Mathematics 7
Unit 9: Geometry: Constructions

G01

SCO G01: Students will be expected to perform geometric constructions, including
- perpendicular line segments
- parallel line segments
- perpendicular bisectors
- angle bisectors

[CN, R, V]

[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation
[T] Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G01.01 Describe examples of parallel line segments, perpendicular line segments, perpendicular bisectors, and angle bisectors in the environment.
G01.02 Identify on a given diagram line segments that are parallel or perpendicular.
G01.03 Draw and construct a line segment perpendicular to another line segment and explain why they are perpendicular.
G01.04 Draw and construct a line segment parallel to another line segment and explain why they are parallel.
G01.05 Draw and construct the bisector of a given angle, using more than one method, and verify that the resulting angles are equal.
G01.06 Draw and construct the perpendicular bisector of a line segment, using more than one method, and verify the construction.

Scope and Sequence

Mathematics 6	**Mathematics 7**	**Mathematics 8**
G01 Students will be expected to construct and compare triangles, including scalene, isosceles, equilateral, right, obtuse, or acute in different orientations.	**G01** Students will be expected to perform geometric constructions, including ▪ perpendicular line segments ▪ parallel line segments ▪ perpendicular bisectors ▪ angle bisectors	**G01** Students will be expected to draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms.

Background

While the terms draw and construct are sometimes used interchangeably, it is important to note that constructions are performed using a compass and straight edge (without markings) as tools. Drawings may use the same tools as constructions, as well as Mira, mirrors, patty paper, and rulers. Measurements can be used in drawings. A sketch is a drawing completed without drawing tools.

"What makes shapes alike and different can be determined by an array of geometric properties. For example, shapes have sides that are parallel, perpendicular, or neither…" (Van de Walle and

Lovin 2006b, 179)

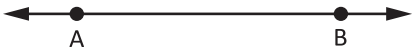
In Mathematics 5, students identified examples of perpendicular and parallel sides, edges, and faces in their study of 2-D shapes and 3-D objects (G01). They also identified examples of perpendicular and parallel line segments in the environment. Because Mathematics 5 students lack experience with measuring angles, the angle formed by perpendicular lines was identified as a right angle. Students measured and drew angles using protractors in Mathematics 6. In Mathematics 7, students will create parallel and perpendicular line segments and bisectors, as well as angle bisectors, using geometric constructions. Students should be exposed to a variety of methods when developing a concept such as perpendicular lines. Constructions, however, will be made with a straight edge and compass. Students should be able to describe how each construction was completed.

Since some of the constructions for parallel line segments involve perpendiculars, an option is to introduce perpendicular line segments and bisectors first. Quite often students don't distinguish between perpendicular line segments and perpendicular bisectors. Lines and angles can be bisected. In the word *bisectors*, *bi* means two and *sect* means to cut. When a line or an angle is bisected, it is cut into two pieces of equal size. We could say it is divided in half or divided down the middle. Students should come to understand the meaning of bisection through reference to familiar words with the same prefix like bicycle, biplane, and bilingual. Students should be able to explain similarities and differences between line bisectors and perpendicular line bisectors. Students also need to know the difference between line segments that intersect each other and those that bisect each other.

Lines, rays, and line segments are made up of sets of points and are straight and one-dimensional. Their only dimension is length.

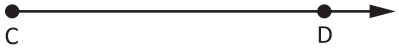
A *line* is a set of points that extends indefinitely in opposite directions.

The line AB:



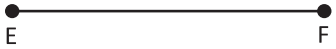
A *ray* is a set of points that extends indefinitely in one direction.

The ray CD:



A *line segment* is a set of points along a line with two finite endpoints.

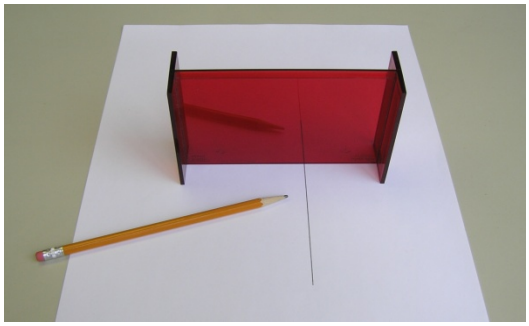
The line segment EF:



In Mathematics 5 and Mathematics 6, students will have explored methods of drawing **perpendicular lines** that include the following:

- *Method 1: Use a right angle.* Draw a line segment using a straightedge. Place the right angle of a right triangle or square on the line segment. Draw a line segment along the other side of the right angle of the triangle or square. Since these line segments intersect at 90° angles, or form a right angle, they are perpendicular to each other.
- *Method 2: Use a protractor.* Draw a line segment using a straightedge. Mark a point on the line segment. Align the point and line segment with the cross lines on the protractor. Mark the 90° measure. Use a straightedge to connect the original point and the 90° mark. Because the angle between the two line segments is 90°, they are perpendicular to each other.

- *Method 3: Use a Mira.* Draw a line segment using a straightedge. Lay the Mira across the line segment and adjust its position until the reflection of the line segment in front of the Mira lines up on top of the line segment behind the Mira. Draw a line segment along the edge of the Mira. That line segment is perpendicular to the original line segment because the angles at the intersection of the line segments are right angles.

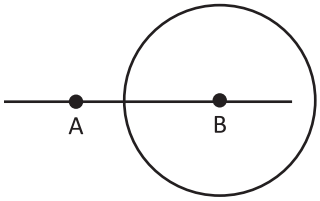


- *Method 4: Use tracing paper.* Use a straightedge to draw a line segment. Carefully fold the paper along the line segment so that the portion of the line segment on top falls directly on the part of the line segment underneath. When the line segments are aligned, crease the fold in the paper. Open it up and use a straightedge to draw the line segment along the crease. The line segments are perpendicular because the angle formed at the intersection measures 90°.

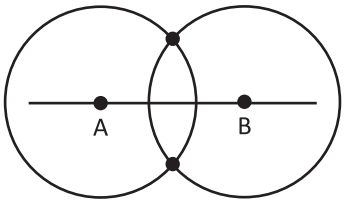
  In this outcome students are being asked to make constructions that require the use of a straightedge and a compass.

- *Method 5: Use a straightedge and a compass.* Draw a line segment using the straightedge. Mark any two points along the line segment.

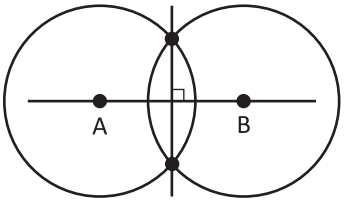
  – Use the compass to draw a circle with one of the points as the centre and a radius greater than one-half the distance between the two points.



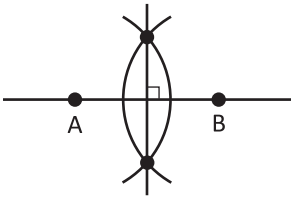
- Use the compass to draw a second circle with the same radius and the other point as centre.



- The circles will intersect at two points. Use the straightedge to connect those two points. The angle between the original line segment and the resulting line segment measures 90°; therefore, the line segments are perpendicular to each other.

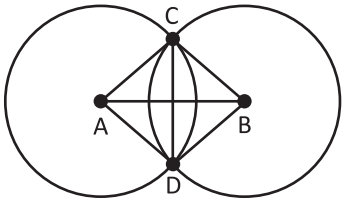


- To get a perpendicular line, it is not necessary to draw the entire circle; only an arc needs to be drawn.

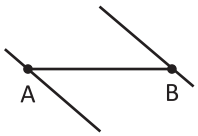


The use of circles emphasizes that the line segments from the centres to the points joined to form the perpendicular line segments are radii of congruent circles, and, therefore, are the same length.

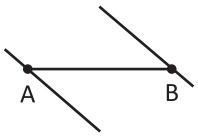
- This method used to construct perpendicular lines actually results in the construction of **perpendicular bisectors**. After drawing the circles, use the endpoints of the line segment, AB, as the centres for the circles. AC, CB, BD, and DA are congruent radii so ACBD is an equilateral quadrilateral. Therefore ACBD must be a rhombus. In a rhombus, the diagonals, in this case AB and CD, are perpendicular bisectors of each other.



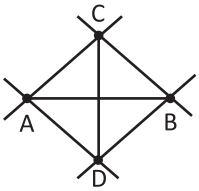
- A perpendicular bisector can also be constructed by creating a rhombus around a line segment. This will create the perpendicular bisector of the segment because the diagonals of a rhombus are perpendicular bisectors of each other.
- Lay a straightedge at an angle across a line segment with the endpoints on opposite sides of the straightedge. Adjust the straightedge until each endpoint touches the side of the straightedge. Trace lines through the endpoints along both sides of the straightedge.



- Rotate the straightedge until the end that was above the line is now below the line, and the end of the straightedge that was below is now above. Once again, adjust the straightedge so each endpoint touches the side of the straightedge, and trace both sides of the straightedge.



- Remove the straightedge. The intersecting lines create a rhombus. The diagonals of the rhombus are perpendicular bisectors.

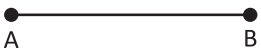


Methods for creating **parallel line segments** may include the following:

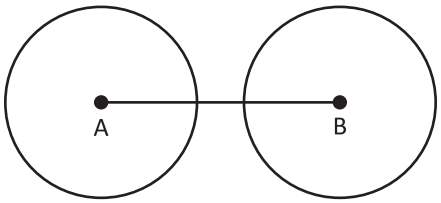
- Trace both edges of a straightedge, such as a ruler.
- Diagonals of a rectangle are the same length and they bisect each other. If the diagonals are connected in the centre to form an X, all four arms of the X are congruent to each other. Use two straws (or geo-strips, cardboard strips, stir sticks, etc.) to represent the diagonals, mark the midpoint of each, and connect them with a push-pin. Lay the X on a sheet of paper. Connect the

endpoints of two of the arms with a straightedge and trace a line segment. Mark the endpoints of the two remaining arms of the X and connect the points with a straightedge. The result is two parallel line segments. Stretch or collapse the X to adjust the distance between the parallel line segments.

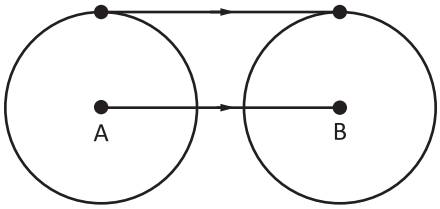
- Place a straightedge on a sheet of paper. Set the base of a right-angle triangle, or some other square corner, on the straightedge. Trace the side that creates a line segment perpendicular to the original line segment. Without moving the straightedge, slide the right angle along the straightedge to any desired position, and trace the same side of the right angle. All the line segments are parallel to one another.
- Draw a line segment. Connect points that are 90° and equidistant from the line segment. Use a right triangle or a protractor to draw two lines that are perpendicular to the original line segment. Measure and mark the same distance up each perpendicular line. Connect the marks to create a parallel line segment. The perpendicular lines are also parallel.
- Use a Mira to draw a line segment. Then use the Mira to draw perpendicular lines by making sure the original line segment is in line with its reflection in the Mira. The perpendicular lines are all parallel to each other.
- Fold a piece of paper carefully with the corners matching. Fold the paper again in the same direction. Crease the folds well. Open the paper and, using a straightedge, trace line segments along the creases. The resulting line segments are parallel.
- Use a compass and a straightedge.
  - Draw a line segment AB.



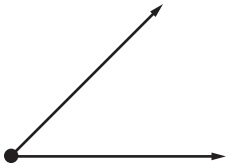
  - Use a compass to draw a circle around point A and a circle around point B with the same radius.



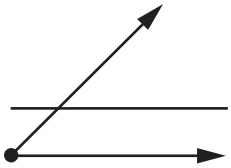
- Mark the highest (or lowest) points of each circle and connect them with a line. This line will be parallel to the original line segment AB.



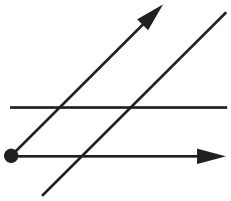
- Methods for generating **angle bisectors** may include the following:
  - Use a protractor.
    - > Use a protractor to measure the original angle.
    - > Divide the measurement in half.
    - > Use the protractor to mark the new measure and draw the bisector.
  - Use tracing paper.
    - > Copy the angle onto tracing paper.
    - > Fold the paper so that the two original rays lie on top of each other.
    - > Crease the paper along the fold.
    - > Open the paper and use a straightedge to trace the crease.
  - Use a Mira.
    - > Place the Mira so that part of its length lies over the vertex of the angle.
    - > Adjust the angle of the Mira until each of the angle rays are reflected on top of each other.
    - > Trace the edge of the Mira.
  - Use a straightedge.
    - > Line up the edge of the straightedge along one of the rays in the angle so that the straightedge lies “inside” the angle.



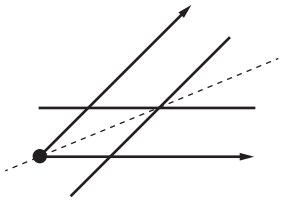
    - > Trace the side of the straightedge not on the ray.



> Trace the side of the straightedge for the other ray.

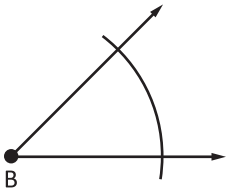


> Connect the vertex of the angle with the intersection of the lines just drawn.

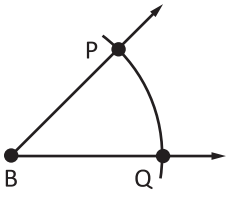


– Use a compass and a straightedge.

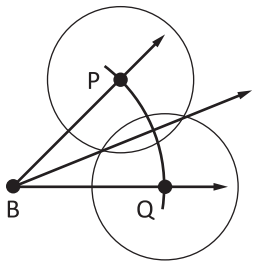
> Place the compass point on the vertex of the angle and draw an arc across the two rays.



> Place the compass point on one of the intersecting points, and draw a circle or an arc around the centre area of the angle. Keeping the same radius setting, draw a circle around the other point where the first arc intersects the other ray. It is not necessary to draw entire circles, just the intersection of the arcs of the circle approximately where the bisector will be.



> Use the straightedge to connect the intersection of the two circles with the vertex of the angle.



Encourage students to think critically about and comment on the ideas that are shared in class. They may acknowledge ideas they agree with, express appreciation for ideas that are explained, ask "how" or "why" questions, or offer further suggestions or support for an idea.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

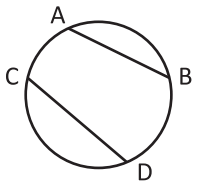
Tasks such as the following could be used to determine students' prior knowledge.

- Using a variety of 2-D shapes, have students identify parallel, intersecting, and perpendicular edges. Observe whether students use correct geometric terminology in their descriptions.
- Ask students to find a pattern block or Power Polygon that illustrates:
  - parallel sides and no right angles
  - parallel sides and right angles
- Have students represent parallel or perpendicular lines, or a variety of angles, including a right angle, using items like geo-strips, toothpicks, or straws. Some suggestions include:
  - parallel sides and no right angles
  - parallel to one another
  - intersecting
  - perpendicular at an end point of one straw
  - perpendicular at endpoints of each straw
  - one straw perpendicular to the other straw, but not at its end points

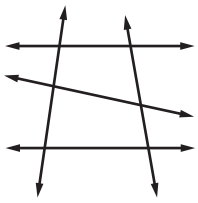
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (which can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Draw a line segment approximately 10 cm in length. Construct the perpendicular bisector of this segment, and explain your method.
- Draw an acute angle. Construct the angle bisector. Explain your method.
- Construct the perpendicular bisectors of line AB and line CD. (These bisectors, if done correctly, should meet at the centre of the circle.)



- Explain the difference between a line bisector and a perpendicular line bisector.
- Describe a situation when an angle bisector and a line bisector are the same thing.
- Draw a line segment approximately 10 cm in length and to construct a parallel line segment. Explain their method.
- Identify the parallel lines in the following diagram.



- Define **perpendicular bisector** and to explain how a perpendicular bisector differs from a perpendicular line segment.
- Complete a project such as the following:
  - Replicate the floor plan for a sport facility (e.g., court, field, rink).
  - Design a map for a community, a fairground, a school campus, or a campground. Design major services to be equidistant from strategic points in the area.
  - Create a design for a fence, lattice, fabric pattern, or piece of artwork and include a certain number of lines that must be included (e.g., circles or half circles, parallel lines, perpendicular lines, perpendicular and angle bisectors)
- Use what you have learned about angles, parallel and perpendicular lines, and bisectors to create your project. Prepare a report on the types of lines included in the project and how you created the lines.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Teach students to construct perpendicular bisectors of line segments and bisectors of angles using a variety of methods, such as paper folding, use of Miras, and compasses.
- Have students practise drawing perpendicular and parallel line segments. Then ask them to make a math journal entry commenting on which method they prefer, which method they believe to be most accurate, and which applications each method is best suited for.
- Discuss why and when people may use a perpendicular bisector. For example, a perpendicular bisector may be used to find the best place to put a single support under a beam, to find the division in a drawing, a design, or a building from which to build formal symmetry, to divide a piece of property into equal portions or to find a line that is the same distance from two points. The last application may be important in a variety of contexts, ranging from planning a city or a meeting spot to setting up a lemonade stand.
- Have students practise using various methods to create perpendicular bisectors. Individuals can create their own line segments or create line segments for their partners, or the teacher can assign line segments. After students have had sufficient practice in using the methods, ask them to make a math journal entry commenting on their preferred method, the method they think is most useful or most accurate, applications for the different methods, and so on.
- Have students practise drawing parallel line segments of different lengths and different distances apart. Partners, teachers, or the toss of a number cube can determine the length or distance, or students can choose their own measurements. After trying several methods to draw designated parallel line segments, students can choose the method they prefer and write about it in their math journals.
- Ask students to explain what an angle bisector is, and how they could test to see whether or not a line actually bisects an angle.
- Have students practise drawing angle bisectors. Supply students with angles or angle measures for which students can create bisectors, or have students generate their own measures. Ask students to identify and justify a preferred technique for bisecting angles and write about it in their math journals. Differentiation: Provide students with a handout outlining and illustrating methods for creating an angle bisector, along with a compass and templates for those students who may need them.
- Have students use geometry software to practice and investigate angle bisectors of different shapes.
- Assemble geometry kits for student use that contain protractors, rulers, compasses, right triangles, squares, Miras, tracing paper, etc.

SUGGESTED LEARNING TASKS

- Give examples of parallel and perpendicular lines from their everyday life. Suggested examples may be:
  - Parallel lines:
    - opposite sides of picture frames
    - railroad or roller coaster tracks

    - > lines on loose-leaf paper
    - > rows of siding on a house
    - > lines of latitude
    - > guitar strings
  - Perpendicular lines:
    - > railway tracks and railway ties
    - > fence posts and fence rails
    - > four way stops
    - > lines of latitude and longitude on a map
    - > a wall and a shelf
- Write the upper case letters of the alphabet that only use line segments. Find examples of bisectors of segments, perpendicular segments, and perpendicular bisectors.
- Make a list of as many pairs of parallel lines as you can find in the classroom in two minutes. After the time is up, pass your list to another student. Then, one at a time, read an entry from the list. Everybody who has that entry on their list will cross it off. At the end, the list with the most remaining entries will be the winner. (Students can be paired up for this activity.) Repeat this activity exploring perpendicular lines.
- Draw a line that is neither vertical nor horizontal. Then, using a method of your choice, draw a second line that is parallel to the first. Repeat this activity exploring perpendicular lines.
- Think of 2-D shapes (excluding quadrilaterals) that have parallel sides. Include diagrams to illustrate your thinking.
- Respond to the following:
  - Can two lines be both parallel and perpendicular?
  - Can a line have more than one line that is perpendicular to it?
  - Explain your reasoning. Provide examples.
- Create artwork composed of coloured lines and angles (perhaps similar to the work of Piet Mondrian) including labelled points. Create a key to go with your artwork that lists the parallel and perpendicular lines. The pictures and keys can be posted for display.
- Build an origami figure or shape, unfold it, trace the lines, and indicate parallel and perpendicular lines in the fold lines.
- Use items like geo-strips, toothpicks or straws to create the following:
  - one straw bisecting the other straw but not perpendicular;
  - each straw bisecting the other straw but not perpendicular;
  - one straw bisected by the other straw and perpendicular;
  - each straw bisecting the other straw and perpendicular.
- Provide students with a diagram of interconnected lines to play a game with a partner. Students each choose a different colour of highlighter or light-coloured pencil crayon. They pick either odd or even numbers on a number cube to represent parallel or perpendicular lines. Students take turns shaking the number cube and marking either a set of parallel lines or a set of perpendicular lines with their selected colour. If students cannot find a set of lines, they forfeit their turn. The player with the greatest number of sets of lines wins.
- Identify parallel and perpendicular lines in photographs (i.e., on paper or using an annotation/presentation app). Create a schematic with symbols to identity these lines.

- Investigate drawing perpendicular bisectors for each side of a triangle. Draw a circle using the point of intersection of the perpendicular bisectors as the centre of the circle and the radius as the distance from the centre to one of the vertices of the triangle. Continue this investigation for a variety of triangles, parallelograms, or other polygons.
- Make a graffiti wall. On a sticky note, write an example in the environment of either a pair of parallel lines, a pair of perpendicular lines, a perpendicular bisector, or an angle bisector. Post your sticky notes on the wall. Then choose a sticky note other than your own and determine which category it falls under. They then place it in the appropriate section under parallel lines, perpendicular lines, perpendicular bisector, or angle bisector.

SUGGESTED MODELS AND MANIPULATIVES

- compass
- computer software such as Geometer's Sketchpad
- dot/grid paper
- geo-boards*
- geo-strips
- Miras
- straight edge
- tracing paper

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ angle	▪ angle
▪ angle bisector	▪ angle bisector
▪ arc	▪ arc
▪ bisect	▪ bisect
▪ bisector	▪ bisector
▪ construct	▪ construct
▪ draw	▪ draw
▪ intersecting lines	▪ intersecting lines
▪ line	▪ line
▪ perpendicular	▪ perpendicular
▪ perpendicular bisector	▪ perpendicular bisector
▪ perpendicular lines	▪ perpendicular lines
▪ parallel	▪ parallel
▪ parallel lines	▪ parallel lines
▪ ray	▪ ray
▪ segment	▪ segment
▪ sketch	▪ sketch

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 8: Geometry (NSSBB #: 2001640)
  - Section 8.1 Parallel Lines
  - Section 8.2 Perpendicular Lines
  - Section 8.3 Constructing Perpendicular Bisectors
  - Section 8.4 Constructing Angle Bisectors
  - Unit Problem: Design the Cover
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages

Modifiable Line Masters