Bottom-up Chart Parsing: the CKY algorithm

Parsing
ISCL-BA-06

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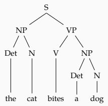
Parsing so far

- Parsing is the task of automatic syntactic analysis
- For most practical purposes, context-free grammars are the most useful formalism for parsing
- We can formulate parsing as
  - Top-down: begin with the start symbol, try to *produce* the input string to be parsed
  - Bottom up: begin with the input, and try to *reduce* it to the start symbol
- Both strategies can be cast as search with backtracking
- Backtracking parsers are inefficient: they recompute sub-trees multiple times

Bottom-up parsing as search



S → NP VP
NP → Det N
VP → V NP
VP → V
Det → a
Det → the
N → cat
N → dog
V → bites
N → bites

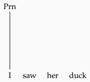
Dealing with ambiguity

I saw her duck

S → NP VP
NP → Prn N
NP → Prn
VP → V NP
VP → V
VP → V S
N → duck
V → duck
V → saw
Prn → I ←
Prn → she
Prn → her

1

Dealing with ambiguity



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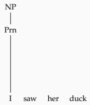

S → NP VP
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NP → Prn
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N → duck
V → duck
V → saw ←
Prn → I
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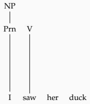
3

Dealing with ambiguity



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VP → V S
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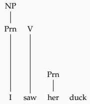
V → saw
Prn → I
Prn → she
Prn → her ←

4

Dealing with ambiguity



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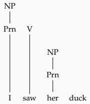
5

Dealing with ambiguity



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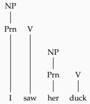
Dealing with ambiguity



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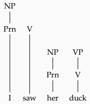
7

Dealing with ambiguity



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NP → Prn
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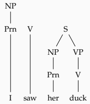
8

Dealing with ambiguity



S → NP VP
NP → Prn N
NP → Prn
VP → V NP
VP → V
VP → V S ←
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V → duck
V → saw
Prn → I
Prn → she
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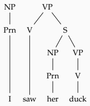
9

Dealing with ambiguity



S $\rightarrow$ NP VP ←
NP $\rightarrow$ Prn N
NP $\rightarrow$ Prn
VP $\rightarrow$ V NP
VP $\rightarrow$ V
VP $\rightarrow$ V S
N $\rightarrow$ duck
V $\rightarrow$ duck
V $\rightarrow$ saw
Prn $\rightarrow$ I
Prn $\rightarrow$ she
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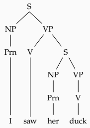
10

Dealing with ambiguity



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NP $\rightarrow$ Prn
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VP $\rightarrow$ V
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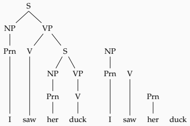
11

Dealing with ambiguity



S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N
NP $\rightarrow$ Prn
VP $\rightarrow$ V NP
VP $\rightarrow$ V
VP $\rightarrow$ V S
N $\rightarrow$ duck ←
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Prn $\rightarrow$ I
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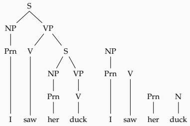
12

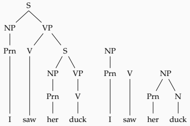
Dealing with ambiguity



S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N ←
NP $\rightarrow$ Prn
VP $\rightarrow$ V NP
VP $\rightarrow$ V
VP $\rightarrow$ V S
N $\rightarrow$ duck
V $\rightarrow$ duck
V $\rightarrow$ saw
Prn $\rightarrow$ I
Prn $\rightarrow$ she
Prn $\rightarrow$ her

13

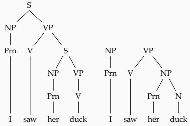
Dealing with ambiguity



S $\rightarrow$ NP VP
NP $\rightarrow$ Prn N
NP $\rightarrow$ Prn
VP $\rightarrow$ V NP ←
VP $\rightarrow$ V
VP $\rightarrow$ V S
N $\rightarrow$ duck
V $\rightarrow$ duck
V $\rightarrow$ saw
Prn $\rightarrow$ I
Prn $\rightarrow$ she
Prn $\rightarrow$ her

14

Dealing with ambiguity



S $\rightarrow$ NP VP ←
NP $\rightarrow$ Prn N
NP $\rightarrow$ Prn
VP $\rightarrow$ V NP
VP $\rightarrow$ V
VP $\rightarrow$ V S
N $\rightarrow$ duck
V $\rightarrow$ duck
V $\rightarrow$ saw
Prn $\rightarrow$ I
Prn $\rightarrow$ she
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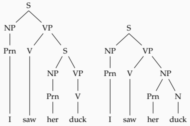
15

Dealing with ambiguity



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NP $\rightarrow$ Prn N
NP $\rightarrow$ Prn
VP $\rightarrow$ V NP
VP $\rightarrow$ V
VP $\rightarrow$ V S
N $\rightarrow$ duck
V $\rightarrow$ duck
V $\rightarrow$ saw
Prn $\rightarrow$ I
Prn $\rightarrow$ she
Prn $\rightarrow$ her

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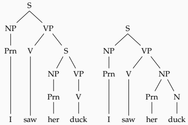
How to represent multiple parses

parse forest grammar



$S_{0:4} \rightarrow NP_{0:1}\ VP_{1:4}$
$NP_{0:1} \rightarrow Prn_{0:1}$
$Prn_{0:1} \rightarrow I_{0:1}$
$VP_{1:4} \rightarrow V_{1:2}\ S_{2:4}$
$V_{1:2} \rightarrow saw_{1:2}$
$S_{2:4} \rightarrow Prn_{2:3}\ V_{3:4}$
$V_{3:4} \rightarrow duck_{3:4}$

$VP_{1:4} \rightarrow V_{1:2}\ NP_{2:4}$
$NP_{2:4} \rightarrow Prn_{2:3}\ N_{3:4}$

CKY algorithm

- The CKY (Cocke–Kasami–Younger) parsing algorithm is a dynamic programming algorithm (Kasami 1965; Younger 1967; Cocke and Schwartz 1970)
- It processes the input *bottom up*, and saves the intermediate results on a *chart*
- Time complexity for *recognition* is $O(n^3)$
- Space complexity is $O(n^2)$
- It requires the CFG to be in *Chomsky normal form* (CNF) (can somewhat be relaxed, but not common)

Chomsky normal form (CNF)

- A CFG is in CNF, if the rewrite rules are in one of the following forms
  - A $\rightarrow$ B C
  - A $\rightarrow$ a

  where A, B, C are non-terminals and a is a terminal
- Any CFG can be converted to CNF
- Resulting grammar is *weakly equivalent* to the original grammar:
  - it generates/accepts the same language
  - but the derivations are different

Converting to CNF: example

S $\rightarrow$ NP VP
S $\rightarrow$ Aux NP VP
NP $\rightarrow$ the N
VP $\rightarrow$ V NP
VP $\rightarrow$ V
N $\rightarrow$ cat
N $\rightarrow$ dog
V $\rightarrow$ bites
N $\rightarrow$ bites

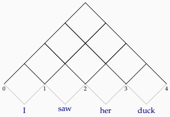
- S $\rightarrow$ Aux NP VP
  S $\rightarrow$ Aux NP VP $\Rightarrow$ S $\rightarrow$ Aux X
  X $\rightarrow$ NP VP
- NP $\rightarrow$ the N
  NP $\rightarrow$ the N $\Rightarrow$ NP $\rightarrow$ X N
  X $\rightarrow$ the
- VP $\rightarrow$ V
  VP $\rightarrow$ V $\Rightarrow$ VP $\rightarrow$ bites

Converting to CNF

1. Eliminate the ϵ rules: if A $\rightarrow \epsilon$ is in the grammar
   - replace any rule B $\rightarrow \alpha$ A β with two rules
     B $\rightarrow \alpha\ \beta$
     B $\rightarrow \alpha$ A′ β
   - add A′ $\rightarrow \alpha$ for all α (except ϵ) whose LHS is A
   - repeat the process for newly created ϵ rules
   - remove the rules with ϵ on the RHS (except S $\rightarrow \epsilon$)
2. Eliminate unit rules: for a rule A $\rightarrow$ B
   - Replace the rule with A $\rightarrow \alpha_1 \mid \ldots \mid \alpha_n$, where $\alpha_1, \ldots, \alpha_n$ are all RHS or rule B
   - Remove the rule A $\rightarrow$ B
   - Repeat the process until no unit rules remain
3. Binarize all the non-binary rules with non-terminal on the RHS: for a rule A $\rightarrow X_1\ X_2 \ldots X_n$:
   - Replace the rule with A $\rightarrow A_1\ X_2 \ldots X_n$, and add $A_1 \rightarrow X_1\ X_2$
   - Repeat the process until all new rules are binary

CKY demonstration

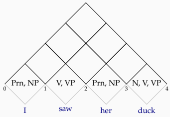
an ambiguous example



S → NP VP
NP → Prn N
VP → V NP
VP → V S
N → duck
VP → duck | saw
V → duck | saw
Prn → I | she | her
NP → I | she | her

CKY demonstration

an ambiguous example



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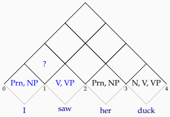
CKY demonstration

an ambiguous example

S → NP VP



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N → duck
VP → duck | saw
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Prn → I | she | her
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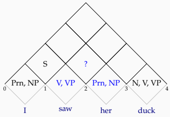
CKY demonstration

an ambiguous example

VP → V NP



S → NP VP
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VP → V S
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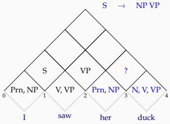
CKY demonstration

an ambiguous example

NP → Prn N
S → NP VP



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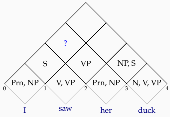
CKY demonstration

an ambiguous example



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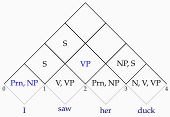
CKY demonstration

an ambiguous example

S → NP VP



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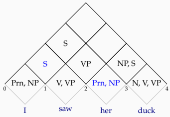
CKY demonstration

an ambiguous example



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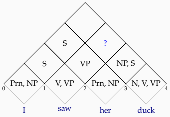
CKY demonstration

an ambiguous example



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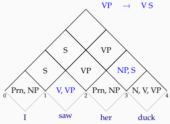
CKY demonstration

an ambiguous example

VP → V NP
VP → V S



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CKY demonstration

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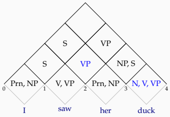

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CKY demonstration

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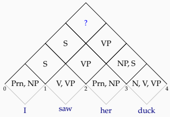

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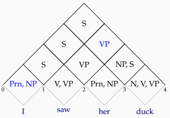
CKY demonstration

an ambiguous example

S → NP VP



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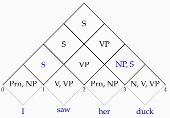

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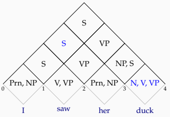

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VP → V S
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VP → duck | saw
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Prn → I | she | her
NP → I | she | her

CKY demonstration: the chart

our chart is a 2D array

NP, Prn	S	S	S
	V, VP	VP	VP
		Prn	NP, S
			V, N, NP

0 she 1 saw 2 her 3 duck 4

Space complexity is $O(n^2)$.

CKY demonstration: the chart

our chart is a 2D array – this is more convenient for programming

S			
S	VP		
S	VP	NP, S	
NP, Prn	V, VP	Prn, NP	V, N, NP

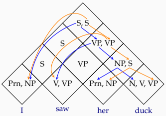
0 she 1 saw 2 her 3 duck 4

Space complexity is $O(n^2)$.

Parsing vs. recognition

- We went through a recognition example
- Note that the algorithm is not directional: it takes the complete input
- Recognition accepts or rejects a sentence based on a grammar
- For parsing, we want to know the derivations that yielded a correct parse
- To recover parse trees, we
  - we follow the same procedure as recognition
  - add back links to keep track of the derivations

Chart parsing example (CKY parsing)



The chart stores a *parse forest* efficiently.

Summary

+ CKY avoids re-computing the analyses by storing the earlier analyses (of sub-spans) in a table
– It still computes lower level constituents that are not allowed by the grammar
– CKY requires the grammar to be in CNF
- CKY has $O(n^3)$ recognition complexity
- For parsing we need to keep track of backlinks
- CKY can efficiently store all possible parses in a chart
- Enumerating all possible parses have exponential complexity (worst case)

Next:

- Top-down chart parsing: Earley algorithm
- Suggested reading: Grune and Jacobs (2007, section 7.2)

Acknowledgments, references, additional reading material

Cocke, John and J. T. Schwartz (1970). *Programming languages and their compilers: preliminary notes.* Tech. rep. Courant Institute of Mathematical Sciences, NYU.

Grune, D. and C.J.H. Jacobs (2007). *Parsing Techniques: A Practical Guide.* second. Monographs in Computer Science. The first edition is available at `http://dickgrune.com/Books/PTAPG_1st_Edition/BookBody.pdf`. Springer New York. ISBN: 9780387689548.

Kasami, Tadao (1965). *An Efficient Recognition and Syntax-Analysis Algorithm for Context-Free Languages.* Tech. rep. DTIC Document.

Younger, Daniel H (1967). "Recognition and parsing of context-free languages in time n 3". In: *Information and control* 10.2, pp. 189–208.