Designing the Most Efficient Beverage Can Packaging

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Gold CREST Award Report

Mathematically Optimising the dimensions of the Soda Can

The standard volume of soda in most soda cans is 330 ml. If 330 ml of soda has to be held inside the can, the can must have approximately 400 ml of total volume. This is because manufacturers leave some space at the top of the can in order to account for changes in pressure during transportation and the volume of the can has to be included.

The volume of a cylinder is given as:

$$V = \pi r^2 h$$

$$400 = \pi r^2 h$$

We can get the height of the cylinder in terms of the radius using some algebra

$$h = \frac{400}{\pi r^2}$$

The surface area gives the area of the aluminium required to make the can. The surface area of the cylinder is given by:

$$A = 2\pi r^2 + 2\pi r h$$

We can use the relationship between the height and the radius we found and substitute h in this equation

$$A = 2\pi r^2 + 2\pi r \,.\, \frac{400}{\pi r^2}$$

$$A = 2\pi r^2 + \frac{800}{r}$$

We can calculate the first derivative to find how the surface area changes with the radius

$$\frac{dA}{dr} = 4\pi r - \frac{800}{r^2}$$

When the first derivative will be 0, the surface area will be minimised or maximised. Hence, we can find the radius which minimises the surface area.

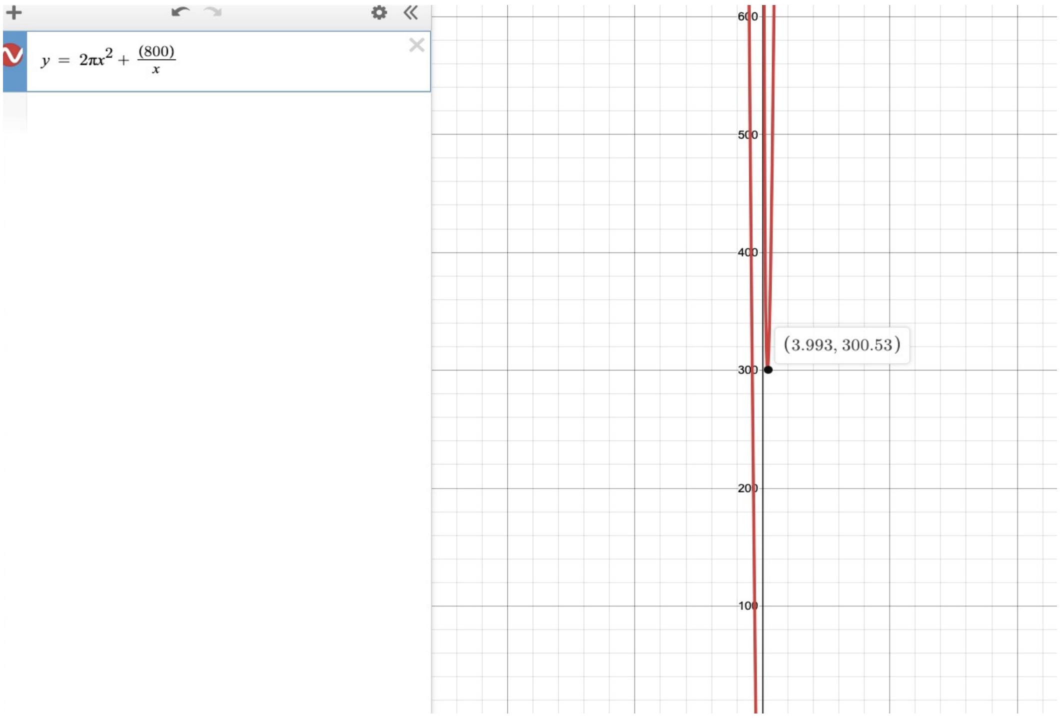
$$0 = 4\pi r - \frac{800}{r^2}$$

$$\frac{800}{r^2} = 4\pi r$$

$$200 = \pi r^3$$

$$r = \sqrt[3]{\frac{200}{\pi}}$$

$$r = 3.99\ cm$$



By graphing the expression for the surface area, we can confirm that the radius of 3.99 cm is a minimum point, where the surface area is minimised.

The height of the can would then be:

$$h = \frac{400}{\pi(3.99)^2}$$

$$h = \frac{330}{\pi(3.99)^2}$$

$$h = 8.00\ cm$$

Therefore, a can with radius 3.99 cm and height 8.00 cm will be optimum to minimise the amount of aluminium needed for soda cans. The aluminium needed would then be:

$$A = 2\pi r^2 + 2\pi rh$$
$$A = 2\pi(3.99)^2 + 2\pi(3.99)(8.00)$$
$$A = 300.6\ \text{cm}^2$$

Comparing the proposed design with current designs in the market

In this section, I compare the design I have proposed with designs from soda manufacturers. Coca Cola and Dr.Pepper were chosen as they are

Coca Cola

Coca Cola's new 330 ml soda can is designed with a height of 15.6 cm and a radius of 2.85 cm. The total volume of this can will then be:

$$V = \pi r^2 h$$

$$V = \pi(2.85)^2(15.6)$$

$$V = 398.1\ \text{cm}^3$$

Hence, the estimate of $400\ \text{cm}^3$ for the total volume of the can was fairly accurate.

The amount of aluminium required to make the Coca Cola can be found by using its surface area:

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 2\pi(2.85)^2 + 2\pi(2.85)(15.6)$$

$$A = 330.4\ \text{cm}^2$$

The amount of aluminium needed to make this can is 29.8 cm^2 more than the proposed can. Therefore, Coca Cola can reduce their aluminium usage by 9.02% by increasing the radius of their cans by 1.14 cm and decreasing the height by 7.6 cm.

Dr. Pepper

Dr. Pepper's 330 ml soda can has differing dimensions from the Coca Cola can, with a height of 11.52 cm and radius of 3.31 cm. The total volume for this can is:

$$V = \pi r^2 h$$

$$V = \pi(3.31)^2(11.52)$$

$$V = 396.5 \text{ cm}^3$$

Therefore, the total volume this can holds is also very close to the 400 cm^3 estimate. The amount of aluminium required to make this can will be:

$$A = 2\pi r^2 + 2\pi rh$$

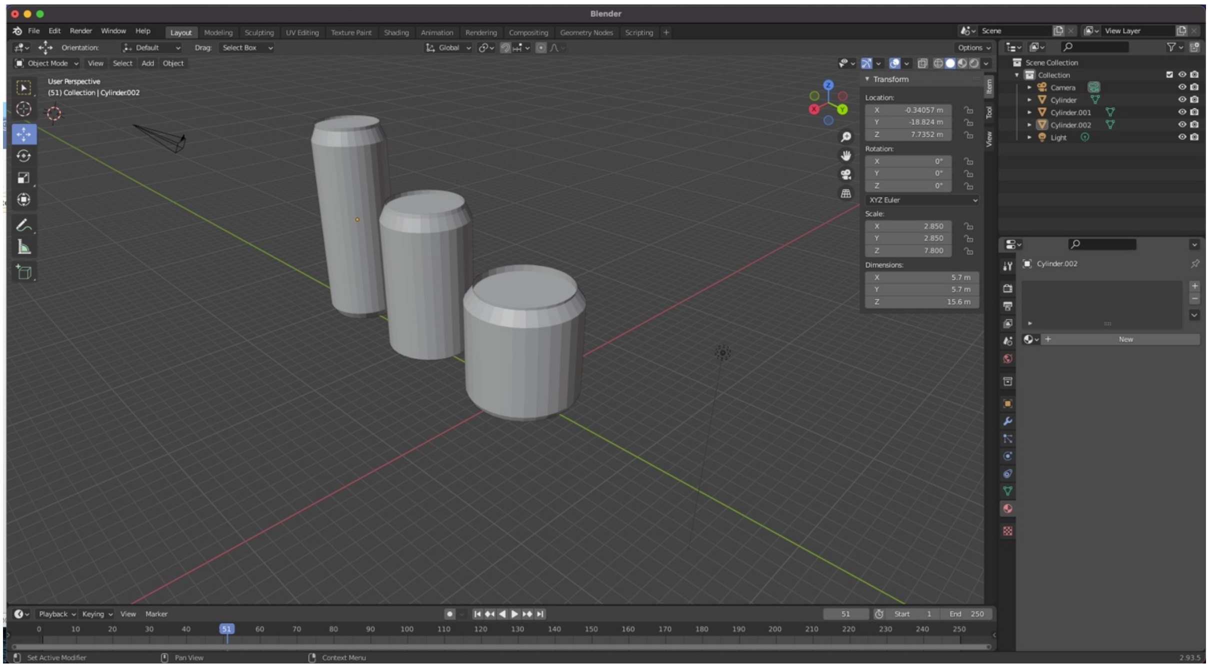
$$A = 2\pi(3.31)^2 + 2\pi(3.31)(11.52)$$

$$A = 308.4 \text{ cm}^2$$

This is significantly less than the amount of aluminium required to make the Coca Cola can, however, it is still more than the proposed can design by 7.8 cm^2. Dr.Pepper can also

increase the radius of their cans by 0.68 cm and decrease the height by 3.52 cm to reduce aluminium use by 2.5%.

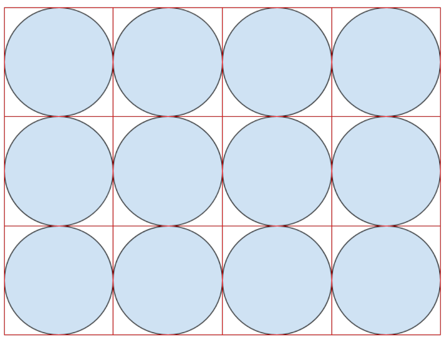
Qualitative Analysis



I used blender to design all three of the cans approximately and compare them. The can to the extreme left is the Coca Cola can. Coca Cola has designed it in order to be thin and tall, making it easy to hold. However, it is very inefficient and requires more aluminium to produce. Dr.Pepper’s can is more efficient that the Coca Cola can and is wider and shorter. The proposed can is even shorter and wider than the Dr.Pepper can, ensuring maximum efficiency. With a width of 7.98 cm, the proposed can would not be very inconvenient to hold and would result in a lot of cost savings and environmental benefits.

Packaging the cans in boxes

The packaging of aluminium cans in usually done by using a tessellation of squares.



Because of this, a lot of the space is wasted.

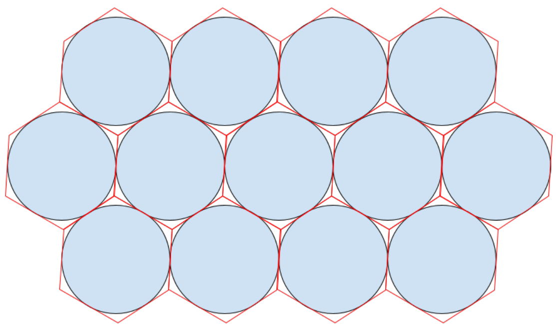
We can calculate the 2-dimensional area wasted for every can be found by subtracting the area of the circle from the area of the square. This can be done as:

$$A_{square} = (2r)^2$$
$$A_{square} = 4r^2$$
$$A_{circle} = \pi r^2$$

$$A_{wasted} = 4r^2 - \pi r^2$$
$$A_{wasted} = (4 - \pi) r^2$$
$$A_{wasted} = 0.858 r^2$$

The area lost for every can in the proposed design, with a radius of 3.99 cm, will be $13.67\ \text{cm}^2$. This is an inefficiency which can be eliminated.

In order to do this, I investigated that the Hexagon tessellation could be used, largely reducing the amount of wasted space.



The 2-dimensional space wasted can be calculated by subtracting the area of the circle from the area of the hexagon. This is done as:

$$A_{hexagon} = \frac{3\sqrt{3}}{2} a^2$$

where a is the side length of the hexagon.

$$\cos 30 = \frac{r}{a}$$

$$\mathrm{a} = \frac{r}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}r$$

$$A_{hexagon} = \frac{3\sqrt{3}}{2}(\frac{2}{\sqrt{3}}r)^2$$

$$A_{hexagon} = 2\sqrt{3}\, r^2$$

$$A_{circle} = \pi r^2$$

$$A_{wasted} = 2\sqrt{3}\, r^2 - \pi r^2$$

$$A_{wasted} = 0.323r^2$$

The area lost for every can in the proposed design, with radius 3.99 cm, would be $5.13\ \mathrm{cm}^2$, which is 62% less than the wastage in square tessellation. Hence, hexagonal packaging could be used to increase efficiency.

In order to gauge how effective my proposed designs are, I had to calculate how they would increase efficiency when packed in Coca Cola trailers. I extensively searched the web for this information, however, it wasn't available anywhere. My mentor works in a logistics company which handles the transportation of Coca Cola cans. Through him, I enquired that Coca Cola transports cans in a 40 ft*7 ft*7ft trailer. I ensured that all the companies the data belonged to were being informed that it was shared with me for a project, avoiding ethical issues related to the unauthorized distribution of data. Therefore, I calculated the number of cans they can transport using a square tessellation:

$$V_{can} = 4r^2 * h$$

$$V_{can} = 4(3.99\ \mathrm{cm})^2 * 8\ \mathrm{cm}$$

$$V_{can} = 509.44\ \mathrm{cm}^3$$

$$V_{trailer} = 1219.2\ \mathrm{cm} * 213.4\ \mathrm{cm} * 213.4\ \mathrm{cm}$$

$$V_{trailer} = 5.55 * 10^7\ \mathrm{cm}^3$$

$$cans = \frac{5.55 * 10^7\ \mathrm{cm}^3}{509.44\ \mathrm{cm}^3}$$

$$cans = 108986$$

Theoretically, with square tessellation, 108986 cans will be transported. However, the actual number will be significantly lesser due to space taken by packaging containers.

$$V_{can} = 2\sqrt{3}\, r^2 * h$$

$$V_{can} = 2\sqrt{3}\, (3.99\ \text{cm})^2 * 8\ \text{cm}$$

$$V_{can} = 441.19\ \text{cm}^3$$

$$cans = \frac{5.55 * 10^7\ \text{cm}^3}{400.12\ \text{cm}^3}$$

$$cans = 125845\ \text{cm}^3$$

Therefore, 16859 extra cans can be delivered using the hexagon tessellation packaging. This is a theoretical estimate, and the actual number of extra cans delivered will be notably less, however, it is evident that the design would definitely improve some efficiency.

Conclusion

In conclusion, my research has shown that beverage packaging with a radius of 3.99 cm and height of 8.00 cm will minimise the amount of aluminium used in production. When compared with cans existing in the market, I also found that the proposed can would use 9.02% less aluminium than the Coca Cola can and 2.5% less aluminium than the Dr.Pepper can. I also proposed that the cans be packaged using hexagonal tessellation instead of square tessellation, largely reducing the space wasted when packaging the cans together.

This means that using the design I have researched and proposed, beverage manufacturing companies could greatly reduce their costs and protect the environment. It is reported that in 1994, 3.1 million tons of aluminium waste was generated and aluminium containers and packaging, such as soft drink and beer cans, contributed 2.1 million tons. Even if adopting efficient packaging allows us to reduce 1% of aluminium waste from packaging, 21,000 tons of waste could be decreased. This is particularly important as aluminium takes 200-500 years to complete degrade in a landfill (District n.d.).

With the help of my mentor, I will set up meetings with officials from beverage companies and explain my proposition, helping them understand how it can help their company and the environment.

Evaluating the Method Used

The method I used, adopting a calculus approach for minimising the surface area of the can, was effective and quickly allowed me to get accurate readings for the optimum dimensions. My mentor also agreed that this method was the best option for me and resulted in accurate readings.

Reflection

I feel this project really helped me develop my scientific and mathematical skillset, while carrying out research which has real world applications. With this project, I got the opportunity of applying calculus to a real problem. I understood the mathematics behind optimization and learnt all about packaging through extensive research. I read numerous research papers related to packaging optimization and learnt a lot from them. I have definitely become more aware of increasing waste being generated due to inefficient packaging, and enjoyed contributing to solve this problem. Over the course of my research, I also developed my critical thinking and communication skills.

In this project, I could have managed my time better, allowing me to comfortably finish the project instead of having to rush through certain parts of it when I had more school work. I could have also analysed the design of the can more comprehensively across different aspects of the supply chain like warehouses. I would also want to learn to use sophisticated modelling software used by engineers to be able to design my can with various physical properties.

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