

0.0 TOPIC

$$x + y \quad \text{some example equation related to the topic} \quad (1)$$

Mynd? Always try to have a visual representation of the equation

Topic: describes. minute details about the equation

Theorem: *Definition of "Topic"* theorem + tab

Uses: What is it be used for in linear algebra

Example: "We can use **this** to calculate **that**", followed by an example

Mynd? Always try to have a visual along with the example

Exercise: Exercise for the reader

Revision

0.1 LINEAR EQUATIONS

$$x + 2 = y \quad (2)$$

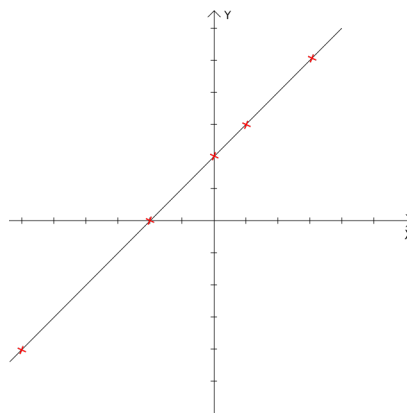
This is an example of a **linear equation**, one that has two variables, x and y , and it describes how the value of one of the variables depends on the value of the other variable.

What makes it **linear** is that every variable is only raised to the first power, so *this*

$$x^2 + 2 = y$$

is **not** a linear equation.

Lets plot (draw) equation (2), for a few diferent values of x , say, -6 , -2 , 1 and 5 .



As you can see, the graph of this equation is a **straight line**, which is true for all linear equations

0.1 LINEAR EQUATION WITH MULTIPLE VARIABLES

$$x + 9y + z = 3 \quad (3)$$

Lets now extend our definition of a linear equation to include more variables. (2) had only x and y , but (3) has 3 variables, x , y and z .

The 9 in front of the y ? That is called a *coefficient*, and the 3 on the right-hand-side is called a *constant*

Because we will (eventually) run out of letters in the alphabet, we write our equations like this:

$$c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n = b$$

Here the c 's are the *coefficients*, the x 's are the *variables* and the b is (spoiler) the *constant*.

It may look complex the first time, but you'll get used to reading equations like this.

Lets look at an example: Jimmy goes to the store to buy cokes, snickers and apples. Jimmy knows that a can of coke is 2.2\$, snickers is 1.6\$ and an apple is 3.0\$

If c_1 is price of coke, c_2 is price of snickers and c_3 is price of apples, our equation would look like this

$$2.2x_1 + 1.6x_2 + 3.0x_3 = b$$

Jimmy needs a couple of cokes (x_1) and four apples (x_3). Jimmy has 20\$.

How many snickers bars can he buy with the leftover money?

$$2.2 \cdot 2 + 1.6x_2 + 3.0 \cdot 4 = 20$$

Do the math and help Jimmy get his snickers by solving for x_2 .

0.1 SYSTEM OF EQUATIONS

$$2x_1 + x_2 = 17$$

$$x_1 + 3x_2 = 26$$

The equations above are an example of a **system of equations**.

We want to solve these systems by finding the correct values for the *variables*, in this case, x_1 and x_2 , so that both equations work out.

We could start by *guessing* that $x_1 = 3$ and $x_2 = 5$, which would give us

$$2 \cdot 3 + 2 = 8 \neq 17$$

$$3 + 3 \cdot 5 = 18 \neq 26$$

This is far from correct, we need the **the substitution method**.

The substitution method consists of **two** steps, that you use over and over again until the system has been solved. These steps are:

1. Isolating a variable

2. Substitution

3. Simplification

Lets take another look at our system

$$2x_1 + x_2 = 17 \tag{4}$$

$$x_1 + 3x_2 = 26 \tag{5}$$

and solve it using the substitution method.

1. isolate the x_1 from (4), $x_1 = 26 - 3x_2$
2. substitute x_1 into (3), $2(26 - 3x_2) + x_2 = 17$
3. now the system looks like

$$2(26 - 3x_2) + x_2 = 17 \quad (6)$$

$$x_1 + 3x_2 = 26 \quad (7)$$

4. which simplifies to

$$x_2 = 7 \quad (8)$$

$$x_1 + 3x_2 = 26 \quad (9)$$

5. now we just insert x_2 into (8) to get

$$x_2 = 7 \quad (10)$$

$$x_1 + 21 = 26 \quad (11)$$

6. so (8) simplifies to $x_1 = 5$

Now its your turn to solve the following

$$6x_1 + 2x_2 = 70$$

$$3x_1 + 3x_2 = 45$$

Vectors

0.2 VECTOR PROPERTIES

$$\vec{v} = \begin{bmatrix} 4 & 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

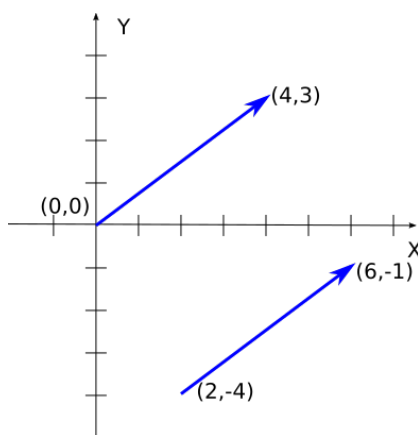
Here we have an example of a **(2-dimensional)** vector, written both as a **row vector** and a **column vector**. All that a vector is is a **length** and a **direction**

The length of a vector is written as $|\vec{v}|$ and found with the Pythagorean-theorem:

$$|\vec{v}| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

and its direction is written $\theta_{\vec{v}}$ and calculated using absolute classic geometry

$$\theta_{\vec{v}} = \tan^{-1} \frac{3}{4}$$



We can easily draw 2-dimensional vectors on an xy-plane. Here the vector \vec{v} appears twice, one starting from $(0,0)$ and the other from $(2,-4)$.