# Bottom-up Chart Parsing: the CKY algorithm

Parsing
ISCL-BA-06

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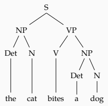
## Parsing so far

- Parsing is the task of automatic syntactic analysis
- For most practical purposes, context-free grammars are the most useful formalism for parsing
- We can formulate parsing as
  - Top-down: begin with the start symbol, try to *produce* the input string to be parsed
  - Bottom up: begin with the input, and try to *reduce* it to the start symbol
- Both strategies can be cast as search with backtracking
- Backtracking parsers are inefficient: they recompute sub-trees multiple times





## Bottom-up parsing as search




S → NP VP
NP → Det N
VP → V NP
VP → V
Det → a
Det → the
N → cat
N → dog
V → bites
N → bites





## Dealing with ambiguity

I saw her duck

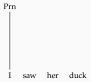
1

S → NP VP
NP → Prn N
NP → Prn
VP → V NP
VP → V
VP → V S
N → duck
V → duck
V → saw
Prn → I ←
Prn → she
Prn → her





## Dealing with ambiguity



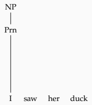

2

S → NP VP
NP → Prn N
NP → Prn ←
VP → V NP
VP → V
VP → V S
N → duck
V → duck
V → saw
Prn → I
Prn → she
Prn → her





## Dealing with ambiguity



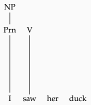

3

S → NP VP
NP → Prn N
NP → Prn
VP → V NP
VP → V
VP → V S
N → duck
V → duck
V → saw ←
Prn → I
Prn → she
Prn → her





## Dealing with ambiguity



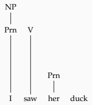

4

S → NP VP
NP → Prn N
NP → Prn
VP → V NP
VP → V
VP → V S
N → duck
V → duck
V → saw
Prn → I
Prn → she
Prn → her ←





## Dealing with ambiguity



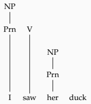

5

S → NP VP
NP → Prn N
NP → Prn ←
VP → V NP
VP → V
VP → V S
N → duck
V → duck
V → saw
Prn → I
Prn → she
Prn → her





## Dealing with ambiguity



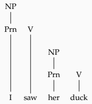

6

S → NP VP
NP → Prn N
NP → Prn
VP → V NP
VP → V
VP → V S
N → duck
V → duck ←
V → saw
Prn → I
Prn → she
Prn → her





## Dealing with ambiguity



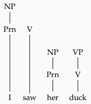

7

S → NP VP
NP → Prn N
NP → Prn
VP → V NP
VP → V ←
VP → V S
N → duck
V → duck
V → saw
Prn → I
Prn → she
Prn → her





## Dealing with ambiguity



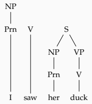

8

S → NP VP ←
NP → Prn N
NP → Prn
VP → V NP
VP → V
VP → V S
N → duck
V → duck
V → saw
Prn → I
Prn → she
Prn → her





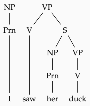
## Dealing with ambiguity




9

S → NP VP
NP → Prn N
NP → Prn
VP → V NP
VP → V
VP → V S ←
N → duck
V → duck
V → saw
Prn → I
Prn → she
Prn → her

## Dealing with ambiguity



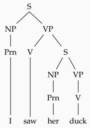

- S → NP VP ←
- NP → Prn N
- NP → Prn
- VP → V NP
- VP → V
- VP → V S
- N → duck
- V → duck
- V → saw
- Prn → I
- Prn → she
- Prn → her

10





## Dealing with ambiguity



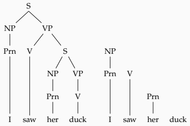

- S → NP VP
- NP → Prn N
- NP → Prn
- VP → V NP
- VP → V
- VP → V S
- N → duck
- V → duck
- V → saw
- Prn → I
- Prn → she
- Prn → her

11





## Dealing with ambiguity



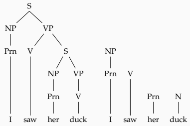

- S → NP VP
- NP → Prn N
- NP → Prn
- VP → V NP
- VP → V
- VP → V S
- N → duck ←
- V → duck
- V → saw
- Prn → I
- Prn → she
- Prn → her

12





## Dealing with ambiguity



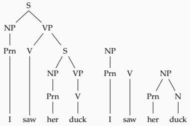

- S → NP VP
- NP → Prn N ←
- NP → Prn
- VP → V NP
- VP → V
- VP → V S
- N → duck
- V → duck
- V → saw
- Prn → I
- Prn → she
- Prn → her

13





## Dealing with ambiguity



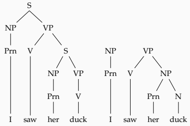

- S → NP VP
- NP → Prn N
- NP → Prn
- VP → V NP ←
- VP → V
- VP → V S
- N → duck
- V → duck
- V → saw
- Prn → I
- Prn → she
- Prn → her

14





## Dealing with ambiguity



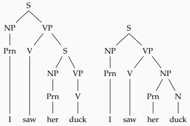

- S → NP VP ←
- NP → Prn N
- NP → Prn
- VP → V NP
- VP → V
- VP → V S
- N → duck
- V → duck
- V → saw
- Prn → I
- Prn → she
- Prn → her

15





## Dealing with ambiguity




- S → NP VP
- NP → Prn N
- NP → Prn
- VP → V NP
- VP → V
- VP → V S
- N → duck
- V → duck
- V → saw
- Prn → I
- Prn → she
- Prn → her

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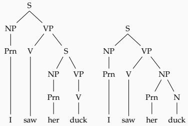
## How to represent multiple parses

parse forest grammar




- $S_{0:4} \rightarrow NP_{0:1}\ VP_{1:4}$
- $NP_{0:1} \rightarrow Prn_{0:1}$
- $Prn_{0:1} \rightarrow I_{0:1}$
- $VP_{1:4} \rightarrow V_{1:2}\ S_{2:4}$
- $V_{1:2} \rightarrow saw_{1:2}$
- $S_{2:4} \rightarrow Prn_{2:3}\ V_{3:4}$
- $V_{3:4} \rightarrow duck_{3:4}$
- $VP_{1:4} \rightarrow V_{1:2}\ NP_{2:4}$
- $NP_{2:4} \rightarrow Prn_{2:3}\ N_{3:4}$





## CKY algorithm

- The CKY (Cocke–Kasami–Younger) parsing algorithm is a dynamic programming algorithm (Kasami 1965; Younger 1967; Cocke and Schwartz 1970)
- It processes the input *bottom up*, and saves the intermediate results on a *chart*
- Time complexity for *recognition* is $O(n^3)$
- Space complexity is $O(n^2)$
- It requires the CFG to be in *Chomsky normal form* (CNF) (can somewhat be relaxed, but not common)





## Chomsky normal form (CNF)

- A CFG is in CNF, if the rewrite rules are in one of the following forms
  - A → B C
  - A → a

  where A, B, C are non-terminals and a is a terminal
- Any CFG can be converted to CNF
- Resulting grammar is *weakly equivalent* to the original grammar:
  - it generates/accepts the same language
  - but the derivations are different





## Converting to CNF: example

- S → NP VP
- S → Aux NP VP
- NP → the N
- VP → V NP
- VP → V
- N → cat
- N → dog
- V → bites
- N → bites

- S → Aux NP VP
  S → Aux NP VP ⇒ S → Aux X
  X → NP VP
- NP → the N
  NP → the N ⇒ NP → X N
  X → the
- VP → V
  VP → V ⇒ VP → bites

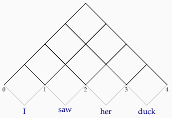




## Converting to CNF

1. Eliminate the $\epsilon$ rules: if A → $\epsilon$ is in the grammar
   - replace any rule B → $\alpha$ A $\beta$ with two rules
     B → $\alpha\ \beta$
     B → $\alpha$ A′ $\beta$
   - add A′ → $\alpha$ for all $\alpha$ (except $\epsilon$) whose LHS is A
   - repeat the process for newly created $\epsilon$ rules
   - remove the rules with $\epsilon$ on the RHS (except S → $\epsilon$)
2. Eliminate unit rules: for a rule A → B
   - Replace the rule with A → $\alpha_1$ | … | $\alpha_n$, where $\alpha_1, \ldots, \alpha_n$ are all RHS or rule B
   - Remove the rule A → B
   - Repeat the process until no unit rules remain
3. Binarize all the non-binary rules with non-terminal on the RHS: for a rule A → $X_1\ X_2 \ldots X_n$:
   - Replace the rule with A → $A_1\ X_2 \ldots X_n$, and add $A_1 \rightarrow X_1\ X_2$
   - Repeat the process until all new rules are binary

## CKY demonstration
an ambiguous example

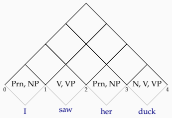



S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N
VP $\rightarrow$ V NP
VP $\rightarrow$ V S
N $\rightarrow$ duck
VP $\rightarrow$ duck | saw
V $\rightarrow$ duck | saw
Prn $\rightarrow$ I | she | her
NP $\rightarrow$ I | she | her





## CKY demonstration
an ambiguous example



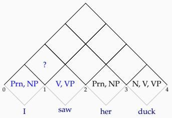

S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N
VP $\rightarrow$ V NP
VP $\rightarrow$ V S
N $\rightarrow$ duck
VP $\rightarrow$ duck | saw
V $\rightarrow$ duck | saw
Prn $\rightarrow$ I | she | her
NP $\rightarrow$ I | she | her





## CKY demonstration
an ambiguous example

S $\rightarrow$ NP VP



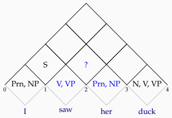

S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N
VP $\rightarrow$ V NP
VP $\rightarrow$ V S
N $\rightarrow$ duck
VP $\rightarrow$ duck | saw
V $\rightarrow$ duck | saw
Prn $\rightarrow$ I | she | her
NP $\rightarrow$ I | she | her





## CKY demonstration
an ambiguous example

VP $\rightarrow$ V NP



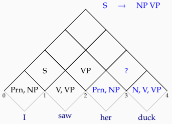

S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N
VP $\rightarrow$ V NP
VP $\rightarrow$ V S
N $\rightarrow$ duck
VP $\rightarrow$ duck | saw
V $\rightarrow$ duck | saw
Prn $\rightarrow$ I | she | her
NP $\rightarrow$ I | she | her





## CKY demonstration
an ambiguous example

NP $\rightarrow$ Prn N
S $\rightarrow$ NP VP

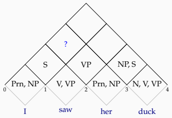



S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N
VP $\rightarrow$ V NP
VP $\rightarrow$ V S
N $\rightarrow$ duck
VP $\rightarrow$ duck | saw
V $\rightarrow$ duck | saw
Prn $\rightarrow$ I | she | her
NP $\rightarrow$ I | she | her





## CKY demonstration
an ambiguous example



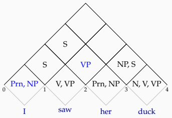

S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N
VP $\rightarrow$ V NP
VP $\rightarrow$ V S
N $\rightarrow$ duck
VP $\rightarrow$ duck | saw
V $\rightarrow$ duck | saw
Prn $\rightarrow$ I | she | her
NP $\rightarrow$ I | she | her





## CKY demonstration
an ambiguous example

S $\rightarrow$ NP VP




S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N
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Prn $\rightarrow$ I | she | her
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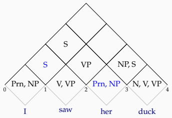
## CKY demonstration
an ambiguous example




S $\rightarrow$ NP VP
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VP $\rightarrow$ duck | saw
V $\rightarrow$ duck | saw
Prn $\rightarrow$ I | she | her
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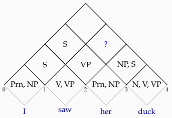
## CKY demonstration
an ambiguous example



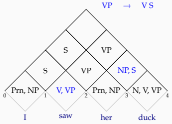

S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N
VP $\rightarrow$ V NP
VP $\rightarrow$ V S
N $\rightarrow$ duck
VP $\rightarrow$ duck | saw
V $\rightarrow$ duck | saw
Prn $\rightarrow$ I | she | her
NP $\rightarrow$ I | she | her





## CKY demonstration
an ambiguous example

VP $\rightarrow$ V NP
VP $\rightarrow$ V S




S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N
VP $\rightarrow$ V NP
VP $\rightarrow$ V S
N $\rightarrow$ duck
VP $\rightarrow$ duck | saw
V $\rightarrow$ duck | saw
Prn $\rightarrow$ I | she | her
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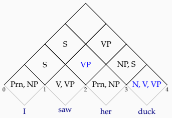
## CKY demonstration
an ambiguous example

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Prn $\rightarrow$ I | she | her
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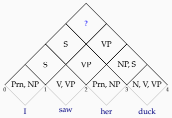
## CKY demonstration
an ambiguous example

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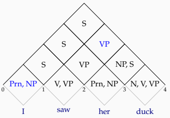
VP $\rightarrow$ V NP
VP $\rightarrow$ V S
N $\rightarrow$ duck
VP $\rightarrow$ duck | saw
V $\rightarrow$ duck | saw
Prn $\rightarrow$ I | she | her
NP $\rightarrow$ I | she | her

## CKY demonstration

an ambiguous example

S → NP VP




S → NP VP
NP → Prn N
VP → V NP
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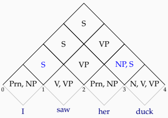
## CKY demonstration

an ambiguous example




S → NP VP
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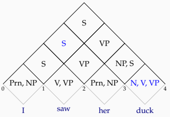
## CKY demonstration

an ambiguous example




S → NP VP
NP → Prn N
VP → V NP
VP → V S
N → duck
VP → duck | saw
V → duck | saw
Prn → I | she | her
NP → I | she | her





## CKY demonstration: the chart

our chart is a 2D array

| | | | |
|---|---|---|---|
| NP, Prn | S | S | S |
| | V, VP | VP | VP |
| | | Prn | NP, S |
| | | | V, N, NP |

0 she 1 saw 2 her 3 duck 4

Space complexity is $O(n^2)$.





## CKY demonstration: the chart

our chart is a 2D array – this is more convenient for programming

| | | | |
|---|---|---|---|
| S | | | |
| S | VP | | |
| S | VP | NP, S | |
| NP, Prn | V, VP | Prn, NP | V, N, NP |

0 she 1 saw 2 her 3 duck 4

Space complexity is $O(n^2)$.





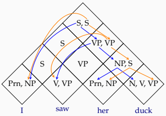
## Parsing vs. recognition

- We went through a recognition example
- Note that the algorithm is not directional: it takes the complete input
- Recognition accepts or rejects a sentence based on a grammar
- For parsing, we want to know the derivations that yielded a correct parse
- To recover parse trees, we
  - we follow the same procedure as recognition
  - add back links to keep track of the derivations





## Chart parsing example (CKY parsing)




The chart stores a *parse forest* efficiently.





## Summary

\+ CKY avoids re-computing the analyses by storing the earlier analyses (of sub-spans) in a table

– It still computes lower level constituents that are not allowed by the grammar

– CKY requires the grammar to be in CNF

- CKY has $O(n^3)$ recognition complexity
- For parsing we need to keep track of backlinks
- CKY can efficiently store all possible parses in a chart
- Enumerating all possible parses have exponential complexity (worst case)

Next:

- Top-down chart parsing: Earley algorithm
- Suggested reading: Grune and Jacobs (2007, section 7.2)



## Acknowledgments, references, additional reading material

Cocke, John and J. T. Schwartz (1970). *Programming languages and their compilers: preliminary notes*. Tech. rep. Courant Institute of Mathematical Sciences, NYU.

Grune, Dick and Ceriel J.H. Jacobs (2007). *Parsing Techniques: A Practical Guide*. second. Monographs in Computer Science. The first edition is available at `http://dickgrune.com/Books/PTAPG_1st_Edition/BookBody.pdf`. Springer New York. ISBN: 9780387689548.

Kasami, Tadao (1965). *An Efficient Recognition and Syntax-Analysis Algorithm for Context-Free Languages*. Tech. rep. DTIC Document.

Younger, Daniel H (1967). "Recognition and parsing of context-free languages in time n 3". In: *Information and control* 10.2, pp. 189–208.