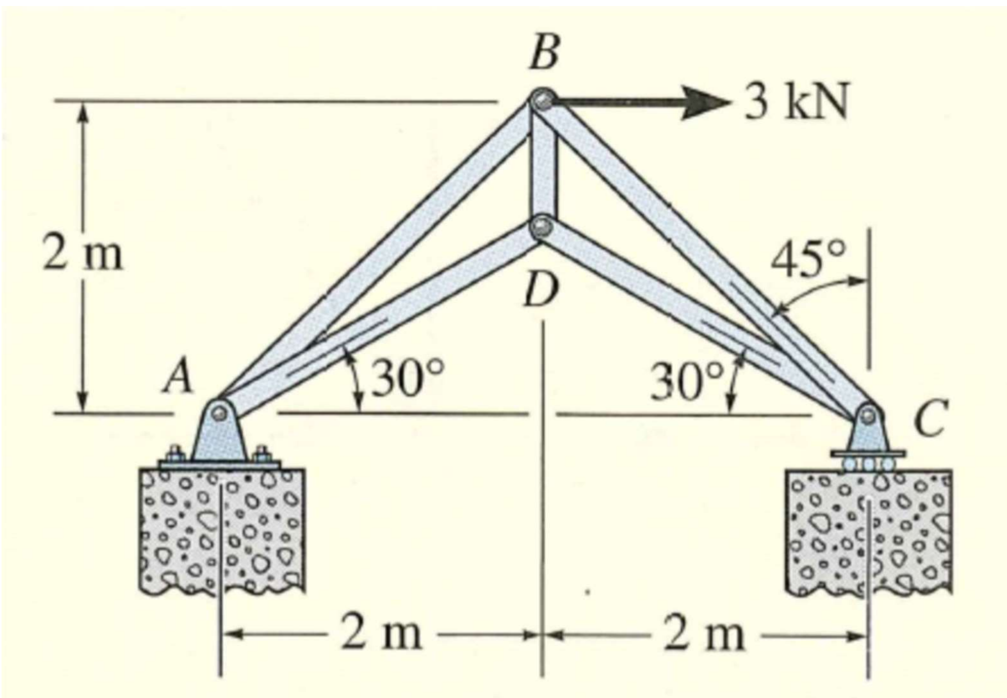
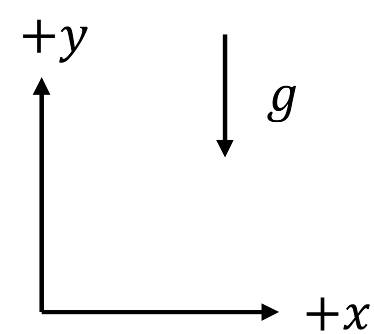
Members: AB, BC, AD, BD, CD (neglect mass of members)
Joints: A (pin support), B, C (roller support), D

Using the method of joints, we will assume members are in tension initially.



Coordinate system:

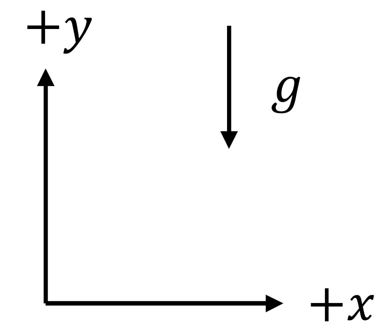


Given:

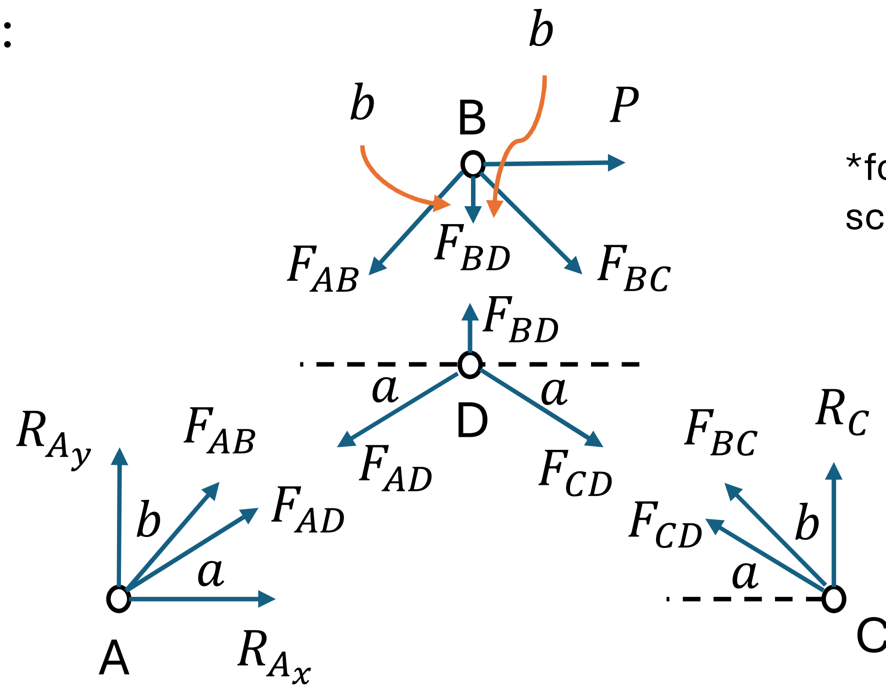
- $P = 3\text{ kN}$ (external force)

Let:

- $a = 30°$
- $b = 45°$



Free Body Diagram:



*force magnitudes not drawn to scale

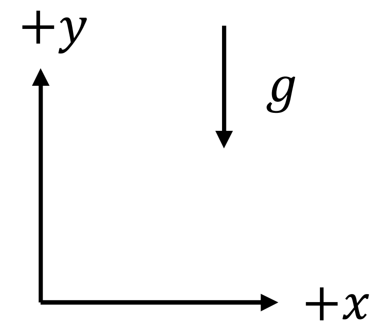
Since the system is in equilibrium:

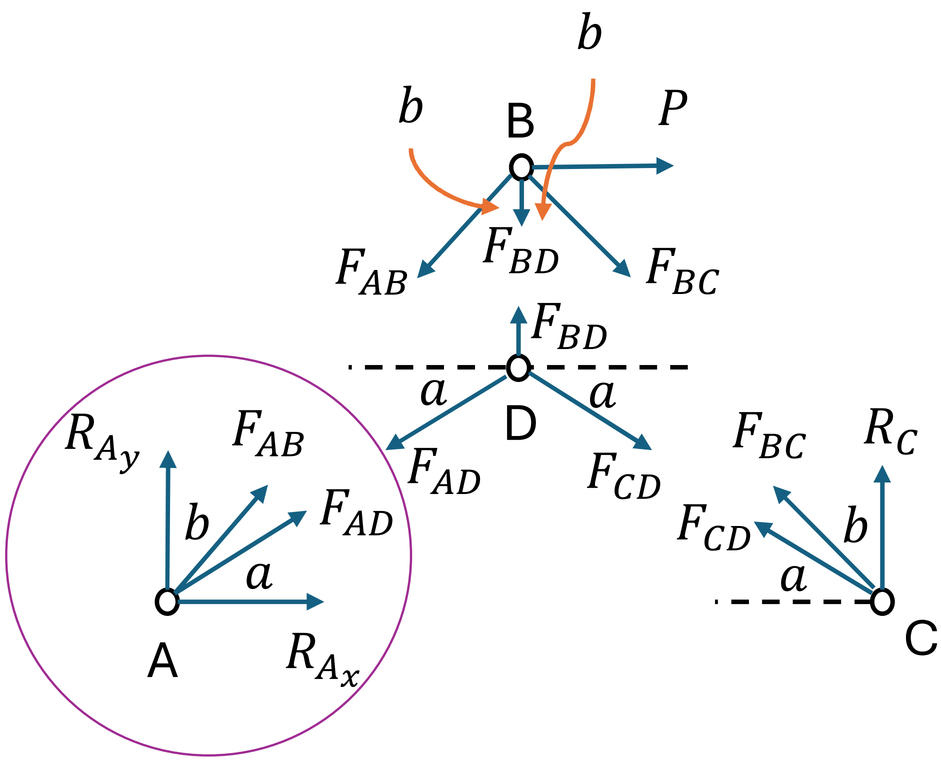
$\sum F_x = 0$ & $\sum F_y = 0$

At joint A:

$$\sum F_x = 0 = R_{A_x} + F_{AD}\cos(a) + F_{AB}\sin(b)$$

$$\sum F_y = 0 = R_{A_y} + F_{AD}\sin(a) + F_{AB}\cos(b)$$

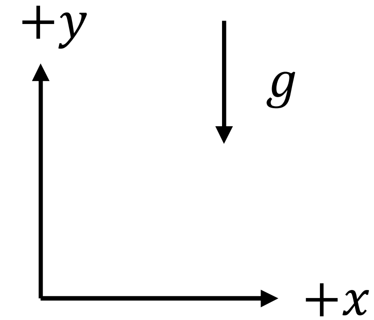


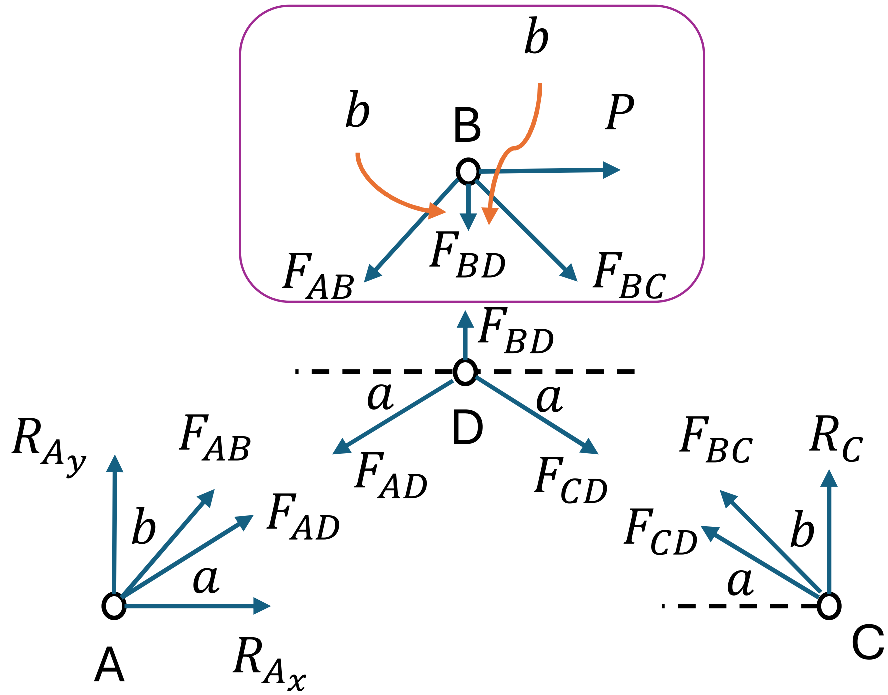


At joint B:

$$\sum F_x = 0 = P + F_{BC}\sin(b) - F_{AB}\sin(b)$$

$$\sum F_y = 0 = -F_{BD} - F_{BC}\cos(b) - F_{AB}\cos(b)$$

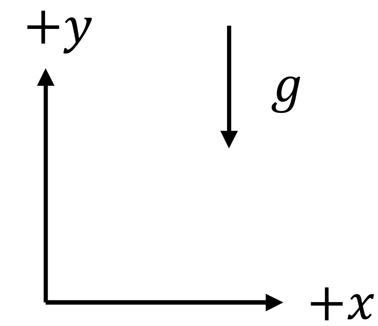


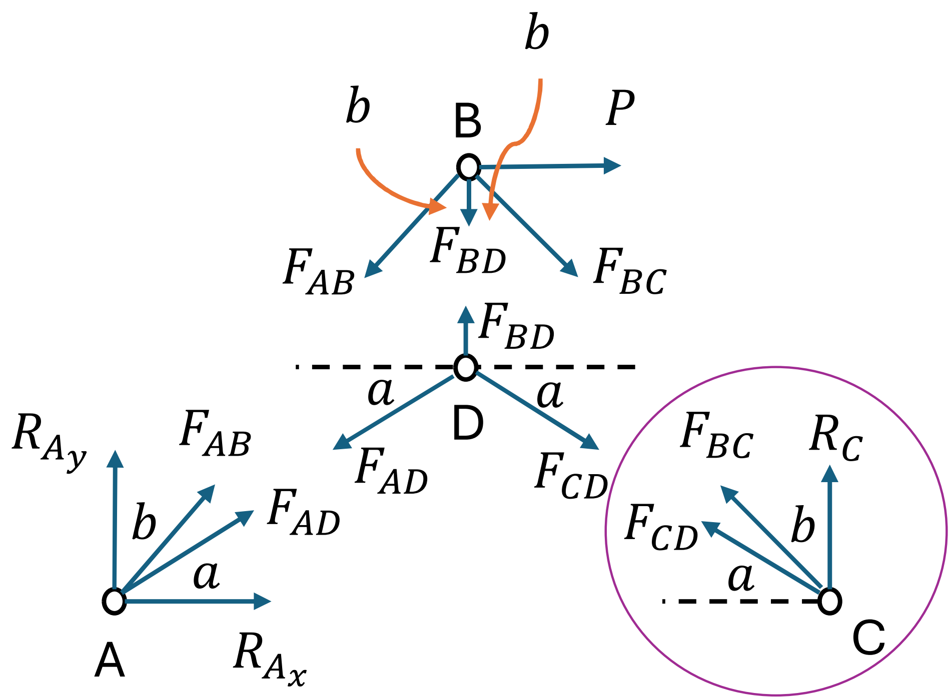


At joint C:

$$\sum F_x = -F_{CD}\cos(a) - F_{BC}\sin(b)$$

$$\sum F_y = 0 = R_C + F_{CD}\sin(a)\,F_{BC}\cos(b)$$

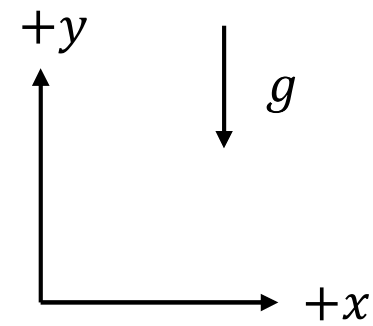


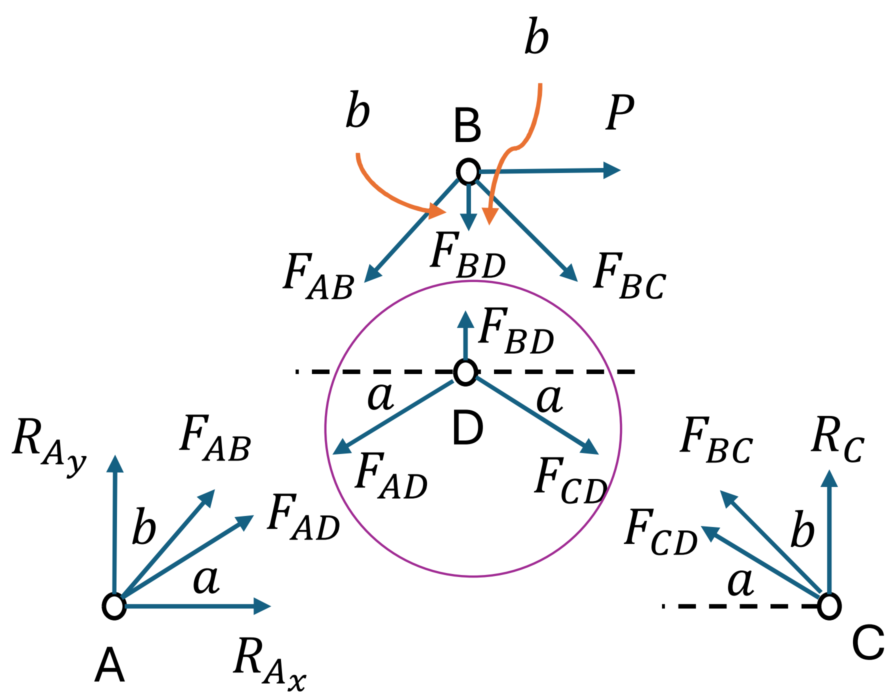


At joint D:

$$\sum F_x = 0 = F_{CD}\cos(a) - F_{AD}\cos(a)$$

$$\sum F_y = 0 = F_{BD} - F_{CD}\sin(a) - F_{AD}\sin(a)$$





Placing the unknown F and R terms on the left and known constants on the right:

$$R_{A_x} + F_{AD}\cos(a) + F_{AB}\sin(b) = 0$$
$$R_{A_y} + F_{AD}\sin(a) + F_{AB}\cos(b) = 0$$

$$F_{BC}\sin(b) - F_{AB}\sin(b) = -P$$
$$-F_{BD} - F_{BC}\cos(b) - F_{AB}\cos(b) = 0$$

$$-F_{CD}\cos(a) - F_{BC}\sin(b) = 0$$
$$R_C + F_{CD}\sin(a)\, F_{BC}\cos(b) = 0$$

$$F_{CD}\cos(a) - F_{AD}\cos(a) = 0$$
$$F_{BD} - F_{CD}\sin(a) - F_{AD}\sin(a) = 0$$

Constructing the vector of unknowns:

$$R_{A_x} + F_{AD}\cos(a) + F_{AB}\sin(b) = 0$$
$$R_{A_y} + F_{AD}\sin(a) + F_{AB}\cos(b) = 0$$

$$F_{BC}\sin(b) - F_{AB}\sin(b) = -P$$
$$-F_{BD} - F_{BC}\cos(b) - F_{AB}\cos(b) = 0$$

$$-F_{CD}\cos(a) - F_{BC}\sin(b) = 0$$
$$R_C + F_{CD}\sin(a)\, F_{BC}\cos(b) = 0$$

$$F_{CD}\cos(a) - F_{AD}\cos(a) = 0$$
$$F_{BD} - F_{CD}\sin(a) - F_{AD}\sin(a) = 0$$

$$\begin{matrix} F_{AB} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{CD} \\ R_C \\ R_{A_x} \\ R_{A_y} \end{matrix}$$

Constructing the vector of constants:

$$R_{A_x} + F_{AD}\cos(a) + F_{AB}\sin(b) = 0$$
$$R_{A_y} + F_{AD}\sin(a) + F_{AB}\cos(b) = 0$$

$$F_{BC}\sin(b) - F_{AB}\sin(b) = -P$$
$$-F_{BD} - F_{BC}\cos(b) - F_{AB}\cos(b) = 0$$

$$-F_{CD}\cos(a) - F_{BC}\sin(b) = 0$$
$$R_C + F_{CD}\sin(a)\, F_{BC}\cos(b) = 0$$

$$F_{CD}\cos(a) - F_{AD}\cos(a) = 0$$
$$F_{BD} - F_{CD}\sin(a) - F_{AD}\sin(a) = 0$$

$$\begin{matrix} 0 \\ 0 \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

Constructing the coefficient matrix:

$$\sin(b)\,F_{AB} + \cos(a)\,F_{AD} + 0F_{BC} + 0F_{BD} + 0F_{DC} + 0R_C + 1R_{A_x} + 0R_{A_y} = 0$$
$$\cos(b)\,F_{AB} + \sin(a)\,F_{AD} + 0F_{BC} + 0F_{BD} + 0F_{DC} + 0R_C + 0R_{A_x} + 1R_{A_y} = 0$$

$$(-\sin(b))F_{AB} + 0F_{AD} + \sin(b)\,F_{BC} + 0F_{BD} + 0F_{DC} + 0R_C + 0R_{A_x} + 0R_{A_y} = -P$$
$$(-\cos(b))F_{AB} + 0F_{AD} + (-\cos(b))F_{BC} + (-1)F_{BD} + 0F_{DC} + 0R_C + 0R_{A_x} + 0R_{A_y} = 0$$

$$0F_{AB} + 0F_{AD} + (-\sin(b))F_{BC} + 0F_{BD} + (-\cos(a))F_{CD} + 0R_C + 0R_{A_x} + 0R_{A_y} = 0$$
$$0F_{AB} + 0F_{AD} + \cos(b)\,F_{BC} + 0F_{BD} + \sin(a)\,F_{CD} + 1R_C + 0R_{A_x} + 0R_{A_y} = 0$$

$$0F_{AB} + (-\cos(a))F_{AD} + 0F_{BC} + 0F_{BD} + \cos(a)\,F_{CD} + 0R_C + 0R_{A_x} + 0R_{A_y} = 0$$
$$0F_{AB} + (-\sin(a))F_{AD} + 0F_{BC} + 1F_{BD} + (-\sin(a))F_{CD} + 0R_C + 0R_{A_x} + 0R_{A_y} = 0$$

$$\begin{matrix}
\sin(b) & \cos(a) & 0 & 0 & 0 & 0 & 1 & 0 \\
\cos(b) & \sin(a) & 0 & 0 & 0 & 0 & 0 & 1 \\
-\sin(b) & 0 & \sin(b) & 0 & 0 & 0 & 0 & 0 \\
-\cos(b) & 0 & -\cos(b) & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -\sin(b) & 0 & -\cos(a) & 0 & 0 & 0 \\
0 & 0 & \cos(b) & 0 & \sin(a) & 1 & 0 & 0 \\
0 & -\cos(a) & 0 & 0 & \cos(a) & 0 & 0 & 0 \\
0 & -\sin(a) & 0 & 1 & -\sin(a) & 0 & 0 & 0
\end{matrix}$$

Writing the system of linear equations in matrix form $\mathbf{AX} = \mathbf{B}$:

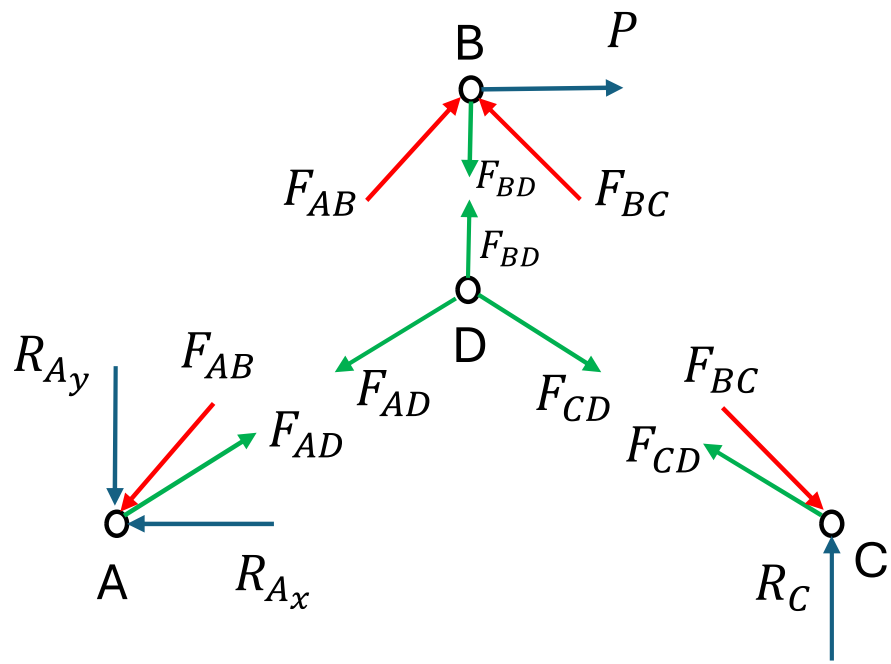
$$\underbrace{\begin{pmatrix} \sin(b) & \cos(a) & 0 & 0 & 0 & 0 & 1 & 0 \\ \cos(b) & \sin(a) & 0 & 0 & 0 & 0 & 0 & 1 \\ -\sin(b) & 0 & \sin(b) & 0 & 0 & 0 & 0 & 0 \\ -\cos(b) & 0 & -\cos(b) & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sin(b) & 0 & -\cos(a) & 0 & 0 & 0 \\ 0 & 0 & \cos(b) & 0 & \sin(a) & 1 & 0 & 0 \\ 0 & -\cos(a) & 0 & 0 & \cos(a) & 0 & 0 & 0 \\ 0 & -\sin(a) & 0 & 1 & -\sin(a) & 0 & 0 & 0 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{CD} \\ R_C \\ R_{A_x} \\ R_{A_y} \end{pmatrix}}_{\mathbf{X}} = \underbrace{\begin{pmatrix} 0 \\ 0 \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{\mathbf{B}}$$

Solving the system of equations using the GaussElim.m function that I created in MATLAB:

```
>> a = 30*pi/180; b = 45*pi/180; P = 3;

>> A = [sin(b) cos(a)   0      0     0     0 1 0;
        cos(b) sin(a)   0      0     0     0 0 1;
       -sin(b)   0    sin(b)  0     0     0 0 0;
       -cos(b)   0   -cos(b) -1     0     0 0 0;
          0      0   -sin(b)  0 -cos(a)  0 0 0;
          0      0    cos(b)  0  sin(a)  1 0 0;
          0   -cos(a)   0      0  cos(a)  0 0 0;
          0   -sin(a)   0      1 -sin(a)  0 0 0];

>> B =  [0; 0; -P; 0; 0; 0; 0; 0];

>> X = GaussElim(A,B)

     → X = -0.7765
            4.0981
           -5.0191
            4.0981
            4.0981
            1.5000
           -3.0000
           -1.5000
```

Summarizing the results:

$$\mathbf{X} = \begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{CD} \\ R_C \\ R_{A_x} \\ R_{A_y} \end{pmatrix} = \begin{pmatrix} -0.7765 \\ 4.0981 \\ -5.0191 \\ 4.0981 \\ 4.0981 \\ 1.5000 \\ -3.0000 \\ -1.5000 \end{pmatrix} \text{ kN}$$



Initially, it was assumed that all members were in tension. It turns out that the forces F_{AB}, F_{BC} and the reactions R_{A_x}, R_{A_y} point in the opposite direction. This results in members AB and BC being in **compression**, while members AD, BD, and CD are in **tension**.