CS 135 Spring 2022: Mid-Term Practice Problems.

Problem 1. Either prove that the following argument is valid or give a counterexample:
If Bob carries an umbrella, then either it is raining, or it is cloudy
If it is cloudy then Bob does not carry an umbrella
Therefore, if it is raining Bob carries an umbrella

Problem 2. Either prove that the following argument is valid or give a counterexample:

$$\begin{array}{l} A \Rightarrow B \vee \neg D \\ \neg C \Rightarrow \neg B \wedge D \\ \hline D \Rightarrow A \vee C \\ \hline \neg A \Rightarrow (D \Rightarrow B) \end{array}$$

Problem 3. Prove (using logic or set identities) that for all sets A, B, C:

$$(B - A) \cup (C - A) \subseteq (B \cup C) - A$$

Problem 4. Determine all amounts of postage that can be formed using only 4-cent and 11-cent stamps. Prove your answer using strong induction. Be sure to show the base case, the inductive hypothesis, and the inductive step.

Problem 5. Consider the following Scheme function:

```
(define (mystery L)
  (if (null? L) 0
    (if (null? (cdr L)) 1
          (+ 1 (mystery (cddr L))))))
```

(a) What is (mystery (list 1 2 3 4))? What is (mystery (list 1 2 3 4 5))?
(b) What is the function computed by mystery? Justify your answer with a proof.
(c) Now change the last line of mystery so the new definition becomes:

```
(define (mystery L)
  (if (null? L) 0
    (if (null? (cdr L)) 1
          (+ (mystery (cdr L)) (mystery (cddr L))))))
```

What function does mystery now compute? Justify your answer with a proof.

Problem 6. Prove by induction the statement $\forall n \geq 0$: $1 + 2 + 2^2 + \cdots 2^n = 2^{n+1} - 1$

a. State and establish the base case.
b. State the inductive hypothesis.
c. Establish the inductive step.

Problem 7. Prove the following equality, for all $n \geq 1$:

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$$

Problem 8. Consider the following Scheme functions:

```
(define (func A)  (myfun A '() ))

(define (myfun A B)
        (if (null? A) B
                (myfun (cdr A) (cons (car A ) B))))
```

a. What is the value returned by (func '(1 2 3))?
b. What is the value returned by func on a list of length N?
c. Prove, by induction on the length of list A, the correctness of your assertion in part b.