

# Probabilistic context-free parsing

Parsing  
ISCL-BA-06

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Seminar für Sprachwissenschaft

Winter Semester 2020/21

# Context-free grammars

## recap

- Context free (CF) grammars are most practically useful grammars in the Chomsky hierarchy
- Most of the parsing theory (and practice) is build on parsing CF languages
- The context-free rules have the form

$$A \rightarrow \alpha$$

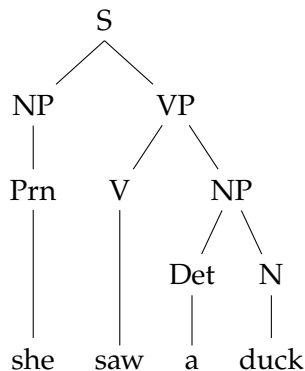
where  $A$  is a single non-terminal symbol and  $\alpha$  is a (possibly empty) sequence of terminal or non-terminal symbols

# An example context-free grammar

$S \rightarrow NP VP$   
 $S \rightarrow Aux NP VP$   
 $NP \rightarrow Det N$   
 $NP \rightarrow Prn$   
 $NP \rightarrow NP PP$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Prp NP$   
 $N \rightarrow duck$   
 $N \rightarrow park$   
 $N \rightarrow parks$   
 $V \rightarrow duck$   
 $V \rightarrow ducks$   
 $V \rightarrow saw$   
 $Prn \rightarrow she | her$   
 $Prp \rightarrow in | with$   
 $Det \rightarrow a | the$

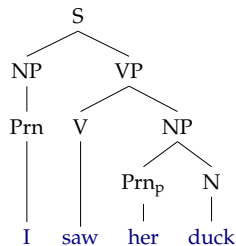
## Derivation of sentence 'she saw a duck'

$S \Rightarrow NP VP$   
 $NP \Rightarrow Prn$   
 $Prn \Rightarrow she$   
 $VP \Rightarrow V NP$   
 $V \Rightarrow saw$   
 $NP \Rightarrow Det N$   
 $Det \Rightarrow a$   
 $N \Rightarrow duck$



# Representations of a context-free parse tree

*A parse tree:*



*A history of derivations:*

- $S \Rightarrow NP VP$
- $NP \Rightarrow Prn$
- $Prn \Rightarrow I$
- $VP \Rightarrow V NP$
- $V \Rightarrow \text{saw}$
- $NP \Rightarrow Prn_p N$
- $Prn_p \Rightarrow \text{her}$
- $N \Rightarrow \text{duck}$

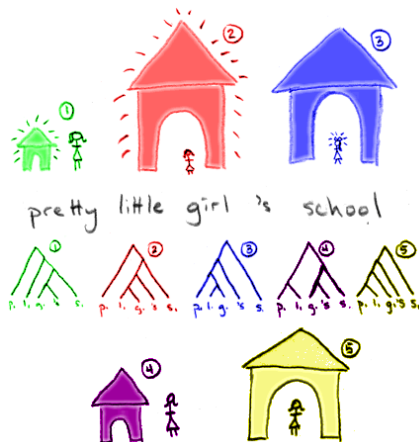
*A sequence with (labeled) brackets*

$$\left[ {}_S \left[ {}_{NP} \left[ {}_{Prn} I \right] \right] \left[ {}_{VP} \left[ V \text{ saw} \right] \left[ {}_{NP} \left[ {}_{Prn_p} \text{ her} \right] \left[ {}_N \text{ duck} \right] \right] \right] \right]$$

# Parsing with context-free grammars

- Parsing can be
  - top down: start from  $S$ , search for derivation that leads to the input
  - bottom up: start from input, try to *reduce* it to  $S$
- Naive search for both recognition/parse is intractable
- Dynamic programming methods allow polynomial time *recognition*
  - CKY bottom-up, requires Chomsky normal form
- Earely top-down (with bottom-up filtering), works with unrestricted grammars
  - $O(n^3)$  time complexity (for recognition)
- Chart parsers are (reasonably) efficient, and they can represent ambiguity in their output
- However, they do not help with *resolving ambiguity*

# Natural languages are ambiguous



# Some types of ambiguities

- Lexical ambiguity
  - She is looking for a match
  - We saw her duck
- Attachment ambiguity
  - I saw the man with a telescope
  - Panda eats bamboo shoots and leaves
- Local ambiguity (garden path sentences)
  - The horse raced past the barn fell
  - The old man the boats
  - Fat people eat accumulates

# Ambiguity and the parsers

- Given a grammar, chart parsers (e.g., CKY, Early) can parse natural language sentences relatively efficiently
- These parsers also represent all possible parse trees in their chart/output efficiently
- However, they have nothing to say about which of these parses are the most likely one.
- The task of selecting the best parse among many is called disambiguation
- In almost all practical uses, parsers are combined with disambiguators



# We do not recognize many ambiguities

- Time flies like an arrow
- Outside of a dog, a book is a man's best friend
- One morning I shot an elephant in my pajamas
- Don't eat the pizza with a knife and fork

A parser, nevertheless, produces multiple parses for these sentences.

# We do not recognize many ambiguities

- Time flies like an arrow; fruit flies like a banana
- Outside of a dog, a book is a man's best friend
- One morning I shot an elephant in my pajamas
- Don't eat the pizza with a knife and fork

A parser, nevertheless, produces multiple parses for these sentences.

# We do not recognize many ambiguities

- Time flies like an arrow; fruit flies like a banana
- Outside of a dog, a book is a man's best friend; inside it's too hard to read
- One morning I shot an elephant in my pajamas
- Don't eat the pizza with a knife and fork

A parser, nevertheless, produces multiple parses for these sentences.

# We do not recognize many ambiguities

- Time flies like an arrow; fruit flies like a banana
- Outside of a dog, a book is a man's best friend; inside it's too hard to read
- One morning I shot an elephant in my pajamas. How he got in my pajamas, I don't know.
- Don't eat the pizza with a knife and fork

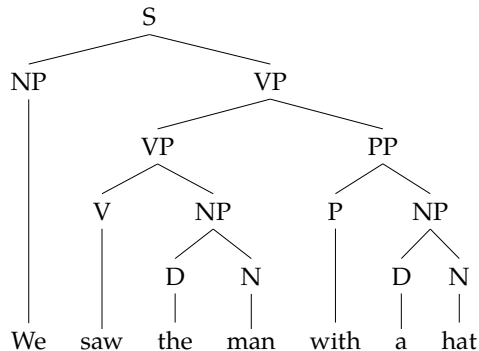
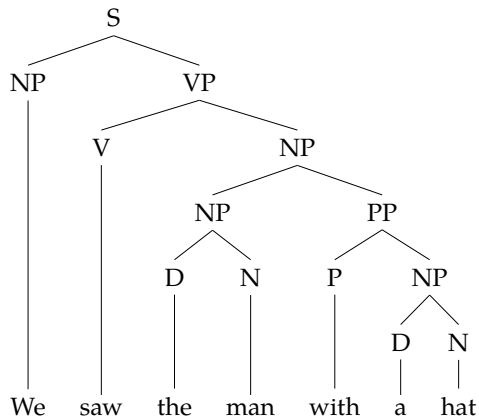
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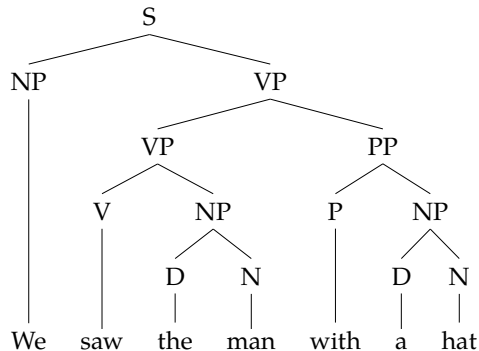
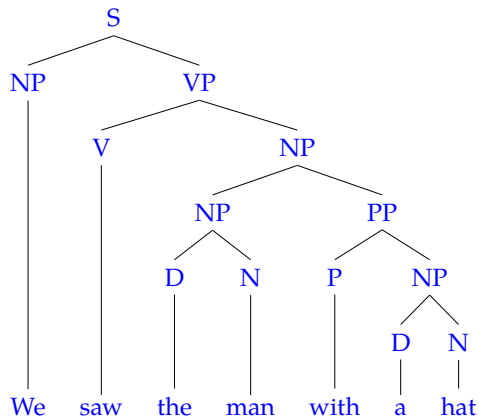
- Time flies like an arrow; fruit flies like a banana
- Outside of a dog, a book is a man's best friend; inside it's too hard to read
- One morning I shot an elephant in my pajamas. How he got in my pajamas, I don't know.
- Don't eat the pizza with a knife and fork; the one with mushrooms is better.

A parser, nevertheless, produces multiple parses for these sentences.

# The task: choosing the most plausible parse



# The task: choosing the most plausible parse



# Statistical parsing

- Find the most plausible parse of an input string given all possible parses
- We need a scoring function, for each parse, given the input
- We typically use probabilities for scoring, task becomes finding the parse (or tree),  $t$ , given the input string  $\mathbf{w}$

$$t_{\text{best}} = \arg \max_t P(t \mid \mathbf{w})$$

- Note that some ambiguities need a larger context than the sentence to be resolved correctly



# Probability refresher (1)

- Probability is a measure of (un)certainty of an event
- We quantify the probability of an event with a number between 0 and 1
  - 0 the event is impossible
  - 0.5 the event is as likely to happen (or happened) as it is not
  - 1 the event is certain
- All possible outcomes of a trial (experiment or observation) is called the *sample space* ( $\Omega$ )

Axioms of probability states that

1.  $P(E) \in \mathbb{R}, P(E) \geq 0$
2.  $P(\Omega) = 1$
3. For *disjoint* events  $E_1$  and  $E_2$ ,  $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

# Probability refresher (2)

## Joint and conditional probabilities, chain rule

- Joint probability of two events is noted as  $P(x, y)$
- The conditional probability is defined as

$$P(x|y) = \frac{P(x,y)}{P(y)} \text{ or } P(x, y) = P(x|y)P(y)$$

- If the events  $x$  and  $y$  are independent,

$$P(x|y) = P(x), P(y|x) = p(y), P(x, y) = P(x)P(y)$$

- For more than two variables (chain rule):

$$P(x, y, z) = P(z|x, y)P(y|x)P(x) = P(x|y, z)P(y|z)P(z) = \dots$$

- If all are independent

$$P(x, y, z) = P(x)P(y)P(z)$$

# Probabilistic context free grammars (PCFG)

- A probabilistic context free grammar augments a CFG by adding a probability value to each rule

$$A \rightarrow \alpha \quad [p]$$

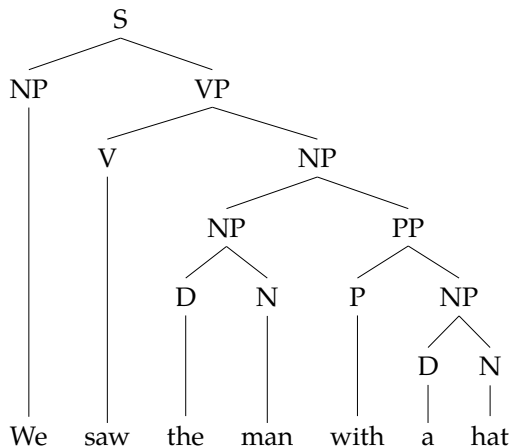
where  $A$  is a non-terminal,  $\alpha$  is string of terminals and non-terminals, and  $p$  is the probability associated with the rule

- Like CFGs, a PCFG accepts a sentence if it can be derived from  $S$  with rules  $R_1 \dots R_k$
- The probability of a parse tree  $t$  of input string  $\mathbf{w}$ ,  $P(t \mid \mathbf{w})$ , corresponding to the derivation  $R_1 \dots R_k$  is

$$P(t \mid \mathbf{w}) = \prod_1^k p(R_i)$$

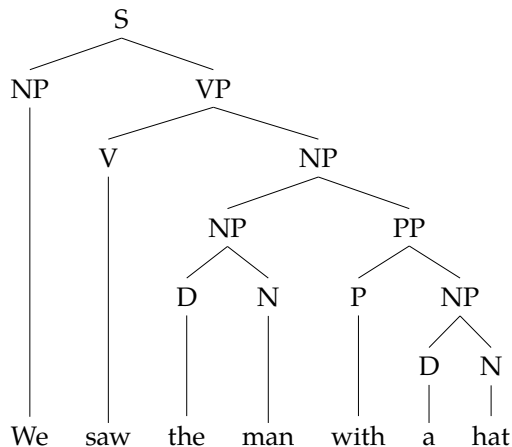
where  $p(R_i)$  is the probability of the rule  $R_i$

## PCFG example (1)



S	→ NP VP	1.0
NP	→ D N	0.7
NP	→ NP PP	0.2
NP	→ We	0.1
VP	→ V NP	0.9
VP	→ VP PP	0.1
PP	→ P NP	1.0
N	→ hat	0.2
N	→ man	0.8
V	→ saw	1.0
P	→ with	1.0
D	→ a	0.6
D	→ the	0.4

## PCFG example (1)

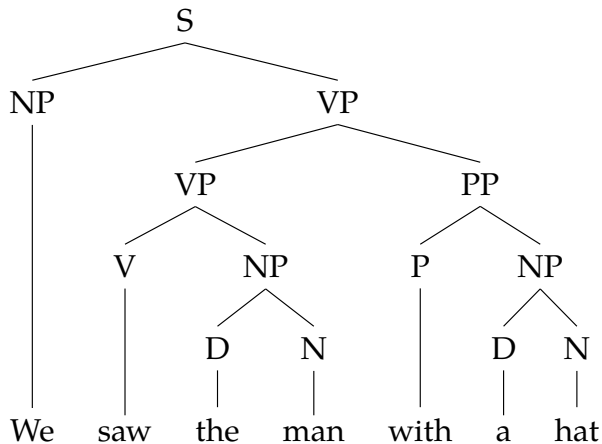


$$P(t) = 1.0 \times 0.1 \times 0.9 \times 1.0 \times 0.2 \times 0.7 \times 0.4 \times 0.8 \times 1.0 \times 1.0 \times 0.7 \times 0.6 \times 0.2$$

$$= 0.000263424$$

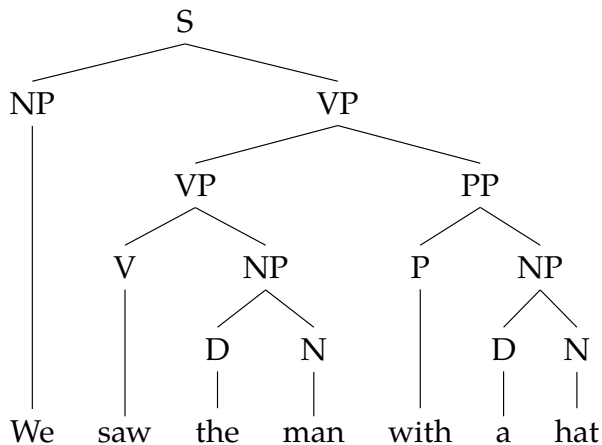
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## PCFG example (2)



S	→ NP VP	1.0
NP	→ D N	0.7
NP	→ NP PP	0.2
NP	→ We	0.1
VP	→ V NP	0.9
VP	→ VP PP	0.1
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## PCFG example (2)



$$\begin{aligned}
 P(t) &= 1.0 \times 0.1 \times \mathbf{0.1} \times 0.9 \times 1.0 \times 0.7 \times 0.4 \times 0.8 \times 1.0 \times 1.0 \times 0.7 \times 0.6 \times 0.2 \\
 &= 0.0001693440
 \end{aligned}$$

S	→ NP VP	1.0
NP	→ D N	0.7
NP	→ NP PP	0.2
NP	→ We	0.1
VP	→ V NP	0.9
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N	→ hat	0.2
N	→ man	0.8
V	→ saw	1.0
P	→ with	1.0
D	→ a	0.6
D	→ the	0.4

# Where do the rule probabilities come from?

- Supervised: estimate from a treebank, e.g., using maximum likelihood estimation
- Unsupervised: expectation-maximization (EM)



## PCFGs - an interim summary

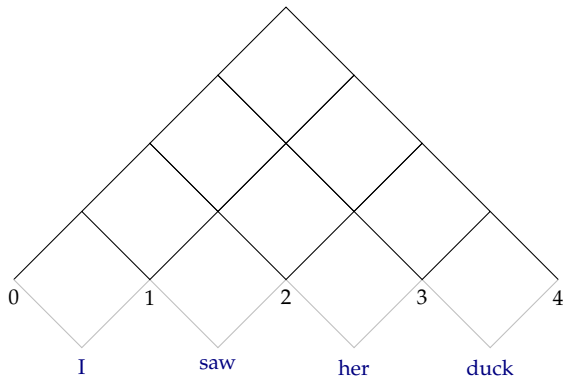
- PCFGs assign probabilities to parses based on CFG rules used during the parse
- PCFGs assume that the rules are independent
- PCFGs are generative models, they assign probabilities to  $P(t, \mathbf{w})$ , we can calculate the probability of a sentence by

$$P(\mathbf{w}) = \sum_t P(t, \mathbf{w}) = \sum_t P(t)$$

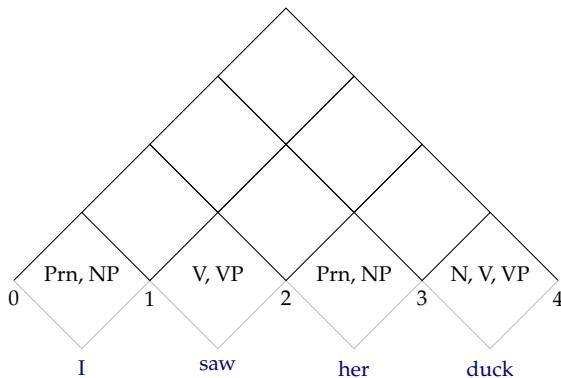
# PCFG chart parsing

- Both CKY and Earley algorithms can be adapted to PCFG parsing
- CKY matches PCFG parsing quite well
  - to get the best parse, store the constituent with the highest probability in every cell of the chart
  - to get n-best best parse (beam search), store the n-best constituents in every cell in the chart

# CKY for PCFG parsing



# CKY for PCFG parsing



$$P(\text{Prn}_{01}) = P(\text{Prn} \rightarrow \text{I})$$

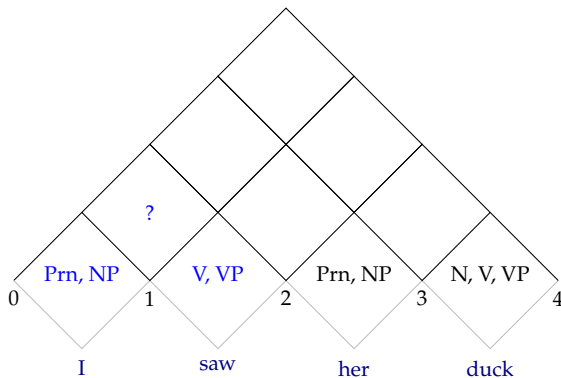
$$P(\text{V}_{12}) = P(\text{V} \rightarrow \text{saw})$$

$$P(\text{NP}_{01}) = P(\text{NP} \rightarrow \text{I})$$

$$P(\text{VP}_{12}) = P(\text{VP} \rightarrow \text{saw})$$

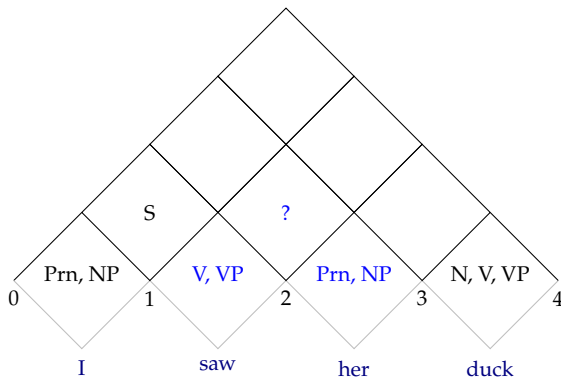
...

# CKY for PCFG parsing

 $S \rightarrow NP VP$ 


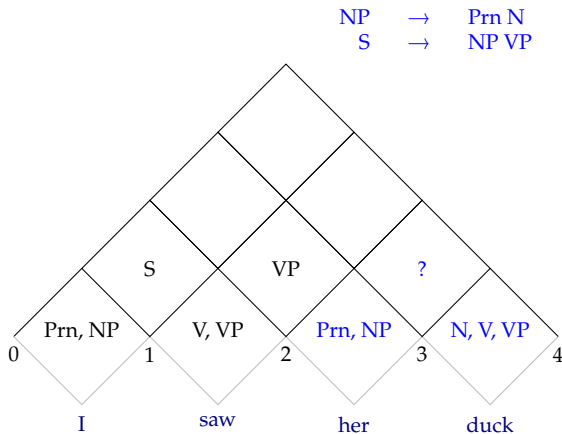
$$P(S_{02} \Rightarrow NP_{01} VP_{12}) = P(NP_{01})P(VP_{12})P(S \rightarrow NP VP)$$

# CKY for PCFG parsing

$$\text{VP} \rightarrow \text{V NP}$$


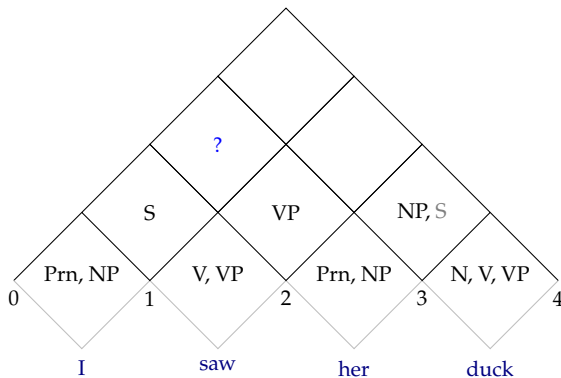
$$P(\text{VP}_{13} \Rightarrow \text{V}_{12}\text{NP}_{23}) = P(\text{V}_{12})P(\text{NP}_{23})P(\text{VP} \rightarrow \text{V NP})$$

# CKY for PCFG parsing



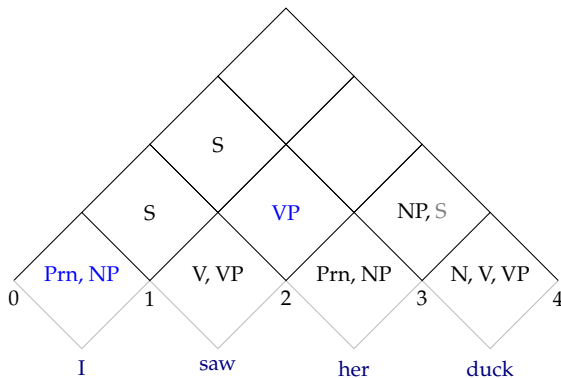
$$\begin{aligned}
 P(\text{NP}_{24} \Rightarrow \text{Prn}_{23} \text{N}_{34}) &= P(\text{Prn}_{23}) P(\text{N}_{34}) P(\text{NP} \rightarrow \text{Prn N}) \\
 &> \\
 P(\text{S}_{24} \Rightarrow \text{NP}_{23} \text{VP}_{34}) &= P(\text{NP}_{23}) P(\text{VP}_{34}) P(\text{S} \rightarrow \text{NP VP})
 \end{aligned}$$

# CKY for PCFG parsing



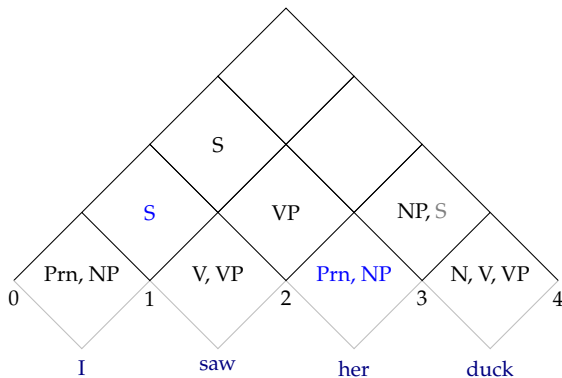


# CKY for PCFG parsing

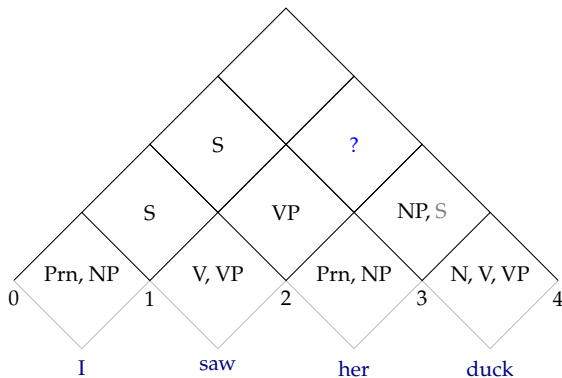
 $S \rightarrow NP VP$ 


$$P(S_{03} \Rightarrow NP_{01} VP_{23}) = P(NP_{01})P(VP_{13})P(S \rightarrow NP VP)$$

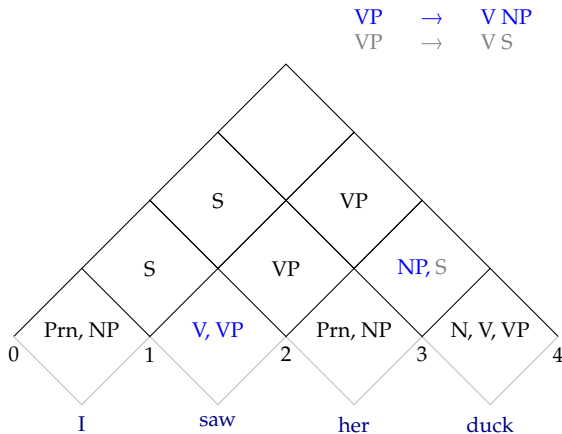
# CKY for PCFG parsing



# CKY for PCFG parsing

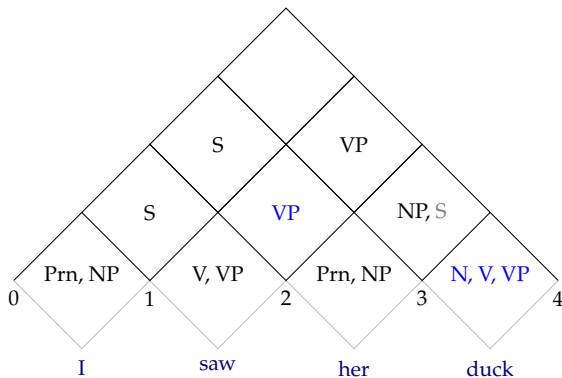


# CKY for PCFG parsing

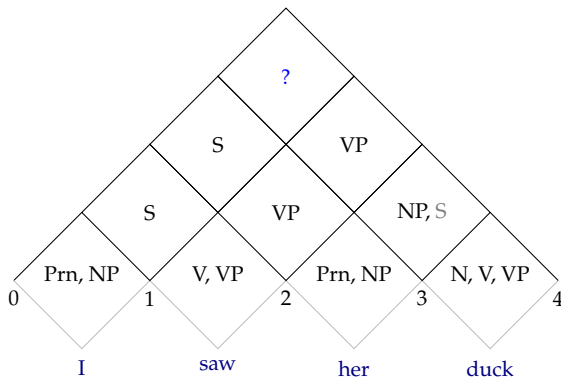


$$P(\text{VP}_{14} \Rightarrow \text{V}_{12}\text{NP}_{24}) = P(\text{V}_{12})P(\text{NP}_{24})P(\text{VP} \rightarrow \text{V NP})$$

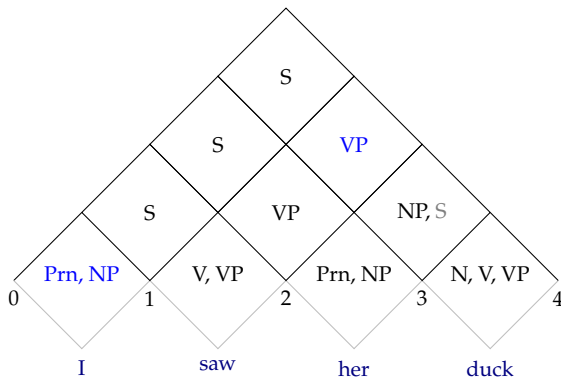
# CKY for PCFG parsing



# CKY for PCFG parsing

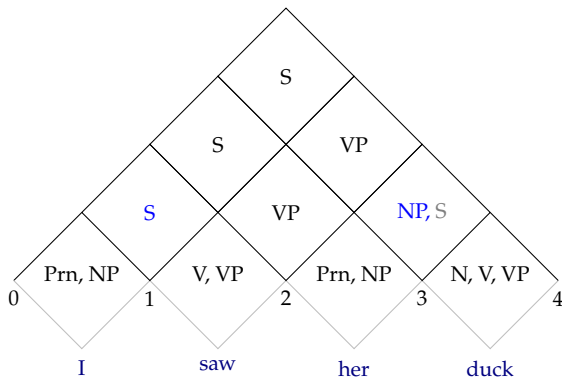


# CKY for PCFG parsing

 $S \rightarrow NP VP$ 


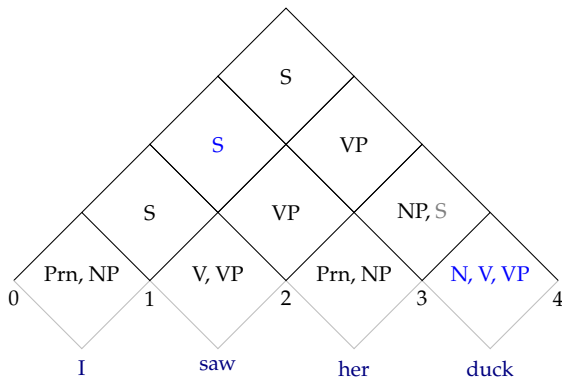
$$P(S_{14} \Rightarrow NP_{01} VP_{14}) = P(NP_{01})P(VP_{14})P(S \rightarrow NP VP)$$

# CKY for PCFG parsing





# CKY for PCFG parsing



# What makes the difference in PCFG probabilities?

S	⇒ NP VP	1.0
NP	⇒ We	0.1
VP	⇒ VP PP	0.1
VP	⇒ V NP	0.8
V	⇒ saw	1.0
NP	⇒ D N	0.7
D	⇒ the	0.4
N	⇒ man	0.8
PP	⇒ P NP	1.0
P	⇒ with	1.0
NP	⇒ D N	0.7
D	⇒ a	0.6
N	⇒ hat	0.2

S	⇒ NP VP	1.0
NP	⇒ We	0.1
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V	⇒ saw	1.0
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P	⇒ with	1.0
NP	⇒ D N	0.7
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N	⇒ hat	0.2

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P	⇒ with	1.0
NP	⇒ D N	0.7
D	⇒ a	0.6
N	⇒ hat	0.2

The parser's choice would not be affected by lexical items!

# What is wrong with PCFGs?

- In general: the assumption of independence
- The parents affect the correct choice for children, for example, in English NP  $\rightarrow$  Prn is more likely in the subject position
- The lexical units affect the correct decision, for example:
  - We eat the pizza with hands
  - We eat the pizza with mushrooms
- Additionally: PCFGs use local context, difficult to incorporate arbitrary/global features for disambiguation

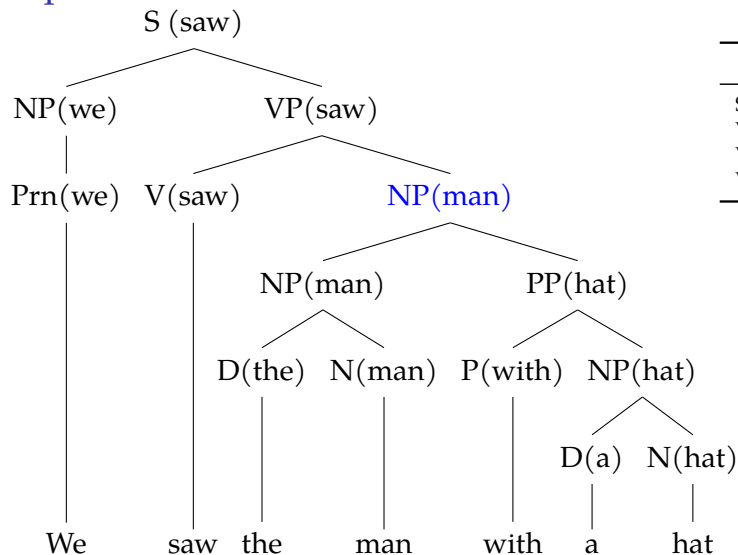
# Solutions to PCFG problems

- Independence assumptions can be relaxed by either
  - Parent annotation
  - Lexicalization
  - Reranking
- To condition on arbitrary/global information: discriminative models
- Most practical PCFG parsers are lexicalized, and often use a re-ranker conditioning on other (global) features

# Lexicalizing PCFGs

- Replace non-terminal  $X$  with  $X(h)$ , where  $h$  is a tuple with the lexical word and its POS tag
- Now the grammar can capture (head-driven) lexical dependencies
- But number of nonterminals grow by  $|V| \times |T|$
- Estimation becomes difficult (many rules, data sparsity)
- Some treebanks (e.g., Penn Treebank) do not annotate heads, they are automatically annotated (based on heuristics)

# Example lexicalized derivation



## Example rules:

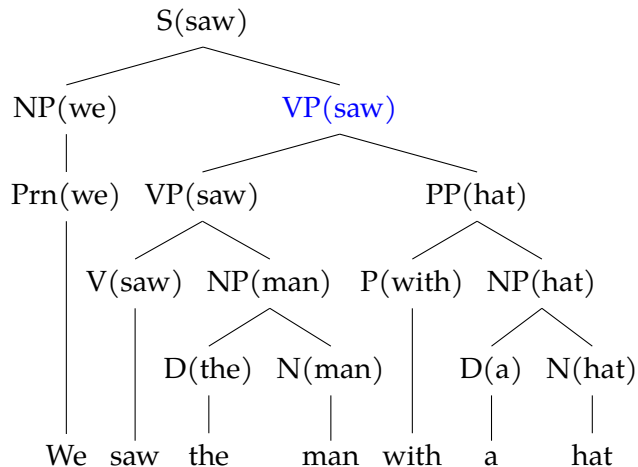
$S(\text{saw}) \rightarrow NP(\text{we}) VP(\text{saw})$

$VP(\text{saw}) \rightarrow V(\text{saw}) NP(\text{man})$

$VP(\text{saw}) \rightarrow VP(\text{saw}) PP(\text{hat})$

$VP(\text{saw}) \rightarrow VP(\text{saw}) PP(\text{telescope})$

# Example lexicalized derivation



## Example rules:

$S(\text{saw}) \rightarrow NP(\text{we}) VP(\text{saw})$

$VP(\text{saw}) \rightarrow V(\text{saw}) NP(\text{man})$

$VP(\text{saw}) \rightarrow VP(\text{saw}) PP(\text{hat})$

$VP(\text{saw}) \rightarrow VP(\text{saw}) PP(\text{telescope})$



# Evaluating the parser output

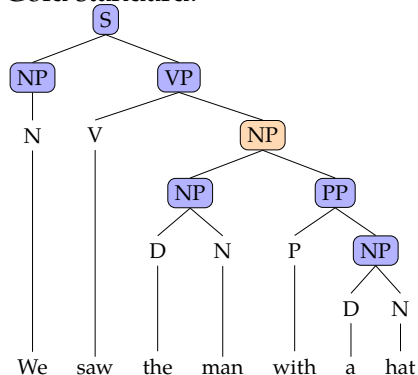
- A parser can be evaluated
  - extrinsically based on its effect on a task (e.g., machine translation) where it is used
  - intrinsically based on the match with ideal parsing
- The typically evaluation (intrinsic) is based on a *gold standard* (GS)
- Exact match is often
  - very difficult to achieve (think about a 50-word newspaper sentence)
  - not strictly necessary (recovering parts of the parse can be useful for many purposes)

## Parser evaluation metrics

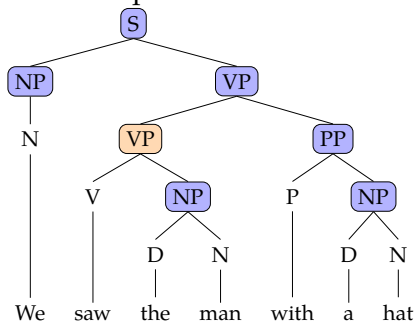
- Common evaluation metrics are (PARSEVAL):
  - precision the ratio of correctly predicted nodes
  - recall the nodes (in GS) that are predicted correctly
  - f-measure harmonic mean of precision and recall  $\left( \frac{2 \times \text{precision} \times \text{recall}}{\text{precision} + \text{recall}} \right)$
- The measures can be
  - unlabeled the spans of the nodes are expected to match
  - labeled the node label should also match
- Crossing brackets (or average non-crossing brackets)
  - ( We ( saw ( them ( with binoculars )))
  - ( We (( saw them ) ( with binoculars )))
- Measures can be averaged per constituent (micro average), or over sentences (macro average)

# PARSEVAL example

Gold standard:



Parser output:



$$\text{precision} = \frac{6}{7} \quad \text{recall} = \frac{6}{7} \quad \text{f-measure} = \frac{6}{7}$$

# Problems with PARSEVAL metrics

- PARSEVAL metrics favor certain type of structures
  - Results are surprisingly well for flat tree structures (e.g., Penn treebank)
  - Results of some mistakes are catastrophic (e.g., low attachment)
- Not all mistakes are equally important for semantic distinctions
- Some alternatives:
  - Extrinsic evaluation
  - Evaluation based on extracted dependencies

# Summary

- PCFG is a simple attempt to augment CFG with probabilities
- PCFG parsing alone is suboptimal: independence assumptions are too strong
- Solutions include (a combination of ) lexicalization, parent annotation and re-ranking
- Reading suggestion: Jurafsky and Martin (2009, Chapter 14)

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Next:

- Dependency grammars and dependency parsing

# Acknowledgments, references, additional reading material



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