely shifted (up or down, left or right) in the *xy* plane



- A particular translation (how far left/right, how far up/down) is specified by a **translation vector**:

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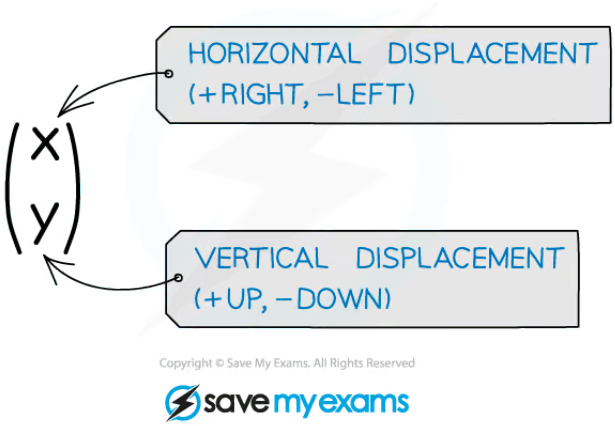

What do I need to know about graph translations?

- The graph of $y = f(x) + a$ is a **vertical** translation of the graph $y = f(x)$ by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$
  - The graph moves **up for positive** values of *a* and **down for negative** values of *a*
  - The *x*-coordinates stay the same

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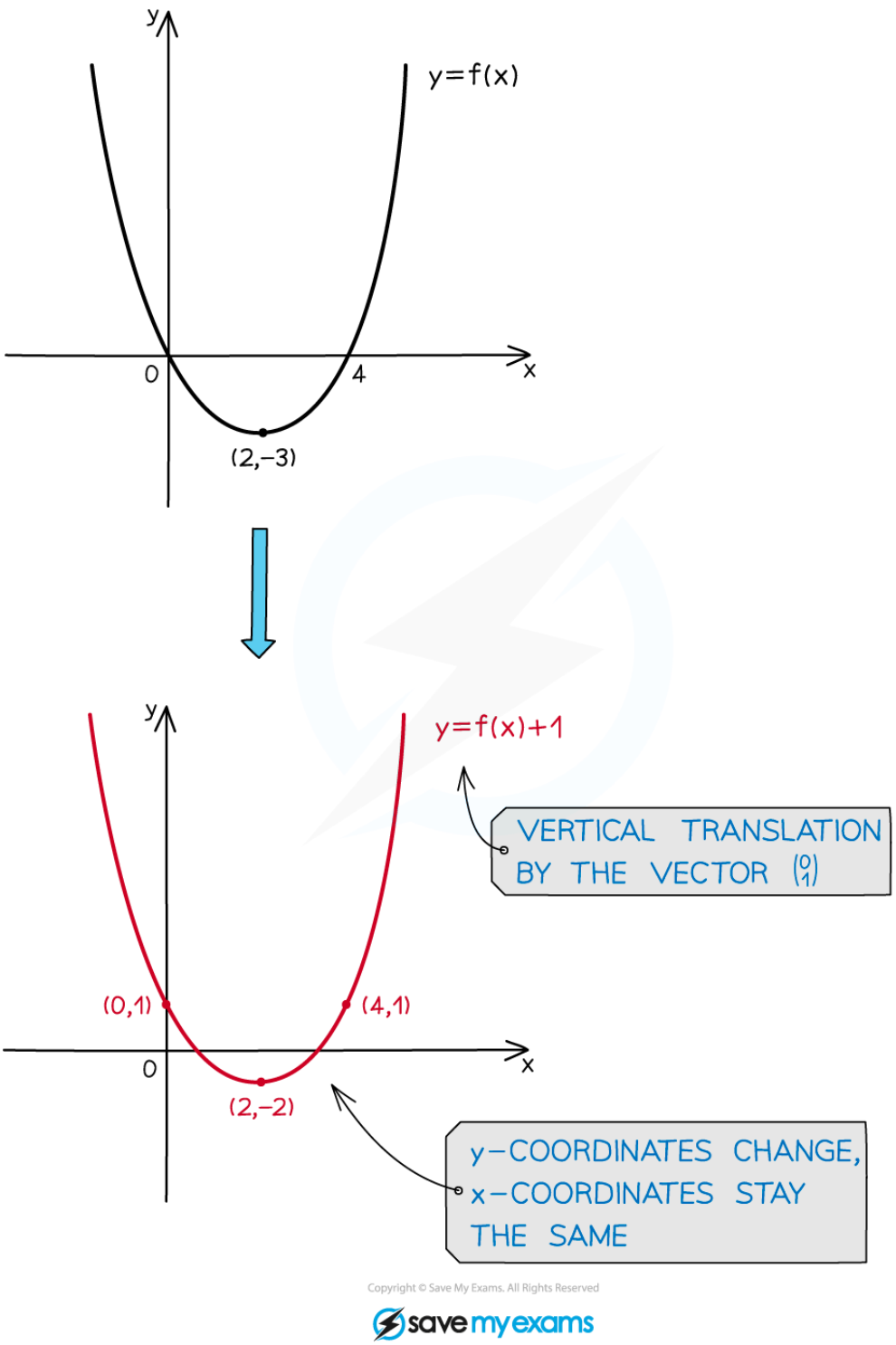

- The graph of $y = f(x + a)$ is a **horizontal** translation of the graph $y = f(x)$ by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
  - The graph moves **left for positive** values of *a* and **right for negative** values of *a*
  - The *y*-coordinates stay the same


Your notes

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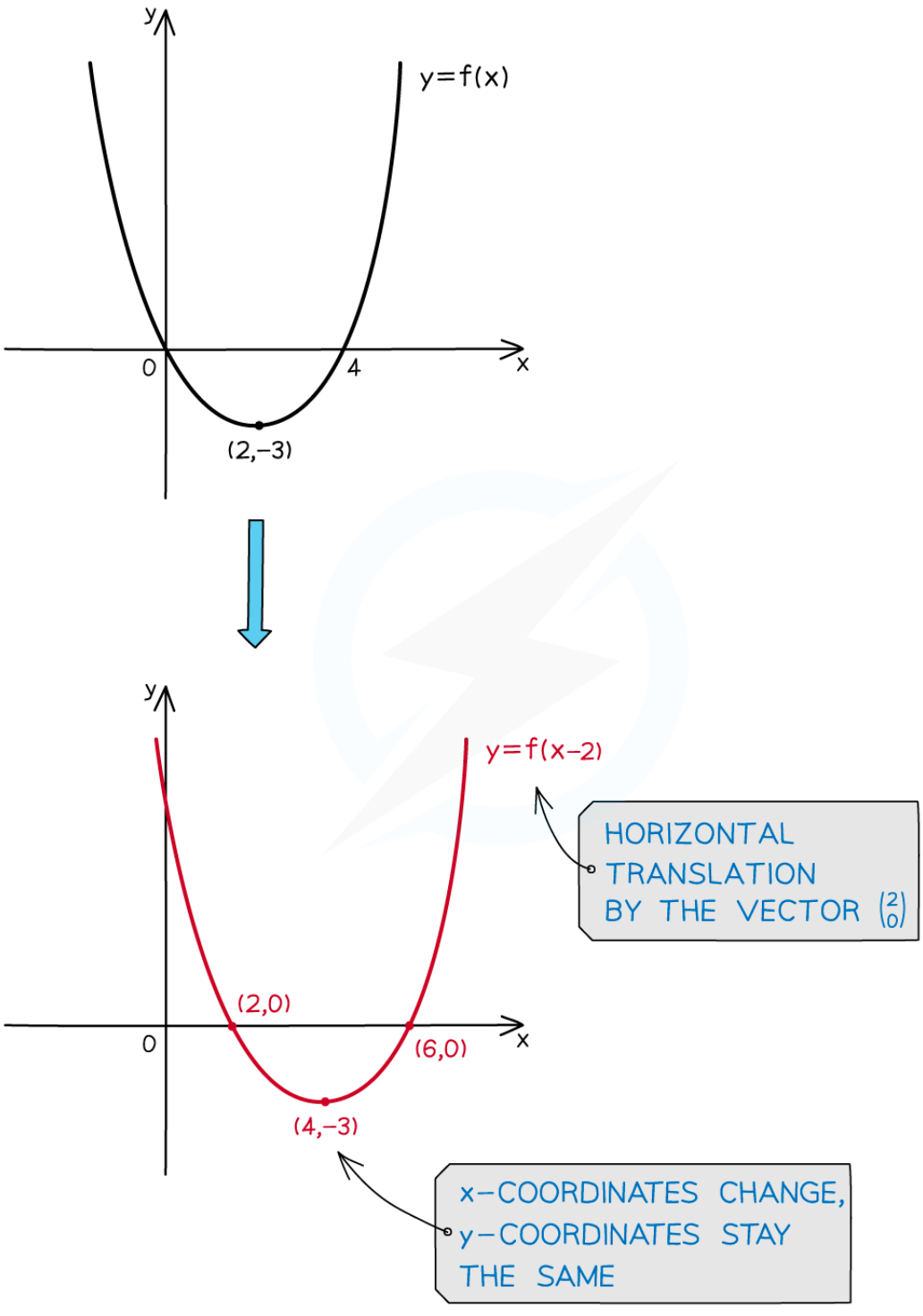

- Any asymptotes of **f(x)** are also translated. If an asymptote is parallel to the direction of translation, however, it will not be affected


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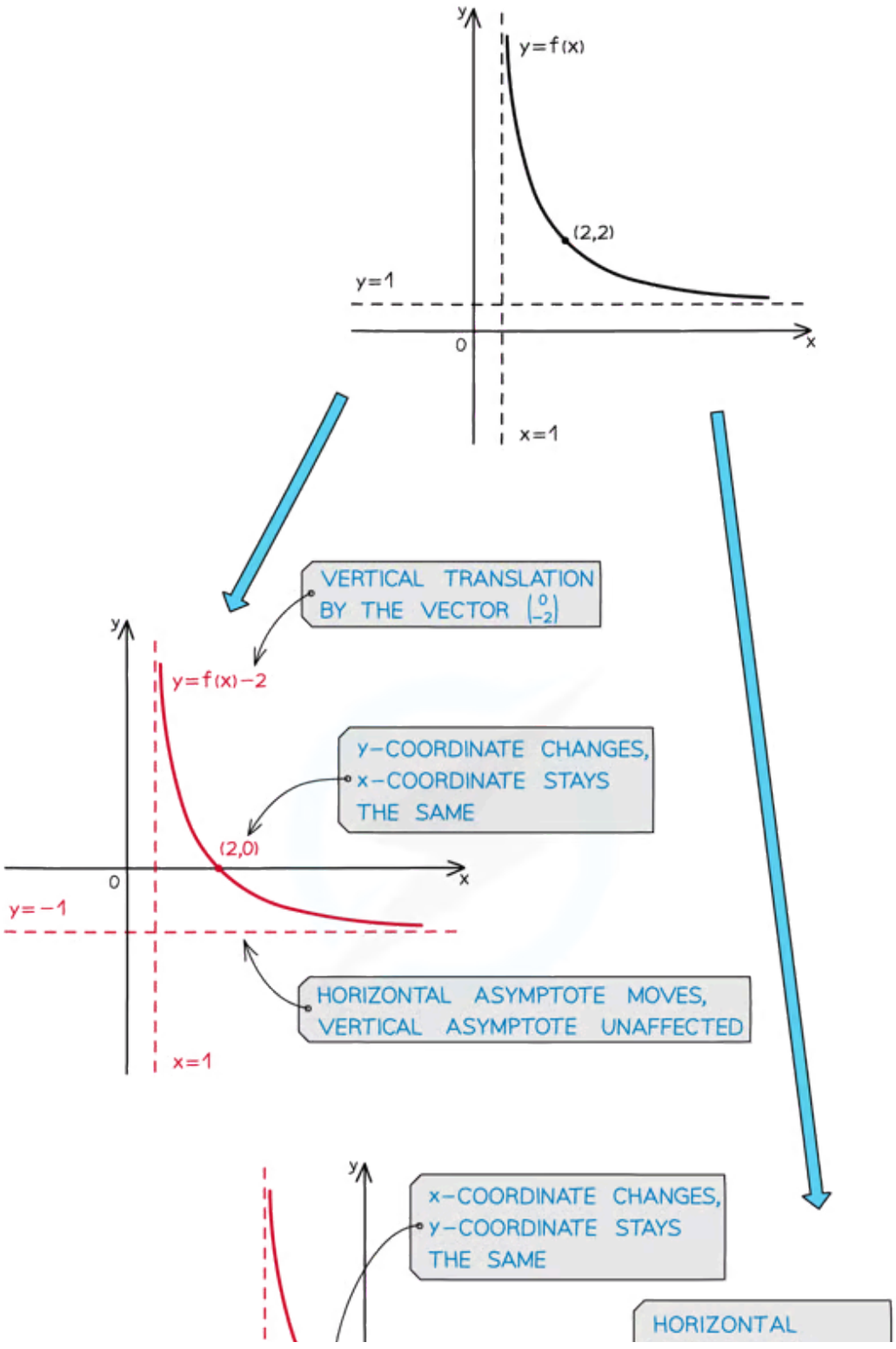

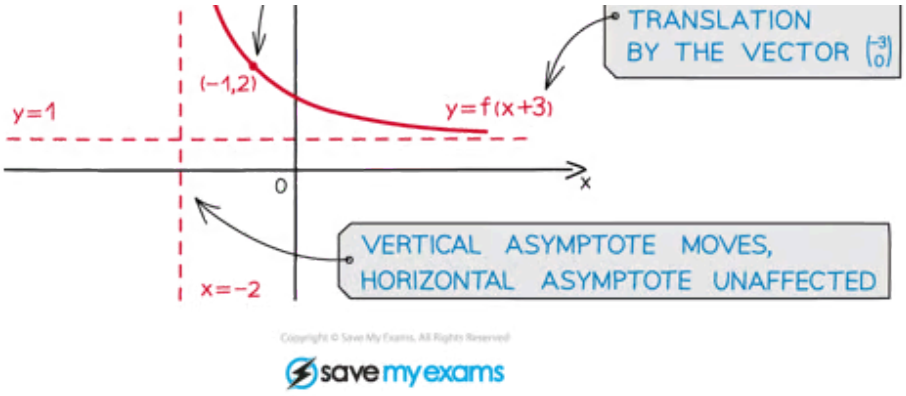

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Worked example

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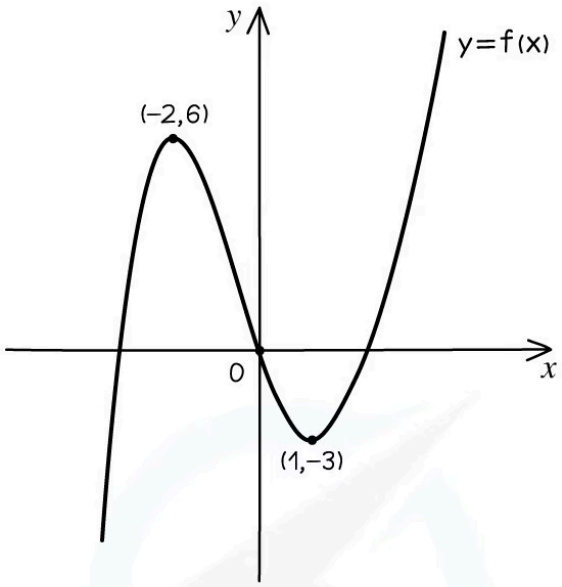
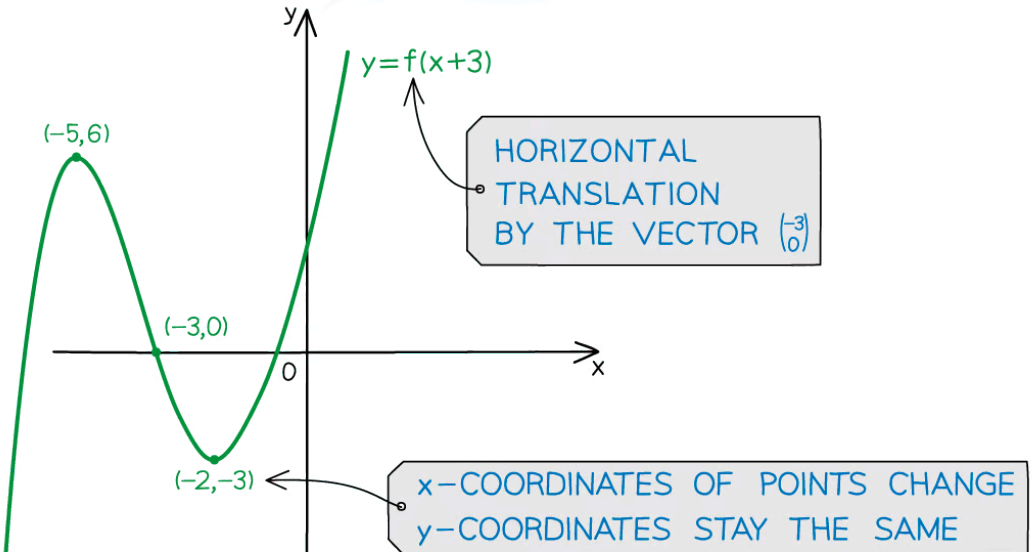
Your notes

The diagram below shows the graph of $y = f(x)$.



Sketch the graph of $y = f(x + 3)$.



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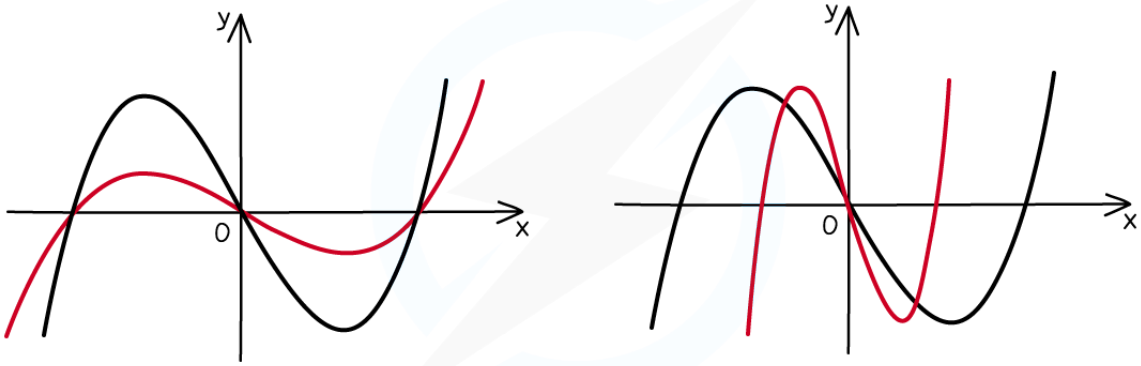
2.9.2 Stretches

Your notes

Stretches

What are graph transformations?

- When you alter a function in certain ways, the effects on the graph of the function can be described by geometrical transformations
- With a **stretch** all the points on the graph are moved towards or away from either the ***x*** or the ***y*** axis by a constant scale factor



What do I need to know about graph stretches?

- The graph of $y = af(x)$ is a **vertical** stretch of the graph $y = f(x)$ by a scale factor of a, centred on the x axis
  - The x coordinates of points stay the same; y coordinates are multiplied by a
  - Points **on the x axis** stay where they are
  - All other points move **parallel to the y axis**, away from ($a > 1$) or towards ($0 < a < 1$) the x axis

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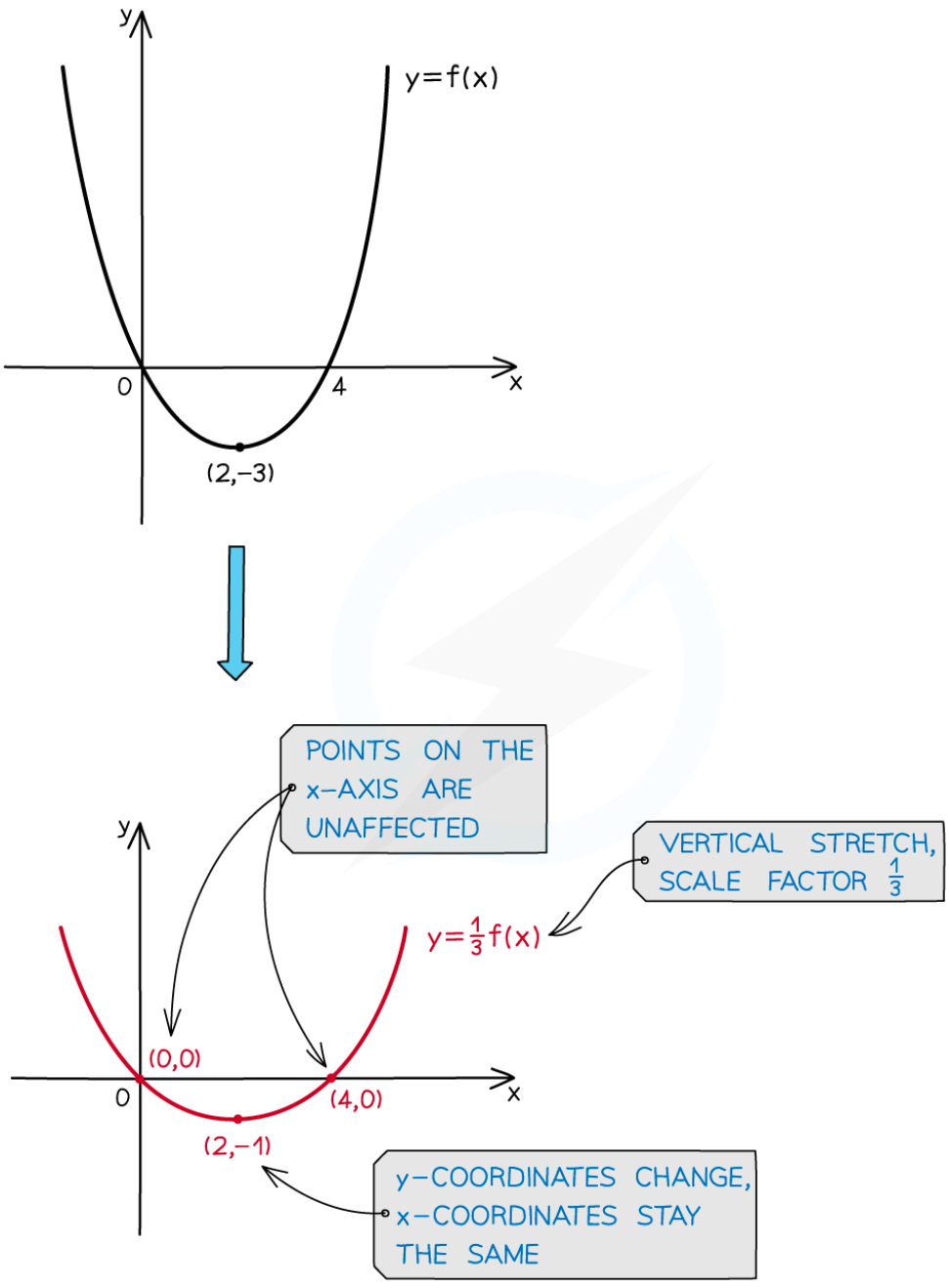


Your notes

- The graph of $y = f(ax)$ is a **horizontal** stretch of the graph $y = f(x)$ by a scale factor of $\frac{1}{a}$, centred on the *y*
  - The y coordinates of points stay the same; x coordinates are multiplied by $\frac{1}{a}$
  - Points **on the *y* axis** stay where they are
  - All other points move **parallel to the *x* axis**, away from ($0 < a < 1$) or towards ($a > 1$) the *y* axis

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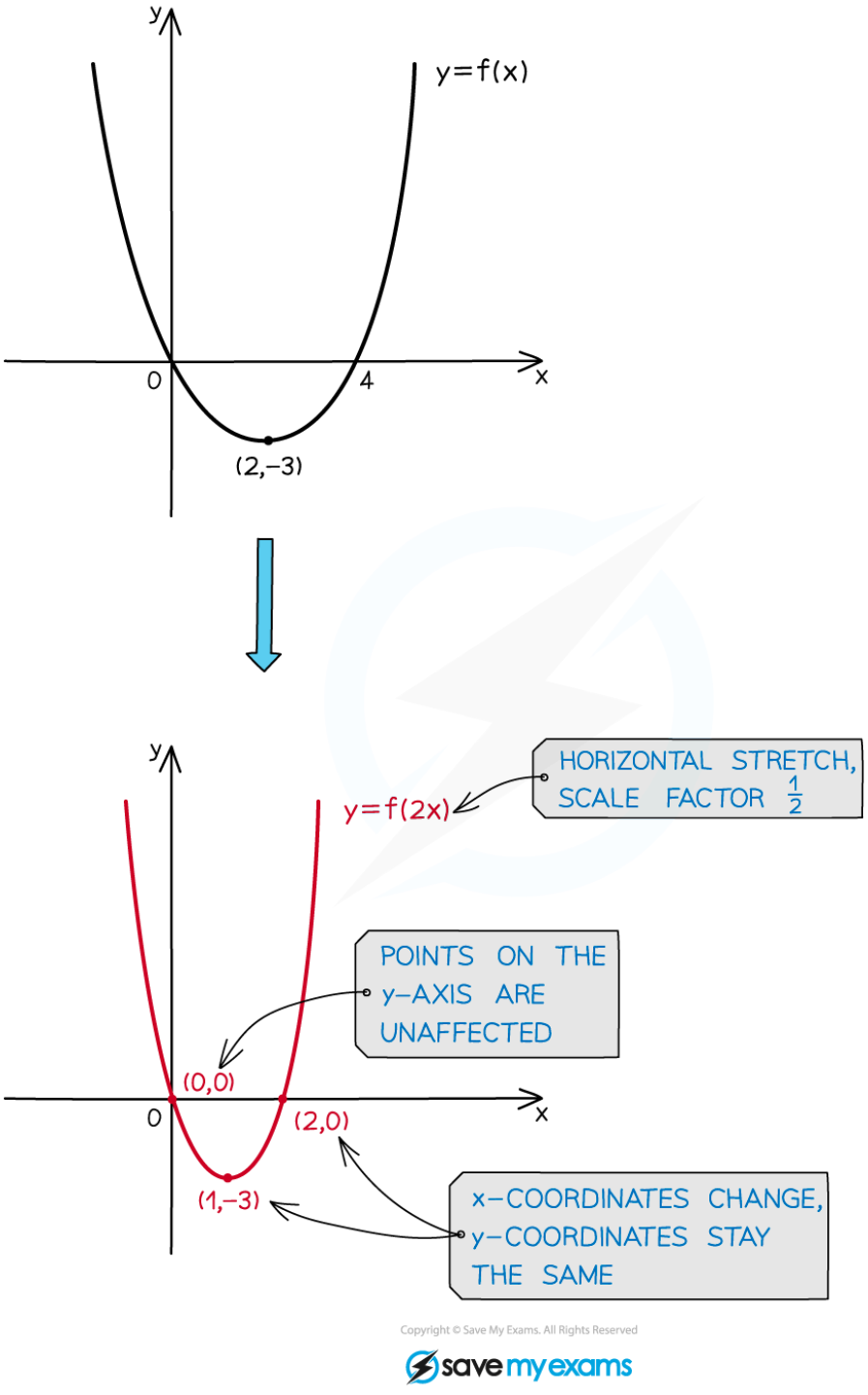

- Any asymptotes of **f(x)** are also affected by the stretch (stretch them as you would stretch the function of a straight line)

- If an asymptote is one of the coordinate axes, or is parallel to the direction of the stretch, however, it will not be affected


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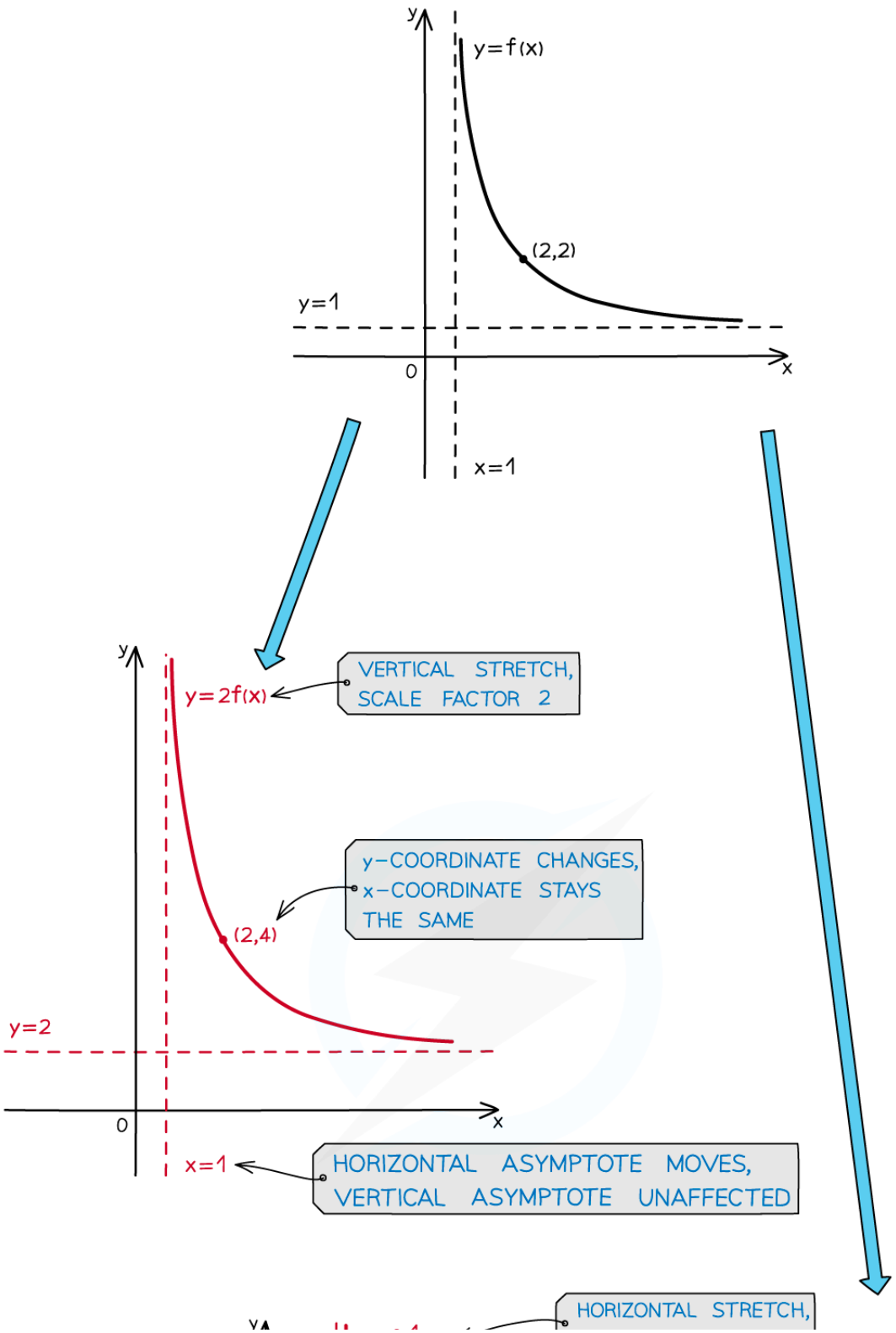

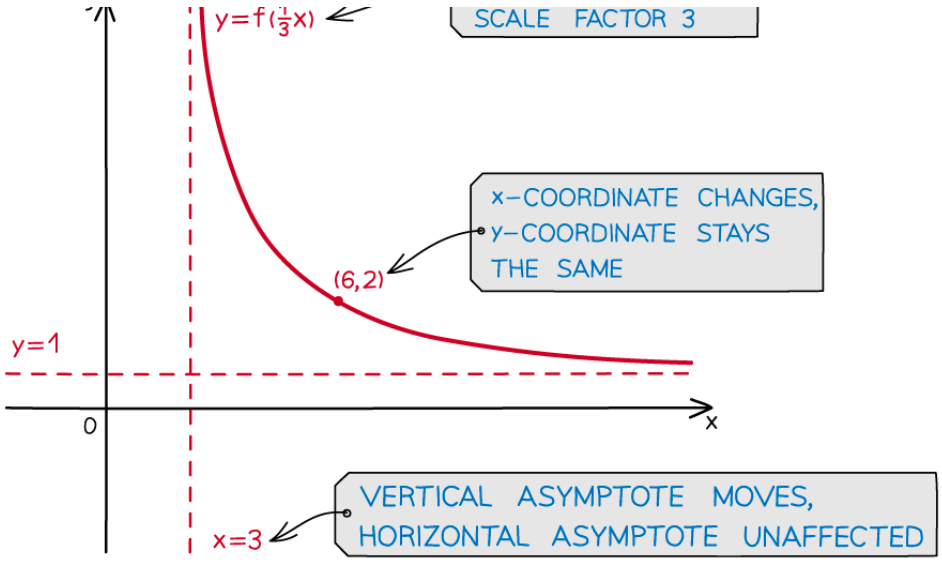

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Examiner Tip

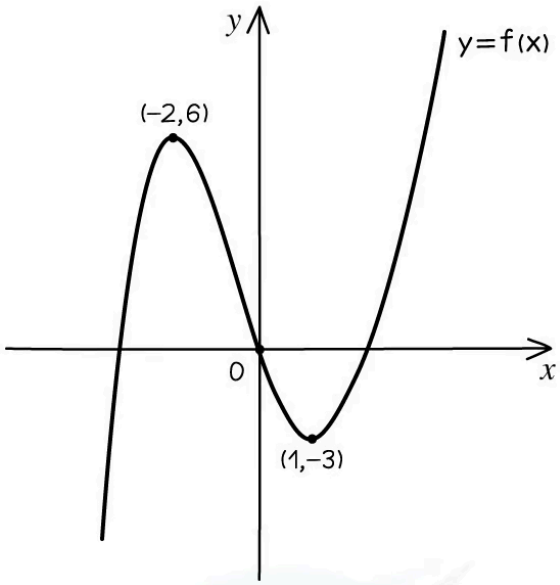
- When you sketch a stretched graph, be sure to indicate the new coordinates of any points that are marked on the original graph.
- Try to indicate the coordinates of points where the stretched graph intersects the coordinate axes (if you don't have the equation of the original function this may not be possible).
- If the graph has asymptotes, don't forget to sketch the asymptotes of the stretched graph as well.

Worked example

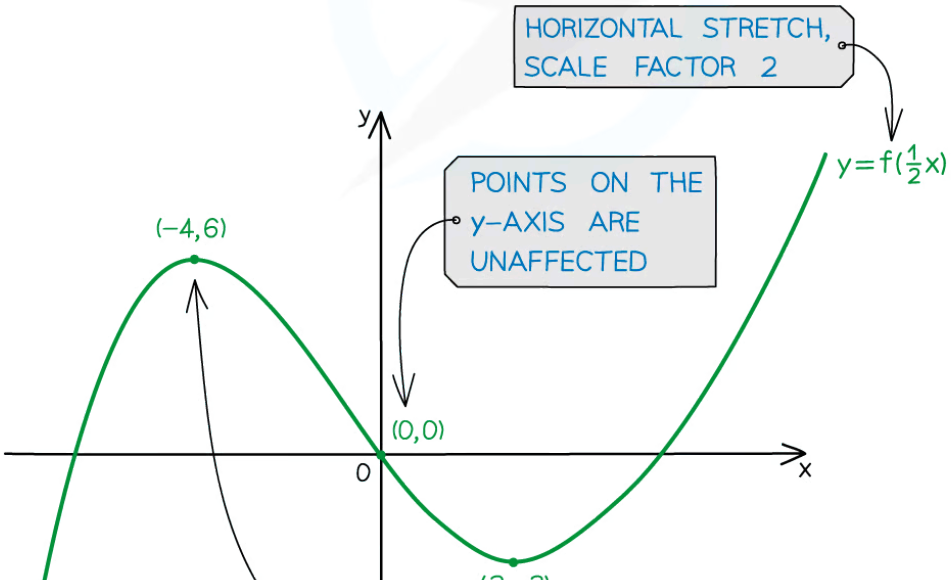
Your notes

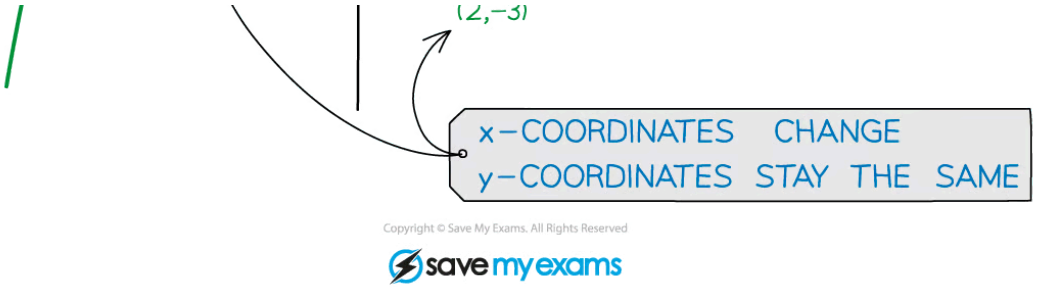
Your notes

The diagram below shows the graph of $y = f(x)$.



Sketch the graph of $y = f(\frac{1}{2}x)$.





Your notes

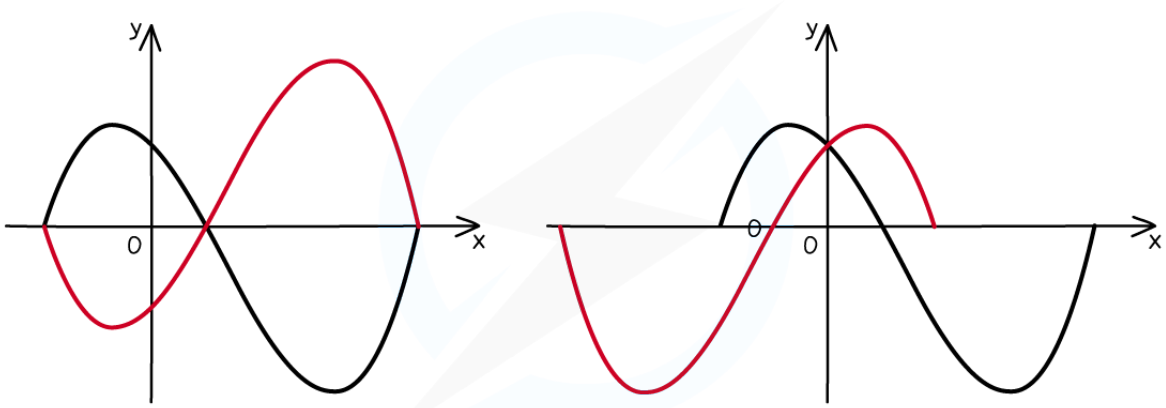
2.9.3 Reflections

Your notes

Reflections

What are graph transformations?

- When you alter a function in certain ways, the effects on the graph of the function can be described by geometrical transformations
- With a **reflection** all the points on the graph are reflected in either the ***x*** or the ***y***



- Any asymptotes of **f(*x*)** are also affected by the reflection (reflect them as you would reflect the function of a straight line)
- If an asymptote is one of the coordinate axes, or is perpendicular to the coordinate axis in which the graph is reflected, it will not be affected

What do I need to know about graph reflections?

- The graph of $y = -f(x)$ is a reflection in the x axis
  - The ***x*** coordinates of points stay the same; ***y*** coordinates have their signs flipped (positive to negative, negative to positive)
  - Points **on the *x* axis** stay where they are
  - All other points are reflected to the other side of the ***x*** axis

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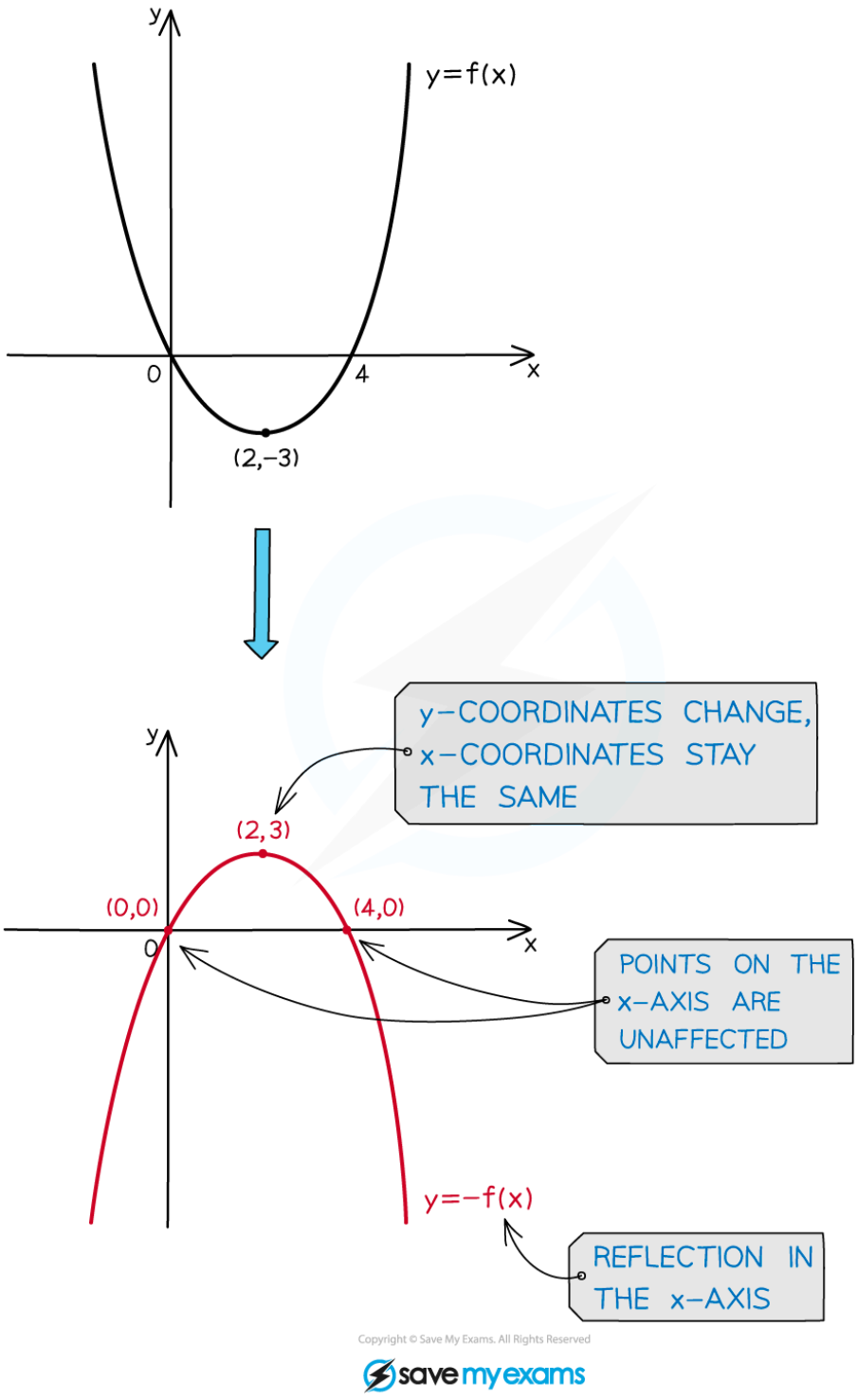

- The graph of $y = f(-x)$ is a reflection in the y axis

- The ***y*** coordinates of points stay the same; ***x*** coordinates have their signs flipped (positive to negative, negative to positive)
- Points **on the *y* axis** stay where they are
- All other points are reflected to the other side of the ***y*** axis


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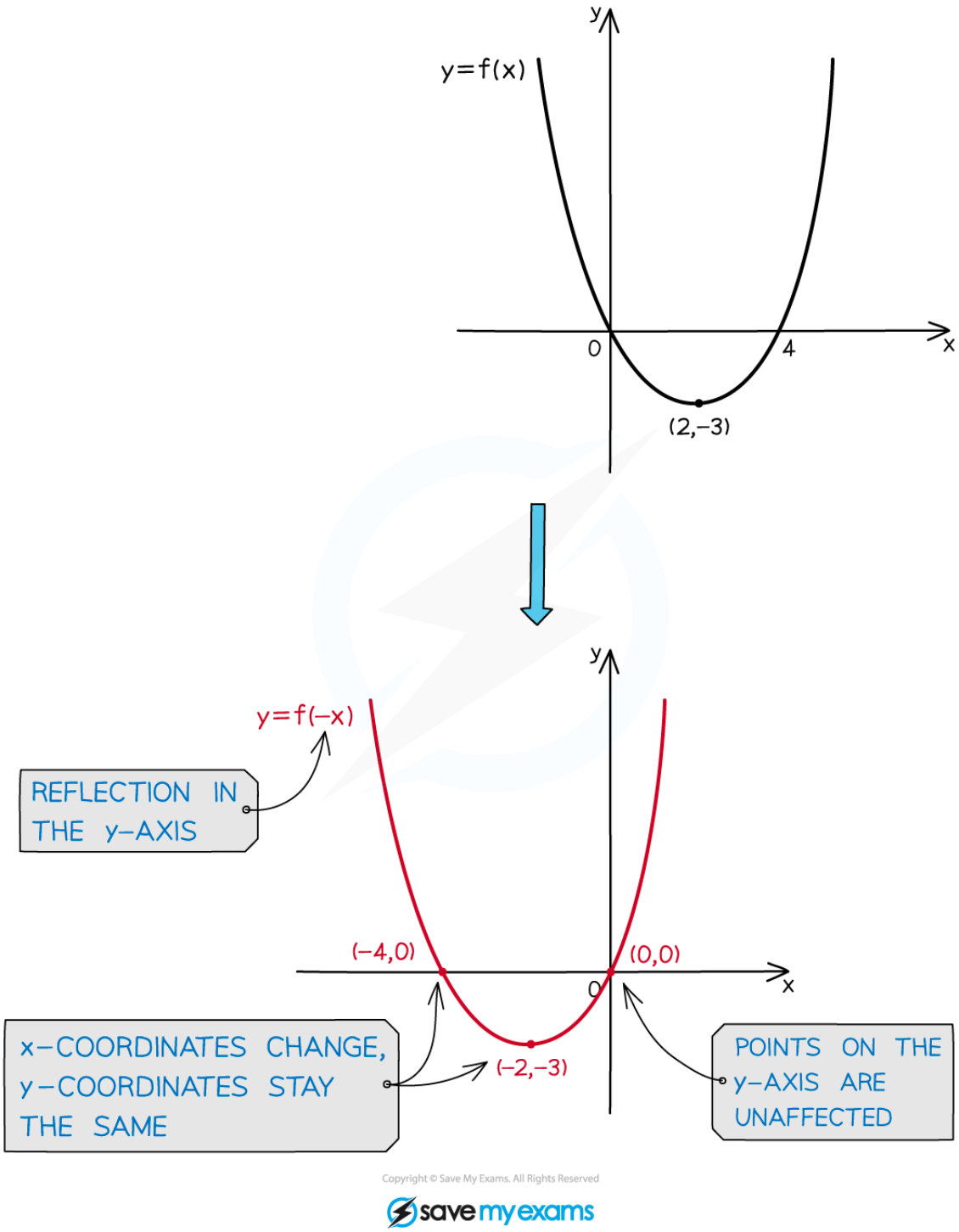
Your notes



- Any asymptotes of **f(x)** are also affected by the reflection (reflect them as you would reflect the function of a straight line)
- If an asymptote is one of the coordinate axes, or is perpendicular to the coordinate axis in which the graph is reflected, it will not be affected


Your notes

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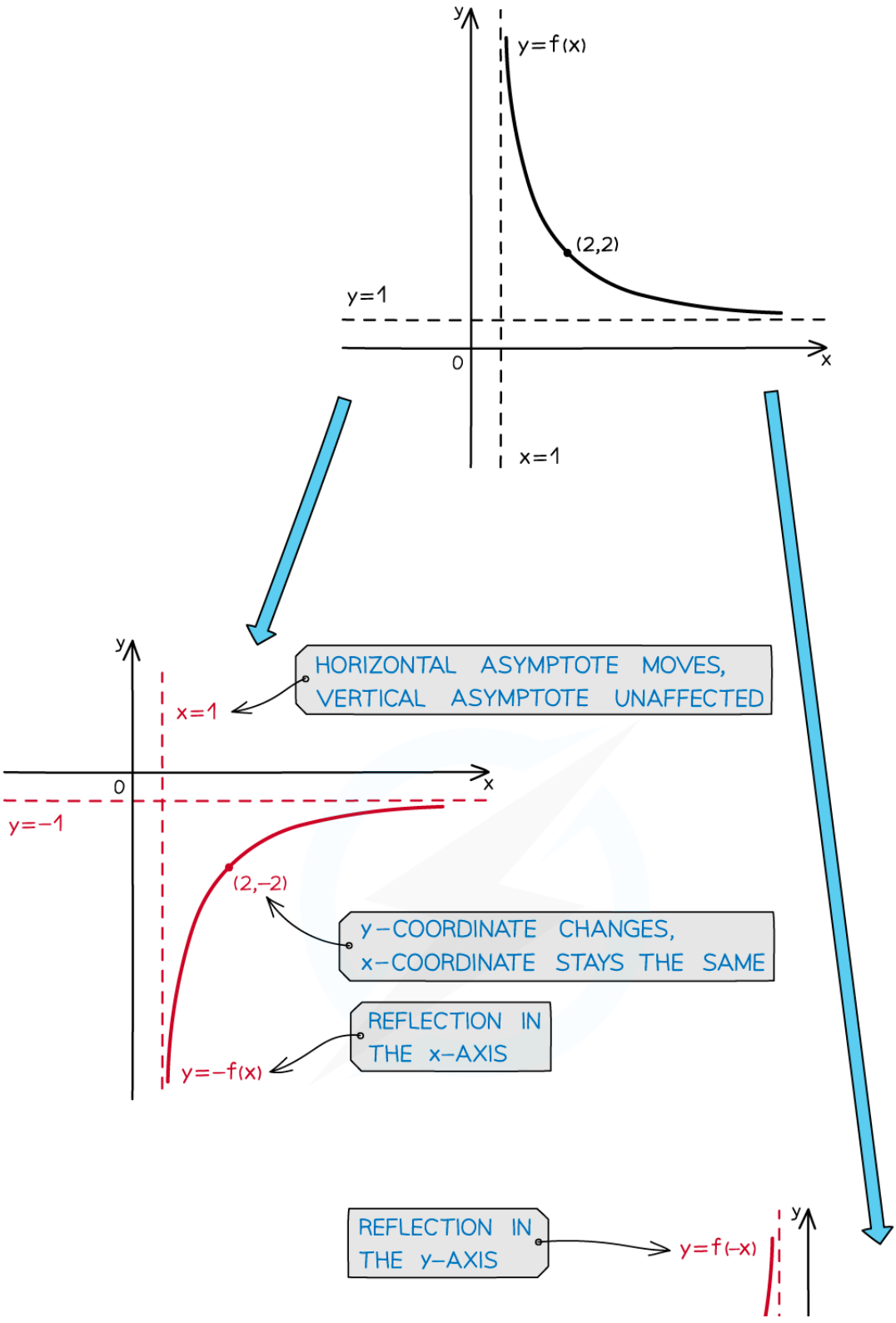

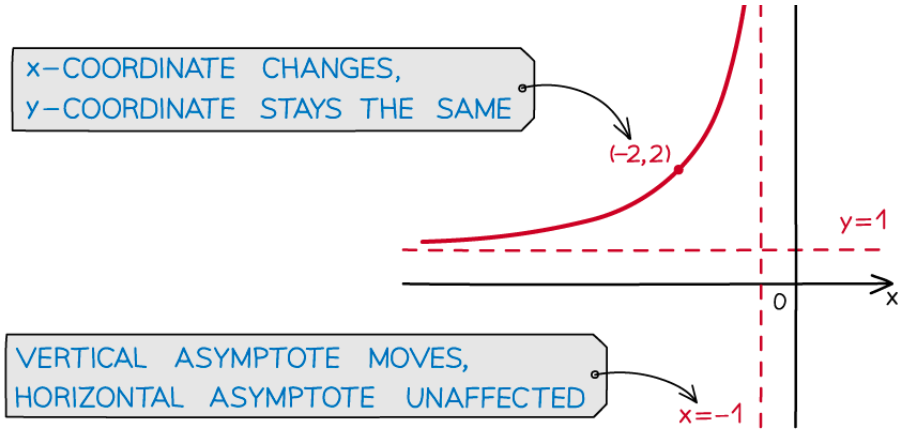

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Worked example

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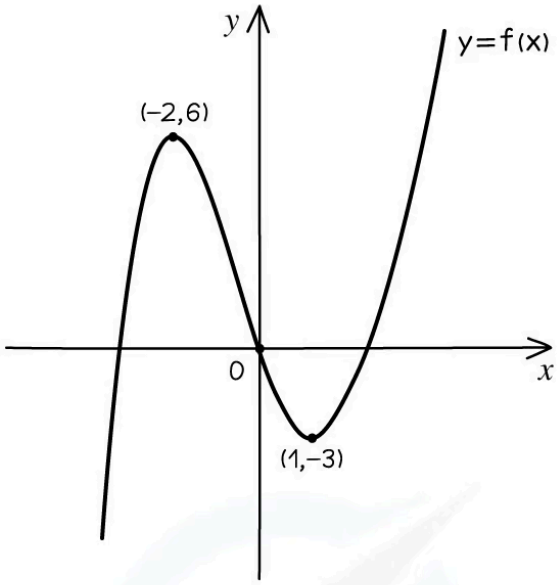
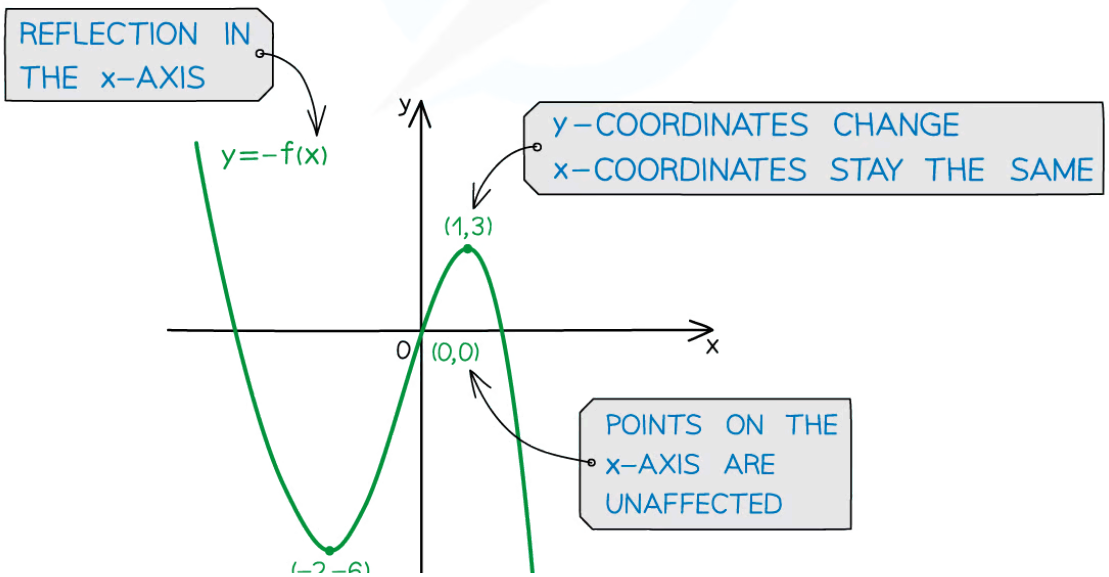
The diagram below shows the graph of $y = f(x)$.



Sketch the graph of $y = -f(x)$.



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