Typing Rule of Baremetalisp

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1 Introduction

In this paper, I will formally describe the typing rule of Baremetalisp, which is a well typed Lisp for trusted execution environment.

2 Notation and Syntax

Table 1 and Fig. 1 shows notation used in this paper and syntax for the typing rule, respectively.

Listing 1: Example of variable and type

```
(defun add (a b) (Pure (-> (Int Int) Int))
    (+ a b))
```

x is a variable. For example, $x \in \{a, b\}$ in Listing 1. $\mathcal{T}$ is a type. For example, $\mathcal{T} \in \{\mathtt{Int}, (\rightarrow\ (\mathtt{Int}\ \mathtt{Int})\ \mathtt{Int})\}$ in Listing 1. $(\rightarrow\ (\mathtt{Int}\ \mathtt{Int})\ \mathtt{Int})$ is a function type which takes 2 integer values and return 1 integer value. `Pure` in Listing 1 denotes the effect of the function but I just ignore it now. Function effects will be described in Sec. 4. $\mathcal{T}$ can be other forms as described in Fig. 1 such as `Bool`, $'(\mathtt{Int})$, $[\mathtt{Bool}\ \mathtt{Int}]$, $(\text{List}\ a)$, $(\text{List}\ \mathtt{Int})$. C is a type constraint, which is a set of pairs of types. For example, $C = \{(\rightarrow\ (t_1\ t_2)\ t) = (\rightarrow\ (\mathtt{Int}\ \mathtt{Int})\ \mathtt{Int})\}$ deduced from Listing 1 means $(\rightarrow\ (t_1\ t_2)\ t)$ and $(\rightarrow\ (\mathtt{Int}\ \mathtt{Int})\ \mathtt{Int})$ are semantically equal and every type variable in C, t_1, t_2, t, is thus `Int`. Γ is a map from variable and label to type. For example, $\Gamma = \{a : t_1, b : t_2, + : (\rightarrow\ (\mathtt{Int}\ \mathtt{Int})\ \mathtt{Int})\}$ in Listing 1. Γ is called context generally, thus I call Γ context in this paper.

Listing 2: Example of user defined data type

```
(data (List a)
    (Cons a (List a))
    Nil)
```

t is a type variable. For example, $t \in \{a\}$ in Listing 2. L is a label for user defined type. For example, $L \in \{\text{Cons}, \text{Nil}\}$ in Listing 2. D is user defined data. For example, $D \in \{\text{List}\}$ in Listing 2. Γ will hold mapping from labels in

Table 1: Notation

$A \Rightarrow B$	logical implication (if A then B)
e	expression
z	integer literal such as 10, -34, 112
x	variable
t	type variable
D	type name of user defined data
L	label of user defined data
E	effect
$E_{\mathcal{T}} : T \rightarrow E$	effect of type
$\mathcal{T}$	type
C	type constraint
$io(C) : C \rightarrow \texttt{Bool}$	does C contain `IO` functions?
Γ	context
$\mathcal{P}$	pattern
$\mathcal{P}_{let}$	pattern of let expression
$\mathcal{T}_1 \equiv_\alpha \mathcal{T}_2$	$\mathcal{T}_1$ and $\mathcal{T}_2$ are α-equivalent
$\mathcal{S} : t \rightarrow \mathcal{T}$	substitution from type variable to type
$\mathcal{T} \cdot \mathcal{S}$	apply $\mathcal{S}$ to $\mathcal{T}$
$\mathcal{X}$	set of t
$FV_{\mathcal{T}} : \mathcal{T} \rightarrow \mathcal{X}$	function from $\mathcal{T}$ to its free variables
$FV_{\Gamma} : \Gamma \rightarrow \mathcal{X}$	function from Γ to its free variables
$Size : L \rightarrow \texttt{Int}$	the number of labels L's type has
$\Gamma \vdash e : \mathcal{T} \mid_{\mathcal{X}} C$	e's type is deduced as $\mathcal{T}$ from Γ under constraint C and type variables $\mathcal{X}$

addition to variables. For example, $\Gamma = \{\text{Cons} : (\text{List } a), \text{Nil} : (\text{List } a), \text{Cons}_{1st} : a, \text{Cons}_{2nd} : (\text{List } a)\}$ in Listing 2.

$FV_{\mathcal{T}}$ and FV_{Γ} are functions, which take $\mathcal{T}$ and Γ and return free variables. For example, $FV_{\mathcal{T}}((\rightarrow\ (t_1\ t_2)\ t)) = \{t_1, t_2, t\}$ and

$$\begin{aligned}
&FV_{\Gamma}(\{a : t_1, b : t_1, + : (\rightarrow\ (\texttt{Int Int})\ \texttt{Int})\}) \\
&= \{FV_{\mathcal{T}}(t_1), FV_{\mathcal{T}}(t_1), FV_{\mathcal{T}}((\rightarrow\ (\texttt{Int Int})\ \texttt{Int}))\} \\
&= \{t_1, t_2\}.
\end{aligned}$$

$\mathcal{T}_1 \equiv_\alpha \mathcal{T}_2$ denotes that $\mathcal{T}_1$ and $\mathcal{T}_2$ are α-equivalent, which means $\mathcal{T}_1$ and $\mathcal{T}_2$ are semantically equal. For example, $(\rightarrow\ (t_1\ t_2)\ t) \equiv_\alpha (\rightarrow\ (t_{10}\ t_{11})\ t_{12})$. $\mathcal{S}$ is a substitution, which is a map from type variable to type, and it can be applied to $\mathcal{T}$ as $\mathcal{T} \cdot \mathcal{S}$. For example, if $\mathcal{S}(t_1) = [\texttt{Bool Int}], \mathcal{S}(t_2) = (\text{List } t_3)$ then $(\rightarrow\ (t_1\ t_2)\ t) \cdot \mathcal{S} = (\rightarrow\ ([\texttt{Bool Int}]\ (\text{List } t_3))\ t)$.

Listing 3: Example of pattern matching

```
(data Dim2 (Dim2 Int Int))

(data (Maybe t)
    (Just t)
```

```
5      Nothing)
6
7  (defun match-let (a) (Pure (-> ((Maybe Dim2)) Int))
8      (match a
9          ((Just val)
10             (let (((Dim2 x y) val))
11                 (+ x y)))
12         (Nothing
13             0)))
```

$\mathcal{P}$ and $\mathcal{P}_{let}$ are pattern in match and let expressions. For example, in listings 3, (Just val) and Nothing at line 9 and 12 are from $\mathcal{P}$ and (Dim2 x y) at line 10 is from $\mathcal{P}_{let}$. $Size$ is a function which takes a label and return the number of labels the label's type has. For example, $Size(\text{Just}) = Size(\text{Nothing}) = 2$ because Maybe type has 2 labels and $Size(\text{Dim2}) = 1$ because Dim2 type has 1 label in listings 3.

3 Typing Rule

In this section, I will introduce the typing rule of Baremetalisp. Before describing the rule, I introduce an assumption that there is no variable shadowing to make it simple. This means that every variable should be properly α-converted by using the De Bruijn index technique or variable shadowing should be handled when implementing the type inference algorithm.

Fig. 2 and 3 are the typing rule of expressions and function definitions.

4 Effect

$\mathcal{C}$	$:=$	$\mathcal{T} = \mathcal{T}, \mathcal{C}$	**type constraint**
	$\mid$	$\varnothing$	
Γ	$:=$		**context**
		$x : \mathcal{T}, \Gamma$	type of variable
	$\mid$	$L : \mathcal{T}, \Gamma$	type of label
	$\mid$	$L_{nth} : \mathcal{T}, \Gamma$	n-th type of label's element
	$\mid$	$\varnothing$	
E	$:=$	`Pure` $\mid$ `IO`	**effect**
$\mathcal{T}$	$:=$		**type**
		`Int`	
	$\mid$	`Bool`	
	$\mid$	$'(\mathcal{T})$	list type
	$\mid$	$[\mathcal{T}+]$	tuple type
	$\mid$	D	user defined type
	$\mid$	$(D\ \mathcal{T}+)$	user defined type with type arguments
	$\mid$	$(E\ (\rightarrow\ (\mathcal{T}*)\ \mathcal{T}))$	function type
	$\mid$	t	type variable
$\mathcal{P}$	$:=$		**pattern**
		x	variable
	$\mid$	L	label
	$\mid$	$(L\ \mathcal{P}+)$	label with patterns
	$\mid$	$'()$	empty list
	$\mid$	$[\mathcal{P}+]$	tuple
$\mathcal{P}_{let}$	$:=$		**patten for let**
		x	variable
	$\mid$	$(L\ \mathcal{P}_{let}+)$	label with patterns
	$\mid$	$[\mathcal{P}_{let}+]$	tuple

Figure 1: Syntax

$$\Gamma \vdash \texttt{true} : \texttt{Bool} \mid_{\varnothing} \varnothing \quad \text{(T-True)} \qquad \Gamma \vdash \texttt{false} : \texttt{Bool} \mid_{\varnothing} \varnothing \quad \text{(T-False)}$$

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T \mid_{\varnothing} \varnothing} \quad \text{(T-Var)} \qquad \Gamma \vdash z : \texttt{Int} \mid_{\varnothing} \varnothing \quad \text{(T-Num)}$$

$$\frac{x : T' \in \Gamma \quad T' \cdot S \equiv_{\alpha} T}{\Gamma \vdash x : T \mid_{FV_{\mathcal{T}}(T)} \varnothing} \quad \text{(T-VarPoly)}$$

$$\frac{\begin{array}{l}\Gamma \vdash \mathcal{P}_{let} : \mathcal{T}_0 \mid_{\mathcal{X}_0} C_0 \quad \Gamma \vdash e_1 : \mathcal{T}_1 \mid_{\mathcal{X}_1} C_1 \quad \Gamma \vdash e_2 : \mathcal{T}_2 \mid_{\mathcal{X}_2} C_2 \\ \mathcal{X}_0 \cap \mathcal{X}_1 \cap \mathcal{X}_2 = \varnothing \quad C = C_0 \cup C_1 \cup C_2 \cup \{\mathcal{T}_0 = \mathcal{T}_1\}\end{array}}{\Gamma \vdash (\texttt{let1}\ \mathcal{P}_{let}\ e_1\ e_2) : \mathcal{T}_2 \mid_{\mathcal{X}_0 \cup \mathcal{X}_1 \cup \mathcal{X}_2} C} \quad \text{(T-Let1)}$$

$$\frac{\begin{array}{l}\Gamma \vdash e_1 : \mathcal{T}_1 \mid_{\mathcal{X}_1} C_1 \quad \Gamma \vdash e_2 : \mathcal{T}_2 \mid_{\mathcal{X}_2} C_2 \quad \Gamma \vdash e_3 : \mathcal{T}_3 \mid_{\mathcal{X}_3} C_3 \\ \mathcal{X}_1 \cap \mathcal{X}_2 \cap \mathcal{X}_3 = \varnothing \quad C = C_1 \cup C_2 \cup C_3 \cup \{\mathcal{T}_1 = \texttt{Bool}, \mathcal{T}_2 = T_3\}\end{array}}{\Gamma \vdash (\texttt{if}\ e_1\ e_2\ e_3) : \mathcal{T}_2 \mid_{\mathcal{X}_1 \cup \mathcal{X}_2 \cup \mathcal{X}_3} C} \quad \text{(T-If)}$$

$$\frac{\begin{array}{l}\Gamma \vdash e_1 : \mathcal{T}_1 \mid_{\mathcal{X}_1} C_1 \quad \Gamma \vdash e_2 : \mathcal{T}_2 \mid_{\mathcal{X}_2} C_2 \wedge \cdots \wedge \Gamma \vdash e_n : \mathcal{T}_n \mid_{\mathcal{X}_n} C_n \\ \{t\} \cap FV_{\Gamma}(\Gamma) = \varnothing \quad \{t\} \cap \mathcal{X}_1 \cap \cdots \cap \mathcal{X}_n = \varnothing \\ \mathcal{X} = \{t\} \cup \mathcal{X}_1 \cup \cdots \cup \mathcal{X}_n \quad E = E_{\mathcal{T}}(\mathcal{T}_1) \\ C = C_1 \cup \cdots \cup C_n \cup \{\mathcal{T}_1 = (E\ (\rightarrow\ (\mathcal{T}_2\ \cdots\ \mathcal{T}_n)\ t))\}\end{array}}{\Gamma \vdash (e_1\ e_2\ \cdots\ e_n) : t \mid_{\mathcal{X}} C} \quad \text{(T-App)}$$

$$\frac{\begin{array}{l}\Gamma \vdash e_0 : \mathcal{T}_0 \mid_{\mathcal{X}_0} C_0 \\ \Gamma \vdash e_1 : \mathcal{T}_{e1} \mid_{\mathcal{X}_{e1}} C_{e1} \wedge \cdots \wedge \Gamma \vdash e_n : \mathcal{T}_{en} \mid_{\mathcal{X}_{en}} C_{en} \\ \Gamma \vdash \mathcal{P}_1 : \mathcal{T}_{p1} \mid_{\mathcal{X}_{p1}} C_{p1} \wedge \cdots \wedge \Gamma \vdash \mathcal{P}_{pn} : \mathcal{T}_{pn} \mid_{\mathcal{X}_{pn}} C_{pn} \\ \mathcal{X}_0 \cap \mathcal{X}_{e1} \cap \cdots \cap \mathcal{X}_{en} \cap \mathcal{X}_{p1} \cap \cdots \cap \mathcal{X}_{pn} = \varnothing \\ \mathcal{X} = \mathcal{X}_0 \cup \mathcal{X}_{e1} \cup \cdots \cup \mathcal{X}_{en} \cup \mathcal{X}_{p1} \cup \cdots \cup \mathcal{X}_{pn} \\ C = C_0 \cup C_{e1} \cup \cdots \cup C_{en} \cup C_{p1} \cup \cdots \cup C_{pn} \cup \\ \qquad \{\mathcal{T}_0 = \mathcal{T}_{p1}, \cdots, \mathcal{T}_0 = \mathcal{T}_{pn}\} \cup \{\mathcal{T}_{e1} = \mathcal{T}_{e2}, \cdots, \mathcal{T}_{e1} = \mathcal{T}_{en}\}\end{array}}{\Gamma \vdash (\texttt{match}\ e_0\ (\mathcal{P}_1\ e_1)\ \cdots\ (\mathcal{P}_n\ e_n)) : T_{e1} \mid_{\mathcal{X}} C} \quad \text{(T-Match)}$$

Figure 2: Typing rule (1/2)

$$\Gamma \vdash \ '() : \ '(T) \mid_{\{T\}} \varnothing \quad \text{(T-Nil)} \qquad \frac{L : \mathcal{T}' \in \Gamma \quad \mathcal{T}' \cdot \mathcal{S} \equiv_\alpha \mathcal{T}}{\Gamma \vdash L : \mathcal{T} \mid_{FV_{\mathcal{T}}(\mathcal{T})} \varnothing} \quad \text{(T-Label0)}$$

$$\frac{\begin{array}{l}\Gamma \vdash e_1 : T_1 \mid_{\mathcal{X}_1} C_1 \wedge \cdots \wedge \Gamma \vdash e_n : T_n \mid_{\mathcal{X}_n} C_n \\ \mathcal{X}_1 \cap \cdots \cap \mathcal{X}_n = \varnothing \quad \mathcal{X} = \mathcal{X}_1 \cup \cdots \cup \mathcal{X}_n \quad C = C_1 \cup \cdots \cup C_n\end{array}}{\Gamma \vdash [e_1 \ \cdots \ e_n] : [T_1 \ \cdots \ T_n] \mid_{\mathcal{X}} C} \quad \text{(T-Tuple)}$$

$$\frac{\begin{array}{l}\Gamma \vdash e_1 : T_1 \mid_{\mathcal{X}_1} C_1 \wedge \cdots \wedge \Gamma \vdash e_n : T_n \mid_{\mathcal{X}_n} C_n \\ \mathcal{X}_1 \cap \cdots \cap \mathcal{X}_n = \varnothing \quad \mathcal{X} = \mathcal{X}_1 \cup \cdots \cup \mathcal{X}_n \\ C = C_1 \cup \cdots \cup C_n \cup \{T_1 = T_2, \cdots, T_1 = T_n\}\end{array}}{\Gamma \vdash \ '(e_1 \ \cdots \ e_n) : \ '(T_1) \mid_{\mathcal{X}} C} \quad \text{(T-List)}$$

$$\frac{\begin{array}{l}\Gamma \vdash e_1 : \mathcal{T}_1 \mid_{\mathcal{X}_1} C_1 \wedge \cdots \wedge \Gamma \vdash e_n : \mathcal{T}_n \mid_{\mathcal{X}_n} C_n \\ L : \mathcal{T}_0' \in \Gamma \quad \mathcal{T}_0' \cdot \mathcal{S} \equiv_\alpha \mathcal{T}_0 \quad FV(\mathcal{T}_0) \cap \mathcal{X}_1 \cap \cdots \cap \mathcal{X}_n = \varnothing \\ FV_{\mathcal{T}}(\mathcal{T}_0) \cap FV_{\Gamma}(\Gamma) = \varnothing \quad \mathcal{X} = FV(\mathcal{T}_0) \cup \mathcal{X}_1 \cup \cdots \cup \mathcal{X}_n \\ L_{1st} : T_1' \in \Gamma \wedge \cdots \wedge L_{nth} : T_n' \in \Gamma \\ C = C_1 \cup \cdots \cup C_n \cup \{T_1' \cdot \mathcal{S} = \mathcal{T}_1, \cdots, T_n' \cdot \mathcal{S} = \mathcal{T}_n\}\end{array}}{\Gamma \vdash (L \ e_1 \ \cdots \ e_n) : \mathcal{T}_0 \mid_{\mathcal{X}} C} \quad \text{(T-Label)}$$

$$\frac{\begin{array}{l}\Gamma \vdash x_1 : t_1 \mid_\varnothing \varnothing \wedge \cdots \wedge \Gamma \vdash x_n : t_n \mid_\varnothing \varnothing \\ \Gamma \vdash e : \mathcal{T}_0 \mid_{\mathcal{X}} C_0 \quad \neg io(C) \\ C = \{\mathcal{T} = (\texttt{Pure} \ (\rightarrow \ (t_1 \ \cdots \ t_n) \ \mathcal{T}_0))\} \cup C_0\end{array}}{\Gamma \vdash (\texttt{lambda} \ (x_1 \ \cdots \ x_n) \ e) : \mathcal{T} \mid_{\mathcal{X}} C} \quad \text{(T-Lambda)}$$

$$\frac{\begin{array}{l}\Gamma \vdash x_1 : \mathcal{T}_1 \mid_\varnothing \varnothing \wedge \cdots \wedge \Gamma \vdash x_n : \mathcal{T}_n \mid_\varnothing \varnothing \\ \Gamma \vdash e : \mathcal{T}_0 \mid_{\mathcal{X}} C_0 \quad E = E_{\mathcal{T}}(\mathcal{T}) \quad (E = \texttt{Pure}) \Rightarrow \neg io(C) \\ C = C_0 \cup \{\mathcal{T} = (E \ (\rightarrow (\mathcal{T}_1 \ \cdots \ \mathcal{T}_n) \ \mathcal{T}_0))\}\end{array}}{\Gamma \vdash (\texttt{defun} \ \text{name} \ (x_1 \ \cdots \ x_n) \ \mathcal{T} \ e) : \mathcal{T} \mid_{\mathcal{X}} C} \quad \text{(T-Defun)}$$

Figure 3: Typing rule (2/2)

$$\Gamma \vdash \texttt{true} : \texttt{Bool} \mid_{\varnothing} \varnothing \quad \text{(P-True)} \qquad \Gamma \vdash \texttt{false} : \texttt{Bool} \mid_{\varnothing} \varnothing \quad \text{(P-False)}$$

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T \mid_{\varnothing} \varnothing} \quad \text{(P-Var)} \qquad \Gamma \vdash z : \texttt{Int} \mid_{\varnothing} \varnothing \quad \text{(P-Num)}$$

$$\Gamma \vdash\ '() :\ '(T) \mid_{\{T\}} \varnothing \quad \text{(P-Nil)} \qquad \frac{L : \mathcal{T}' \in \Gamma \quad \mathcal{T}' \cdot \mathcal{S} \equiv_{\alpha} \mathcal{T}}{\Gamma \vdash L : \mathcal{T} \mid_{FV_{\mathcal{T}}(\mathcal{T})} \varnothing} \quad \text{(P-Label0)}$$

$$\frac{\begin{array}{l} \Gamma \vdash \mathcal{P}_1 : \mathcal{T}_1 \mid_{\mathcal{X}_1} C_1 \wedge \cdots \wedge \Gamma \vdash \mathcal{P}_n : \mathcal{T}_n \mid_{\mathcal{X}_n} C_n \\ L : \mathcal{T}_0' \in \Gamma \quad \mathcal{T}_0' \cdot \mathcal{S} \equiv_{\alpha} \mathcal{T}_0 \quad FV(\mathcal{T}_0) \cap \mathcal{X}_1 \cap \cdots \cap \mathcal{X}_n = \varnothing \\ FV_{\mathcal{T}}(\mathcal{T}_0) \cap FV_{\Gamma}(\Gamma) = \varnothing \quad \mathcal{X} = FV(\mathcal{T}_0) \cup \mathcal{X}_1 \cup \cdots \cup \mathcal{X}_n \\ L_{1st} : T_1' \in \Gamma \wedge \cdots \wedge L_{nth} : T_n' \in \Gamma \\ C = C_1 \cup \cdots \cup C_n \cup \{T_1' \cdot \mathcal{S} = \mathcal{T}_1, \cdots, T_n' \cdot \mathcal{S} = \mathcal{T}_n\} \\ Size(L) = 1 \text{ for only } P_{let} \end{array}}{\Gamma \vdash (L\ \mathcal{P}_1\ \cdots\ \mathcal{P}_n) : \mathcal{T}_0 \mid_{\mathcal{X}} C} \quad \text{(P-Label)}$$

$$\frac{\begin{array}{l} \Gamma \vdash \mathcal{P}_1 : \mathcal{T}_1 \mid_{\mathcal{X}_1} C_1 \wedge \cdots \wedge \Gamma \vdash \mathcal{P}_n : \mathcal{T}_n \mid_{\mathcal{X}_n} C_n \\ \mathcal{X}_1 \cap \cdots \cap \mathcal{X}_n = \varnothing \quad \mathcal{X} = \mathcal{X}_1 \cup \cdots \cup \mathcal{X}_n \quad C = C_1 \cup \cdots \cup C_n \end{array}}{\Gamma \vdash [\mathcal{P}_1\ \cdots\ \mathcal{P}_n] : [\mathcal{T}_1 \cdots \mathcal{T}_n] \mid_{\mathcal{X}} C} \quad \text{(P-Tuple)}$$

Figure 4: Typing rule of pattern