Bottom-up Chart Parsing: the CKY algorithm Parsing ISCL-BA-06

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University of Tübingen Seminar für Sprachwissenschaft

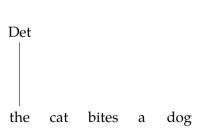
Winter Semester 2020/21

Parsing so far

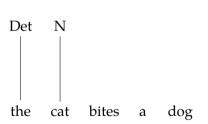
- Parsing is the task of automatic syntactic analysis
- For most practical purposes, context-free grammars are the most useful formalism for parsing
- We can formulate parsing as
 - Top-down: begin with the start symbol, try to *produce* the input string to be parsed
 - Bottom up: begin with the input, and try to reduce it to the start symbol
- Both strategies can be cast as search with backtracking
- Backtracking parsers are inefficient: they recompute sub-trees multiple times

the cat bites a dog

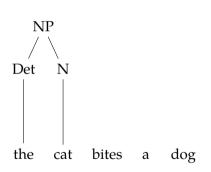
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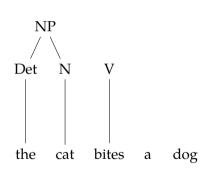
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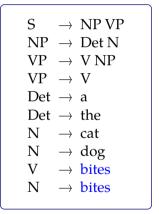


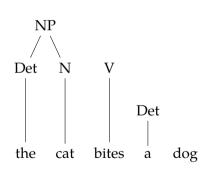
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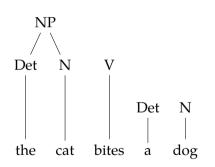
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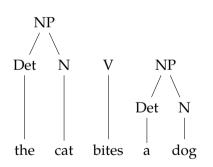




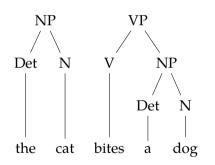
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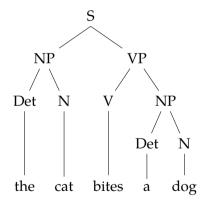
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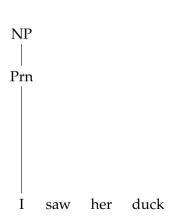
 $NP \rightarrow Prn$ $VP \rightarrow V NP$ $VP \ \to V$ $VP \rightarrow VS$ $N \rightarrow duck$ $V \rightarrow duck$ \rightarrow saw $Prn \rightarrow I$ $Prn \rightarrow she$ $Prn \rightarrow her$

 $\begin{array}{ccc} S & \rightarrow NP \ VP \\ NP & \rightarrow Prn \ N \end{array}$

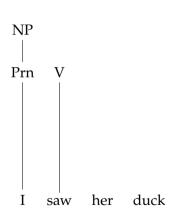
I saw her duck

Prn her duck saw

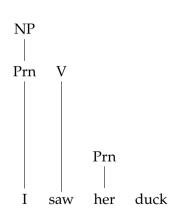
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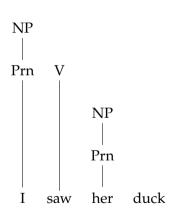
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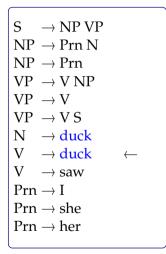


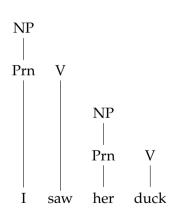
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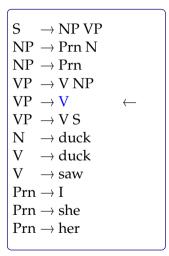


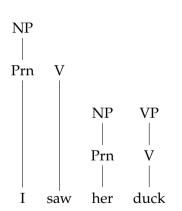
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Prn \rightarrow her
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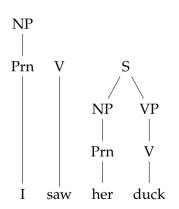


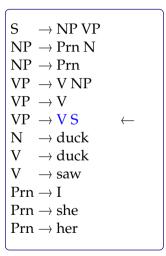


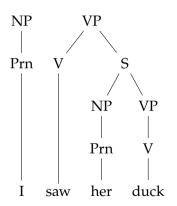


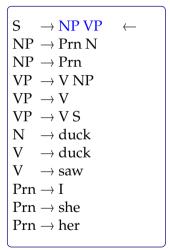


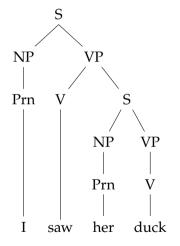
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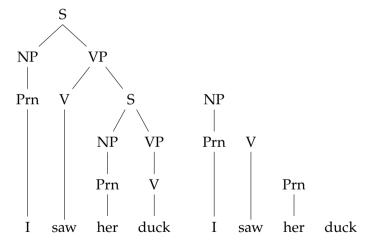


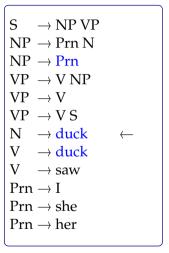


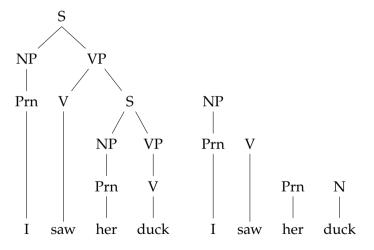




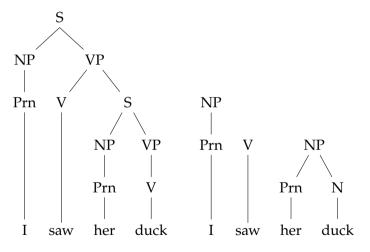
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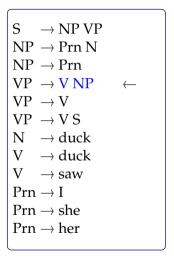


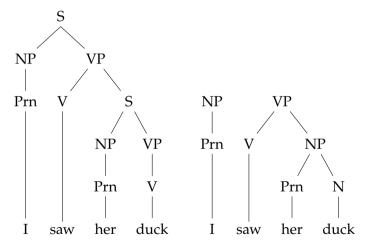


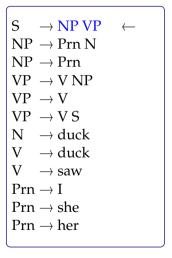


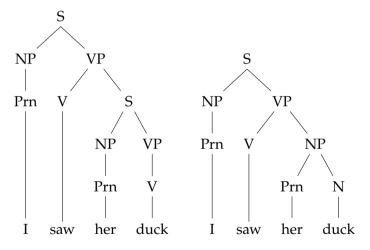
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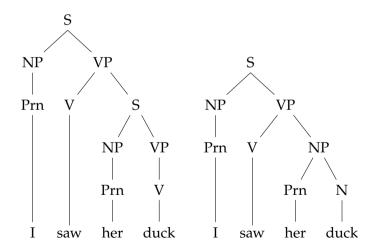




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How to represent multiple parses

parse forest grammar



 $S_{0.4} \rightarrow NP_{0.1} VP_{1.4}$ $NP_{0:1} \rightarrow Prn_{0:1}$ $Prn_{0:1} \rightarrow I_{0:1}$ $VP_{1\cdot4} \rightarrow V_{1\cdot2} S_{2\cdot4}$ $V_{1:2} \rightarrow saw_{1:2}$ $S_{2\cdot4} \longrightarrow Prn_{2\cdot3} V_{3\cdot4}$ $V_{3\cdot4} \rightarrow duck_{3\cdot4}$ $VP_{1:4} \rightarrow V_{1:2} NP_{2:4}$ $NP_{2:4} \rightarrow Prn_{2:3} N_{3:4}$

CKY algorithm

- The CKY (Cocke–Kasami–Younger) parsing algorithm is a dynamic programming algorithm (Kasami 1965; Younger 1967; Cocke and Schwartz 1970)
- It processes the input *bottom up*, and saves the intermediate results on a *chart*
- Time complexity for *recognition* is $O(n^3)$
- Space complexity is $O(n^2)$
- It requires the CFG to be in *Chomsky normal form* (CNF) (can somewhat be relaxed, but not common)

Chomsky normal form (CNF)

- A CFG is in CNF, if the rewrite rules are in one of the following forms
 - $-A \rightarrow BC$
 - $-A \rightarrow a$

where A, B, C are non-terminals and a is a terminal

- Any CFG can be converted to CNF
- Resulting grammar is *weakly equivalent* to the original grammar:
 - it generates/accepts the same language
 - but the derivations are different

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```
S \rightarrow NP VP
S \rightarrow Aux NP VP
NP \rightarrow the N
VP \rightarrow VNP
VP \rightarrow V
N \rightarrow cat
N \rightarrow dog
   \rightarrow bites
N \rightarrow bites
```

```
\begin{array}{cccc} S & \rightarrow & NP \ VP \\ S & \rightarrow & Aux \ NP \ VP \\ NP & \rightarrow & the \ N \\ VP & \rightarrow & V \ NP \\ VP & \rightarrow & V \\ N & \rightarrow & cat \\ N & \rightarrow & dog \\ V & \rightarrow & bites \\ N & \rightarrow & bites \\ \end{array}
```

```
\begin{array}{ccc} \bullet \ S \ \rightarrow Aux \ NP \ VP \\ S \ \rightarrow Aux \ NP \ VP \end{array} \quad \Rightarrow \quad \begin{array}{ccc} S \ \rightarrow Aux \ X \\ X \ \rightarrow NP \ VP \end{array}
```

```
\begin{array}{ccc} S & \rightarrow & NP \ VP \\ S & \rightarrow & Aux \ NP \ VP \\ NP & \rightarrow & the \ N \\ VP & \rightarrow & V \ NP \\ VP & \rightarrow & V \\ N & \rightarrow & cat \\ N & \rightarrow & dog \\ V & \rightarrow & bites \\ N & \rightarrow & bites \\ \end{array}
```

```
\begin{array}{ccc} \bullet & S & \rightarrow Aux \; NP \; VP \\ S & \rightarrow Aux \; NP \; VP & \Rightarrow & S & \rightarrow Aux \; X \\ & & X & \rightarrow NP \; VP \end{array}
```

```
\begin{array}{ccc} \bullet & NP \to the \ N \\ NP \to the \ N & \Rightarrow & NP \to X \ N \\ & & X \to the \end{array}
```

```
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\begin{array}{ccc} \bullet & S & \rightarrow Aux \; NP \; VP \\ S & \rightarrow Aux \; NP \; VP & \Rightarrow & S & \rightarrow Aux \; X \\ & & X & \rightarrow NP \; VP \end{array}
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- $\begin{array}{ccc} \bullet & NP \to the \ N \\ NP \to the \ N & \Rightarrow & NP \to X \ N \\ & & X \to the \end{array}$
- $\begin{array}{cccc} \bullet & VP & \to V \\ & VP & \to V & \Rightarrow & VP & \to bites \end{array}$

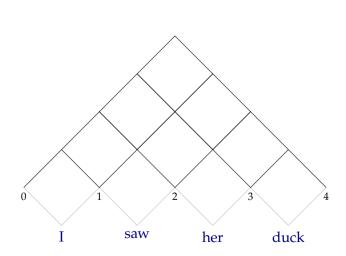
Converting to CNF

- 1. Eliminate the ϵ rules: if $A \to \epsilon$ is in the grammar
 - replace any rule B $\,\rightarrow \alpha$ A β with two rules

$$\begin{array}{c} B \ \rightarrow \alpha \ \beta \\ B \ \rightarrow \alpha \ A' \ \beta \end{array}$$

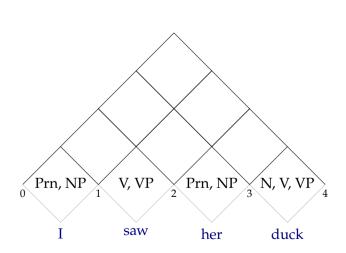
- add A' $\rightarrow \alpha$ for all α (except ε) whose LHS is A
- repeat the process for newly created ϵ rules
- remove the rules with ε on the RHS (except S $\rightarrow \varepsilon$)
- 2. Eliminate unit rules: for a rule $A \rightarrow B$
 - Replace the rule with A $\rightarrow \alpha_1 \mid ... \mid \alpha_n$, where $\alpha_1, ..., \alpha_n$ are all RHS or rule B
 - Remove the rule $A \rightarrow B$
 - Repeat the process until no unit rules remain
- 3. Binarize all the non-binary rules with non-terminal on the RHS: for a rule $A \rightarrow X_1 X_2 ... X_n$:
 - Replace the rule with A \rightarrow A₁ X₂...X_n, and add A₁ \rightarrow X₁ X₂
 - Repeat the process until all new rules are binary

an ambiguous example

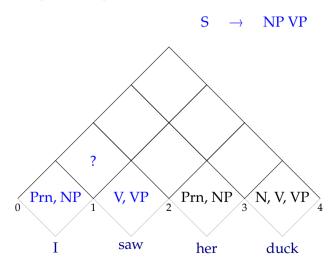


 $\begin{array}{lll} S & \rightarrow NP \ VP \\ NP & \rightarrow Prn \ N \\ VP & \rightarrow V \ NP \\ VP & \rightarrow V \ S \\ N & \rightarrow duck \\ VP & \rightarrow duck \ | \ saw \\ V & \rightarrow duck \ | \ saw \\ Prn & \rightarrow I \ | \ she \ | \ her \\ NP & \rightarrow I \ | \ she \ | \ her \end{array}$

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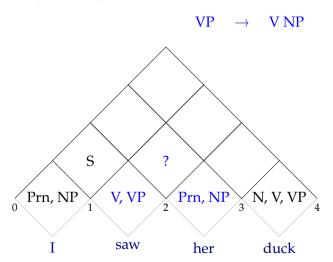


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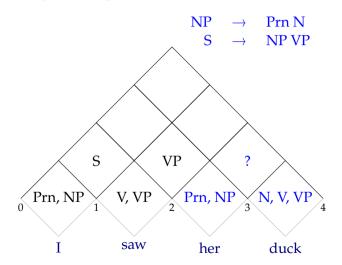


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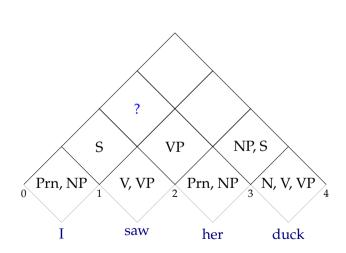
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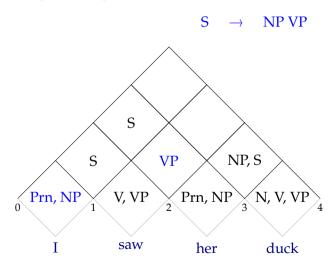


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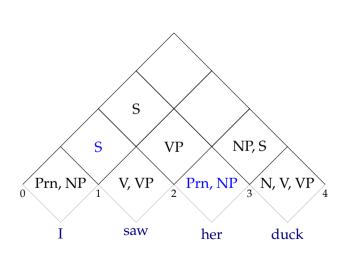


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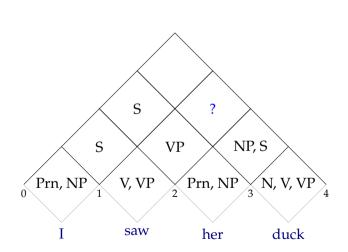


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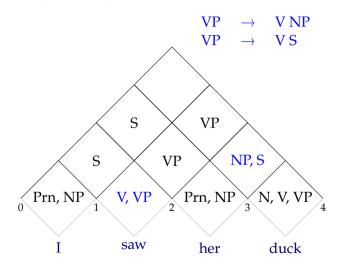
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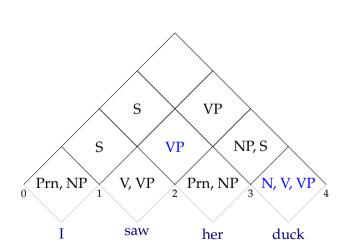


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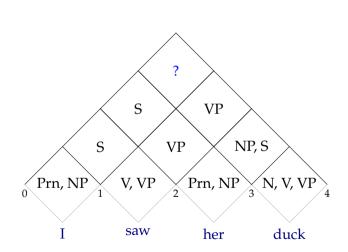


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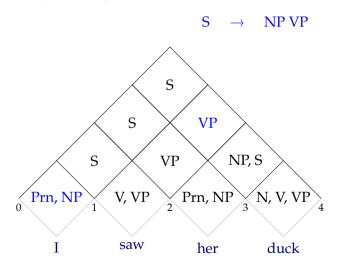
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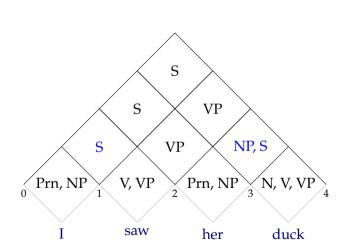


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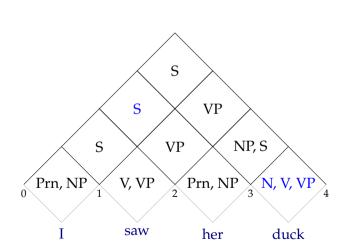


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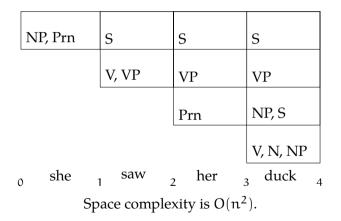
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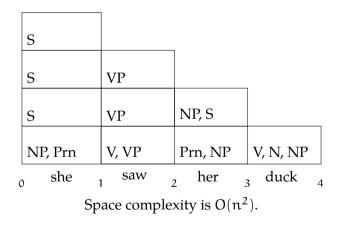
CKY demonstration: the chart

our chart is a 2D array



CKY demonstration: the chart

our chart is a 2D array - this is more convenient for programming



Parsing vs. recognition

- We went through a recognition example
- Note that the algorithm is not directional: it takes the complete input
- Recognition accepts or rejects a sentence based on a grammar
- For parsing, we want to know the derivations that yielded a correct parse
- To recover parse trees, we
 - we follow the same procedure as recognition
 - add back links to keep track of the derivations

Chart parsing example (CKY parsing)

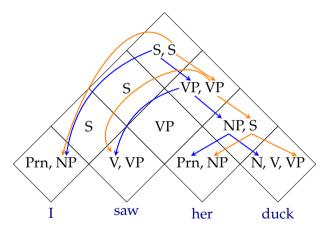
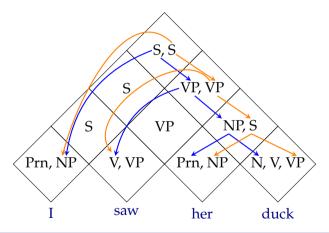


Chart parsing example (CKY parsing)



The chart stores a parse forest efficiently.

Summary

- + CKY avoids re-computing the analyses by storing the earlier analyses (of sub-spans) in a table
- It still computes lower level constituents that are not allowed by the grammar
- CKY requires the grammar to be in CNF
- CKY has O(n³) recognition complexity
- For parsing we need to keep track of backlinks
- CKY can efficiently store all possible parses in a chart
- Enumerating all possible parses have exponential complexity (worst case)

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Next:

- Top-down chart parsing: Earley algorithm
- Suggested reading: Grune and Jacobs (2007, section 7.2)

Acknowledgments, references, additional reading material



Cocke, John and J. T. Schwartz (1970). *Programming languages and their compilers: preliminary notes*. Tech. rep. Courant Institute of Mathematical Sciences. NYU.



Grune, Dick and Ceriel J.H. Jacobs (2007). Parsing Techniques: A Practical Guide. second. Monographs in Computer Science. The first edition is available at http://dickgrune.com/Books/PTAPG_1st_Edition/BookBody.pdf. Springer New York. ISBN: 9780387689548.



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