EGR 5110: Homework #4

Due on April 20, 2024 at 11:59pm

Professor Nissenson

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Background

A long rectangular fin is attached to a heat source. The fin is much longer (into the page) than its other dimensions, so heat flow is approximately two-dimensional. Its left side is subjected to a constant base temperature of 100 °C and the other three sides experience convection. The fin's initial temperature is 40 °C and the free stream air temperature is 25 °C.

Below is a cross sectional view of the fin:

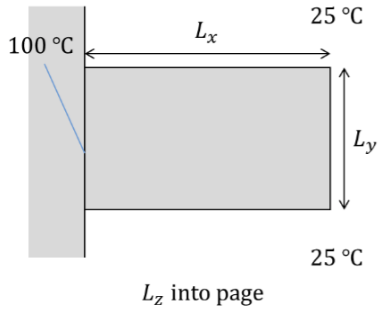

Figure 1: Long Rectangular Fin Attached to Heat Source

The time-dependent temperature distribution is governed by the 2D heat diffusion equation

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{1}$$

where T is temperature and α is the thermal diffusivity coefficient.

Goal: Solve Equation (1) from an initial time t_0 to a final time t_f for the temperature distribution across the 2D rectangular fin in Figure 1 (as a function of time) using a finite-difference method.

Deriving Node Equations

Scenarios

Let's analyze each scenario based on the values of thermal conductivity (k_{cond}), thermal diffusivity (α), and convection coefficient (h) listed in the table below:

Table 1: Five Scenarios Using an Explicit Finite-Difference Method

Scenario	k_{cond} $\left(\frac{\text{W}}{\text{m}\,^\circ\text{C}}\right)$	α $\left(\frac{\text{m}^2}{\text{s}}\right)$	h $\left(\frac{\text{W}}{\text{m}^2\,^\circ\text{C}}\right)$	t_{ss} (min)	T_{avg} tip 1D eqn* (°C)	T_{avg} tip sim* (°C)	$\dot{Q}$ 1D eqn* (W)	$\dot{Q}$ sim* (W)
Pure Al, fan high	240	97×10^{-6}	100	0.93	93.94	94.16	133.31	126.32
Pure Al, fan low	240	97×10^{-6}	10	0.99	99.35	99.37	14.02	13.27
AISI 302	15	4×10^{-6}	100	11.23	52.54	53.57	78.49	74.32
Low k, high α	3	100×10^{-6}	100	0.055	28.77	29.40	37.61	34.43
High k, low α	100	3×10^{-6}	100	27.51	86.72	87.18	124.08	117.68

* The average tip temperature and heat rate are the values at the end of the simulation, which are well past the time when the contour lines stop moving.

We'll look at how adjusting these parameters affects the temperature distribution, time to reach steady-state, and heat rate into the fin.

Scenario 1: Pure Aluminum, Fan High

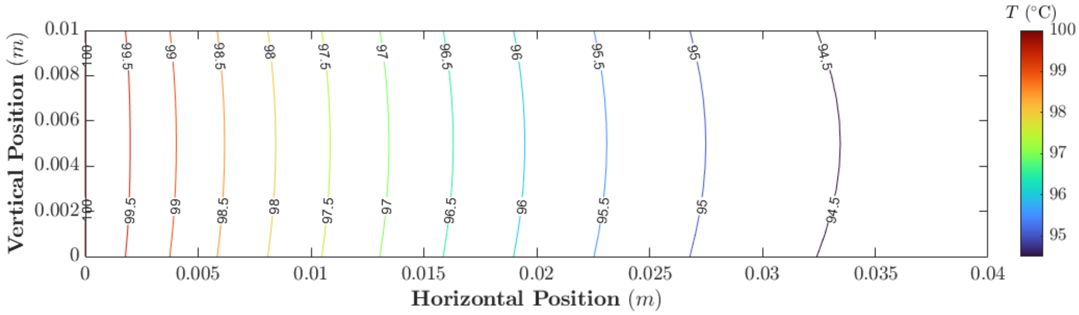

Figure 2: Steady-State Temperature Distrubution for Scenario 1

Simulation Parameters:

- dt = 0.0012
- N_t = 500,000
- $B = 4.167 \times 10^{-4}$
- $\lambda = 0.1164$

Effect of Adjustments:

- **Temperature Distrubution**:
- **Time to Steady State** (t_{ss}):
- **Heat Rate** ($\dot{Q}$):

Scenario 2: Pure Aluminum, Fan Low

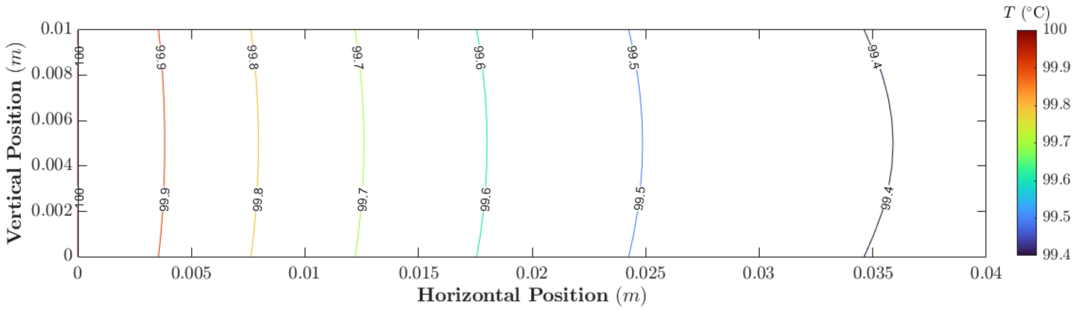

Figure 3: Steady-State Temperature Distrubution for Scenario 2

Simulation Parameters:

- dt = 0.00015
- N_t = 200,000
- $B = 4.167 \times 10^{-4}$
- $\lambda = 0.1455$

Effect of Adjustments:

- **Temperature Distrubution**:
- **Time to Steady State** (t_{ss}):
- **Heat Rate** ($\dot{Q}$):

Scenario 3: Stainless Steel, AISI 302

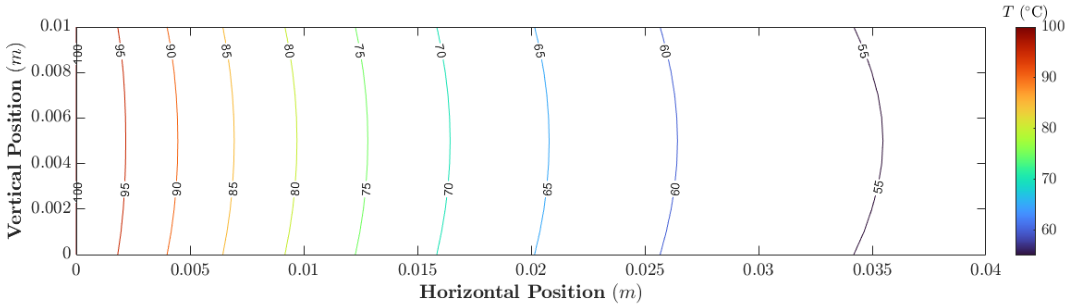

Figure 4: Steady-State Temperature Distrubution for Scenario 3

Simulation Parameters:

- dt = 0.0015
- N_t = 100,000
- $B = 6.667 \times 10^{-3}$
- $\lambda = 0.06$

Effect of Adjustments:

- **Temperature Distrubution**:
- **Time to Steady State** (t_{ss}):
- **Heat Rate** ($\dot{Q}$):

Scenario 4: Low k, high α

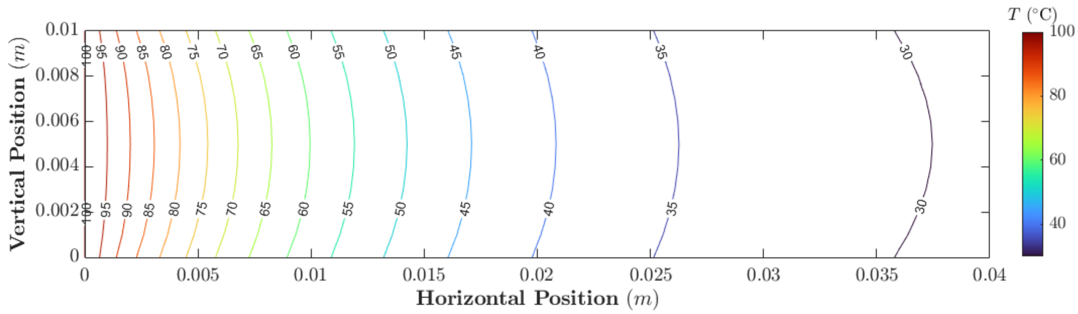

Figure 5: Steady-State Temperature Distrubution for Scenario 4

Simulation Parameters:

- dt = 0.0015
- N_t = 200,000
- B = 0.0333
- λ = 0.15

Effect of Adjustments:

- **Temperature Distrubution**:
- **Time to Steady State** (t_{ss}):
- **Heat Rate** ($\dot{Q}$):

Scenario 5: High k, low α

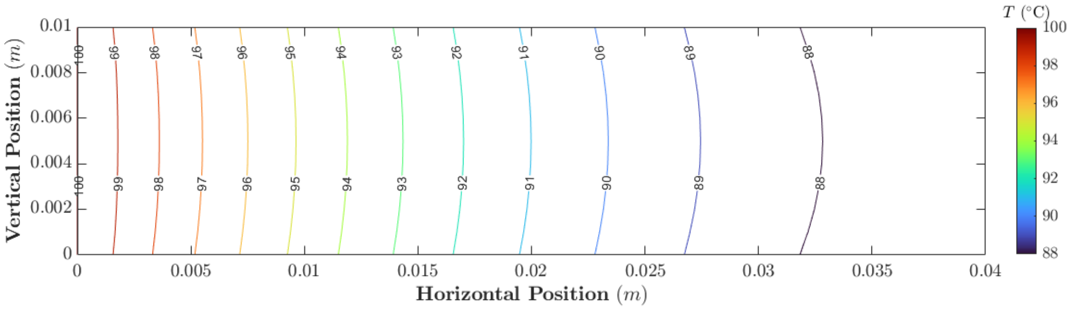

Figure 6: Steady-State Temperature Distrubution for Scenario 5

Simulation Parameters:

- dt = 0.0120
- N_t = 200,000
- B = 0.036
- λ = 0.01

Effect of Adjustments:

- **Temperature Distrubution**:
- **Time to Steady State** (t_{ss}):
- **Heat Rate** ($\dot{Q}$):