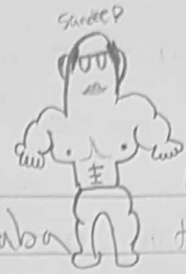
ax + by = gcd(a, b)
x, y from pulverizer



Issac Zhang
Rohan Kulkarni

3/7
wo de er zi, wo hen ai ni! - Chen baba

I pledge my honor that I have abided by the Stevens Honor System

4/5 Problem Set 8

1) Regular Year: 365 days
leap year (every 3 reg. years) 366 days
12 13's in a year, 52 weeks = 52 fridays in a year.
let 0 = monday, etc...
31 / 30 / 28 days in a reg. year; 31 % 7 to get the day the next month starts.
30 % 7
Prove: every month starts 28 % 7
on a k to k+6

cover base cases; starting each year on sun/mon/etc...

FINISH NEXT PAGE →

Chinese Remainder Theorem
3, 5, 11 are pairwise relatively prime

2) $x = 2020 \pmod 3 \equiv 1 \pmod 3$
$x = 2018 \pmod 5 \equiv 3 \pmod 5$
$x = 2021 \pmod{11} \equiv 8 \pmod{11}$
next arrival of all 3 within $\overset{3 \cdot 5 \cdot 11}{165}$ years

① $m = 165$
② $M_1 = \frac{m}{m_1} = 55$, $M_2 = 33$, $M_3 = 15$
③ $y_1 \equiv M_1^{-1} \equiv 55 \equiv 1 \pmod 3$ → pulverizer & inverse modulo
$y_2 \equiv M_2^{-1} \equiv 33 \equiv 2 \pmod 5$
$y_3 \equiv M_3^{-1} \equiv 15 \equiv 3 \pmod{11}$
④ $x = (1 \cdot 1 \cdot 55) + (2 \cdot 3 \cdot 33) + (3 \cdot 8 \cdot 15)$ ← $x = a_1 y_1 M_1 + a_2 y_2 M_2 + \ldots + a_n y_n M_n$
$= 55 + 297 + 480 = 613 \equiv 118 \pmod{165}$ ⇒ every 165 years after year 0, they arrive together on the 118th year

⇒ The next arrival occurs after 2021 and is 118 plus a multiple of 165; earliest multiple of 165 right before 2021 is 12 ⇒ $165 \cdot 12 = 1980$
$1980 + 118 = \boxed{2098}$

4/5 Problem Set 8

1). • Let 0 = monday, 1 = tuesday, ..., 6 = sunday. (the numbers don't really matter, just that we represent the seven days of the week as a number k to k+6).

• Given that the current month starts on weekday k, the next month will start on k plus the number of days in the current month mod 7.

Reg

Jan: 31

Feb: 28 leap 29

Mar: 31

Apr: 30

May: 31

Jun: 30

Jul: 31

Aug: 31

Sep: 30

Oct: 31

Nov: 30

Dec: 31

• Any given normal year will start on a weekday k. The year starts off on a January, so the following month, February, starts on weekday $k + (31 \% 7) = k+3$. The following months of the year follows this rule:

March starts on $(k+3) + (\overset{0}{28 \% 7}) = k+3$

April starts on $(k+3) + (\overset{3}{31 \% 7}) = k+6$

May, $(k+6) + (\overset{2}{30 \% 7}) = k+8 \equiv k + (8 \% 7) = k+1$

June, $(k+1) + (\overset{3}{31 \% 7}) = k+4$

July, $(k+4) + (\overset{2}{30 \% 7}) = k+6$

Aug, $(k+6) + (\overset{3}{31 \% 7}) = k+9 \equiv k + (9 \% 7) = k+2$

Sep, $(k+2) + (\overset{3}{31 \% 7}) = k+5$

Oct, $(k+5) + (\overset{2}{30 \% 7}) = k+7 \equiv k + (7 \% 7) = k$

Nov, $k + (\overset{3}{31 \% 7}) = k+3$

Dec, $(k+3) + (\overset{2}{30 \% 7}) = k+5$

$\Rightarrow$ It is shown, with there being at least one month in a reg. year that starts on each weekday $k, k+1, \ldots, k+6$, that at least one month of the year starts on a Sunday, and a month that starts on a Sunday (6) the 1st, will have a Friday (4) the 13th since 12 days away from sunday is a friday ($(6+12) \% 7 = 4$).

• With the same logic, we can prove that there exists a Friday the 13th in any given leap year starting on weekday k:

Jan, k

Feb, $k + (\overset{3}{31 \% 7}) = k+3$

Mar, $(k+3) + (\overset{1}{29 \% 7}) = k+4$

Apr, $(k+4) + (\overset{3}{31 \% 7}) = k+7 \equiv k$

May, $k + (\overset{2}{30 \% 7}) = k+2$

Jun, $(k+2) + (\overset{3}{31 \% 7}) = k+5$

Jul, $(k+5) + (\overset{2}{30 \% 7}) = k+7 = k$

Aug, $k + (\overset{3}{31 \% 7}) = k+3$

Sep, $(k+3) + (\overset{3}{31 \% 7}) = k+6$

Oct, $(k+6) + (\overset{2}{30 \% 7}) = k+8 \equiv k+1$

Nov, $(k+1) + (\overset{3}{31 \% 7}) = k+4$

Dec, $(k+4) + (\overset{2}{30 \% 7}) = k+6$

~~$33a \equiv 1 \pmod 5$ $\quad a=2$~~

~~$339 \% 5 = 1$~~

~~$55(x) \% 3 = 1$~~

~~118 165 1184 165 ... 165 2021 1884 2165~~

4/5 Problem Set 8

E(1) $\gcd(m_1, m_2) = d$; $am_1 + bm_2 = d$

$x \equiv r_1 \pmod{m_1}$

$x \equiv r_2 \pmod{m_2}$

$\Leftrightarrow x = am_1 + r_1$ $\Big\}$ This can be done assuming

$x = bm_2 + r_2$ $\Big\}$ that $r_1 \equiv r_2 \pmod d$

$am_1 + r_1 = bm_2 + r_2$

$0 = am_1 + r_1 - bm_2 - r_2 = \underbrace{am_1 - bm_2}_{-d} + r_1 - r_2$

$d = r_1 - r_2$

$\Rightarrow r_1 = r_2 + d \equiv r_2 \pmod d$.

E(2) for $j = a_i \ldots a_{i+98}$, $j \% n^3 = 0$, $n > 1$

~~too big a range, no induction, PRIME #S → pairwise coprime w/ a cube~~

We can choose 99 prime numbers $p_0, p_2, \ldots, p_{98}$

- Given this, we can use the CRT

$x \equiv -i \pmod{p_i^3}$

for $i = 0, 1, \ldots, 98$

$\Leftrightarrow$ $x \equiv 0 \pmod{p_0^3}$

$x \equiv -1 \pmod{p_1^3}$

$x \equiv -2 \pmod{p_2^3}$

$\ldots$

$x \equiv -98 \pmod{p_{98}^3}$

- Since $p_0 \ldots p_{99}$ are prime, the mods are pairwise co-prime, and there exists a solution x.

$\Rightarrow$ Given this, there exists 99 consecutive integers x_i such that $x_i = x + i$, and each x_i is divisible by p_i^3, where p_i is a prime greater than 1.