ETH zürich

Autonomous Systems Lab

Robot Dynamics Exercise: Modeling and Control of Micro Aerial Vehicles (MAVs)

Mike Allenspach
Maximilian Brunner

1st December 2021, Zürich

MAV Dynamic Model
Equations of Motion (EoM)

Generic EoM

$$\mathbf{M}(\vec{\varphi})\ddot{\vec{\varphi}} + \vec{b}\left(\vec{\varphi}, \dot{\vec{\varphi}}\right) + \vec{g}(\vec{\varphi}) + \mathbf{J}_{ex}^{T}\vec{F}_{ex} = \mathbf{S}^{T}\vec{\tau}_{act}$$

Change of Momentum and Spin in Body Frame

$$\begin{bmatrix} mE_{3x3} & 0 \\ 0 & \boldsymbol{I} \end{bmatrix}\begin{bmatrix} {}_{B}\dot{\boldsymbol{v}} \\ {}_{B}\dot{\boldsymbol{\omega}} \end{bmatrix} + \begin{bmatrix} {}_{B}\boldsymbol{\omega} \times m\ {}_{B}\boldsymbol{v} \\ {}_{B}\boldsymbol{\omega} \times \boldsymbol{I}\ {}_{B}\boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} {}_{B}\boldsymbol{F} \\ {}_{B}\boldsymbol{M} \end{bmatrix}$$

E_{3x3}: Identity matrix

Torque!!

Change of Position and Attitude in World Frame

$${}_{E}\dot{\boldsymbol{x}} = \boldsymbol{C}_{EB}\,{}_{B}\boldsymbol{v}$$

$$E_r\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = {}_{B}\boldsymbol{\omega}$$

MAV Dynamic Model
Forces and Moments

$$ {}_B\boldsymbol{F} = {}_B\boldsymbol{F}_G + {}_B\boldsymbol{F}_{Aero} \qquad {}_B\boldsymbol{M} = {}_B\boldsymbol{M}_{Aero} $$

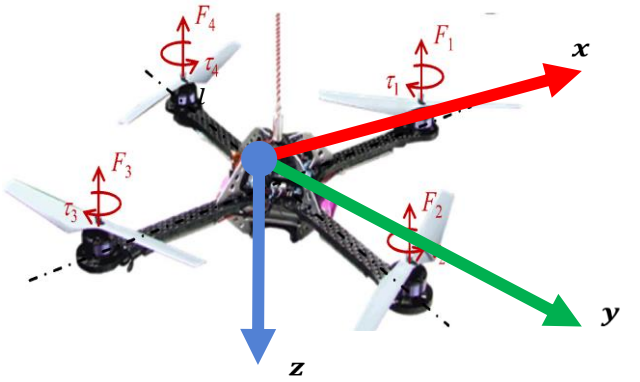
$$ {}_B\boldsymbol{F}_G = \boldsymbol{C}_{EB}^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}_E $$

Aerodynamic Propeller Forces (Thrust)

$$ U_1 = \sum_{i=1}^{4} F_i \Longrightarrow {}_B\boldsymbol{F}_{Aero} = -\begin{pmatrix} 0 \\ 0 \\ U_1 \end{pmatrix} $$

Aerodynamic Propeller Moments

$$ \begin{cases} U_2 = l(F_4 - F_2) \\ U_3 = l(F_1 - F_3) \\ U_4 = -\tau_1 + \tau_2 - \tau_3 + \tau_4 \end{cases} \Rightarrow {}_B\boldsymbol{M}_{Aero} = \begin{pmatrix} U_2 \\ U_3 \\ U_4 \end{pmatrix} $$



$$ F_i = b\omega_{p,i}^2 \qquad \tau_i = \pm d\omega_{p,i}^2 $$

b : thrust constant

d : drag constant

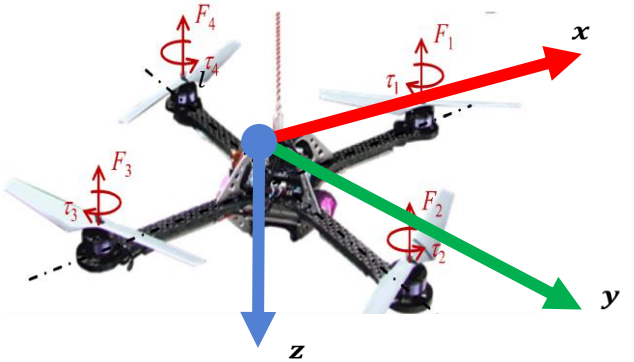
l : distance of propeller from the CoG

$\omega_{p,i}$: rotational speed of propellor i

MAV Dynamic Model
Control Allocation Quadcopter

$$U_1 = \sum_{i=1}^{4} F_i \Rightarrow {}_B\boldsymbol{F}_{Aero} = -\begin{pmatrix} 0 \\ 0 \\ U_1 \end{pmatrix}$$

$$\begin{cases} U_2 = l(F_4 - F_2) \\ U_3 = l(F_1 - F_3) \\ U_4 = -\tau_1 + \tau_2 - \tau_3 + \tau_4 \end{cases} \Rightarrow {}_B\boldsymbol{M}_{Aero} = \begin{pmatrix} U_2 \\ U_3 \\ U_4 \end{pmatrix}$$



Allocation Matrix

$$\boldsymbol{A}_{quad} = \begin{pmatrix} b & b & b & b \\ 0 & -lb & 0 & lb \\ lb & 0 & -lb & 0 \\ -d & d & -d & d \end{pmatrix} \quad \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \boldsymbol{A}_{quad} \begin{pmatrix} \omega_{p,1}^2 \\ \omega_{p,2}^2 \\ \omega_{p,3}^2 \\ \omega_{p,4}^2 \end{pmatrix}$$

$$F_i = b\omega_{p,i}^2 \qquad \tau_i = \pm d\omega_{p,i}^2$$

b : thrust constant

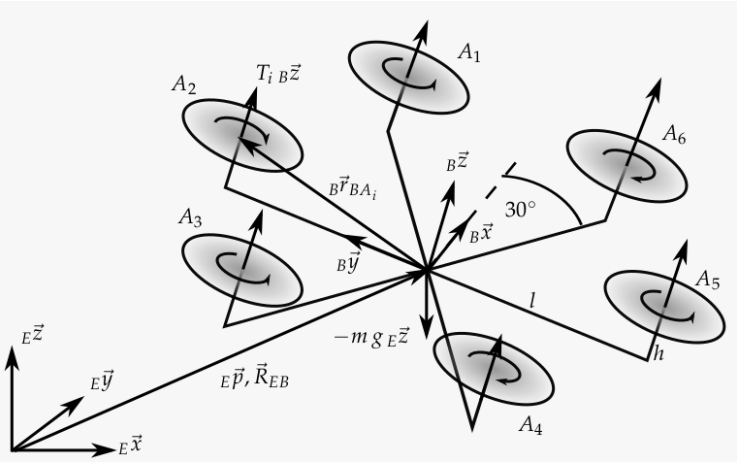
d : drag constant

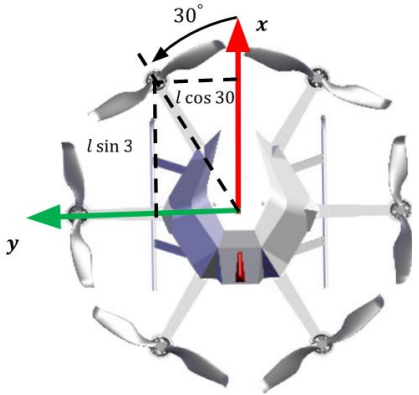
l : distance of propeller from the CoG

$\omega_{p,i}$: rotational speed of propellor i

MAV Dynamic Model
Control Allocation Hexacopter

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \omega_{p,1}^2 \\ \vdots \\ \omega_{p,n}^2 \end{pmatrix}$$





$$F_i = b\omega_{p,i}^2 \qquad \tau_i = \pm d\omega_{p,i}^2$$

b : thrust constant

d : drag constant

l : distance of propeller from the CoG

$\omega_{p,i}$: rotational speed of propellor i

Mike Allenspach
Doctorate at D-MAVT
mike.allenspach@mavt.ethz.ch

ETH Zurich
Autonomous Systems Lab
LEE J 304
Leonhardstrasse 21
8092 Zürich, Switzerland

www.asl.ethz.ch

Autonomous Systems Lab