EGR 5110: Homework #4

Due on April 20, 2024 at 11:59pm

Professor Nissenson

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Background

A long rectangular fin is attached to a heat source. The fin is much longer (into the page) than its other dimensions, so heat flow is approximately two-dimensional. Its left side is subjected to a constant base temperature of 100 °C and the other three sides experience convection. The fin's initial temperature is 40 °C and the free stream air temperature is 25 °C.

Below is a cross sectional view of the fin:

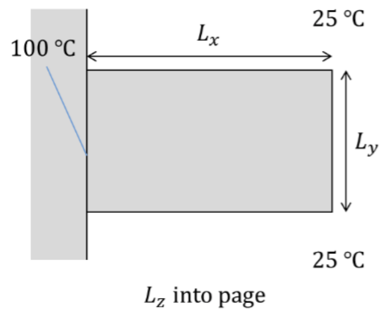

Figure 1: Long Rectangular Fin Attached to Heat Source

The time-dependent temperature distribution is governed by the 2D heat diffusion equation

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{1}$$

where T is temperature and α is the thermal diffusivity coefficient.

Goal: Solve Equation (1) from an initial time t_0 to a final time t_f for the temperature distribution across the 2D rectangular fin in Figure 1 (as a function of time) using a finite-difference method.

Deriving Node Equations

Scenarios

Table 1: Five Scenarios Using an Explicit Finite-Difference Method

Scenario	k_{cond} $\left(\frac{\text{W}}{\text{m}\,^\circ\text{C}}\right)$	α $\left(\frac{\text{m}^2}{\text{s}}\right)$	h $\left(\frac{\text{W}}{\text{m}^2\,^\circ\text{C}}\right)$	t_{ss} (min)	T_{avg} tip 1D eqn* (°C)	T_{avg} tip sim* (°C)	$\dot{Q}$ 1D eqn* (W)	$\dot{Q}$ sim* (W)
Pure Al, fan high	240	97×10^{-6}	100	0.93	93.94	94.16	133.31	126.32
Pure Al, fan low	240	97×10^{-6}	10	0.99	99.35	99.37	14.02	13.27
AISI 302	15	4×10^{-6}	100	11.23	52.54	53.57	78.49	74.32
Low k, high α	3	100×10^{-6}	100	0.055	28.77	29.40	37.61	34.43
High k, low α	100	3×10^{-6}	100	27.51	86.72	87.18	124.08	117.68

* The average tip temperature and heat rate are the values at the end of the simulation, which are well past the time when the contour lines stop moving.

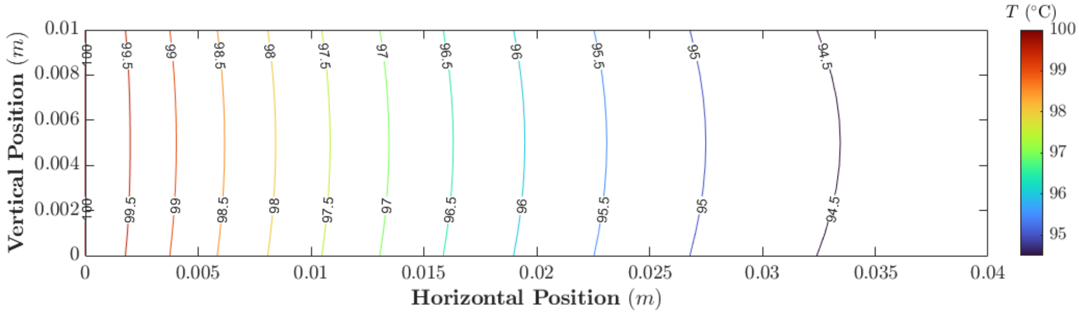

Figure 2: Steady-State Temperature Distrubution for Scenario 1

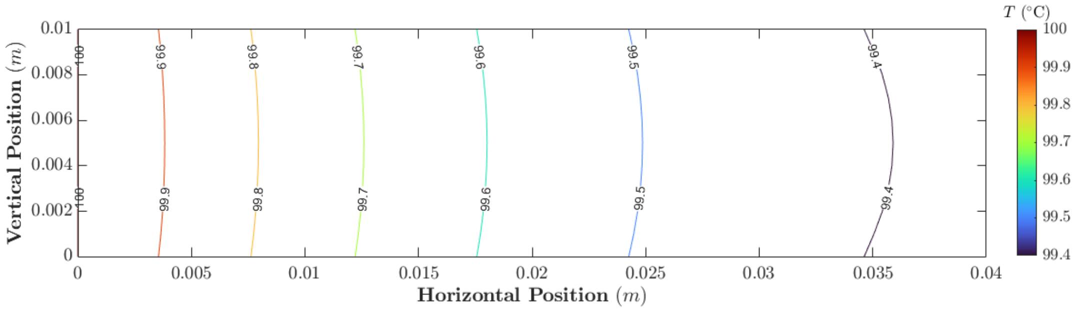

Figure 3: Steady-State Temperature Distrubution for Scenario 2

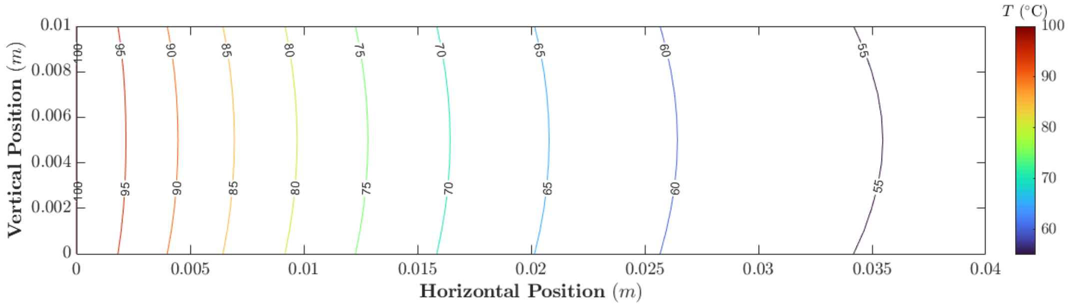

Figure 4: Steady-State Temperature Distrubution for Scenario 3

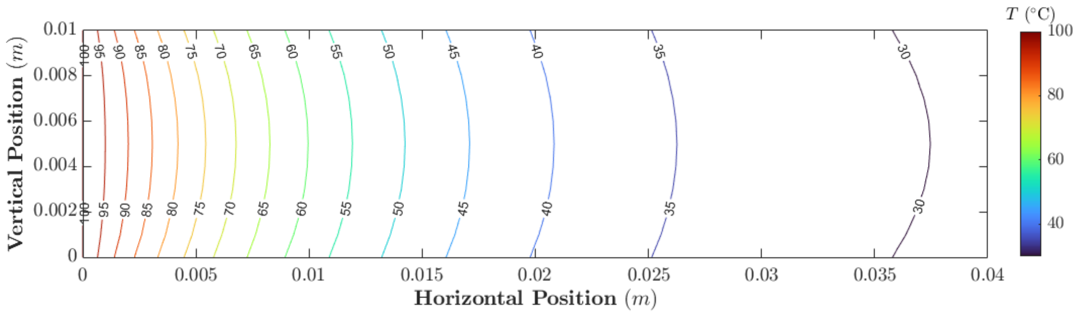

Figure 5: Steady-State Temperature Distrubution for Scenario 4

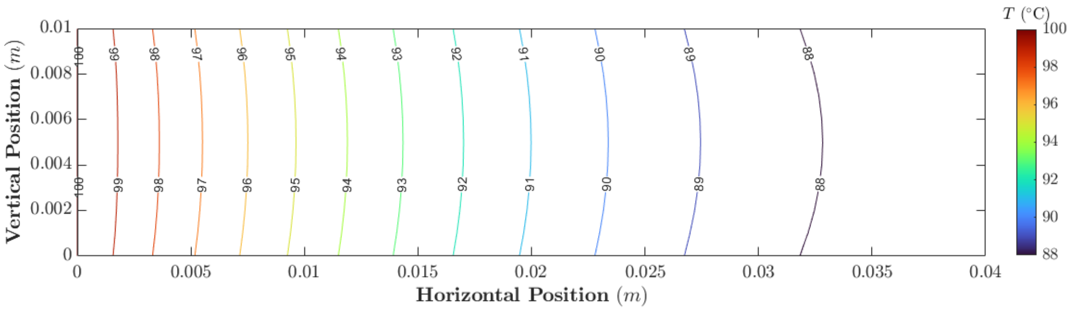

Figure 6: Steady-State Temperature Distrubution for Scenario 5