0.0 Topic

$$x + y \quad \text{some example equation related to the topic} \tag{1}$$

Mynd? Always try to have a visual representation of the equation

Topic: describes. minute details about the equation

Theorem: *Definition of "Topic"* theorem + tab

Uses: What is it be used for in linear algebra

Example: "We can use **this** to calculate **that**", followed by an example

Mynd? Always try to have a visual along with the example

Exercise: Exercise for the reader

Revision

0.1 LINEAR EQUATIONS

$$x + 2 = y \tag{2}$$

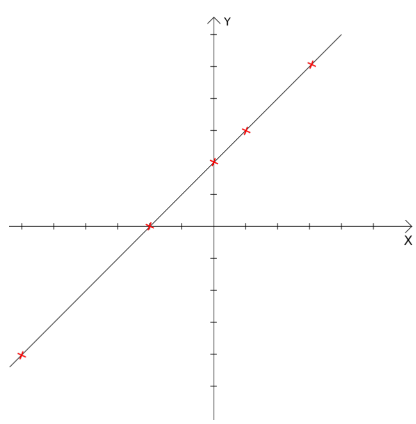
This is an example of a **linear equation**, one that has two variables, x and y, and it describes how the value of one of the variables depends on the value of the other variable.

What makes it **linear** is that every variable is only raised to the first power, so *this*

$$x^2 + 2 = y$$

is **not** a linear equation.

Lets plot (draw) equation (2), for a few diferent values of x, say, -6, -2, 1 and 5.



As you can see, the graph of this equation is a **straight line**, which is true for all linear equations

0.1 Linear equation with multiple variables

$$x + 9y + z = 3 \tag{3}$$

Lets now extend our definition of a linear equation to include more variables. (2) had only x and y, but (3) has 3 variables, x, y and z.
The 9 in front of the y? That is called a *coefficient*, and the 3 on the right-hand-side is called a *constant*

Because we will (eventually) run out of letters in the alphabet, we write our equations like this:

$$c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n = b$$

Here the c's are the *coefficients*, the x's are the *variables* and the b is (spoiler) the *constant* .
It may look complex the first time, but you'll get used to reading equations like this.

Lets look at an example: Jimmy goes to the store to buy cokes, snickers and apples. Jimmy knows that a can of coke is $2.2\$$, snickers is $1.6\$$ and an apple is $3.0\$$
If c_1 is price of coke, c_2 is price of snickers and c_3 is price of apples, our equation would look like this

$$2.2x_1 + 1.6x_2 + 3.0x_3 = b$$

Jimmy needs a couple of cokes (x_1) and four apples (x_3). Jimmy has $20\$$.
How many snickers bars can he buy with the leftover money?

$$2.2 \cdot 2 + 1.6x_2 + 3.0 \cdot 4 = 20$$

Do the math and help Jimmy get his snickers by solving for x_2.

0.1 System of equations

$$2x_1 + x_2 = 17$$
$$x_1 + 3x_2 = 26$$

The equations above are an example of a **system of equations**.

We want to solve these systems by finding the correct values for the *variables*, in this case, x_1 and x_2, so that both equations work out.

We could start by *guessing* that $x_1 = 3$ and $x_2 = 5$, which would give us

$$2 \cdot 3 + 2 = 8 \neq 17$$
$$3 + 3 \cdot 5 = 18 \neq 26$$

This is far from correct, we need the **the substitution method**.

The substitution method consists of **two** steps, that you use over and over again until the system has been solved. These steps are:

1. Isolating a variable
2. Substitution
3. Simplification

Lets take another look at our system

$$2x_1 + x_2 = 17 \tag{4}$$
$$x_1 + 3x_2 = 26 \tag{5}$$

and solve it using the substitution method.

1. isolate the x_1 from (4), $x_1 = 26 - 3x_2$
2. substitute x_1 into (3), $2(26 - 3x_2) + x_2 = 17$
3. now the system looks like

$$2(26 - 3x_2) + x_2 = 17 \tag{6}$$
$$x_1 + 3x_2 = 26 \tag{7}$$

4. which simplifies to

$$x_2 = 7 \tag{8}$$
$$x_1 + 3x_2 = 26 \tag{9}$$

5. now we just insert x_2 into (8) to get

$$x_2 = 7 \tag{10}$$
$$x_1 + 21 = 26 \tag{11}$$

6. so (8) simplifies to $\mathbf{x_1 = 5}$

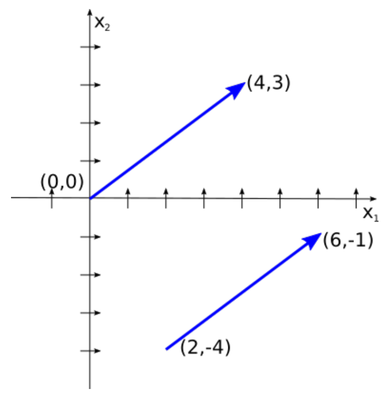
Now its your turn to solve the following

$$6x_1 + 2x_2 = 70$$
$$3x_1 + 3x_2 = 45$$

Vectors

0.2 Vector properties

$$\vec{v} = \begin{bmatrix} 4 & 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$



Vectors: Its easy to think of vectors as a **length** and a **direction**, usually denoted $v = [x_1 x_2 x_3 \dots x_n]$.

Above we have an example of a (**2-dimensional**) vector, written both as a row vector and a column vector. The vector represents a "travel" by 4 steps along the x_1 axis and 3 steps along the x_2 axis. So if you find yourself positioned at the point $(2, -4)$ and someone "applies" this vector to you, you'll be moved to $(2 + 4, -4 + 3) = (6, -1)$.

Vector operations: The length of a vector is written as $|\vec{v}|$ and found with the Pythagorean-theorem:

$$|\vec{v}| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

and its direction is written $\theta_{\vec{v}}$ and calculated using absolute classic geometry

$$\theta_{\vec{v}} = \tan^{-1}\frac{3}{4}$$

To **transpose** a vector, $\vec{v}^{\mathsf{T}}$ is simply to change it from a row vector to a column vector and vice-versa

$$\begin{bmatrix} 4 & 3 \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

Example:

Exercise: asd

0.2 Elementary vector operations

$10 \cdot [3 \quad 5]$	scalar multiplication
$[3 \quad 5]^{\mathsf{T}}$	transpose
$[2 \quad 1] + [1 \quad 2]$	addition
$[2 \quad 1] - [1 \quad 2]$	subtraction

Operations: These 4 operations are the most elementary and common operations you will be using in linear algebra. They are

- Scalar multiplication simply multiplies every element of the vector with the scalar

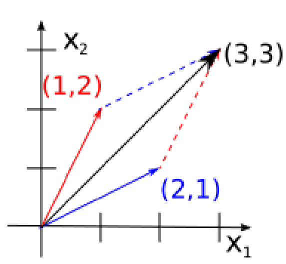
$$10 \cdot [3 \quad 5] = [30 \quad 50]$$

- Transposing a vector is simply converting it from a row-vector to a colum-vector, or a column-vector back to a row-vector

$$[3 \quad 5]^{\mathsf{T}} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

- Adding two vectors is exactly how you would think

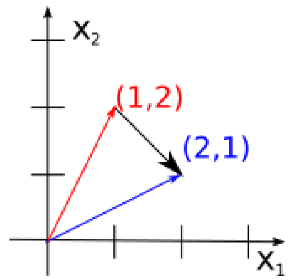
$$[1 \quad 2] + [2 \quad 1] = [3 \quad 3]$$



- Subtraction, like addition, is just how you'd think

$$[2 \quad 1] - [1 \quad 2] = [1 \quad -1]$$

however, drawing subtraction is a bit more involved than addition:



Exercice: Here are 3 vectors, $u = [3 \quad 5], v = [8 \quad 10], p = [10 \quad 1, 1]$, can you add them together?