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Tutorial - 1

Assignment

Q1. What do you understand by asymptotic notations? Define different asymptotic notation with example?

Asymptotic notations describes the running time of an algorithm. How much time an algo takes with the given input.

There are three different types of asymptotic notations

1. Big - O notation
2. Omega notation
3. Theta notation

Big - O notation (O - notation)
Big - O notation represent the upper bound of the running time of an algorithm. Thus,

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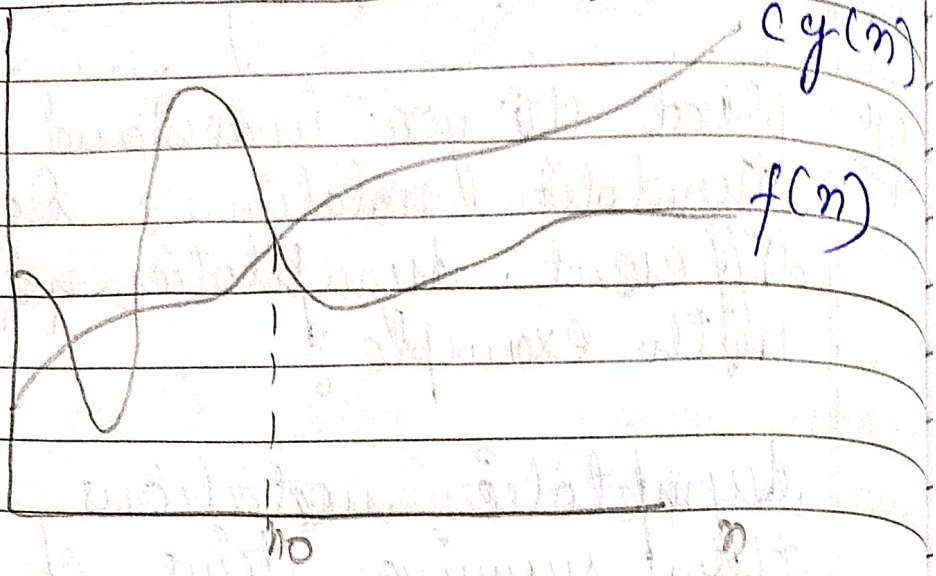
It gives the worst case of complexity of an algorithm.



$$f(n) = O(g(n))$$

if

$$f(n) \leq cg(n)$$
$$\forall\ n \geq n_0$$

Hence Big O gives the upper bound of a function

Omega notation (Ω- notation)

Omega notation represent the lower bound of the running time of an algorithm. Thus, it provides the best case complexity of an algorithm

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Q2:- what should be time complexity of

```
for(i=1 to n)
    { i = i * 2; }
```

$\sum_{i=1, i=i*2}^{n}$ $2 + 4 + 8 + 16 + 32 \cdots$

$\sum_{i=1}^{n}$ $2^1 + 2^2 + 2^3 + 2^4 + 2^5 \cdots 2^n$

$\sum_{i=1}^{n}$ 2^n

T.C = $O(2^n)$.

③ $T(n) = \begin{cases} 3T(n-1) & \text{if } n>0, \\ 1 & \text{otherwise} \end{cases}$

$T(n) = 3T(n-1)$ — ①

$T(0) = 1$

Let $n = n-1$

$T(n-1) = 3T(n-2)$ — ②

From eq^n ① & ②

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$T(n) = 3[3 \cdot T(n-2)]$ — (3)

Let $n = n-2$

$T(n-2) = 3T(n-3)$ — (4)

Put $T(n-2)$ in eq^n (3)

$T(n) = 3[3[3T(n-3)]]$

$T(n) = 3^3 \cdot T(n-3)$

$T(K) = 3^k T(n-k)$

$n - k = 0$

$n = 1 + k \Rightarrow k = n$

$T(n) = 3^n T(n-n)$

$T(n) = 3^n T(0)$

$T(n) = 3^n$

$T(n) = 3^n$

$T.C = O(3^n)$ Ans

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④ $T(n) = \begin{cases} 2T(n-1) - 1 \\ T(0) = 1 \end{cases}$

Sol^n :-

$T(n) = 2T(n-1) - 1$ — ①

Let $n = n-1$

$T(n-1) = 2T(n-2) - 1$ — ②

~~Let $n = n-2$~~ put $T(n-1)$ in eq^n ①

$T(n) = 2[2T(n-2) - 1] - 1$ — ③

Let $n = n-2$

from eq^n ①

$T(n-2) = 2T(n-3) - 1$ — ④

from eq^n ③ & ④

$T(n) = 2[2[2.[T(n-3) - 1] - 1] - 1$

$T(n) = 2^3 T(n-3) - 2^2 - 2^1 - 2$ — ⑤

Generalizing

$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - 2^{k-3} - \cdots - 2^0$

Let $n - k = 0$

$n = k$

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$$T(n) = 2^n T(n-n) - 2^{n-1} - 2^{n-2} - 2^{n-3} - \cdots - 2^0$$

$$T(n) = 2^n T(0) - 2^{n-1} - 2^{n-2} - 2^{n-3} - \cdots - 2^0$$

$$T(n) = 2^n \cdot 1 - 2^{n-1} - 2^{n-2} - 2^{n-3} - \cdots - 2^0$$

$$T(n) = 2^n - 2^{n-1} - 2^{n-2} - 2^{n-3} - \cdots - 2^0$$

$$T(n) = 2^n - [2^n - 1]$$

$$2^n - 2^n + 1$$

$$\boxed{T(n) = O(1)}$$

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Assignment

Q5. int i=1, s=1
while (s<=n)
{
i++;
s = s+i;
printf ("#");
}

i = 1	s = 1
i = 2	s = 3 = 1+2
i = 3	s = 6 = 1+2+3
i = 4	s = 10 = 1+2+3+4

$\sum_{s=1}^{?}$ s=s+i

1 + 3 + 6 + 10 ---- k

$$\frac{k(k+1)}{2} > n \quad [\because s <= n]$$

$$\frac{k(k+1)}{2} > n$$

$$\frac{k^2}{2} + \frac{k}{2} > n$$

lower limit just ignore

Constant → (2)

$$k^2 > n$$

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$k \geq \sqrt{n}$

T.C of the above is $O\sqrt{n}$
$T(n) = O(\sqrt{n})$

⑥ Time complexity of -

```
void function (int n)
{
  int i, count = 0;
  for(i = 1; i*i <= n; i++)
      count++;
}
```

Sol^n

$i = 1$	$1^2 = n$
$i = 2$	$4^2 = n$
$i = 3$	$9^2 = n$
$i = 4$	$16^2 = n$

$\sum_{i=1}^{n} 1 + 2^2 + 3^2 + 4^2 \cdots \; k^2 = n$

$i^2 i^2 i^2$

$\frac{k(k+1)}{2} \; 2 = n$

$k^2 \; 2 = n$

or $k\sqrt{2} = \sqrt{n}$

$T.C = O(\sqrt{n})$ Ans

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(7) Time complexity of

```
void function(int n)
{ int i, j, k, count = 0;
for(i = n/2; i <= n; i++)
{
for(j = 1; j <= n; j = j * 2)
for(k = 1; k <= n; k = k * 2)
    count++;
}
```

i	j	k
n/2	$\log n$	$(\log n)^2$
n/2+1	$\log n$	$(\log(n))^2$
⋮	⋮	⋮
upto n times	⋮	⋮
n	$\log n$	$[\log(n)]^2$

$\left(\frac{n}{2}+1\right)$ times

$$O(i*k) = O\left(\frac{n}{2}+1\right) * \left[\log_2 n\right]^2$$

$$T(n) = O\left(n(\log_2 n)^2\right)$$ Ans

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(8) Time complexity

```
function (int n)
{
  if (n == 1) return;
  for (i = 1 to n)
  {
    for (j = 1 to n)
    {
      printf ("**");
    }
  }
  3
  3
  3
  function (n-3);
}
```

Ans:- $T(n) = T(n-3) + n^2$ — (1)

let $n = n-3$

$T(n-3) = T(n-6) + (n-3)^2$ — (2)

from eqn (1) & (2)

$T(n) = T(n-6) + (n-3)^2 + n^2$ — (3)

let $n = n-6$

from eqn (1)

$T(n-6) = T(n-9) + (n-6)^2$ — (4)

from eqn (3) & (4)

$T(n) = T(n-9) + (n-6)^2 + (n-3)^2 + n^2$

$T(n) = T(n-3k) + (n^2 + (n-3)^2 +$

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$(n-6)^2 + \cdots [n-(3(k-1)]^2$

Let $n-3k=1$

$n = 1+3k$

$k = (n-1)/3$

$T(n) = T(1) + n^2 + (n-3)^2 + (n-6)^2 \cdots [n-n+1)^2]$

$T(n) = 1 + n^2 + (n-3)^2 + (n-6)^2 \cdots 1$

$T(n) = cn^2 + k$ where c & k are constant.

$\boxed{T(n) = O(n^2)}$

(9) void function (int n)

{

for (i = 1 to n)

{

for (j = 1; j <= n; j = j + i)

printf ("GG");

}

}

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$i = 1$, $j = 1, 2, 3, 4 \; \ldots \; n$ times

$i = 2$, $j = 1, 3, 5, 7 \; \ldots \; n/2$ times

$i = 3$ $j = 1, 4, 7, 11 \; \ldots \; n/3$ times

$i = n$ $j = 1 \; \ldots$ 1 times

i : n times

j : $n/2$, $n/3$, ... 1 times

$n(\log n)$

(10) For the function n^k and c^n n, what is the asymptotic relationship b/w these functions? Assume that $k >= 1$ and $c > 1$ are constants. Find out the values of c and n0 for which relation holds

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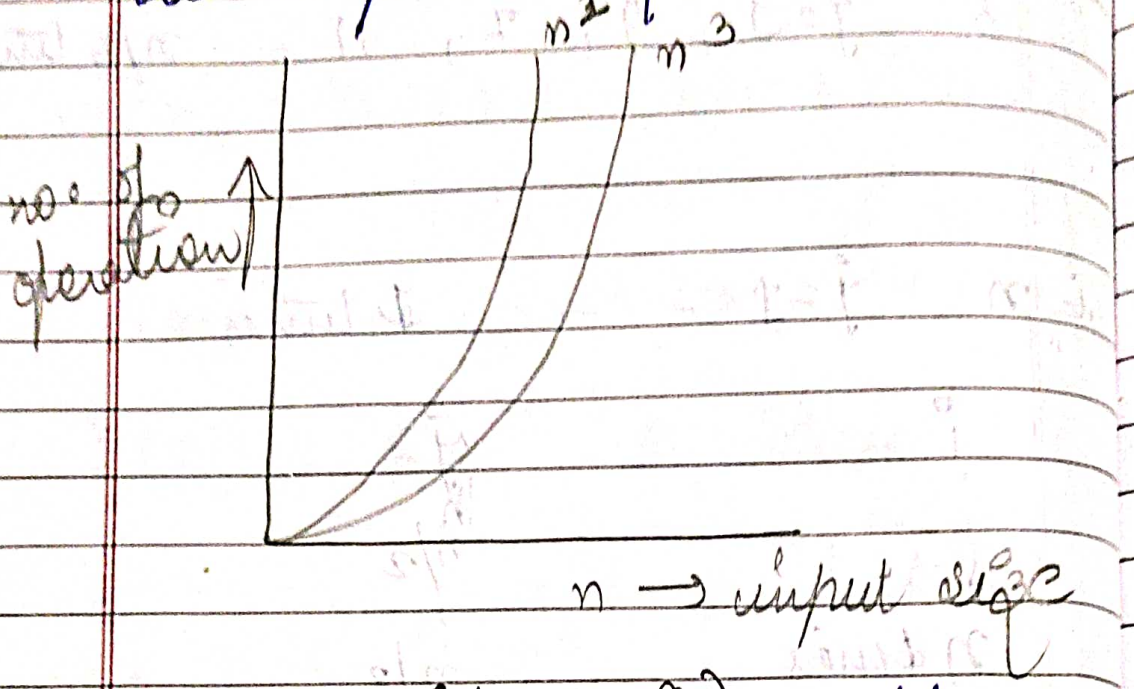
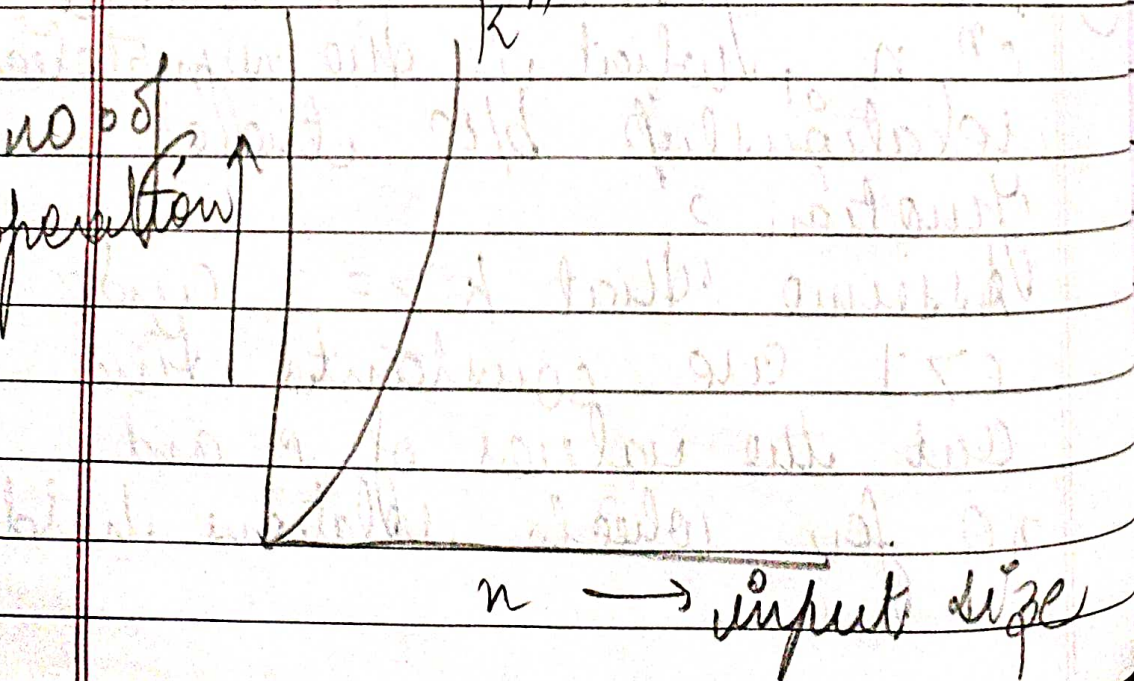
Def^n:- Polynomial (n^k):- When complexity grows proportion to the power of k of the input size



Exponential (k^n):- At each step the no. of operation becomes multiplied by k exponentially



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Relation b/w these two functions:-

n^k & c^n

Relation b/w n^k & c^n

$n^k = O(c^n)$

$n^k \leq a c^n$

$\forall\ n \geq n_0$ & some constant $a > 0$.

for

$n_0 = 1$

$c = 2$

$1^k \leq a\, 2^1$

$n_0 = 1$ & $c = 2$