Issac Zheng

I pledge my honor that I have abided by the Stevens Honor System

2/10 Problem Set 3

1) a) no

b) no

c) yes; Ella trades with someone every round ≠ Ella trades with everyone during some rounds.

d) yes; there exists a person Ella trades w/ every round ≠ every round, Ella trades w/ someone.

e) yes; Ella trades w/ someone every round ≠ Ella trades with everyone during some rounds.

f) yes; $\neg(\exists x \forall y: Trade(Ella, x, y)) = \forall x \exists y: \neg Trade(Ella, x, y) \neq \forall x \exists y: Trade(Ella, x, y)$

2) a) $A \Rightarrow B$

$\exists x (P(x) \to Q(x)) \to (\forall x P(x) \to \exists y Q(y))$	Conditional Identities
$\exists x (\neg P(x) \vee Q(x)) \to (\neg \forall x P(x) \vee \exists y Q(y))$	↓
$\neg \exists x (\neg P(x) \vee Q(x)) \vee (\neg \forall x P(x) \vee \exists y Q(y))$	De Morgan's Law
$\neg \exists x (\neg P(x) \vee Q(x)) \vee (\exists x \neg P(x) \vee \exists y Q(y))$	(c is a particular element) Existential Instantiation
$\neg \exists x (\neg P(x) \vee Q(x)) \vee (\neg P(c) \vee Q(c))$	Existential Generalization
$\neg \exists x (\neg P(x) \vee Q(x)) \vee \exists x (\neg P(x) \vee Q(x))$	Complement Law
T	

b) $B \Rightarrow A$

$(\forall x P(x) \to \exists y Q(y)) \to \exists x (P(x) \to Q(x))$	Conditional Identities x2
$(\neg \forall x P(x) \vee \exists y Q(y)) \to \exists x (\neg P(x) \vee Q(x))$	De Morgan's Law
$(\exists x \neg P(x) \vee \exists y Q(y)) \to \exists x (\neg P(x) \vee Q(x))$	Existential Instantiation (c is a particular element)
$(\neg P(c) \vee Q(c)) \to \exists x (\neg P(x) \vee Q(x))$	Existential Generalization
$\exists x (\neg P(x) \vee Q(x)) \to \exists x (\neg P(x) \vee Q(x))$	Conditional Identities
$\neg \exists x (\neg P(x) \vee Q(x)) \vee \exists x (\neg P(x) \vee Q(x))$	Complement Law
T	

3) 1).

$E \cup F = E \cup (\bar{E} \cap F)$	Distributive Law
$= (E \cup \bar{E}) \cap (E \cup F)$	Complement Law
$= U \cap (E \cup F)$	Identity Law
$= E \cup F$	

2)

$(A - B) - C \subseteq (A - C)$	Set Subtraction Law
↳ $(A \cap \bar{B}) \cap \bar{C}$	Commutative Law
$= (\bar{B} \cap A) \cap \bar{C}$	Associative Law
$= \bar{B} \cap (A \cap \bar{C}) \subseteq (A \cap \bar{C}) = (A - C)$	$A \cap B \subseteq A$

2/11 Problem Set 3 cont

4) a) The domain for x is all people capable of loving, and the domain for d is all days of all time.

b) $\forall d \, \exists x : Loves(x, Juliet, d)$

c) $\forall d : \neg Loves(Iago, Iago, d)$

d) $\exists x \exists y \forall d : Loves(x, y, d) \wedge \neg EQ(x, y)$

e) $\forall x \, \exists d_1 \exists y \, \forall z \, \forall d_2 : (Loves(x, y, d_1) \wedge Loves(y, z, d_2))$
$\rightarrow (Loves(x, y, d_2) \wedge Future(d_1, d_2))$

f) $\forall x \forall y \, \exists d \, \forall z : (\neg Loves(z, x, d) \wedge \neg EQ(x, z))$
$\rightarrow \neg Loves(x, y, d)$

g) $\exists x \exists y \exists d_1 \exists d_2 \exists z \, \forall d_3 : (Loves(x, y, d_1) \wedge Future(d_1, d_2) \wedge$
$\wedge Loves(x, z, d_2) \wedge \neg EQ(y, z))$
$\rightarrow (\neg Loves(y, x, d_3) \wedge Future(d_2, d_3))$