

# Top-down Chart Parsing: the Earley algorithm

Parsing  
ISCL-BA-06

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# Parsing so far

- We can formulate parsing as
  - Top-down: begin with the start symbol, try to *produce* the input string to be parsed
  - Bottom up: begin with the input, and try to *reduce* it to the start symbol
- Another aspect of a parser is its directionality. Two choices are:
  - Directional: parses processes the input left to right (right to left is also possible, but rarely used)
  - Non-directional: order is not important, typically require all input to be in memory before processing

# Top-down parsing as search

the cat bites a dog

S → NP VP

NP → Det N

VP → V NP

VP → V

Det → a

Det → the

N → cat

N → dog

V → bites

# Top-down parsing as search

S

the cat bites a dog

S → NP VP

NP → Det N

VP → V NP

VP → V

Det → a

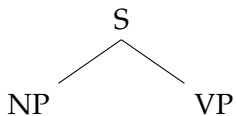
Det → the

N → cat

N → dog

V → bites

# Top-down parsing as search



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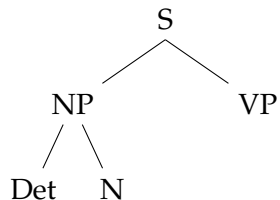
Det → the

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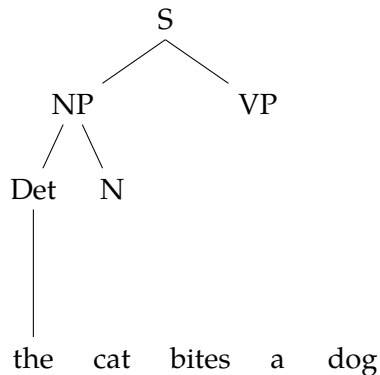
Det → the

N → cat

N → dog

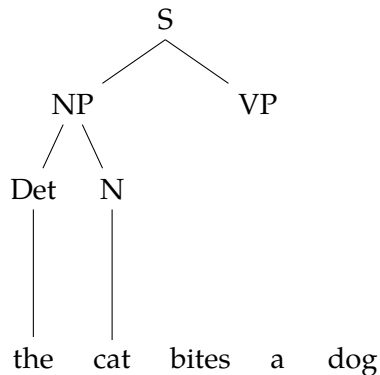
V → bites

# Top-down parsing as search



S	→	NP VP
NP	→	Det N
VP	→	V NP
VP	→	V
Det	→	a
Det	→	the
N	→	cat
N	→	dog
V	→	bites

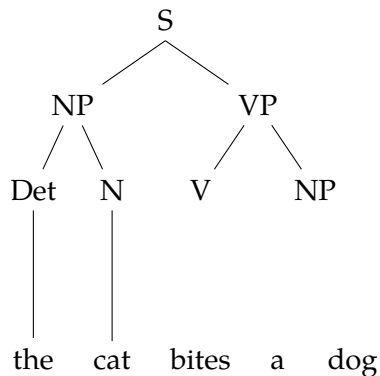
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N	→	cat
N	→	dog
V	→	bites

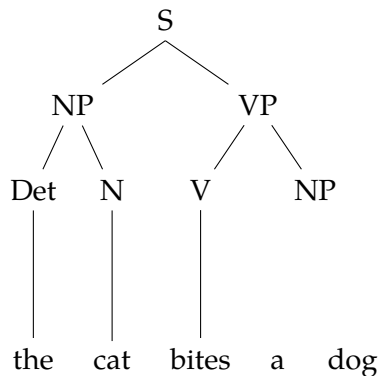


# Top-down parsing as search



S → NP VP  
 NP → Det N  
 VP → V NP  
 VP → V  
 Det → a  
 Det → the  
 N → cat  
 N → dog  
 V → bites

# Top-down parsing as search



S → NP VP

NP → Det N

VP → V NP

VP → V

Det → a

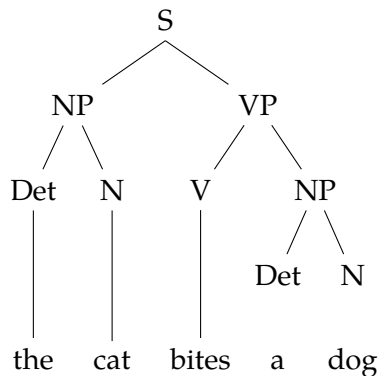
Det → the

N → cat

N → dog

V → bites

# Top-down parsing as search



S → NP VP

NP → Det N

VP → V NP

VP → V

Det → a

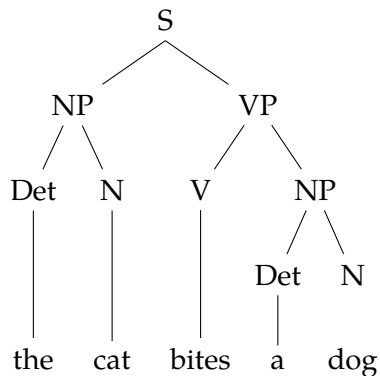
Det → the

N → cat

N → dog

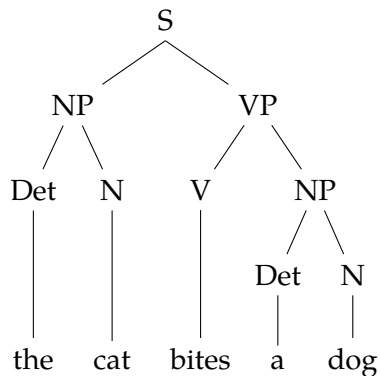
V → bites

# Top-down parsing as search



S → NP VP  
 NP → Det N  
 VP → V NP  
 VP → V  
 Det → a  
 Det → the  
 N → cat  
 N → dog  
 V → bites

# Top-down parsing as search



S → NP VP  
NP → Det N  
VP → V NP  
VP → V  
Det → a  
Det → the  
N → cat  
N → dog  
V → bites

# Earley algorithm

- Earley algorithm is a top down (and left-to-right) parsing algorithm (Earley 1970)
- It allows arbitrary CFGs
- Keeps record of constituents that are
  - predicted using the grammar (top-down)
  - in-progress with partial evidence
  - completed based on input seen so farat every position in the input string
- Time complexity is  $O(n^3)$

## Earley chart entries (states or items)

Earley chart entries are CF rules with a 'dot' on the RHS representing the state of the rule

- $A \rightarrow \bullet \alpha[i, i]$  predicted without any evidence (yet)
- $A \rightarrow \alpha \bullet \beta[i, j]$  partially matched
- $A \rightarrow \alpha \beta \bullet [i, j]$  completed, the non-terminal  $A$  is found in the given span

# Earley algorithm: an informal sketch

1. Start at position 0, predict S
2. Predict all possible states (rules that apply)
3. Read a word
4. Update the table, advance the dot if possible
5. Go to step 2
6. If we have a completed S production at the end of the input, the input is recognized



## Earley algorithm: three operations

Predictor adds all rules that are possible at the given state

Completer adds states from the earlier chart entries that match the completed state to the chart entry being processed, and advances their dot

Scanner adds a completed state to the next chart entry if the current category is a POS tag, and the word matches

# Earley parsing example (chart[0])

0	she	1	saw	2	a	3	duck	4
state	rule	position	operation					
0	$\gamma \rightarrow \bullet S$	[0,0]	initialization					
1	$S \rightarrow \bullet NP VP$	[0,0]	predictor					
2	$S \rightarrow \bullet Aux NP VP$	[0,0]	predictor					
3	$NP \rightarrow \bullet Det N$	[0,0]	predictor					
4	$NP \rightarrow \bullet NP PP$	[0,0]	predictor					
5	$NP \rightarrow \bullet Prn$	[0,0]	predictor					

Note: the chart[0] is independent of the input.

$S \rightarrow NP VP$   
 $S \rightarrow Aux NP VP$   
 $NP \rightarrow Det N$   
 $NP \rightarrow Prn$   
 $NP \rightarrow NP PP$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Prp NP$   
 $N \rightarrow duck$   
 $N \rightarrow park$   
 $V \rightarrow duck$   
 $V \rightarrow ducks$   
 $V \rightarrow saw$   
 $Prn \rightarrow she | her$   
 $Prp \rightarrow in | with$   
 $Det \rightarrow a | the$   
 $Aux \rightarrow does | has$

# Earley parsing example (chart[1])

0	she	1	saw	2	a	3	duck	4
state	rule	position	operation					
6	Prn $\rightarrow$ she •	[0,1]	scanner					
7	NP $\rightarrow$ Prn •	[0,1]	completer					
8	S $\rightarrow$ NP •VP	[0,1]	completer					
9	NP $\rightarrow$ NP •PP	[0,1]	completer					
10	VP $\rightarrow$ •V NP	[1,1]	predictor					
11	VP $\rightarrow$ •V	[1,1]	predictor					
12	VP $\rightarrow$ •VP PP	[1,1]	predictor					
13	PP $\rightarrow$ •Prp NP	[1,1]	predictor					

S  $\rightarrow$  NP VP  
 S  $\rightarrow$  Aux NP VP  
 NP  $\rightarrow$  Det N  
 NP  $\rightarrow$  Prn  
 NP  $\rightarrow$  NP PP  
 VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V  
 VP  $\rightarrow$  VP PP  
 PP  $\rightarrow$  Prp NP  
 N  $\rightarrow$  duck  
 N  $\rightarrow$  park  
 V  $\rightarrow$  duck  
 V  $\rightarrow$  ducks  
 V  $\rightarrow$  saw  
 Prn  $\rightarrow$  she | her  
 Prp  $\rightarrow$  in | with  
 Det  $\rightarrow$  a | the  
 Aux  $\rightarrow$  does | has

# Earley parsing example (chart[2])

0	she	1	saw	2	a	3	duck	4
state	rule	position	operation					
14	$V \rightarrow \text{saw} \bullet$	[1,2]	scanner					
15	$VP \rightarrow V \bullet NP$	[1,2]	completer					
16	$VP \rightarrow V \bullet$	[1,2]	completer					
17	$S \rightarrow NP VP \bullet$	[0,2]	completer					
18	$NP \rightarrow \bullet \text{Det } N$	[2,2]	predictor					
19	$NP \rightarrow \bullet NP PP$	[2,2]	predictor					
20	$NP \rightarrow \bullet \text{Prn}$	[2,2]	predictor					

$S \rightarrow NP VP$   
 $S \rightarrow \text{Aux } NP VP$   
 $NP \rightarrow \text{Det } N$   
 $NP \rightarrow \text{Prn}$   
 $NP \rightarrow NP PP$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow \text{Prp } NP$   
 $N \rightarrow \text{duck}$   
 $N \rightarrow \text{park}$   
 $V \rightarrow \text{duck}$   
 $V \rightarrow \text{ducks}$   
 $V \rightarrow \text{saw}$   
 $\text{Prn} \rightarrow \text{she} \mid \text{her}$   
 $\text{Prp} \rightarrow \text{in} \mid \text{with}$   
 $\text{Det} \rightarrow \text{a} \mid \text{the}$   
 $\text{Aux} \rightarrow \text{does} \mid \text{has}$

# Earley parsing example (chart[3])

0	she	1	saw	2	a	3	duck	4
state				rule		position	operation	
21	Det $\rightarrow$ a •			[2,3]		scanner		
22	NP $\rightarrow$ Det • N			[2,3]		completer		

S  $\rightarrow$  NP VP  
 S  $\rightarrow$  Aux NP VP  
 NP  $\rightarrow$  Det N  
 NP  $\rightarrow$  Prn  
 NP  $\rightarrow$  NP PP  
 VP  $\rightarrow$  V NP  
 VP  $\rightarrow$  V  
 VP  $\rightarrow$  VP PP  
 PP  $\rightarrow$  Prp NP  
 N  $\rightarrow$  duck  
 N  $\rightarrow$  park  
 V  $\rightarrow$  duck  
 V  $\rightarrow$  ducks  
 V  $\rightarrow$  saw  
 Prn  $\rightarrow$  she | her  
 Prp  $\rightarrow$  in | with  
 Det  $\rightarrow$  a | the  
 Aux  $\rightarrow$  does | has

# Earley parsing example (chart[4])

0	she	1	saw	2	a	3	duck	4
state	rule	position	operation					
23	N → duck •	[3,4]	scanner					
24	V → duck •	[3,4]	scanner					
25	NP → Det N •	[2,4]	completer					
26	VP → V NP •	[1,4]	completer					
27	S → NP VP •	[0,4]	completer					

S → NP VP  
 S → Aux NP VP  
 NP → Det N  
 NP → Prn  
 NP → NP PP  
 VP → V NP  
 VP → V  
 VP → VP PP  
 PP → Prp NP  
 N → duck  
 N → park  
 V → duck  
 V → ducks  
 V → saw  
 Prn → she | her  
 Prp → in | with  
 Det → a | the  
 Aux → does | has

# Earley parsing: summary

- Complexity (asymptotic) is the same as CKY
  - time complexity :  $O(n^3)$
  - space complexity:  $O(n^2)$
- Our example shows recognition, we need to maintain back links for parsing
- Again, Earley chart stores a parse forest compactly, but extracting all trees may require exponential time

# Summary

- The early parser is a top-down parser with bottom-up filtering (or, you can also view it the other way around)
- The parser improves over a backtracking parser by
  - dynamic programming: not re-computing the subtrees
  - filtering: not generating hypotheses (predictor) that does not lead to useful
- It can process any CFG (no need for CNF)
- There is a nice relation between CKY and Earley (next week)



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  - filtering: not generating hypotheses (predictor) that does not lead to useful
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Next:

Mon Exercises / assignment / discussion

Thu Deterministic parsing

# An exercise


Construct the CKY and Earley charts for the sentence below

The duck she saw is in the park

Recommended grammar:

$S \rightarrow NP VP$	$PP \rightarrow Prp NP$
$NP \rightarrow Det N$	$N \rightarrow \text{park}$
$NP \rightarrow Prn$	$N \rightarrow \text{duck}$
$NP \rightarrow NP PP$	$V \rightarrow \text{is}$
$NP \rightarrow NP S$	$V \rightarrow \text{saw}$
$VP \rightarrow V NP$	$Prn \rightarrow \text{she}$
$VP \rightarrow V$	$Prp \rightarrow \text{in}$
$VP \rightarrow VP PP$	$Det \rightarrow \text{the}$

# Acknowledgments, references, additional reading material

 Earley, Jay (Feb. 1970). "An Efficient Context-free Parsing Algorithm". In: *Commun. ACM* 13.2, pp. 94–102. ISSN: 0001-0782. DOI: 10.1145/362007.362035. URL: <http://doi.acm.org/10.1145/362007.362035>.