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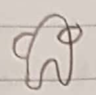

Countable: all points can be visited without repetition

$\mathbb{Q}$, $|\mathbb{Z}|$, $\mathbb{N}$, $\mathbb{N} \times \mathbb{N}$

I pledge my honor that I have abided by the Stevens Honor System

2/23 Problem Set 5

1) a) $(a,b) \in R$; $(x,y) \in R \wedge (x,z) \in R \rightarrow (y,z) \in R$

$(a,b) \in R \wedge (a,a) \in R \rightarrow (b,a) \in R$

$aRb \wedge aRa \rightarrow bRa$ reflexive; $aRa = T$

$\Rightarrow aRb \rightarrow bRa$

$\therefore R$ is symmetric

b) $(a,b) \in R \rightarrow (b,a) \in R$ symmetry

$(b,a) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

$\Rightarrow aRb \wedge bRc \rightarrow aRc$

$\therefore R$ is transitive

$\mathbb{N}: \{0, 1, 2, \dots\}$

2) injective: every x maps to a y exactly once $f: S \rightarrow \mathbb{N}$

well-ordered: the set itself & every nonempty subset has a "smallest" element.

- Let S be the well-ordered set consisting of integers from zero to infinity. Note that $\mathbb{N}$ is also well-ordered. Then, the range of f is $[0, \infty)$, the set of all natural numbers, equal to the range of $\mathbb{N}$. If $f: S \rightarrow \mathbb{N}$ is injective, then every element of S maps to another element (unique) of $\mathbb{N}$ in a one-to-one ratio, implying that $f: S \rightarrow \mathbb{N}$ is bijective. Given that $f: S \rightarrow \mathbb{N}$ is bijective, then $g: \mathbb{N} \rightarrow S$ is also bijective.

$X = \{a, b, c\}$

$g(0) = \min(X) = a$

$g(1) = \min(X - g\{0\})$

$\{b, c\}$

$g(2) = \min(X - \{g(0\} - \{g\{1\}\})$

There is an infinite minimum, as one domain is assigned to a range there is a new minimum is created & repeated infinitely.

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3) monotonically non-decreasing ; always constant or increasing.

a) $\forall x \forall y \; x \leq y \Rightarrow f(x) \leq f(y)$

$\forall a \forall b \; a \leq b \Rightarrow g(a) \leq g(b)$

let $a = f(x)$, $b = f(y)$; $\forall x \forall y \; x \leq y \Rightarrow f(x) \leq f(y) \Rightarrow a \leq b \Rightarrow g(a) \leq g(b)$

$\therefore g \circ f$ is also monotonically non-decreasing.

b) $g \circ f$ can be monotonically non-decreasing if g remains constant no matter the value of x ; therefore, f can be non monotonically non-decreasing while $g \circ f$ remains constant, which is still monotonically non-decreasing.

4) $f: A \to B$

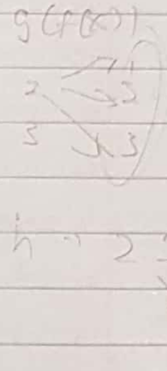
$g: B \to C$

$h: A \to C = g(f(x))$ for $x \in A$

surjective: all elements in the target are mapped to ; range = target.

a) since $h(x)$ is the function g and inherits the range of g, then if $h(x)$ is surjective, then $g(x)$ must also be surjective. The converse is true as well.

$\Rightarrow$ Since $A \to C$ is surjective, and h is a transitive function $A \to B$ and $B \to C$, then there must be a step such that $B \to C$ is surjective.

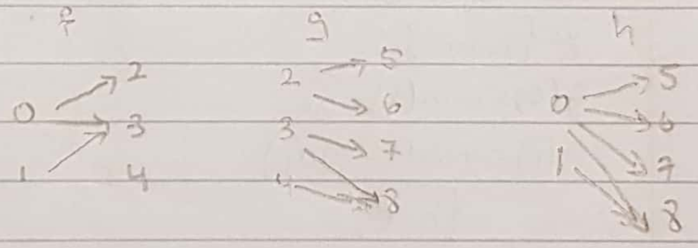


b) No.

$A = \{0, 1\}$

$B = \{2, 3, 4\}$

$C = \{5, 6, 7, 8\}$



f does not have to be surjective for $h(x)$ to be surjective. Thus, $h(x)$ being surjective does not imply $f(x)$ being surjective.

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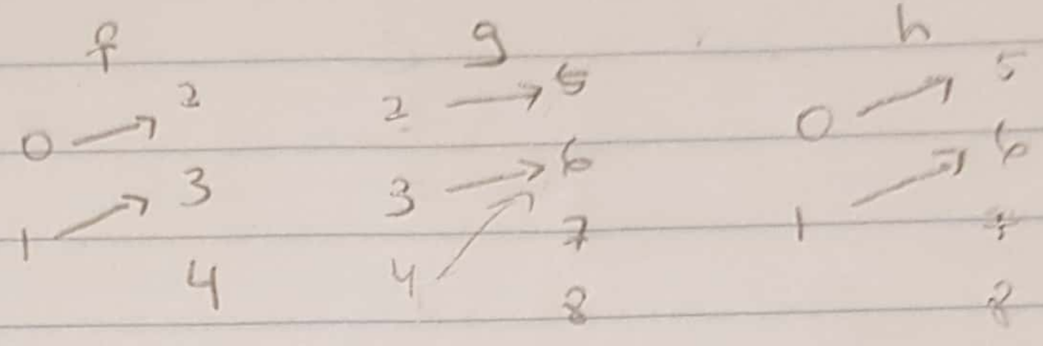
4)

c) No.

$A = \{0, 1\}$

$B = \{2, 3, 4\}$

$C = \{5, 6, 7, 8\}$



If $h(x)$ is injective, $g(x)$ does not have to be injective.

d) Every element in A must map to some value C if $h(x)$ is injective. Therefore, every element in A must map to some value in B so that these values in B can map to the C values transitively that those in A are mapped to in $h(x)$. Thus, if $h(x)$ is injective, then $f(x)$ must be injective.