SEIRD model:

Building a compartmental model:

S: Susceptible (healthy individuals, who can be infected)

E: Exposed (individuals who have been exposed to the infection/encountered the infected person)

I: Infected

R: Recovered (had been infected, cannot be infected again (assumed))

D: Deceased (individuals who died due to the infection)

We first derive the basic SIR model, using the following example:

Suppose we have a disease X. Let the probability that an infected individual can infect a healthy person be 0.2. Let the average number of people a person encounters within a day be 10. Thus, the expected number of people an infected person can infect in a day is $10 \cdot 0.2 = 2$. This is $\beta$, the expected number of people an infected person infects per day.

Let the number of days an infected person has and can spread the disease be D. Suppose, D is 5. Thus, an infected person can infect $\beta D = 2 \cdot 5 = 10$ in this duration (D). This is the reproduction number $R_0$, the total number of people an infected person can infect during D.

The rate of recovery ($\gamma$) is the inverse of the number of days the infection lasts. It is the proportion of the infected that are recovering per day. Thus, $\gamma = 1/D$.

Deriving S(t), I(t), R(t):

S(t) is the number of individuals who are susceptible on day t

I(t) is the number of individuals who are infected on day t

R(t) is the number of individuals who have recovered on day t

We first derive the change in S, I and R per day:

$\beta = 2$; $D = 10$; $\gamma = 0.1$;

Suppose $N = 100$, $I(t) = 25$, $S(t) = 30$, $R(t) = 45$ on day t;

Probability that people can be infected/ are susceptible = 30/100;

Thus, amount of people who can be infected on day t = $20 \cdot 2 \cdot (3/8) = 15$

(An infected person can infect $\beta = 2$ people per day on an average, only $\frac{S}{N} \cdot 100\% = 30\%$ of the total population can be infected, thus I = 25 infected people can infect $\beta I \frac{S}{N} = (25 \cdot 2 \cdot (3/10))$ people per day.)

Thus, change of S(t) = $\frac{d(S(t))}{dt} = -\ 15 \;\Rightarrow\; S' = \frac{d(S(t))}{dt} = -\ \beta I \frac{S}{N}$

change in I(t) = $\frac{d(I(t))}{dt} = \ 15 \Rightarrow\; I' = \frac{d(I(t))}{dt} = \beta I \frac{S}{N}$

We also need to find out the change in R(t) and its relations with I(t) and S(t). R(t) affects I(t), it does not impact S(t). Recovered individuals are not susceptible to diseases.

I(t) = 25, $\gamma = 0.1$ ($\gamma$ is the rate of recovery, that is, the proportion of infected people that are recovering per day (the duration is the number of days the infection lasts))

Thus, the number of people recovering on day t = $\gamma I = 25 \cdot 0.1 = 2.5$;

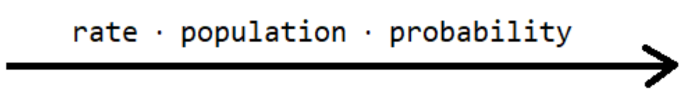
Hence:

change in I(t) = $\frac{d(I(t))}{dt} = \ 15 - 2.5 = 12.5 \Rightarrow\; I' = \frac{d(I(t))}{dt} = \beta I \frac{S}{N} - \gamma I$

change in R(t) = $\frac{d(R(t))}{dt} = \ 2.5 \Rightarrow R' = \frac{d(R(t))}{dt} = \gamma I$

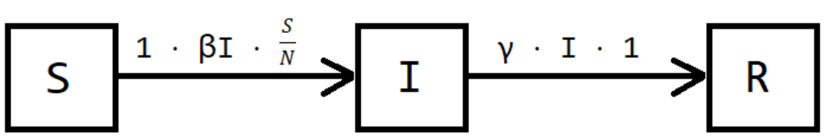
We get the following state transitions:

Compartments are boxes which represent states, transitions from one compartment to another are represented using arrows:




- rate: describes the duration transition takes
- population: the group of individuals that this transition applies to
- probability: the probability of transition taking place for an individual

Thus:

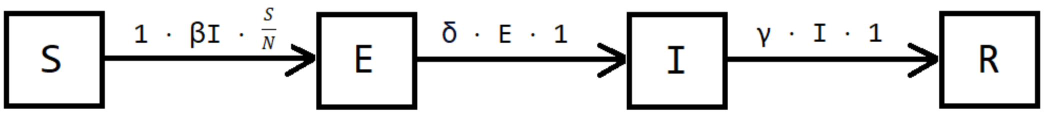



We now derive the exposed compartment:

As all susceptible can be exposed (implies that the probability remains same), exposition happens immediately (rate is 1, that is, rate remains same) and infectious individuals expose $\beta$ individuals per day (implies that population is the same), the rate, probability and population of the S → E will be the same as those for S → I. We get a new transition E → I. If we assume that everyone that's exposed becomes infected, then the probability of exposed becoming infected is 1, the population this transition applies to is E, and the rate at which the exposed are becoming infected is denoted by $\delta$. Thus, Change in E(t) = $E' = \frac{d(E(t))}{dt} = \beta I \frac{S}{N} - \delta E$

Thus, the new change in I(t) = $I' = \frac{d(I(t))}{dt} = \delta E - \gamma I$

There will be no change to the transition I → R.



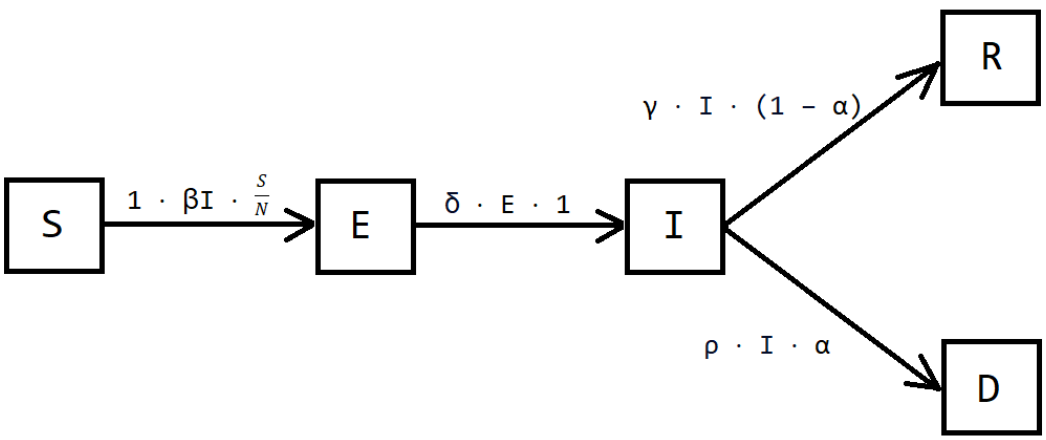

We now derive the deceased compartment:

Infected people either recover or die. I → D transition is introduced and I → R transition is affected. The rate at which the people die is ρ (suppose it takes 7 days to die after infection, then ρ = 1/7). γ does not change. The probability of the infected dying is denoted by α. α is the death/fatality rate. The probability for I → D is α and thus, the probability of I → R is (1 – α).

Thus, change in D(t) = $D' = \frac{d(D(t))}{dt} = \rho\alpha I$

New change in R(t) = $R' = \frac{d(R(t))}{dt} = \gamma(1 - \alpha)I$

New change in I(t) = $I' = \frac{d(I(t))}{dt} = \delta I - \rho\alpha I - \gamma(1 - \alpha)I$




Thus, the change in S, E, I, R and D per day has been obtained. We will solve these ODEs using odeint which was imported from scipy.integrate.

Note:

1) Reproduction Number:
   There are two example implementations for reproduction number in the code.
   Example 1: Constant reproduction number:
   ```
   R_0 = 5.0
   ```

   Example 2: Time-dependent reproduction number:

```
# Chosen a time dependent variable R_0 such that after day 'Ld', a lockdown is enforced,
# pushing R_0 to 0.9, from 15.0

Ld = 40
def R_0(t):
  if(t <= Ld):
    return 15.0
  else:
    return 0.9
```

Based on the example, β is found:

Example 1:

```
beta = R_0/D1
```

Example 2:

```
def beta(t):
  return R_0(t)/D1
```

2) Fatality rate:

There are two example implementations for fatality rate in the code.

Example 1: Constant fatality rate:

```
alpha = 0.2
```

Example 2: Age dependent fatality rate:

Here, we calculate the overall average fatality rate, which is used as α.

```
alpha_ag = {"0-35": 0.02, "35-59": 0.04, "60-79": 0.1, "79+": 0.2} # alpha according to age groups
proportion_ag = {"0-35": 0.26, "35-59": 0.25, "60-79": 0.34, "79+": 0.15} # proportion of total population that is in the age groups
alpha = sum((alpha_ag[i] * proportion_ag[i]) for i in list(alpha_ag.keys())) # overall average fatality rate
```

We then calculate the probability of the infected recovering as $(1 - \alpha)$.

3) The rates (δ and ρ) are derived from the incubation period (D2) and days after infection till death (D3), respectively, in a similar manner as γ from D (D1).