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I pledge my honor that I have abided by the Stevens Honor System.

3/1 Problem Set 6

1) $fac(0) = 1 \quad \forall n > 0: fac(n) = n \times fac(n-1)$

prove: $1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n+1)! - 1 \quad \to \quad n! \cdot n - 0!$

base: $n=1$; $fac(1) = 1 \cdot fac(0)$
$= 1 \cdot 1 = 1$
$1 \cdot 1! = 1 = (1+1)! - 1$
$= 2! - 1$
$= 2 - 1 = 1$

hyp: $1 \cdot 1! + 2 \cdot 2! + \ldots + k \cdot k! = (k+1)! - 1$

step: $1 \cdot 1! + 2 \cdot 2! + \ldots + (k+1) \cdot (k+1)!$
$= 1 \cdot 1! + 2 \cdot 2! + \ldots + k \cdot k! + (k+1) \cdot (k+1)!$
$= (k+1)! - 1 + (k+1)(k+1)!$
$= (k+1)! + (k+1)!(k+1) - 1$
$= (k+1)!(1 + k + 1) - 1$
$\therefore = (k+1)!(k+2) - 1 = (k+2)! - 1$

goal: $(k+2)! - 1$

2) prove: $\forall n \geq 1: (A_1 - B) \cup (A_2 - B) \cup \ldots \cup (A_n - B) = (A_1 \cup A_2 \cup \ldots \cup A_n) - B$

base: $n=1$; $A_1 - B = A_1 - B$

hyp: $(A_1 - B) \cup (A_2 - B) \cup \ldots \cup (A_k - B) = (A_1 \cup A_2 \cup \ldots \cup A_k) - B$

step: $(A_1 - B) \cup (A_2 - B) \cup \ldots \cup (A_{k+1} - B)$
$= (A_1 - B) \cup (A_2 - B) \cup \ldots \cup (A_k - B) \cup (A_{k+1} - B)$ inductive hypothesis
$= ((A_1 \cup A_2 \cup \ldots \cup A_k) - B) \cup (A_{k+1} - B)$ inductive hypothesis
$= ((A_1 \cup A_2 \cup \ldots \cup A_k) \cup A_{k+1}) - B$
$= (A_1 \cup A_2 \cup \ldots \cup A_k \cup A_{k+1}) - B$
$\therefore = (A_1 \cup A_2 \cup \ldots A_{k+1}) - B$

3) a) No, he has not; he has only proven every other P(k), but not every P(k).

b) Ben must first establish the inductive hypothesis before attempting the inductive step to complete the inductive proof.

4) $\forall n \geq 5: 2^n > n^2$

base: $n=5$; $2^5 > 5^2$; $32 > 25$ ✓

hyp: $2^k > k^2$

step: $2^{k+1} > (k+1)^2$
$2^k + 2^k > k^2 + 2k + 1$
$2^k + \cancel{2^k} > \cancel{k^2} + 2k + 1$
$\therefore 2^k > 2k + 1$

prove: $\forall n \geq 5, \; 2^n > 2n + 1$

base: $n \geq 5$, $32 > 11$ ✓

hyp: $2^k > 2k + 1$

step: $2^{k+1} > 2(k+1) + 1$
$2^k + 2^k > 2k + 3$
$(2k+1) + (2k+1) > 2k + 3$
$\therefore 4k + 2 > 2k + 3$ is true for all $k \geq 5$