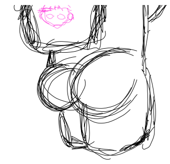

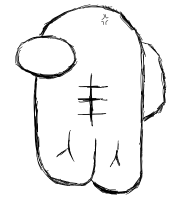

① a. $(c|a \wedge c|b) \Rightarrow c|\gcd(a,b))$

- Let s & t be integers such that the $\gcd(a,b) = sa + tb$
- if $c|a \wedge c|b$, then $c|(sa+bt)$ for any integers x&y

$\therefore$ $c|\gcd(a,b)$ (substitution)

b. $\forall k>0: \gcd(ka, kb) = k \cdot \gcd(a,b)$

- Let s & t be integers such that the $\gcd(a,b) = sa + tb$
- $\gcd(ka, kb) = ska + tkb$
- factor: $k(sa+tb) = k \cdot (sa+tb)$

$\therefore$ $\forall k>0: \gcd(ka, kb) = k \cdot \gcd(a,b)$ by substitution

c. $(\gcd(a,b) = 1 \wedge \gcd(a,c) = 1) \Rightarrow \gcd(a,bc) = 1$

$\gcd(a,b) = 1$ $sa + tb = 1$
$\gcd(a,c) = 1$ $va + uc = 1$

$sa + tb = va + uc = 1$

$(sa+tb)(va+uc) = 1$

$sva^2 + tvab + suac + tubc = 1$

$a(sva + tvb + suc) + (tu)bc = 1$

$a(x) + (y)bc = 1$

$\gcd(a,bc) = 1$

d. $(a|bc \wedge \gcd(a,b) = 1) \Rightarrow a|c$

Since $\gcd(a,b) = 1$, $sa + tb = 1$ then the extended euclidean algorithim states that a & b are relatively prime, therefore $a \nmid b$. If $a|bc$ is true & $a \nmid b$ is true then $a|c$ must be true.

e. $\gcd(a,b) = \gcd(b, \text{rem}(a,b))$

$sb + t(\text{rem}(a,b))$

$sb + t(a - bx)$

$sb + ta - tbx$

$b(s - tx) + a(t)$

$\gcd(a,b) \mid \gcd(\text{rem}(a,b))$

$a = bx + \text{rem}(a,b)$ $\quad \text{rem}(a,b) = a - bx$

$\gcd(a,b) = sa + tb \Rightarrow R$

$s(bx - \text{rem}(a,b)) + tb$

$sbx - sR + tb$

$b(sx + t) - R(s)$

$\text{rem}(a,b) \mid \gcd(a,b)$

2 a. $t : s > 0$ and $m = sa + tb$

$m = (s+b)a + (t-a)b$

$m = (sa + ba) + (tb - ba)$

$m = sa + tb$

b. Round 1: $(0,0) \rightarrow (7,0) \rightarrow (0,7)$

Round 2: $(0,7) \rightarrow (7,7) \rightarrow (2,12) \rightarrow (2,0) \rightarrow (0,2)$

Round 3: $(0,2) \rightarrow (7,2) \rightarrow (0,9)$

Round 4: $(0,9) \rightarrow (7,9) \rightarrow (4,12) \rightarrow (4,0) \rightarrow (0,4)$

Round 5: $(0,4) \rightarrow (7,4) \rightarrow (0,11)$

Round 6: $(0,11) \rightarrow (7,11) \rightarrow (6,12) \rightarrow (6,0) \rightarrow (0,6)$

Round 7: $(0,6) \rightarrow (7,6) \rightarrow (1,12) \rightarrow (1,0) \rightarrow (0,1)$

Round 8: $(0,1) \rightarrow (7,1) \rightarrow (0,8)$

Round 9: $(0,8) \rightarrow (7,8) \rightarrow (3,12) \rightarrow (3,0) \rightarrow (0,3)$

c. sa is the number of times the small jug is filled x capacity of the jug

$sa = b$(how many times the larger jug is emptied) + (remaining water)

$sa = b(x) + w$

$sa + tb = m$ (given)

$sa = n - tb$

$b(x) + w = m - tb$ substitution

$w = m$ since they are constants

since $m = sa + tb$, which is $\gcd(a,b)$

$w = \gcd(a,b)$

Therefore water remaining is = to $\gcd(a,b)$

I pledge my honor that I have abided by the Stevens honor system.

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