Tutorial - 4

Q1°- $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

Sol^n:- $a = 3$, $b = 2$

$c = \log_b a = \log_2 3$

$f(n) = n^2$

$n^c = n^{\log_2 3}$

$n^c = n^{1.58} \propto n^2$ [$\because \log_2 3 = 1.58$]

$f(n) > n^c$

$\boxed{T(n) = \Theta(n^2)}$ Ans

(2) $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

Sol^n:- $a = 4$, $b = 2$

$c = \log_b a = \log_2 4 = \log_2 (2)^2$

$= 2$

$n^c = n^2$

$f(n) = n^2$

$n^c = f(n)$

$T.C = \Theta(n^2 \log n)$ Ans

(3) $T(n) = T\left(\frac{n}{2}\right) + 2^n$

Sol^n:- $a = 1$, $b = 2$ $c = \log_b a$

$n^c = n^0 = 1$ $c = \log_2 1 =$

$f(n) = 2^n$

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$f(n) > n^c$

T.C = 2^n

④ $T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$

$a = 2 \quad b = 2$

$c = \log_b a \quad = \log_2 2 = 1$

$f(n) = n^n$

$T.C = \frac{f(n) > c}{O(n^n)}$ Does not apply as a is not constant.

③ $T(n) = 16\, T\left(\frac{n}{4}\right) + n$

$a = 16 \quad b = 4$

$c = \log_b a = \frac{\log_4 16}{\log_4 (4)^2} = \log_4 4^2 = 2$

$n^c = n^2$

$f(n) = n$

$f(n) < n^c$

$T.C = O(n^2)$ thus

⑤ $T(n) = 2T\left(\frac{n}{2}\right) + n \log n$

Sol^n: $a = 2 \quad b = 2$

$c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n$

$f(n) = n \log n$

$f(n) > n^c$

$n \log n > n$

T.C = $O(n \log n)$

(7) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

Sol: $a = 2$

$b = 2$

$c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n$

$f(n) = \frac{n}{\log n}$

$f(n) < n^c$

T.C = $O(n)$ Ans

(8) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

Sol: $a = 2$, $b = 4$

$c = \log_b a = \log_4 2$

$f(n) > n^c$

$n^{0.51} > n^{\log_4 2}$

T.C = $O(n^{0.51})$ Ans

(9) $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

Sol: $a = 0.5$, $b = 2$

$c = \log_b a$

$n^c = n^{\log_2 (1/2)}$

$f(n) = \frac{1}{n}$

$f(n)$

Does not apply as it is

10. $T(n) = 16\,T\left(\frac{n}{4}\right) + n!$

Sol:

$a = 16 \quad | \quad b = 4$

$c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

$f(n) = n!$

$f(n) > n^c$

$T.C = O(n!)$

11. $T(n) = 4\,T\left(\frac{n}{2}\right) + \log n$

Sol:

$a = 4$

$b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$f(n) < n^c$

$T.C = O(n^2)$

12. $T(n) = \text{Equt}(n)\, T\left(\frac{n}{a}\right) + \log n$

Sol:- $T(n) = \sqrt{n}\, T\left(\frac{n}{2}\right) + \log n$

Can not apply Master's theorem.

13. $T(n) = 3T\left(\frac{n}{2}\right) + n$

Sol:- $a = 3$

$b = 2$

$c = \log_b a = \log 2^3 = 1.58$

$n^c = n^{1.58}$

$f(n) = n$

$n^c > f(n)$

$T.C = O(n^{\log 3})$ Ans

14. $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

Sol:- $a = 3, \quad b = 3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = \sqrt{n}$

$n^c = n^1$

$f(n) < n^c$

$T.C = O(n)$ Ans

15° $T(n) = 4T\left(\frac{n}{2}\right) + cn$

Sol^n :-

$a = 4 \quad b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = n$

$f(n) < n^c$

T.C = $O(n^2)$

16° $T(n) = 3T\left(\frac{n}{4}\right) + n \log n$

Sol^n :- $a = 3 \quad , b = 4$

$c \log_b a = \log_4 3 = 0$

$n^c = n^0 = 1$

$f(n) = n \log n$

$f(n) > n^c$

T.C = $O(n \log n)$

17° * $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

Sol^n :-

$a = 3 \quad b = 3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = \frac{n}{2}$

$n^c = n$

$f(n) < n^c \quad$ T.C = $O(n)$

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18. $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

Sol:

$a = 6, \quad b = 3$

$c = \log_b a = \log_3 6$

$f(n) = n^2 \log n$

$f(n) \Omega n^{\log_3 6}$

$T.C = \Theta(n^2 \log n)$

19. $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

Sol:

$a = 4, \quad b = 2 \quad c = \log_b a = \log_2 4$

$c = 2$

$n^c = n^2$

$f(n) = \frac{n}{\log n}$

$f(n) \Omega n^c$

$T.C = O(n^2)$

20. $T(n) = 64T\left(\frac{n}{8}\right) + n^2 \log n$

$a = 64$

$b = 8$

$c = \log_b a = \log_8 64 = 2$

$n^c = n^2$

$f(n) = (n^2 \log n)$ $f(n)$ is not

Positive so we can not apply.

Q1: $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

Sol^n:-

$a = 7$

$b = 3$

$c = \log_b a = \log_3 7$

$f(n) = n^2$

$f(n) > n^{\log_3 7}$ $\quad [\because \log_3 7 = 1.7]$

$n^2 > n^{1.7}$

T.C $= O(n^2)$

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Q2: $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

Sol^n:

$a = 1$

$b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) = n(2 - \cos n)$

$f(n) > n^c$

T.C $= O(n(2 - \cos n))$