

xLR(k): deterministic bottom-up parsing

Parsing
ISCL-BA-06

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Recap: bottom-up parsing

- Start from the input symbols, try to *reduce* the input to the start symbol
 - Unlike top-down parsing where *productions* drive the parsing, in bottom-up parsing *reduction* is the main operation
 - Reduction matches RHS of a grammar rule, and replaces it with its LHS
 - A typical bottom-up parser has two basic operations
- reduce replace one more more symbols in the sentential form with their LHS
non-terminal
- shift move the next unprocessed symbol from the input to the sentential form

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------|
| | dnvan | shift |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------|
| d | nvan | shift |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|-----------------------|
| dn | van | r: $AN \rightarrow n$ |
| dn | van | shift |

shift/reduce conflict

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|-----------------------|
| dn | van | r: $AN \rightarrow n$ |
| dnv | an | shift |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|-----------------------|
| dn | van | r: $AN \rightarrow n$ |
| dnva | n | shift |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|-----------------------|
| dn | van | r: $AN \rightarrow n$ |
| dnvan | | r: $AN \rightarrow n$ |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| dn | van | r: $AN \rightarrow n$ |
| dnva AN | | r: $AN \rightarrow a AN$ |
| dnva AN | | r: $NP \rightarrow AN$ |

reduce/reduce conflict

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| dn | van | r: $AN \rightarrow n$ |
| dnva AN | | r: $AN \rightarrow a AN$ |
| dnva NP | | reject |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| dn | van | r: $AN \rightarrow n$ |
| dnv AN | | r: $AN \rightarrow a AN$ |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|------------------------|
| dn | van | r: $AN \rightarrow n$ |
| dnv AN | | r: $NP \rightarrow AN$ |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| dn | van | r: $AN \rightarrow n$ |
| dnv NP | | r: $VP \rightarrow v NP$ |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|-----------------------|
| dn | van | r: $AN \rightarrow n$ |
| dn VP | | reject |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|-----------------------|
| dn | van | r: $AN \rightarrow n$ |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| d AN | van | r: $NP \rightarrow d AN$ |
| d AN | van | r: $NP \rightarrow AN$ |
| d AN | van | shift |

shift/reduce conflict

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| d AN | van | r: $NP \rightarrow d AN$ |
| d AN | van | r: $NP \rightarrow AN$ |
| d AN v | an | shift |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| d AN | van | r: $NP \rightarrow d AN$ |
| d AN | van | r: $NP \rightarrow AN$ |
| d AN va | n | shift |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| d AN | van | r: $NP \rightarrow d AN$ |
| d AN | van | r: $NP \rightarrow AN$ |
| d AN van | | r: $AN \rightarrow N$ |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| d AN | van | r: $NP \rightarrow d AN$ |
| d AN | van | r: $NP \rightarrow AN$ |
| d AN va AN | | r: $AN \rightarrow a AN$ |
| d AN va AN | | r: $NP \rightarrow AN$ |

reduce/reduce conflict

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| d AN | van | r: $NP \rightarrow d AN$ |
| d AN | van | r: $NP \rightarrow AN$ |
| d AN va AN | | r: $AN \rightarrow a AN$ |
| d AN va NP | | reject |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| d AN | van | r: $NP \rightarrow d AN$ |
| d AN | van | r: $NP \rightarrow AN$ |
| d AN v NP | | r: $VP \rightarrow v NP$ |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| d AN | van | r: $NP \rightarrow d AN$ |
| d AN | van | r: $NP \rightarrow AN$ |
| d AN VP | | reject |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| d AN | van | r: $NP \rightarrow d AN$ |
| d NP | van | reject |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------|
| NP | van | shift |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------|
| NP v | an | shift |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------|
| NP va | n | shift |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|-----------------------|
| NP van | | r: $AN \rightarrow n$ |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| NP va AN | | r: $AN \rightarrow a AN$ |
| NP va AN | | r: $NP \rightarrow AN$ |

reduce/reduce conflict

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| NP va AN | | r: $AN \rightarrow a AN$ |
| NP va NP | | reject |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|------------------------|
| NP v AN | | r: $NP \rightarrow AN$ |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| NP v NP | | r: $VP \rightarrow v NP$ |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------------------------|
| NP VP | | r: $S \rightarrow NP VP$ |

Bottom-up (shift-reduce) parsing: an example

$S \rightarrow NP VP$ $NP \rightarrow d AN$ $NP \rightarrow AN$
 $VP \rightarrow v NP$ $AN \rightarrow a AN$ $AN \rightarrow n$

| SENT. FORM | INPUT | ACTION |
|------------|-------|--------|
| S | | accept |

Two issues with a backtracking shift-reduce parser

- Obvious one: reduce/reduce and shift/reduce conflicts mean non-determinism
- Not-so-obvious one: recognizing ‘handles’:
 - The rule that we locate at the right edge of the active sentential form is called a *handle*
 - For variable RHS, we need to search the grammar to determine which rule applies (if any)
- In a efficient parser we want to avoid both

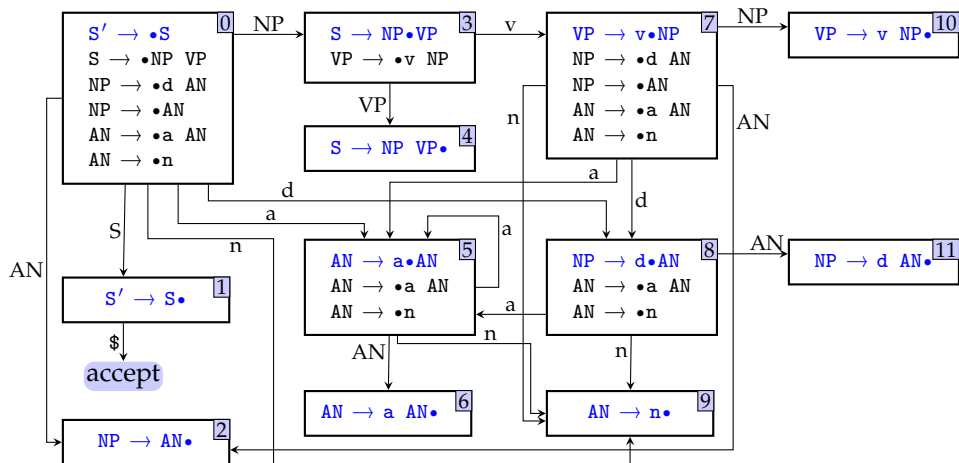
Table driven bottom-up parsing

- The extra work done by a backtracking shift-reduce parser can be eliminated for a large class of grammars
- The general idea is the same with LL(k) grammars: preprocess the grammar to construct a table
- The class of LR(k) (scanning from *Left-to-right*, producing a *Rightmost derivation*) grammars can be parsed deterministically using k lookahead symbols
- $k = 1$ is most common, LR(0) parser are also useful in some cases, larger k allows expressive grammars
- LL(k) grammars are a subset of LR(k) grammars
- Most practical programming language compilers are LR(1) parsers
- LR(k) parsers are difficult to build manually, but tools that take a CF grammar and construct an LR(1) parser are in common use (e.g., yacc)

Dotted rules, or ‘items’, (again) and augmented grammars

- An LR parser keeps a set of states (actually a finite-state automaton) to represent the current parser state during parsing
- An LR parser’s states are sets of ‘dotted rules’ similar to Early or chart parsers we discussed earlier
 - $A \rightarrow \bullet \alpha$
 - $A \rightarrow \alpha \bullet \beta$
 - $A \rightarrow \alpha \bullet$
- We also introduce a new start symbol, with a single production $S' \rightarrow S$
- This rule helps parser to determine when to stop: the parser accepts the input only when reducing S to S'

LR(0) automaton



Shift-reduce parsing with LR(0) automaton

- The simplest version of the LR parsers uses LR(0) automaton to guide the parsing decisions
 - Use a stack to keep track of active states
 - Start with state 0
 - If there is an outgoing edge labeled with the current input, shift: push the target state to the stack
 - Otherwise reduce based on contents of the current state. For example, if the current state contains $S \rightarrow NP\ VP\bullet$,
 - pop two symbols (for NP and VP) from the stack
 - push the state reachable through S from the state on the top of the stack

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

Example

Parsing with LR(0) automaton 1

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|-------|------------|--------------|--------|
| 0 | | d n v a n \$ | shift |

Example

Parsing with LR(0) automaton 2

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|-------|------------|------------|--------|
| 0 8 | d | n v a n \$ | shift |

Example

Parsing with LR(0) automaton 3

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|-------|------------|----------|--------------------|
| 0 8 9 | d n | v a n \$ | AN \rightarrow n |

Example

Parsing with LR(0) automaton 4

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|--------|------------|----------|-----------------------|
| 0 8 11 | d AN | v a n \$ | NP \rightarrow d AN |

Example

Parsing with LR(0) automaton 5

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|-------|------------|----------|--------|
| 0 3 | NP | v a n \$ | shift |

Example

Parsing with LR(0) automaton 6

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|-------|------------|--------|--------|
| 0 3 7 | NP v | a n \$ | shift |

Example

Parsing with LR(0) automaton 7

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|---------|------------|-------|--------|
| 0 3 7 5 | NP v a | n \$ | shift |

Example

Parsing with LR(0) automaton 8

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|-----------|------------|-------|--------|
| 0 3 7 5 9 | NP v a n | \$ | shift |

Example

Parsing with LR(0) automaton 9

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|-----------|------------|-------|--------------------|
| 0 3 7 5 6 | NP v a AN | \$ | AN \rightarrow n |

Example

Parsing with LR(0) automaton 10

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|---------|------------|-------|-----------------------|
| 0 3 7 2 | NP v AN | \$ | AN \rightarrow a AN |

Example

Parsing with LR(0) automaton 11

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|----------|------------|-------|---------------------|
| 0 3 7 10 | NP v NP | \$ | NP \rightarrow AN |

Example

Parsing with LR(0) automaton 12

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|-------|------------|-------|-----------------------|
| 0 3 4 | NP VP | \$ | VP \rightarrow v NP |

Example

Parsing with LR(0) automaton 13

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|-------|------------|-------|-----------------------|
| 0 1 | S | \$ | $S \rightarrow NP VP$ |

Example

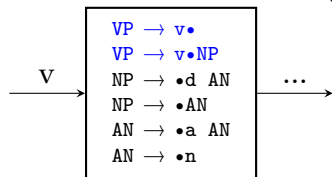
Parsing with LR(0) automaton 14

| state | ACTION | GOTO | | | | | | | |
|-------|------------------------------|------|---|---|---|---|----|----|----|
| | | a | d | n | v | S | NP | VP | AN |
| 0 | shift | 5 | 8 | 9 | e | 1 | 3 | e | 2 |
| 1 | reduce $S' \rightarrow S$ | | | | | | | | |
| 2 | reduce $NP \rightarrow AN$ | | | | | | | | |
| 3 | shift | e | e | e | 7 | e | | 4 | |
| 4 | reduce $S \rightarrow NP VP$ | | | | | | | | |
| 5 | shift | 5 | e | 9 | e | | | | 6 |
| 6 | reduce $AN \rightarrow a AN$ | | | | | | | | |
| 7 | shift | 5 | 8 | 9 | e | | 10 | | 2 |
| 8 | shift | 5 | e | 9 | e | | | | 11 |
| 9 | reduce $AN \rightarrow n$ | | | | | | | | |
| 10 | reduce $VP \rightarrow v NP$ | | | | | | | | |
| 11 | reduce $NP \rightarrow d AN$ | | | | | | | | |

| STACK | SENT. FORM | INPUT | ACTION |
|-------|------------|-------|--------|
| 0 1 | S | \$ | accept |

Limitations of LR(0)

- Assume we have an additional rule: $VP \rightarrow v$
- This would lead to a LR(0) automaton entry



- We have a shift/reduce conflict
- A simple solution (SLR): shift if possible, otherwise reduce
- In general LR(0) parsers/grammars are limited, for most purposes we need more powerful parsers

LR parsers with lookahead

- LR(k): parsers augment the chart entries (items) with lookahead
- Lookahead allows LR(k) parser to parse a larger class of grammars
- The disadvantage is much larger chart sizes
- Another option is the LALR(k) parsers which use a smaller automaton
- LALR(1) parsers and parser generators are commonly used in practice

Why use xLR(k) parsers?

- LR(k) parsers general, efficient (non-backtracking) shift-reduce parsers
- LR(k) parsers can be constructed for (almost) any formal/programming language constructs
- In general LR(k) grammars are more expressive. LL(k) is a subset of LR(k)
- LR(k) parsers can detect syntax errors as soon as it is possible to detect them

LR grammars and ambiguity

- LR(k) parsers cannot handle ambiguity
- If a grammar is ambiguous we cannot construct an LR(k) parse table for it
- In general, determining whether a grammar is ambiguous is intractable
- This is sometimes used for a test for ambiguity:
 - If we can build a LR(k) parser for a grammar, then it is not ambiguous
 - If we cannot, it is inconclusive

What about natural language parsing

- Natural languages are inherently ambiguous
- As a result, we cannot use these parsers for parsing natural languages
- Nevertheless, the techniques are useful
 - We can use LR-like parsers to reduce the non-determinism: GLR parsers (also known as Tomita parser)
 - Instead of a table-driven parser, we can predict the action with a machine learning method: transition-based dependency parsers do that

Summary

- xLR(k) parsers are powerful bottom-up deterministic parsers
- LR grammars are more general than LL grammars
- These parsers are difficult to build manually, but automatic parser generators exist
- Although they cannot handle ambiguity, the similar ideas are also used in natural language parsers to reduce the non-determinism
- Understanding the concepts here is useful for building parser generators and understanding the related natural language parsers
- Reading suggestion: Grune and Jacobs (2007, ch.9), Aho et al. (2007, Section 4.5–4.7)

Acknowledgments, references, additional reading material



Aho, Alfred V., Monica S. Lam, Ravi Sethi, and Jeffrey D. Ullman (2007). *Compilers: Principles, Techniques, & Tools*. Pearson/Addison Wesley. ISBN: 9780321486813.



Grune, Dick and Ceriel J.H. Jacobs (2007). *Parsing Techniques: A Practical Guide*. second. Monographs in Computer Science. The first edition is available at http://dickgrune.com/Books/PTAPG_1st_Edition/BookBody.pdf. Springer New York. ISBN: 9780387689548.

