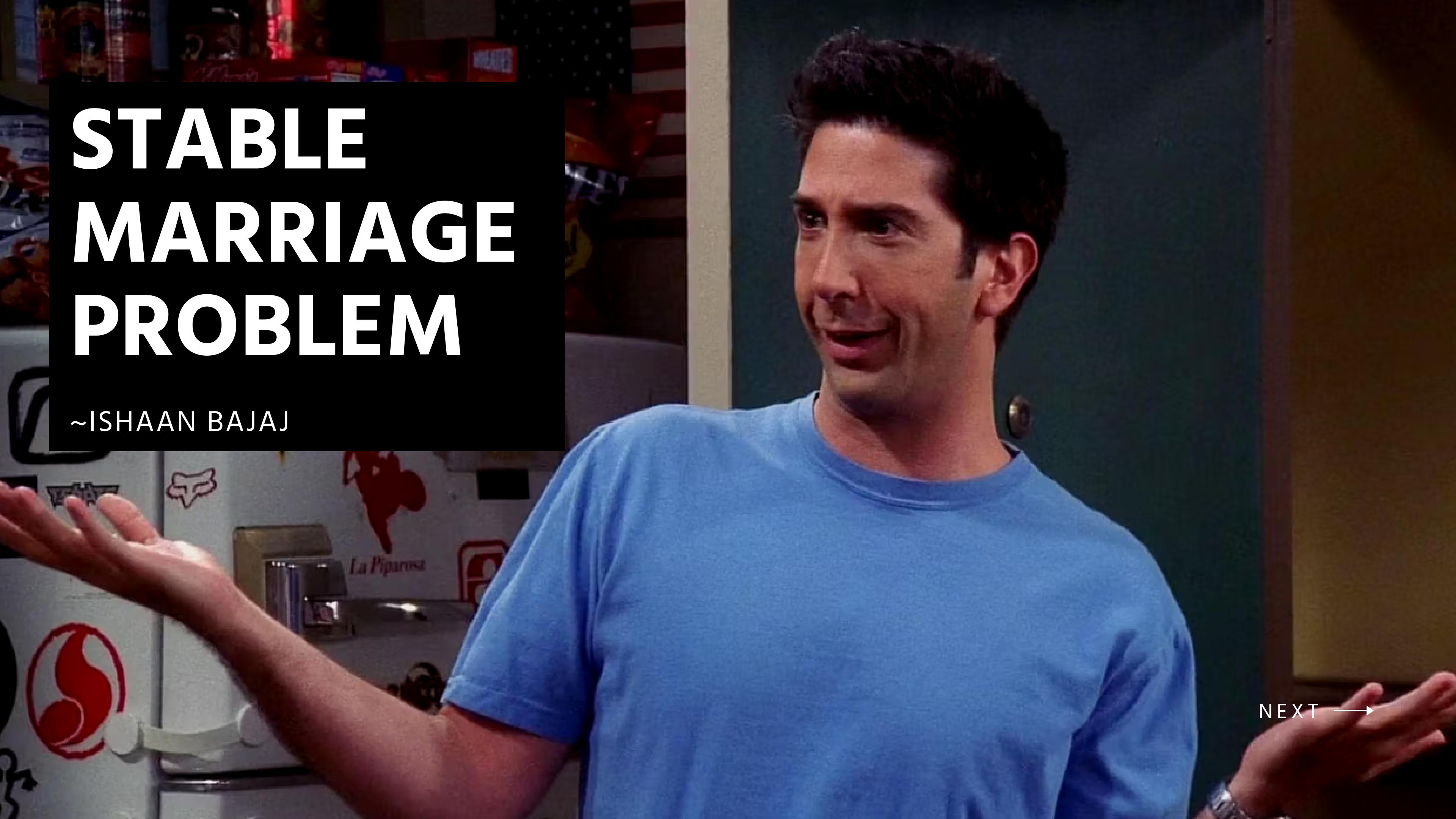


# STABLE MARRIAGE PROBLEM

~ISHAAN BAJAJ



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# Stable Matching Problem

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- There are  $n$  men and  $n$  women, all unmarried.
- Each has a preference list giving a relative preference of each person of the opposite gender.
- Find a matching between the men and the women such that
  - Each man is matched to exactly one woman and each woman is matched to exactly one man (perfect matching).
  - There is no blocking pair (an unmatched pair of a man and a woman who both prefer each other over whoever they are assigned to in the matching).
- Stable matching - perfect matching with no blocking pair.

# Example Preferences



<b>ALBERT</b>	DIANE	EMILY	FERGIE
<b>BRADLEY</b>	EMILY	DIANE	FERGIE
<b>CHARLES</b>	DIANE	EMILY	FERGIE

<b>DIANE</b>	BRADLEY	ALBERT	CHARLES
<b>EMILY</b>	ALBERT	BRADLEY	CHARLES
<b>FERGIE</b>	ALBERT	BRADLEY	CHARLES

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# Example Matching #1

<b>ALBERT</b>	DIANE	EMILY	FERGIE
<b>BRADLEY</b>	EMILY	DIANE	FERGIE
<b>CHARLES</b>	DIANE	EMILY	FERGIE

**IS THIS  
STABLE?**

<b>DIANE</b>	BRADLEY	ALBERT	CHARLES
<b>EMILY</b>	ALBERT	BRADLEY	CHARLES
<b>FERGIE</b>	ALBERT	BRADLEY	CHARLES

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ALBERT

EMILY

FERGIE

# Example Matching #1

ALBERT	DIANE	EMILY	FERGIE
BRADLEY	EMILY	DIANE	FERGIE
CHARLES	DIANE	EMILY	FERGIE

NO.  
Albert and  
Emily form  
a blocking  
pair.

DIANE	BRADLEY	ALBERT	CHARLES
EMILY	ALBERT	BRADLEY	CHARLES
FERGIE	ALBERT	BRADLEY	CHARLES

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# Example Matching #2

<b>ALBERT</b>	DIANE	EMILY	FERGIE
<b>BRADLEY</b>	EMILY	DIANE	FERGIE
<b>CHARLES</b>	DIANE	EMILY	FERGIE

<b>DIANE</b>	BRADLEY	ALBERT	CHARLES
<b>EMILY</b>	ALBERT	BRADLEY	CHARLES
<b>FERGIE</b>	ALBERT	BRADLEY	CHARLES

IS THIS  
STABLE?

NEXT →

# Example Matching #2

<b>ALBERT</b>	DIANE	EMILY	FERGIE
<b>BRADLEY</b>	EMILY	DIANE	FERGIE
<b>CHARLES</b>	DIANE	EMILY	FERGIE

<b>DIANE</b>	BRADLEY	ALBERT	CHARLES
<b>EMILY</b>	ALBERT	BRADLEY	CHARLES
<b>FERGIE</b>	ALBERT	BRADLEY	CHARLES

**YES.**

**There is no  
blocking  
pair.**

NEXT →

# Some Questions

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- Does a stable solution to the marriage problem always exist?
- Can we compute such a solution efficiently?
- Can we compute the best stable solution efficiently?

# Gale - Shapley [1962]

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- Everyone is unmatched
- While some man  $m$  is unmatched:
  - –  $w := m$ 's most-preferred woman to whom he has not proposed yet
  - – If  $w$  is also unmatched:
    - $w$  and  $m$  are engaged
  - – Else if  $w$  prefers  $m$  to her current match  $m'$ 
    - $w$  and  $m$  are engaged,  $m'$  is unmatched
    - – Else:  $w$  rejects  $m$
- Return matched pair

**CLAIM:**

**GS terminates in polynomial time (at most  $n^2$  iterations of the outer loop)**

**PROOF:**

- **Each iteration, one man proposes to someone to whom he has never proposed before.**
- **$n$  men,  $n$  women --  $n \times n$  possible events.**

**CLAIM:**

**GS results in a perfect matching.**

**PROOF (by contradiction):**

- Suppose BWOC that  $m$  is unmatched at termination
- $n$  men,  $n$  women:  $w$  is unmatched, too
- Once a woman is matched, she is never unmatched; she only swaps partners. Thus, nobody proposed to  $w$
- $m$  proposed to everyone (by def. of GS): ><

## **CLAIM:**

**GS results in a stable matching (i.e., there are no blocking pairs).**

## **PROOF (by contradiction):**

- Assume m and w form a blocking pair

**Case #1: m never proposed to w**

- GS: men propose in order of preferences
- m prefers current partner  $w' > w$
- so, m and w are not blocking

## **PROOF (by contradiction):**

- **Case #2: m proposed to w**
- **w rejected m at some point**
- **GS: women only reject for better partners**
- **w prefers current partner  $m' > m$**
- **m and w are not blocking**

**Case #1 and #2 exhaust space.**

# Recap: Some Questions

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- Does a stable solution to the marriage problem always exist? 
- Can we compute such a solution efficiently? 
- Can we compute **the best** stable solution efficiently?

# Man Optimality/Pessimality

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- Let  $S$  be the set of stable matchings.
- $m$  is a valid partner of  $w$  if there exists some stable matching in  $S$  where they are paired.
- A matching is **man optimal** if each man receives his best valid partner – Is this a perfect matching? Stable?
- A matching is **man pessimal** if each man receives his worst valid partner.

**CLAIM:**

**GS – with the man proposing – results in a  
man- optimal matching.**

# Example Matching #2

<b>ALBERT</b>	DIANE	EMILY	FERGIE
<b>BRADLEY</b>	EMILY	DIANE	FERGIE
<b>CHARLES</b>	DIANE	EMILY	FERGIE

<b>DIANE</b>	BRADLEY	ALBERT	CHARLES
<b>EMILY</b>	ALBERT	BRADLEY	CHARLES
<b>FERGIE</b>	ALBERT	BRADLEY	CHARLES

This  
matching is  
man  
optimal.

NEXT →

# Example Matching #2

<b>ALBERT</b>	DIANE	EMILY	FERGIE
<b>BRADLEY</b>	EMILY	DIANE	FERGIE
<b>CHARLES</b>	DIANE	EMILY	FERGIE

<b>DIANE</b>	BRADLEY	ALBERT	CHARLES
<b>EMILY</b>	ALBERT	BRADLEY	CHARLES
<b>FERGIE</b>	ALBERT	BRADLEY	CHARLES

This  
matching is  
woman  
optimal.

NEXT →

# Extensions of the Stable Marriage Problem

# References

- [http://www.cs.cmu.edu/~ariel/proj/15896s16  
/slides/896s16-16.pdf](http://www.cs.cmu.edu/~ariel/proj/15896s16/slides/896s16-16.pdf)

**PRESENTATION FINISHED**



**PLEASE DON'T ASK DIFFICULT  
QUESTIONS**