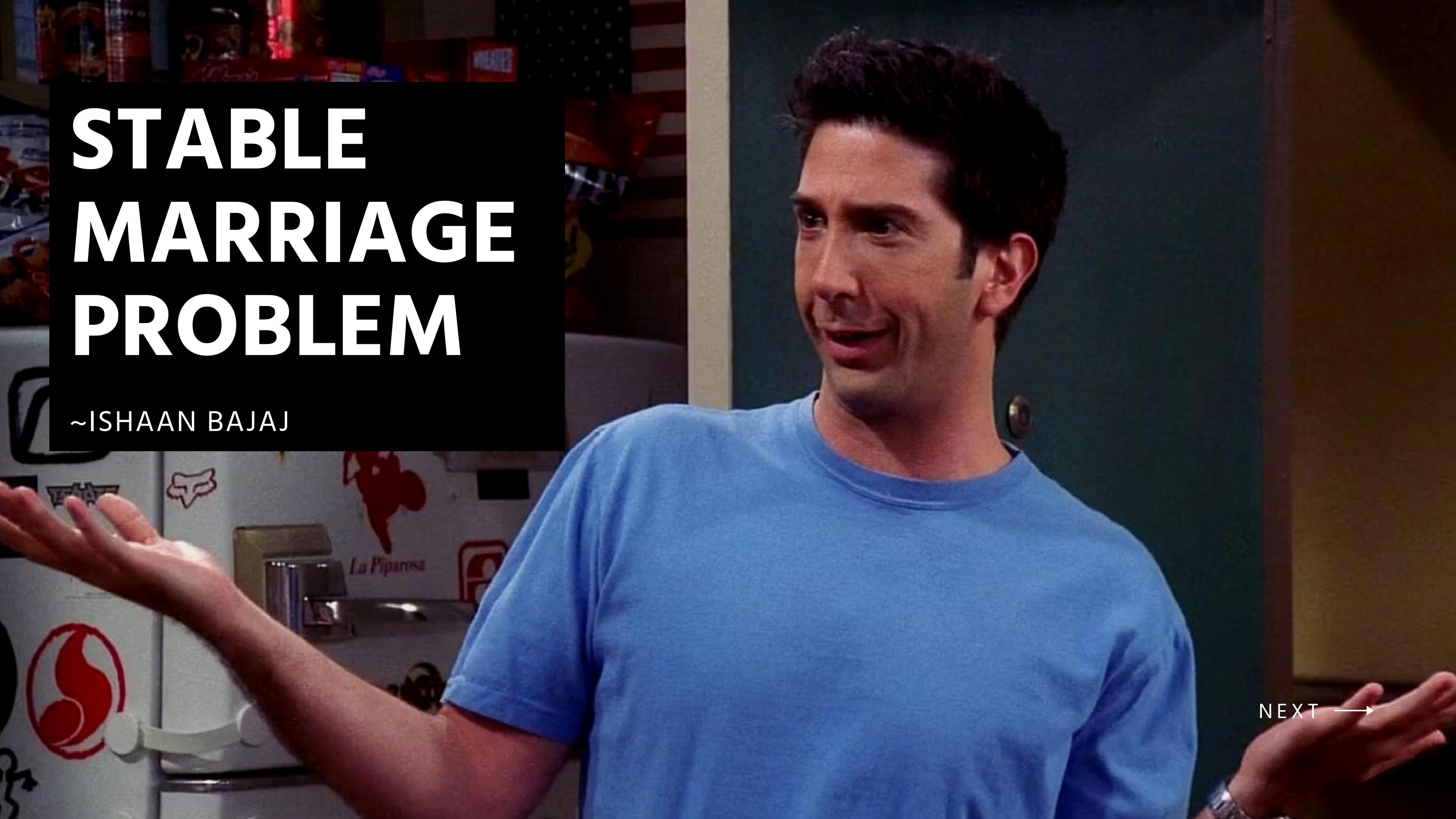


STABLE MARRIAGE PROBLEM

~ISHAAN BAJAJ



NEXT →

Stable Matching Problem

- There are n men and n women, all unmarried.
- Each has a preference list giving a relative preference of each person of the opposite gender.
- Find a matching between the men and the women such that
 - Each man is matched to exactly one woman and each woman is matched to exactly one man (perfect matching).
 - There is no blocking pair (an unmatched pair of a man and a woman who both prefer each other over whoever they are assigned to in the matching).
- Stable matching - perfect matching with no blocking pair.

Example Preferences



ALBERT	DIANE	EMILY	FERGIE
BRADLEY	EMILY	DIANE	FERGIE
CHARLES	DIANE	EMILY	FERGIE

DIANE	BRADLEY	ALBERT	CHARLES
EMILY	ALBERT	BRADLEY	CHARLES
FERGIE	ALBERT	BRADLEY	CHARLES

NEXT →

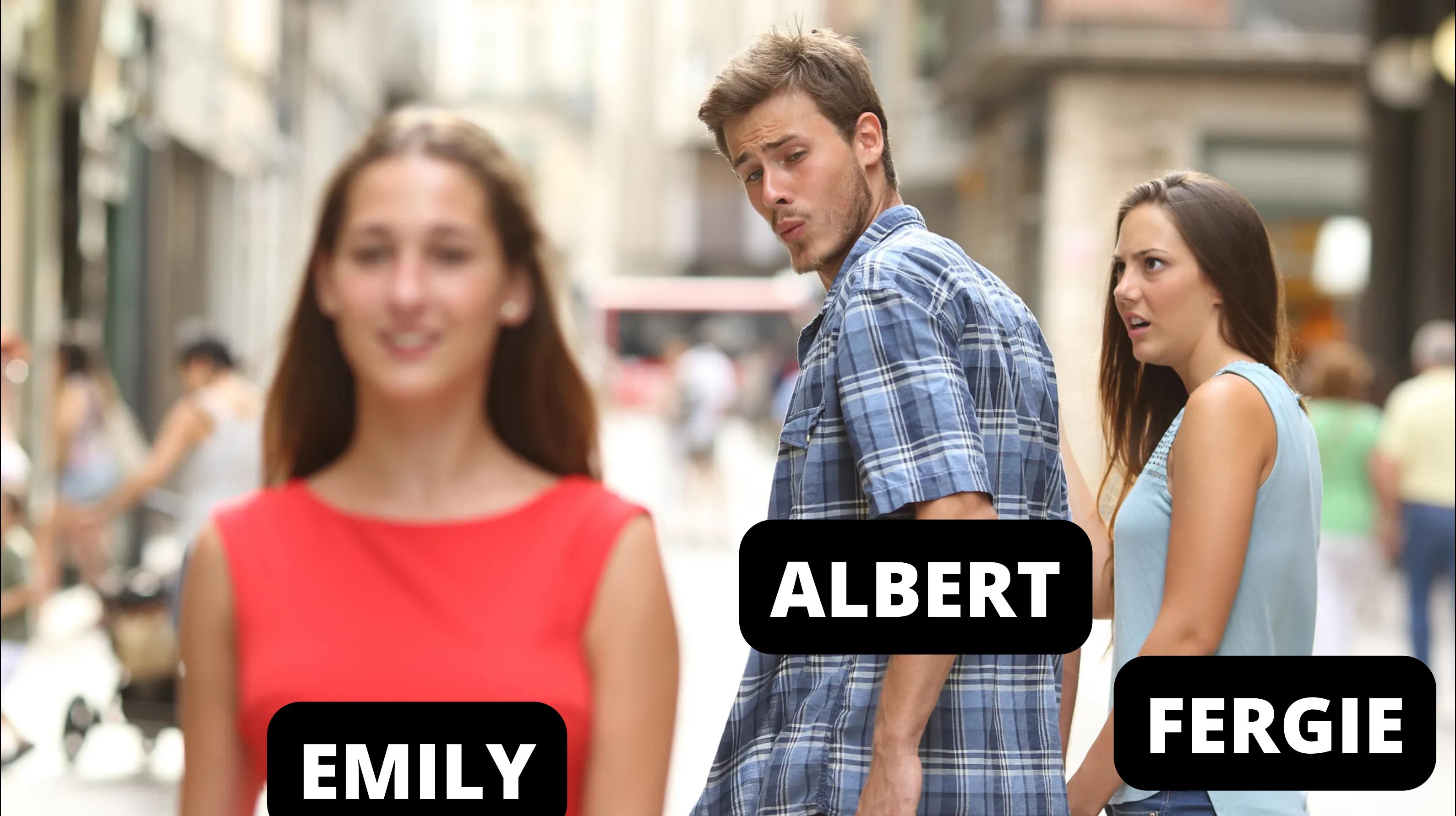
Example Matching #1

ALBERT	DIANE	EMILY	FERGIE
BRADLEY	EMILY	DIANE	FERGIE
CHARLES	DIANE	EMILY	FERGIE

**IS THIS
STABLE?**

DIANE	BRADLEY	ALBERT	CHARLES
EMILY	ALBERT	BRADLEY	CHARLES
FERGIE	ALBERT	BRADLEY	CHARLES

NEXT →



ALBERT

EMILY

FERGIE

Example Matching #1

ALBERT	DIANE	EMILY	FERGIE
BRADLEY	EMILY	DIANE	FERGIE
CHARLES	DIANE	EMILY	FERGIE

NO.
Albert and
Emily form
a blocking
pair.

DIANE	BRADLEY	ALBERT	CHARLES
EMILY	ALBERT	BRADLEY	CHARLES
FERGIE	ALBERT	BRADLEY	CHARLES

NEXT →

Example Matching #2

ALBERT	DIANE	EMILY	FERGIE
BRADLEY	EMILY	DIANE	FERGIE
CHARLES	DIANE	EMILY	FERGIE

DIANE	BRADLEY	ALBERT	CHARLES
EMILY	ALBERT	BRADLEY	CHARLES
FERGIE	ALBERT	BRADLEY	CHARLES

IS THIS
STABLE?

NEXT →

Example Matching #2

ALBERT	DIANE	EMILY	FERGIE
BRADLEY	EMILY	DIANE	FERGIE
CHARLES	DIANE	EMILY	FERGIE

DIANE	BRADLEY	ALBERT	CHARLES
EMILY	ALBERT	BRADLEY	CHARLES
FERGIE	ALBERT	BRADLEY	CHARLES

YES.

**There is no
blocking
pair.**

NEXT →

Some Questions

- Does a stable solution to the marriage problem always exist?
- Can we compute such a solution efficiently?
- Can we compute the best stable solution efficiently?

Gale - Shapley [1962]

- Everyone is unmatched
- While some man m is unmatched:
 - – $w := m$'s most-preferred woman to whom he has not proposed yet
 - – If w is also unmatched:
 - w and m are engaged
 - – Else if w prefers m to her current match m'
 - w and m are engaged, m' is unmatched
 - – Else: w rejects m
- Return matched pair

CLAIM:

GS terminates in polynomial time (at most n^2 iterations of the outer loop)

PROOF:

- **Each iteration, one man proposes to someone to whom he has never proposed before.**
- **n men, n women -- $n \times n$ possible events.**

CLAIM:

GS results in a perfect matching.

PROOF (by contradiction):

- Suppose BWOC that m is unmatched at termination
- n men, n women: w is unmatched, too
- Once a woman is matched, she is never unmatched; she only swaps partners. Thus, nobody proposed to w
- m proposed to everyone (by def. of GS): ><

CLAIM:

GS results in a stable matching (i.e., there are no blocking pairs).

PROOF (by contradiction):

- Assume m and w form a blocking pair

Case #1: m never proposed to w

- GS: men propose in order of preferences
- m prefers current partner $w' > w$
- so, m and w are not blocking

PROOF (by contradiction):

- **Case #2: m proposed to w**
- **w rejected m at some point**
- **GS: women only reject for better partners**
- **w prefers current partner $m' > m$**
- **m and w are not blocking**

Case #1 and #2 exhaust space.

Recap: Some Questions

- Does a stable solution to the marriage problem always exist? 
- Can we compute such a solution efficiently? 
- Can we compute **the best** stable solution efficiently?

Man Optimality/Pessimality

- Let S be the set of stable matchings.
- m is a valid partner of w if there exists some stable matching in S where they are paired.
- A matching is **man optimal** if each man receives his best valid partner – Is this a perfect matching? Stable?
- A matching is **man pessimal** if each man receives his worst valid partner.

CLAIM:

**GS – with the man proposing – results in a
man-optimal matching.**

Example Matching #2

ALBERT	DIANE	EMILY	FERGIE
BRADLEY	EMILY	DIANE	FERGIE
CHARLES	DIANE	EMILY	FERGIE

DIANE	BRADLEY	ALBERT	CHARLES
EMILY	ALBERT	BRADLEY	CHARLES
FERGIE	ALBERT	BRADLEY	CHARLES

This
matching is
man
optimal.

NEXT →

Example Matching #2

ALBERT	DIANE	EMILY	FERGIE
BRADLEY	EMILY	DIANE	FERGIE
CHARLES	DIANE	EMILY	FERGIE

DIANE	BRADLEY	ALBERT	CHARLES
EMILY	ALBERT	BRADLEY	CHARLES
FERGIE	ALBERT	BRADLEY	CHARLES

This
matching is
woman
optimal.

NEXT →

Extensions of the Stable Marriage Problem

References

- [http://www.cs.cmu.edu/~ariel/proj/15896s16
/slides/896s16-16.pdf](http://www.cs.cmu.edu/~ariel/proj/15896s16/slides/896s16-16.pdf)

PRESENTATION FINISHED



**PLEASE DON'T ASK DIFFICULT
QUESTIONS**