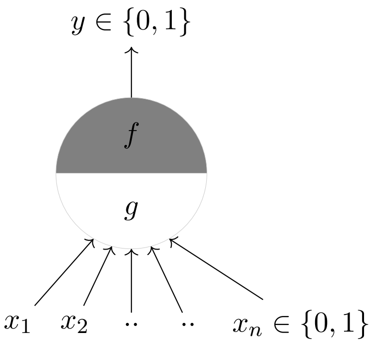
Module 2.2: McCulloch Pitts Neuron



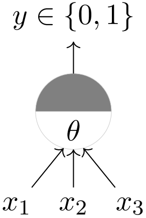
- McCulloch (neuroscientist) and Pitts (logician) proposed a highly simplified computational model of the neuron (1943)
- g aggregates the inputs and the function f takes a decision based on this aggregation
- The inputs can be excitatory or inhibitory
- $y = 0$ if any x_i is inhibitory, else

$$g(x_1, x_2, ..., x_n) = g(\mathbf{x}) = \sum_{i=1}^{n} x_i$$

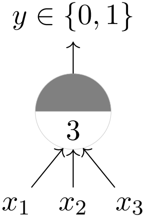
$$\begin{aligned} y = f(g(\mathbf{x})) &= 1 \quad if \quad g(\mathbf{x}) \geq \theta \\ &= 0 \quad if \quad g(\mathbf{x}) < \theta \end{aligned}$$

- θ is called the thresholding parameter
- This is called Thresholding Logic

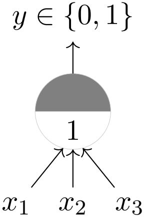
Let us implement some boolean functions using this McCulloch Pitts (MP) neuron ...



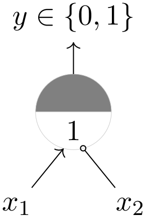
A McCulloch Pitts unit



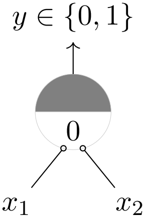
AND function



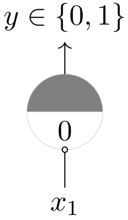
OR function



x_1 AND $!x_2$*



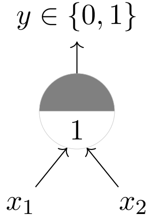
NOR function



NOT function

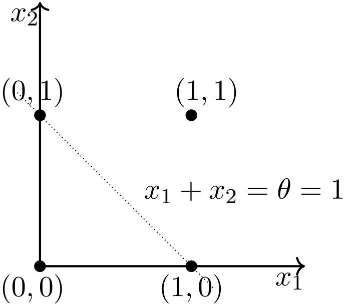
*circle at the end indicates inhibitory input: if any inhibitory input is 1 the output will be 0

- Can any boolean function be represented using a McCulloch Pitts unit ?
- Before answering this question let us first see the geometric interpretation of a MP unit ...

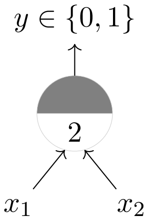


OR function

$x_1 + x_2 = \sum_{i=1}^{2} x_i \geq 1$

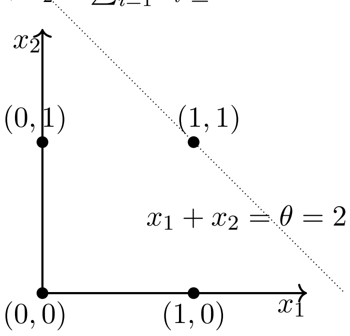


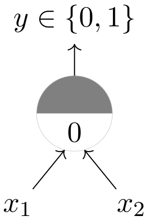
- A single MP neuron splits the input points (4 points for 2 binary inputs) into two halves
- Points lying on or above the line $\sum_{i=1}^{n} x_i - \theta = 0$ and points lying below this line
- In other words, all inputs which produce an output 0 will be on one side ($\sum_{i=1}^{n} x_i < \theta$) of the line and all inputs which produce an output 1 will lie on the other side ($\sum_{i=1}^{n} x_i \geq \theta$) of this line
- Let us convince ourselves about this with a few more examples (if it is not already clear from the math)



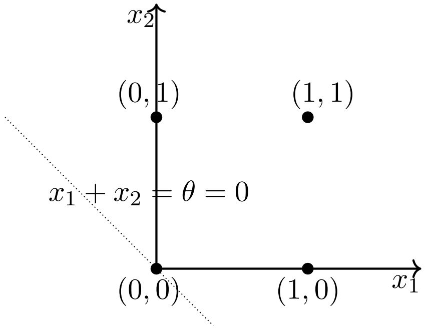
AND function

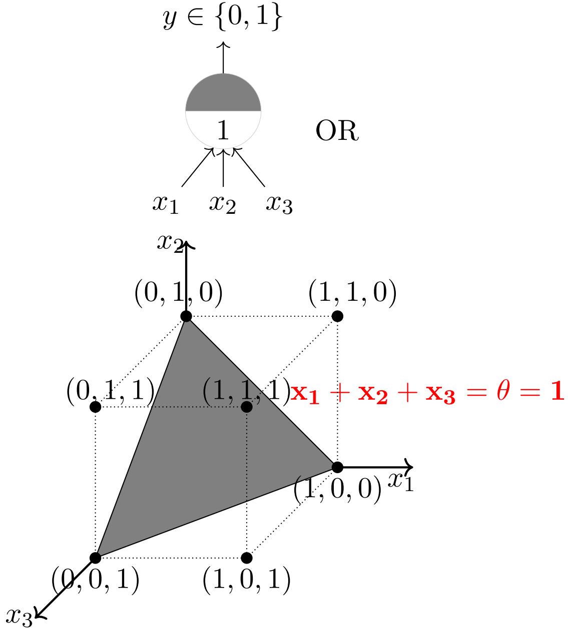
$x_1 + x_2 = \sum_{i=1}^{2} x_i \geq 2$





Tautology (always ON)





- What if we have more than 2 inputs?
- Well, instead of a line we will have a plane
- For the OR function, we want a plane such that the point (0,0,0) lies on one side and the remaining 7 points lie on the other side of the plane

The story so far ...

- A single McCulloch Pitts Neuron can be used to represent boolean functions which are linearly separable
- Linear separability (for boolean functions) : There exists a line (plane) such that all inputs which produce a 1 lie on one side of the line (plane) and all inputs which produce a 0 lie on other side of the line (plane)