$ax + by = \gcd(a, b)$

x, y from pulverizer

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I pledge my honor that I have abided by the Stevens Honor System

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wo de er zi, wo hen ai ni! - Chen baba

4/5 Problem Set 3

1) Regular Year: 365 days

cover base cases; starting each year on sun/mon/ etc...

leap Year (every 3 reg. years) 366 days

12 '13's in a year, 52 weeks = 52 fridays in a year.

let 0=monday, etc..

31/30/28 days in a reg year; 31 % 7 to get the day the next month starts.

30 % 7

Prove: every month starts on a k to k+6

28 % 7

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Chinese Remainder Theorem

3, 5, 11 are pairwise relatively prime

2) $x = 2020 \pmod 3 \equiv 1 \pmod 3$

$x = 2018 \pmod 5 \equiv 3 \pmod 5$

$x = 2021 \pmod{11} \equiv 8 \pmod{11}$

next arrival of all 3 within $\overset{3 \cdot 5 \cdot 11}{165}$ years

① $m = 165$

② $M_1 = \frac{m}{m_1} = 55$, $M_2 = 33$, $M_3 = 15$

③ $y_1 \equiv M_1^{-1} \equiv 55 \equiv 1 \pmod 3$ → pulverizer & inverse modulo

$y_2 \equiv M_2^{-1} \equiv 33 \equiv 2 \pmod 5$

$y_3 \equiv M_3^{-1} \equiv 15 \equiv 3 \pmod{11}$

④ $x = (1 \cdot 1 \cdot 55) + (2 \cdot 3 \cdot 33) + (3 \cdot 8 \cdot 15) \Leftarrow x = a_1 y_1 M_1 + a_2 y_2 M_2 + \dots + a_n y_n M_n$

$= 55 + 297 + 480 = 613 \equiv 118 \pmod{165}$ ⇒ every 165 years after year 0, they arrive together on the 118th year

⇒ The next arrival occurs after 2021 and is 118 plus a multiple of 165; earliest multiple of 165 right before 2021 is 12 ⇒ $165 \cdot 12 = 1980$

$1980 + 118 = \boxed{2098}$

4/5 Problem Set 8

Reg	leap
Jan: 31	
Feb: 28	29
Mar: 31	
Apr: 30	
May: 31	
Jun: 30	
Jul: 31	
Aug: 31	
Sep: 30	
Oct: 31	
Nov: 30	
Dec: 31	

1) • Let 0 = monday, 1 = tuesday, ..., 6 = sunday. (the numbers don't really matter, just that we represent the seven days of the week as a number k to k+6).

• Given that the current month starts on weekday k, the next month will start on k plus the number of days in the current month mod 7.

• Any given normal year will start on a weekday k. The year starts off on a January, so the following month, February, starts on weekday $k + (31 \% 7) = k+3$. The following months of the year follows this rule:

- March starts on $(k+3) + (\overset{0}{28 \% 7}) = \underline{k+3}$
- April starts on $(k+3) + (\overset{3}{31 \% 7}) = \underline{k+6}$
- May, $(k+6) + (\overset{2}{30 \% 7}) = k+8 \equiv k + (8 \% 7) = \underline{k+1}$
- June, $(k+1) + (\overset{3}{31 \% 7}) = \underline{k+4}$
- July, $(k+4) + (\overset{2}{30 \% 7}) = k+6$
- Aug, $(k+6) + (\overset{3}{31 \% 7}) = k+9 \equiv k + (9 \% 7) = \underline{k+2}$
- Sep, $(k+2) + (\overset{3}{31 \% 7}) = \underline{k+5}$
- Oct, $(k+5) + (\overset{2}{30 \% 7}) = k+7 \equiv k + (7 \% 7) = \underline{k}$
- Nov, $k + (\overset{3}{31 \% 7}) = k+3$
- Dec, $(k+3) + (\overset{2}{30 \% 7}) = k+5$

⇒ It is shown, with there being at least one month in a reg. year that starts on each weekday $k, k+1, \ldots, k+6$, that at least one month of the year starts on a Sunday, and a month that starts on a Sunday (6) the 1^{st}, will have a Friday (4) the 13^{th} since 12 days away from sunday is a friday $((6+12) \% 7 = 4)$.

• With the same logic, we can prove that there exists a Friday the 13^{th} in any given leap year starting on weekday k:

- Jan, $\underline{k}$
- Feb, $k + (\overset{3}{31 \% 7}) = \underline{k+3}$
- Mar, $(k+3) + (\overset{1}{29 \% 7}) = \underline{k+4}$
- Apr, $(k+4) + (\overset{3}{31 \% 7}) = k+7 \equiv k$
- May, $k + (\overset{2}{30 \% 7}) = \underline{k+2}$
- Jun, $(k+2) + (\overset{3}{31 \% 7}) = \underline{k+5}$
- Jul, $(k+5) + (\overset{2}{30 \% 7}) = k+7 = k$
- Aug, $k + (\overset{3}{31 \% 7}) = k+3$
- Sep, $(k+3) + (\overset{3}{31 \% 7}) = \underline{k+6}$
- Oct, $(k+6) + (\overset{2}{30 \% 7}) = k+8 \equiv \underline{k+1}$
- Nov, $(k+1) + (\overset{3}{31 \% 7}) = k+4$
- Dec, $(k+4) + (\overset{2}{30 \% 7}) = k+6$