

# xLR(k): deterministic bottom-up parsing

Parsing  
ISCL-BA-06

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## Recap: bottom-up parsing

- Start from the input symbols, try to *reduce* the input to the start symbol
  - Unlike top-down parsing where *productions* drive the parsing, in bottom-up parsing *reduction* is the main operation
  - Reduction matches RHS of a grammar rule, and replaces it with its LHS
  - A typical bottom-up parser has two basic operations
- reduce replace one more more symbols in the sentential form with their LHS  
non-terminal
- shift move the next unprocessed symbol from the input to the sentential form

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
	dnvan	shift

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
d	nvan	shift

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
dn	van	r: $AN \rightarrow n$
dn	van	shift

shift/reduce conflict

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
dn	van	r: $AN \rightarrow n$
dnv	an	shift

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
dn	van	r: $AN \rightarrow n$
dnva	n	shift

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
dn	van	r: $AN \rightarrow n$
dnvan		r: $AN \rightarrow n$



# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
dn	van	r: $AN \rightarrow n$
dnva AN		r: $AN \rightarrow a AN$
dnva AN		r: $NP \rightarrow AN$

reduce/reduce conflict

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
dn	van	r: $AN \rightarrow n$
dnva AN		r: $AN \rightarrow a AN$
dnva NP		reject

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
dn	van	r: $AN \rightarrow n$
dnv AN		r: $AN \rightarrow a AN$

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
dn	van	r: $AN \rightarrow n$
dnv AN		r: $NP \rightarrow AN$

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
dn	van	r: $AN \rightarrow n$
dnv NP		r: $VP \rightarrow v NP$

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
dn	van	r: $AN \rightarrow n$
dn VP		reject

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
dn	van	r: $AN \rightarrow n$

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
d AN	van	r: $NP \rightarrow d AN$
d AN	van	r: $NP \rightarrow AN$
d AN	van	shift

shift/reduce conflict



# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
d AN	van	r: $NP \rightarrow d AN$
d AN	van	r: $NP \rightarrow AN$
d AN v	an	shift

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
d AN	van	r: $NP \rightarrow d AN$
d AN	van	r: $NP \rightarrow AN$
d AN va	n	shift

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
d AN	van	r: $NP \rightarrow d AN$
d AN	van	r: $NP \rightarrow AN$
d AN van		r: $AN \rightarrow N$

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
d AN	van	r: $NP \rightarrow d AN$
d AN	van	r: $NP \rightarrow AN$
d AN va AN		r: $AN \rightarrow a AN$
d AN va AN		r: $NP \rightarrow AN$

reduce/reduce conflict

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
d AN	van	r: $NP \rightarrow d AN$
d AN	van	r: $NP \rightarrow AN$
d AN va AN		r: $AN \rightarrow a AN$
d AN va NP		reject

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
d AN	van	r: $NP \rightarrow d AN$
d AN	van	r: $NP \rightarrow AN$
d AN v NP		r: $VP \rightarrow v NP$

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
d AN	van	r: $NP \rightarrow d AN$
d AN	van	r: $NP \rightarrow AN$
d AN VP		reject

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
d AN	van	r: $NP \rightarrow d AN$
d NP	van	reject



# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
NP	van	shift

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
NP v	an	shift

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
NP va	n	shift

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
NP van		r: $AN \rightarrow n$

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
NP va AN		r: $AN \rightarrow a AN$
NP va AN		r: $NP \rightarrow AN$

reduce/reduce conflict

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
NP va AN		r: $AN \rightarrow a AN$
NP va NP		reject

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
NP v AN		r: $NP \rightarrow AN$

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
NP v NP		r: $VP \rightarrow v NP$



# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
NP VP		r: $S \rightarrow NP VP$

# Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$     $NP \rightarrow d AN$     $NP \rightarrow AN$   
 $VP \rightarrow v NP$     $AN \rightarrow a AN$     $AN \rightarrow n$

SENT. FORM	INPUT	ACTION
S		accept

## Two issues with a backtracking shift-reduce parser

- Obvious one: reduce/reduce and shift/reduce conflicts mean non-determinism
- Not-so-obvious one: recognizing ‘handles’:
  - The rule that we locate at the right edge of the active sentential form is called a *handle*
  - For variable RHS, we need to search the grammar to determine which rule applies (if any)
- In a efficient parser we want to avoid both

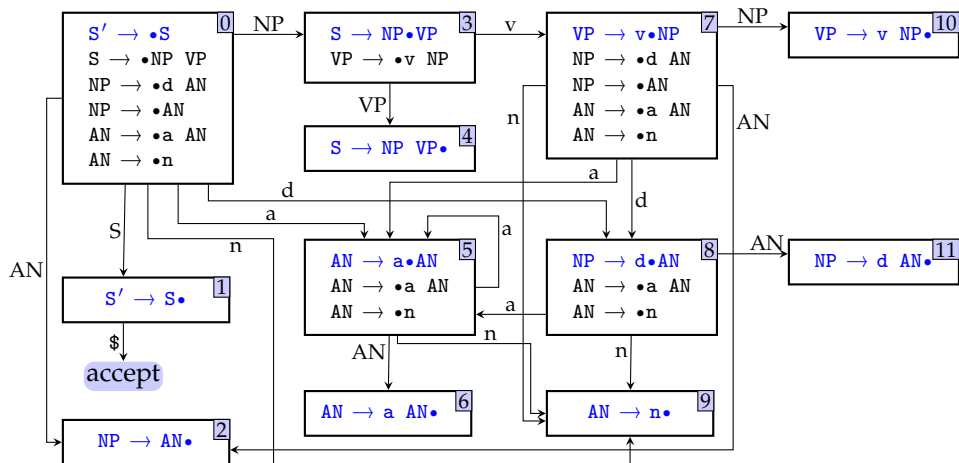
## Table driven bottom-up parsing

- The extra work done by a backtracking shift-reduce parser can be eliminated for a large class of grammars
- The general idea is the same with LL(k) grammars: preprocess the grammar to construct a table
- The class of LR(k) (scanning from *Left-to-right*, producing a *Rightmost derivation*) grammars can be parsed deterministically using k lookahead symbols
- $k = 1$  is most common, LR(0) parser are also useful in some cases, larger k allows expressive grammars
- LL(k) grammars are a subset of LR(k) grammars
- Most practical programming language compilers are LR(1) parsers
- LR(k) parsers are difficult to build manually, but tools that take a CF grammar and construct an LR(1) parser are in common use (e.g., yacc)

## Dotted rules, or ‘items’, (again) and augmented grammars

- An LR parser keeps a set of states (actually a finite-state automaton) to represent the current parser state during parsing
- An LR parser’s states are sets of ‘dotted rules’ similar to Early or chart parsers we discussed earlier
  - $A \rightarrow \bullet \alpha$
  - $A \rightarrow \alpha \bullet \beta$
  - $A \rightarrow \alpha \bullet$
- We also introduce a new start symbol, with a single production  $S' \rightarrow S$
- This rule helps parser to determine when to stop: the parser accepts the input only when reducing  $S$  to  $S'$

# LR(0) automaton



# Shift-reduce parsing with LR(0) automaton

- The simplest version of the LR parsers uses LR(0) automaton to guide the parsing decisions
  - Use a stack to keep track of active states
  - Start with state 0
  - If there is an outgoing edge labeled with the current input, shift: push the target state to the stack
  - Otherwise reduce based on contents of the current state. For example, if the current state contains  $S \rightarrow NP\ VP\bullet$ ,
    - pop two symbols (for NP and VP) from the stack
    - push the state reachable through S from the state on the top of the stack

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								



# Example

## Parsing with LR(0) automaton 1

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0		d n v a n \$	shift

# Example

## Parsing with LR(0) automaton 2

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 8	d	n v a n \$	shift

# Example

## Parsing with LR(0) automaton 3

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 8 9	d n	v a n \$	AN $\rightarrow$ n

# Example

## Parsing with LR(0) automaton 4

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 8 11	d AN	v a n \$	NP $\rightarrow$ d AN

# Example

## Parsing with LR(0) automaton 5

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 3	NP	v a n \$	shift

# Example

## Parsing with LR(0) automaton 6

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 3 7	NP v	a n \$	shift

# Example

## Parsing with LR(0) automaton 7

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 3 7 5	NP v a	n \$	shift

# Example

## Parsing with LR(0) automaton 8

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 3 7 5 9	NP v a n	\$	shift



# Example

## Parsing with LR(0) automaton 9

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 3 7 5 6	NP v a AN	\$	AN $\rightarrow$ n

# Example

## Parsing with LR(0) automaton 10

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 3 7 2	NP v AN	\$	AN $\rightarrow$ a AN

# Example

## Parsing with LR(0) automaton 11

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 3 7 10	NP v NP	\$	NP $\rightarrow$ AN

# Example

## Parsing with LR(0) automaton 12

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 3 4	NP VP	\$	VP $\rightarrow$ v NP

# Example

## Parsing with LR(0) automaton 13

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 1	S	\$	$S \rightarrow NP VP$

# Example

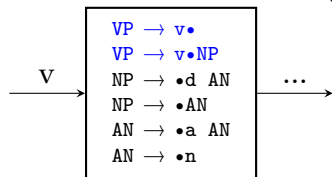
## Parsing with LR(0) automaton 14

state	ACTION	GOTO							
		a	d	n	v	S	NP	VP	AN
0	shift	5	8	9	e	1	3	e	2
1	reduce $S' \rightarrow S$								
2	reduce $NP \rightarrow AN$								
3	shift	e	e	e	7	e		4	
4	reduce $S \rightarrow NP VP$								
5	shift	5	e	9	e				6
6	reduce $AN \rightarrow a AN$								
7	shift	5	8	9	e		10		2
8	shift	5	e	9	e				11
9	reduce $AN \rightarrow n$								
10	reduce $VP \rightarrow v NP$								
11	reduce $NP \rightarrow d AN$								

STACK	SENT. FORM	INPUT	ACTION
0 1	S	\$	accept

## Limitations of LR(0)

- Assume we have an additional rule:  $VP \rightarrow v$
- This would lead to a LR(0) automaton entry



- We have a shift/reduce conflict
- A simple solution (SLR): shift if possible, otherwise reduce
- In general LR(0) parsers/grammars are limited, for most purposes we need more powerful parsers

# LR parsers with lookahead

- LR(k): parsers augment the chart entries (items) with lookahead
- Lookahead allows LR(k) parser to parse a larger class of grammars
- The disadvantage is much larger chart sizes
- Another option is the LALR(k) parsers which use a smaller automaton
- LALR(1) parsers and parser generators are commonly used in practice



## Why use xLR(k) parsers?

- LR(k) parsers general, efficient (non-backtracking) shift-reduce parsers
- LR(k) parsers can be constructed for (almost) any formal/programming language constructs
- In general LR(k) grammars are more expressive. LL(k) is a subset of LR(k)
- LR(k) parsers can detect syntax errors as soon as it is possible to detect them

# LR grammars and ambiguity

- LR(k) parsers cannot handle ambiguity
- If a grammar is ambiguous we cannot construct an LR(k) parse table for it
- In general, determining whether a grammar is ambiguous is intractable
- This is sometimes used for a test for ambiguity:
  - If we can build a LR(k) parser for a grammar, then it is not ambiguous
  - If we cannot, it is inconclusive

# What about natural language parsing

- Natural languages are inherently ambiguous
- As a result, we cannot use these parsers for parsing natural languages
- Nevertheless, the techniques are useful
  - We can use LR-like parsers to reduce the non-determinism: GLR parsers (also known as Tomita parser)
  - Instead of a table-driven parser, we can predict the action with a machine learning method: transition-based dependency parsers do that

# Summary

- xLR(k) parsers are powerful bottom-up deterministic parsers
- LR grammars are more general than LL grammars
- These parsers are difficult to build manually, but automatic parser generators exist
- Although they cannot handle ambiguity, the similar ideas are also used in natural language parsers to reduce the non-determinism
- Understanding the concepts here is useful for building parser generators and understanding the related natural language parsers
- Reading suggestion: Grune and Jacobs (2007, ch.9), Aho et al. (2007, Section 4.5–4.7)

# Acknowledgments, references, additional reading material



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